Conference of the Middle European Cooperation in Statistical Physics, MECO37,

March 18-22, 2012, Tatranské Matliare, Slovak Republik

Gauge/gravity duality: an overview

Z. Bajnok

Theoretical Physics Research Group of the

Hungarian Academy of Sciences, Eötvös University, Budapest

Conference of the Middle European Cooperation in Statistical Physics, MECO37,

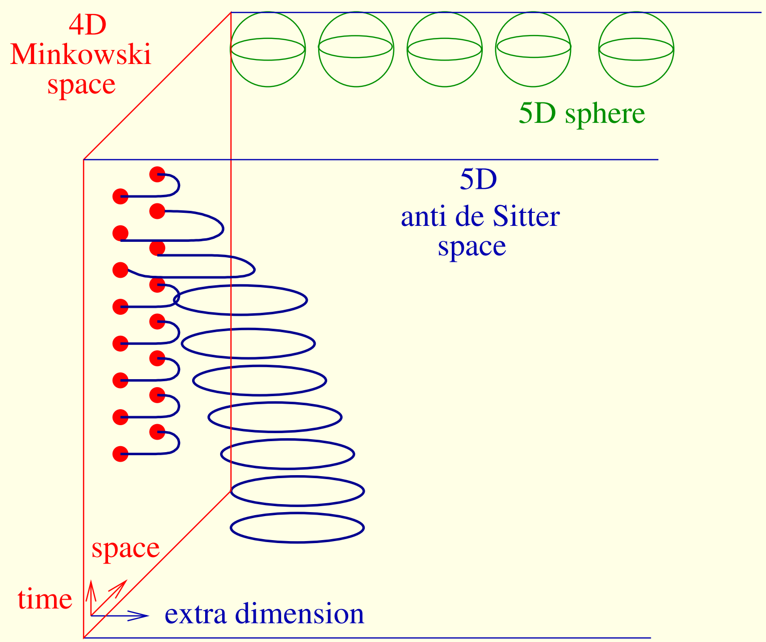
March 18-22, 2012, Tatranské Matliare, Slovak Republik

Gauge/gravity duality: an overview

Z. Bajnok

Theoretical Physics Research Group of the

Hungarian Academy of Sciences, Eötvös University, Budapest



Alias: AdS/CFT correspondencce $\subset$ gauge/gravity duality

Motivation: AdS=CFT

J. Polchinski: TASI lectures, arXiv:1010.6134: Physics World, reader poll:

What is the GREATEST EQUATION EVER?

Motivation: AdS=CFT

J. Polchinski: TASI lectures, arXiv:1010.6134: Physics World, reader poll:

What is the GREATEST EQUATION EVER?

Leonhard Euler (1707-1783)

$e^{i\cdot\pi} + 1 = 0$

$i, \pi, e, 1, 0$ and $+, \cdot, \hat{}$

James Clerk Maxwell (1831-1879)

$d \star F = j \quad ; \quad dF = 0$

unifies: electric+magnetic int.

Motivation: AdS=CFT

J. Polchinski: TASI lectures, arXiv:1010.6134: Physics World, reader poll:

What is the GREATEST EQUATION EVER?

Leonhard Euler (1707-1783)

$e^{i\cdot\pi} + 1 = 0$

$i, \pi, e, 1, 0$ and $+, \cdot, \hat{}$

James Clerk Maxwell (1831-1879)

$d \star F = j \quad ; \quad dF = 0$

unifies: electric+magnetic int.

Juan Martín Maldacena (1968-)

$AdS = CFT$

unifies: all forces+quantum theory

Motivation: AdS=CFT

J. Polchinski: TASI lectures, arXiv:1010.6134: Physics World, reader poll:

What is the GREATEST EQUATION EVER?

Leonhard Euler (1707-1783)

$e^{i\cdot\pi}+1=0$

$i, \pi, e, 1, 0$ and $+, \cdot, \hat{}$

James Clerk Maxwell (1831-1879)

$d\star F=j \quad ; \quad dF=0$

unifies: electric+magnetic int.

