2. ZH megoldások - A csoport

(1) Mozgás előírt görbe mentén

$K = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

$L = K - V$

$V = mgy$

kényszerfeltétel: $y = f(x) \Rightarrow \dot{y} = f'(x)\dot{x}$

$$K = \frac{1}{2} m (\dot{x}^2 + f'(x)^2 \dot{x}^2) = \frac{1}{2} m (1 + f'(x)^2)\dot{x}^2$$

$$V = mg f(x)$$

$$L = K - V = \frac{1}{2} m (1 + f'(x)^2)\dot{x}^2 - mg f(x)$$

Kis rezgések: sorfejtés másodrendig

$\dot{x}^2$ eleve másodrendű

$$f(x) \approx f(x_0) + \underbrace{f'(x_0)}_{0}(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

$$f'(x) \approx \underbrace{f'(x_0)}_{0} + f''(x_0)(x - x_0) + \frac{1}{2} f'''(x_0)(x - x_0)^2$$

és mivel $\dot{x}^2$ eleve másodrendű

$$K = \frac{1}{2} m (1 + f'(x)^2)\dot{x}^2 \approx \frac{1}{2} m \dot{x}^2 + \text{magasabb rend}$$

$$V = mg f(x_0) + \frac{1}{2} mg f''(x_0)(x - x_0)^2$$

$$L = K - V = \frac{1}{2} m \dot{x}^2 \underbrace{- mg f(x_0)}_{\text{áll.}} - \frac{1}{2} mg f''(x_0)(x - x_0)^2$$

$\xi = x - x_0$-val felírva

$$L = \frac{1}{2} m \dot{\xi}^2 - \frac{1}{2} mg f''(x_0) \xi^2 - mg f(x_0)$$

mozgásegyenlet: $m\ddot{\xi} + mg f''(x_0)\xi = 0 \Rightarrow \omega^2 = g f''(x_0)$

(2) $L = \frac{1}{2} m \dot{\underline{r}}^2 + \frac{e}{c} \underline{A}\dot{\underline{r}} - e\phi$

$$\underline{p} = \frac{\partial L}{\partial \dot{\underline{r}}} = m\dot{\underline{r}} + \frac{e}{c}\underline{A} \neq m\dot{\underline{r}}$$

valóban eltér a $\underline{p}$ kanonikus impulzus az $m\dot{\underline{r}}$ kinetikus impulzustól

$$\dot{\underline{r}} = \frac{1}{m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)$$

$$H = \underline{p}\dot{\underline{r}} - L = \frac{1}{m}\,\underline{p}\,\underbrace{\left(\underline{p} - \frac{e}{c}\underline{A}\right)}_{\dot{\underline{r}}} - \frac{1}{2m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)^2 - \frac{e}{c}\underline{A}\underbrace{\dot{\underline{r}}}_{\frac{1}{m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)} + e\phi$$

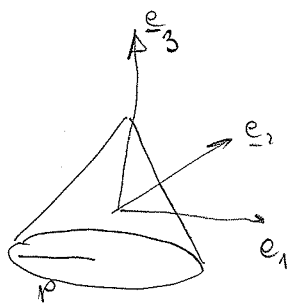
összevonva az első és a harmadik tagot:

$$H = \frac{1}{m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)^2 - \frac{1}{2m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)^2 + e\phi$$

$$= \frac{1}{2m}\left(\underline{p} - \frac{e}{c}\underline{A}\right)^2 + e\phi \qquad \left(= \frac{1}{2} m \dot{\underline{r}}^2 + e\phi\right)$$

5)

Kúp tehetetlenségi nyomaték tenzora:

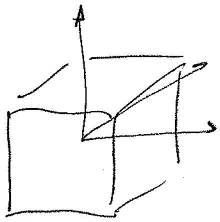


$\Theta_1 = \Theta_2$ mindig teljesül

$$\Theta_1 = \Theta_2 = \frac{3}{80} M_k h^2 + \frac{3}{20} M_k R^2$$

$$\Theta_3 = \frac{3}{10} M_k R^2 \qquad V_k = \frac{1}{3} \pi R^2 h$$

Kocka tehetetlenségi nyomaték tenzora:



$$\Theta_1 = \Theta_2 = \Theta_3 = \frac{1}{6} M_c a^2 \qquad V_c = a^3$$

Kell: a kúpnál $\Theta_1 = \Theta_2 = \Theta_3$ legyen

$$\frac{3}{80} M_k h^2 + \frac{3}{20} M_k R^2 = \frac{3}{10} M_k R^2$$

$$\frac{\cancel{3}}{80} \cancel{M_k} h^2 = \frac{\cancel{3}}{20} \cancel{M_k} R^2$$

$$\frac{h^2}{8} = \frac{R^2}{2} \qquad \boxed{h^2 = 4R^2}$$

$$h = 2R$$

azonos tömeg:

$$\underbrace{\frac{1}{3} \pi R^2 h}_{V_k} \, \rho_k = \underbrace{a^3}_{V_c} \, \rho_c$$

$$\frac{2}{3} \pi R^3 \rho_k = a^3 \rho_c$$

$$\frac{1}{3} \pi R^2 h = \frac{2}{3} \pi R^3$$

és $\Theta_k = \Theta_c$

$$\frac{3}{10} M_k R^2 = \frac{1}{6} M_c a^2$$

azonos

$$\frac{3}{10} R^2 = \frac{1}{6} a^2$$

$$\boxed{\frac{18}{10} R^2 = a^2} \qquad a = \sqrt{\frac{18}{10}}\, R$$

$$\frac{2}{3}\pi \cancel{R^3} \varrho_k = a^3 \varrho_c = \left(\frac{18}{10}\right)^{3/2} \cancel{R^3} \varrho_c$$

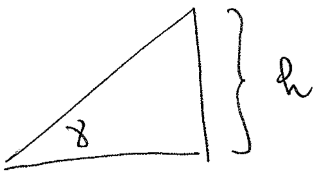
$$\boxed{\frac{2}{3}\left(\frac{10}{18}\right)^{3/2} \pi \varrho_k = \varrho_c}$$

így

$$\boxed{\frac{h}{R} = 2 \qquad \frac{a}{R} = \left(\frac{18}{10}\right)^{1/2} \qquad \frac{\varrho_k}{\varrho_c} = \frac{3}{2}\left(\frac{18}{10}\right)^{3/2}\frac{1}{\pi}}$$

(4) $\Theta_{kerék} = MR^2$

a leírt elhanyagolásokkal



$$\omega = \frac{v}{R}$$

$$E_{lent} = \frac{1}{2} m v^2 + \frac{1}{2}\Theta\omega^2 = \frac{1}{2} m \underbrace{\left(1 + \frac{\Theta}{MR^2}\right)}_{2} v^2 = m v^2$$

$$E_{fent} = mgh$$

$$mgh = mv^2 \qquad \boxed{h = \frac{v^2}{g}}$$

5) v_1', v_2' meghatározása

imp. megm.: $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

ε def:

$$-\varepsilon (v_1 - v_2) = v_1' - v_2'$$

innen

$$v_1' = \frac{m_1 - \varepsilon m_2}{m_1 + m_2} v_1 + \frac{(1+\varepsilon) m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{(1+\varepsilon) m_1}{m_1 + m_2} v_1 + \frac{m_2 - \varepsilon m_1}{m_1 + m_2} v_2$$

elvesző mech. energia (~ hő):

$$Q = -\Delta T = T - T' \qquad T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$T' = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

ide behelyettesítve

$$Q = \frac{1-\varepsilon^2}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2$$