Juan Martín Maldacena (1968-)

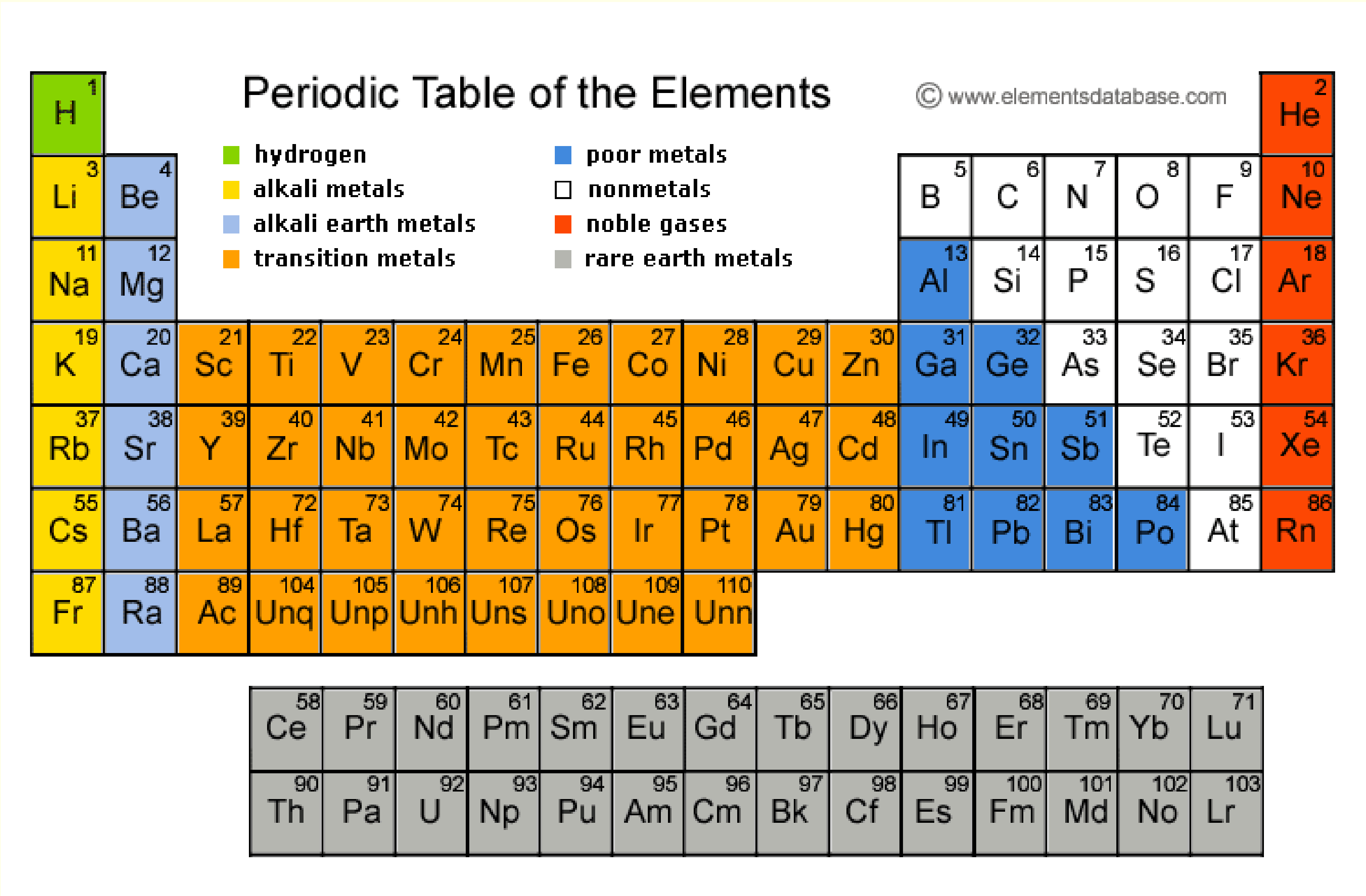
$AdS=CFT$

unifies: all forces+quantum theory

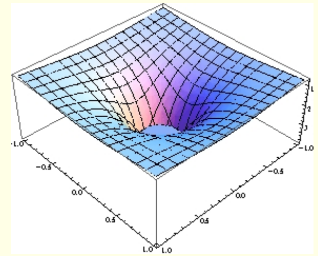
J. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252: more than 8000 citations so far

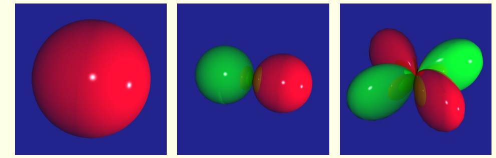
Motivation: Organizing matter

Motivation: Organizing matter



Motivation: Organizing matter

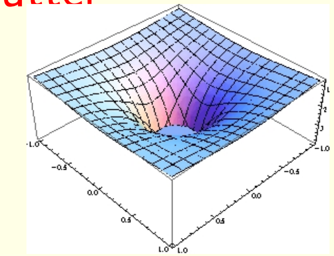


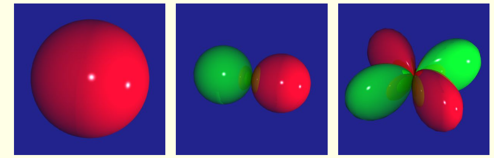


Electric interaction (potential $\Phi(r) = k\frac{Zq}{r}$)

Quantum mechanics (Schrödinger eq.) $H\Psi = (-\frac{(\hbar\nabla)^2}{2m} + \Phi(r))\Psi = E\,\Psi$

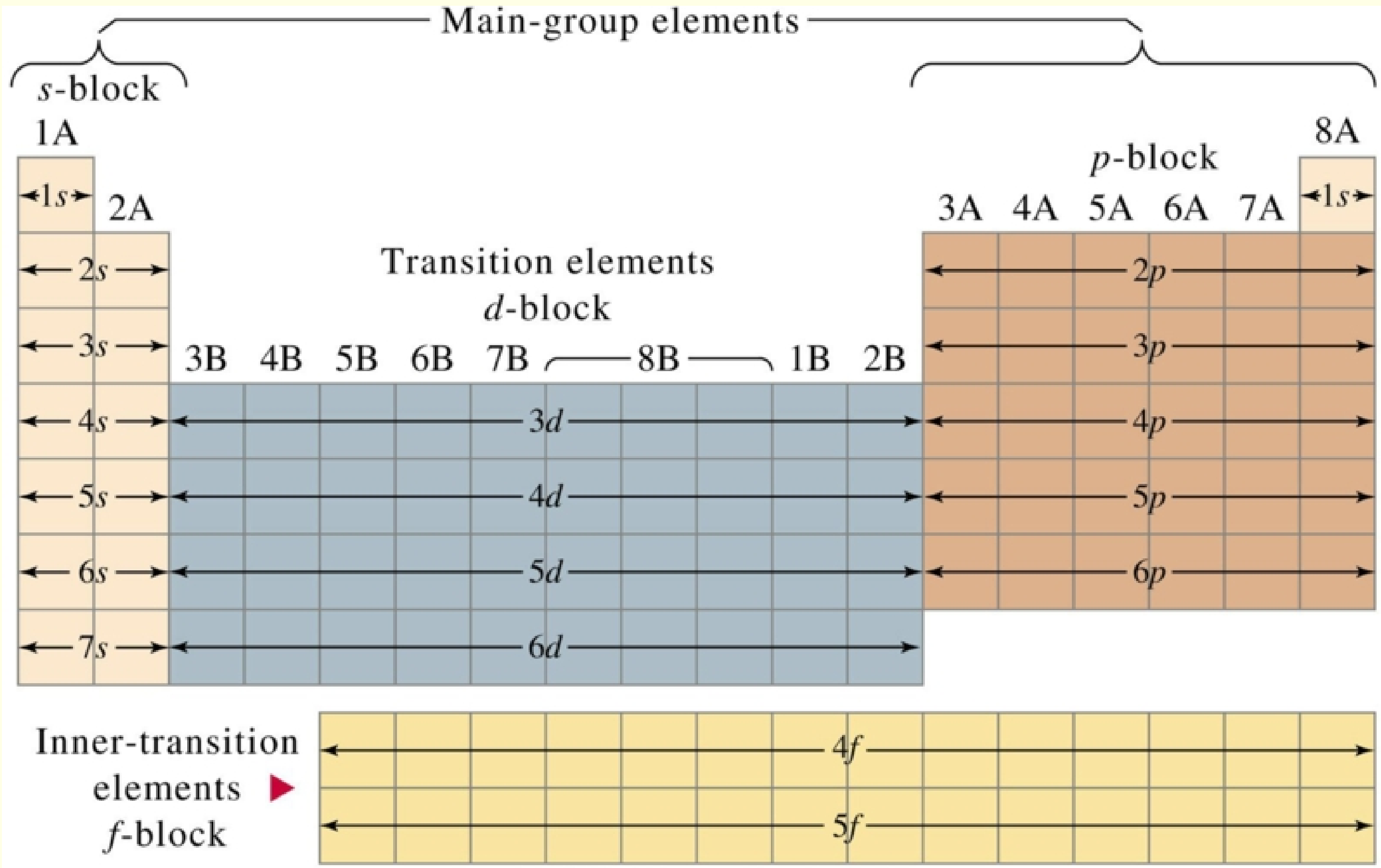
Motivation: Organizing matter



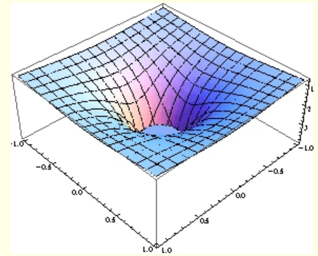


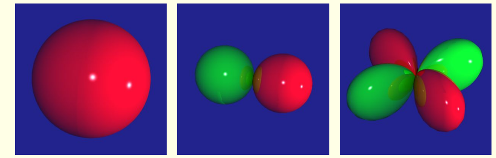
Electric interaction (potential $\Phi(r) = k\frac{Zq}{r}$)

Quantum mechanics (Schrödinger eq.) $H\Psi = (-\frac{(\hbar\nabla)^2}{2m} + \Phi(r))\Psi = E\,\Psi$



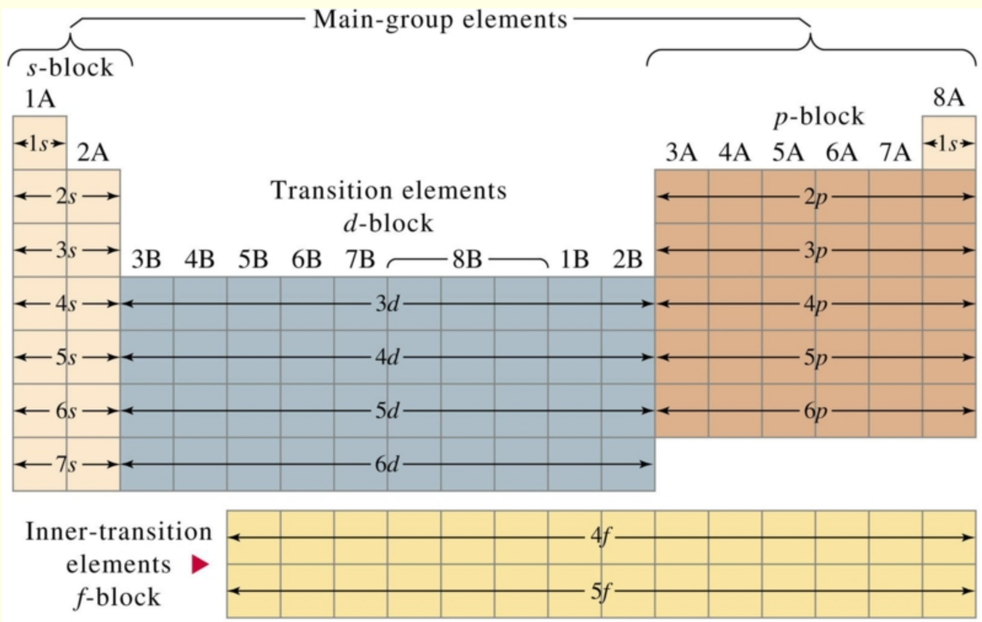
Motivation: Organizing matter

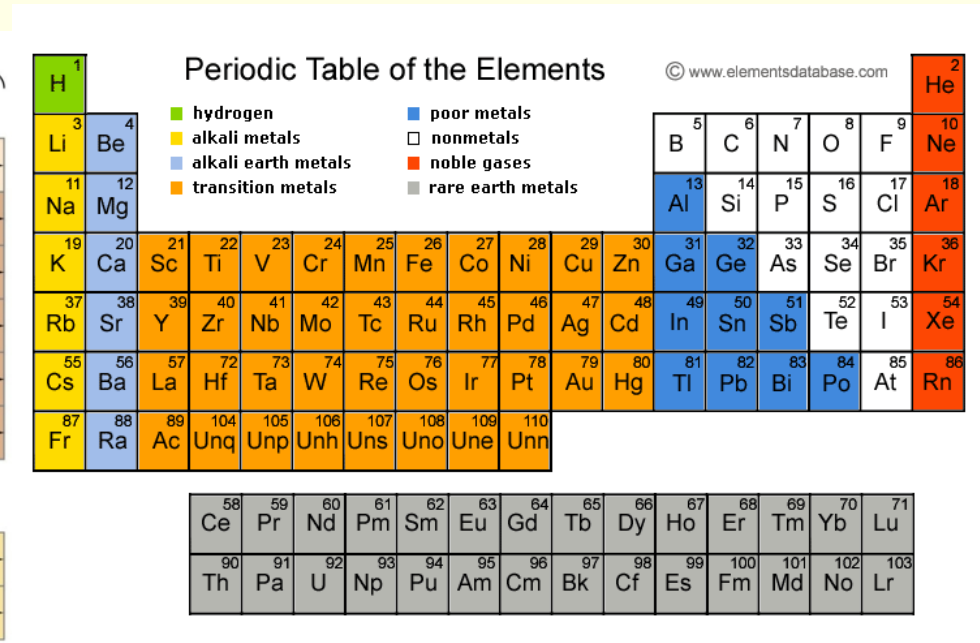




Electric interaction (potential $\Phi(r) = k\frac{Zq}{r}$)

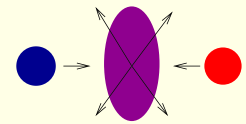
Quantum mechanics (Schrödinger eq.) $H\Psi = (-\frac{(\hbar\nabla)^2}{2m} + \Phi(r))\Psi = E\Psi$



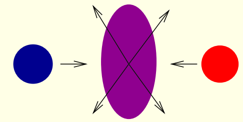


Organizing matter II

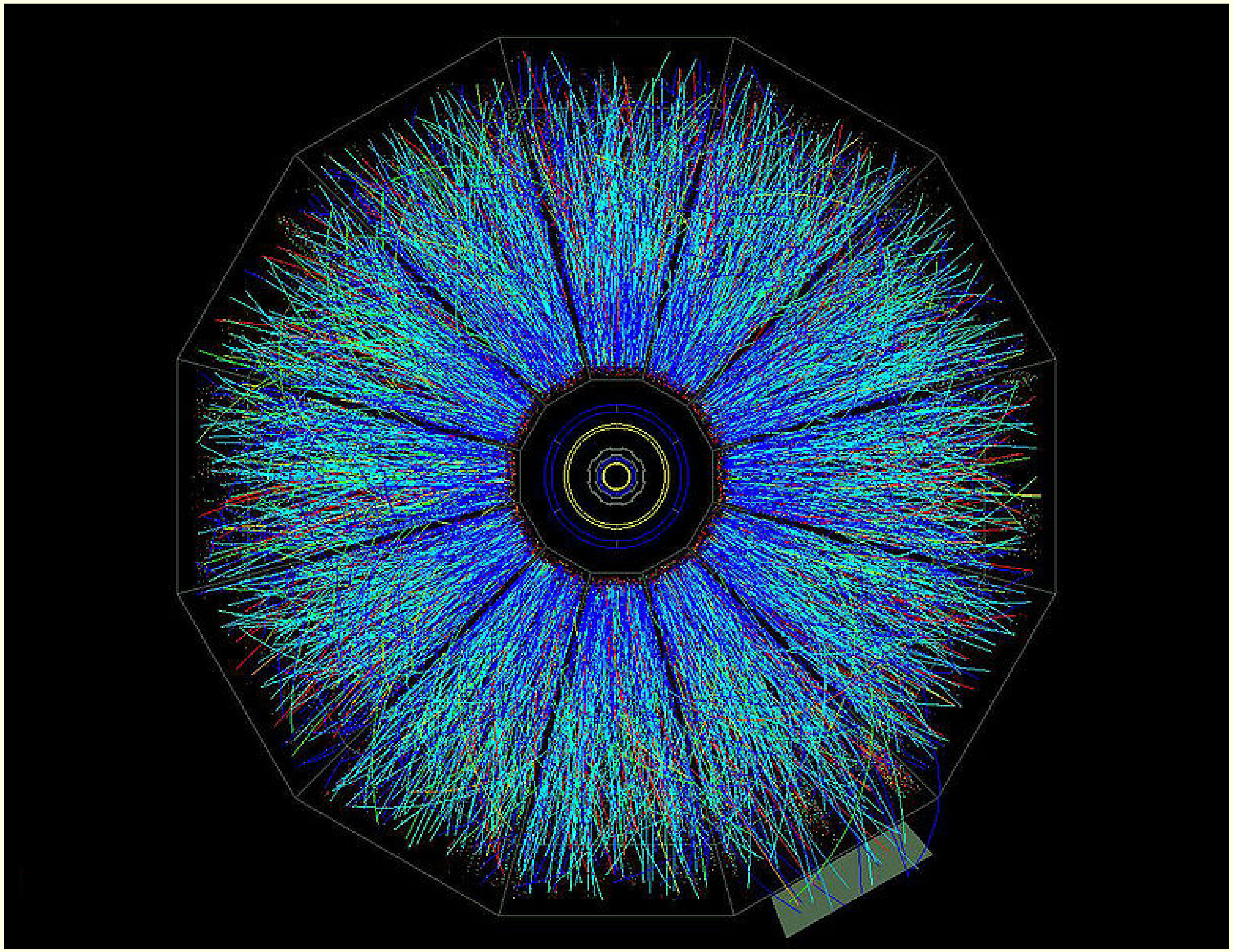
Brookhaven: Relativistic heavy ion collider (gold ion)



Organizing matter II

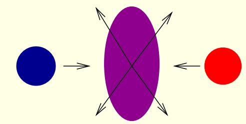


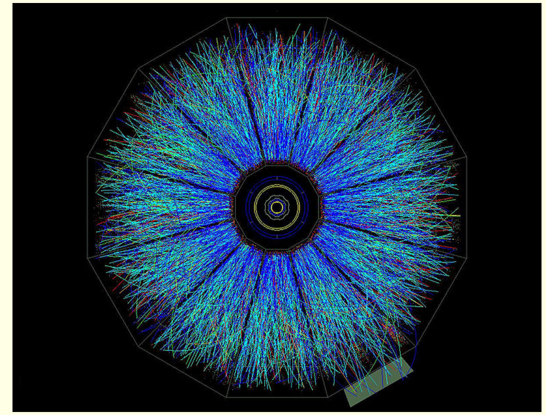
Brookhaven: Relativistic heavy ion collider (gold ion)



Organizing matter II

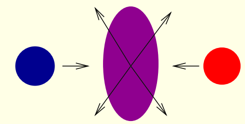
Brookhaven: Relativistic heavy ion collider (gold ion)

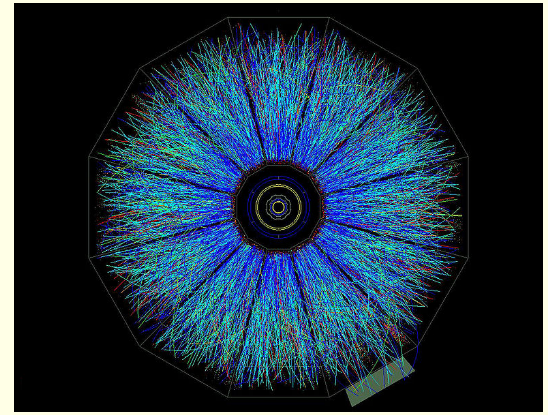




.

Organizing matter II

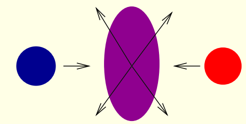


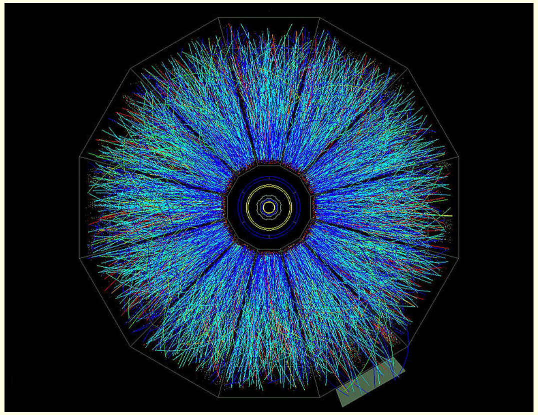


Brookhaven: Relativistic heavy ion collider (gold ion)

Number of elementary particles $>$ number of atoms $\longrightarrow$ classification

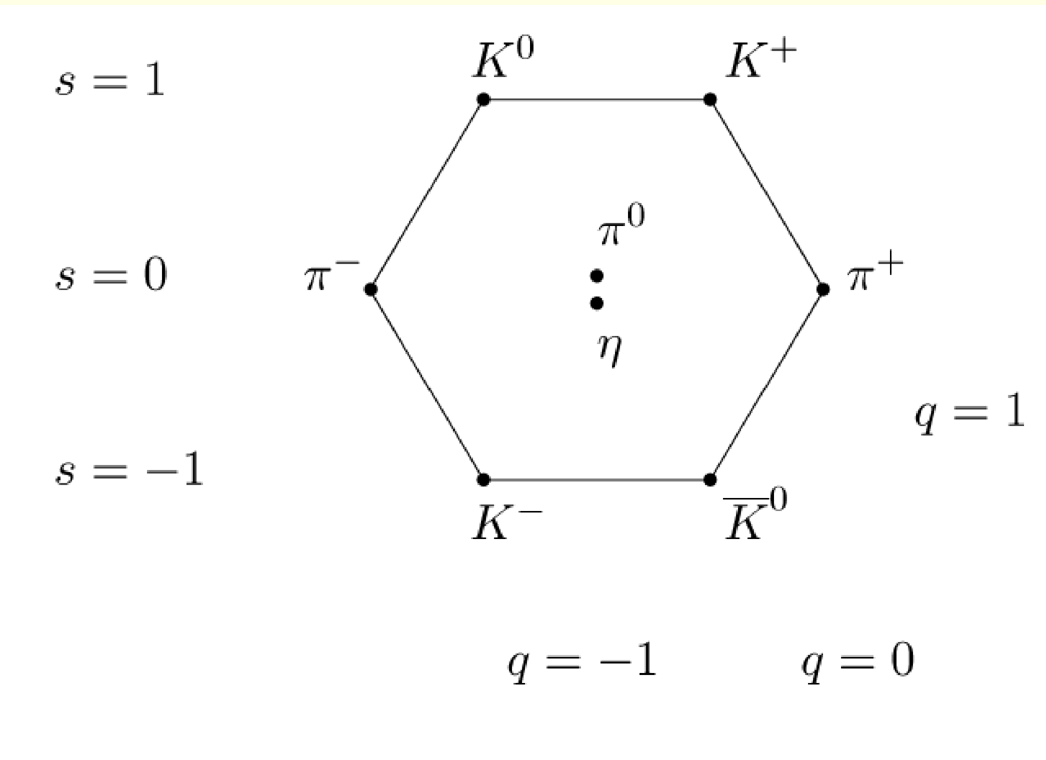
Organizing matter II

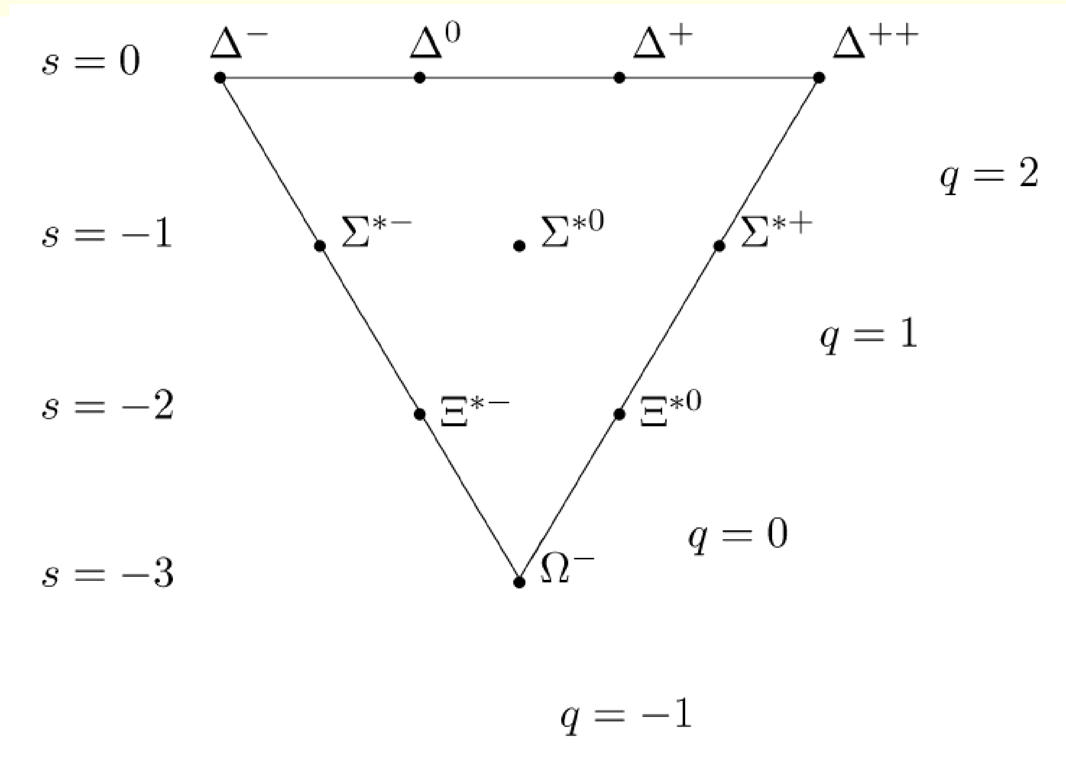




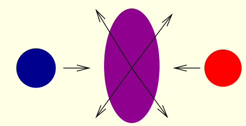
Brookhaven: Relativistic heavy ion collider (gold ion)

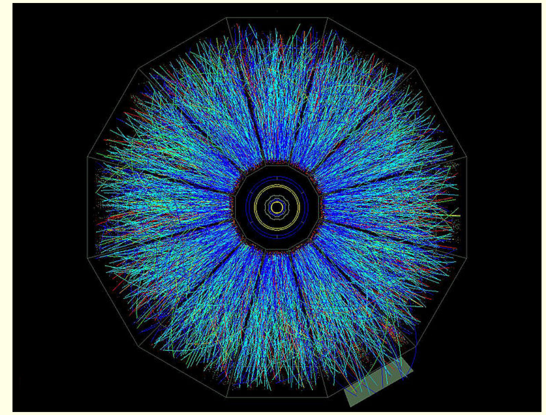
Number of elementary particles $>$ number of atoms $\rightarrow$ classification





Organizing matter II

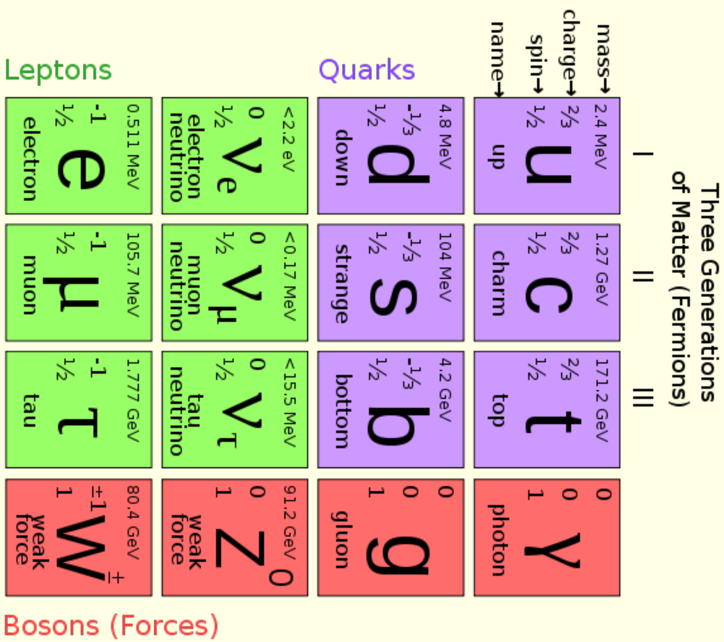




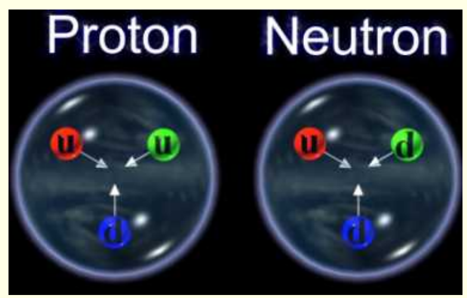
Brookhaven: Relativistic heavy ion collider (gold ion)

Number of elementary particles $>$ number of atoms $\rightarrow$ classification

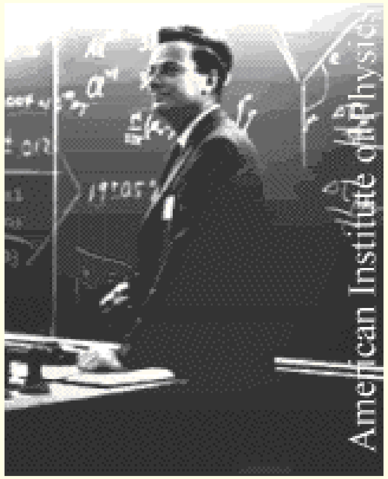
Decay $\rightarrow$ strong, weak interaction $\rightarrow$ Standard Model



Interaction			
γ	photon	electromagnetic	$I.$
$W^{\pm}, Z$	weak bosons	weak	
g	gluon	strong	$II.$
gr	graviton	gravitational	



Quantum electrodynamics



Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

Quantum

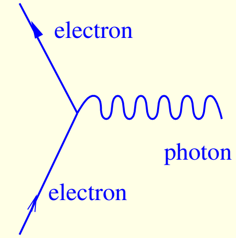
gauge theory

Relativity theory: $A_\mu = (\Phi, \underline{A})$

electric + magnetic int: $F_{\mu\nu}$

+Quantum theory→ QED

$U(1)$ gauge theory: $A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x)$



$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - e\bar{\Psi}A\!\!\!/\Psi$$

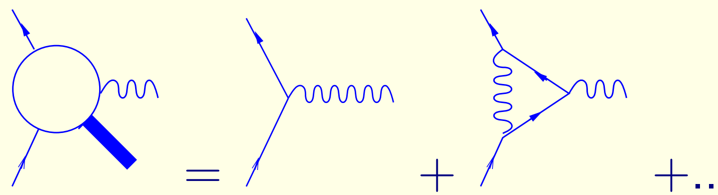
experiment: $\underline{\mu} = g\frac{e\hbar}{2mc}\underline{s}$ where $g = 2(1+a)$

Gabrielse et.al.: $a = 1159652180.85(.76) \times 10^{-12}$

perturbation theory:

Feynman graphs

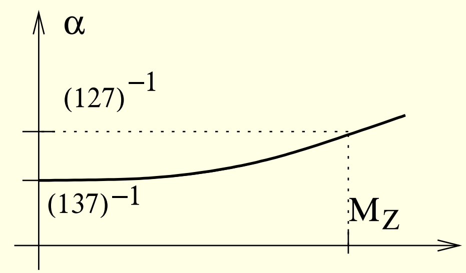
$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$



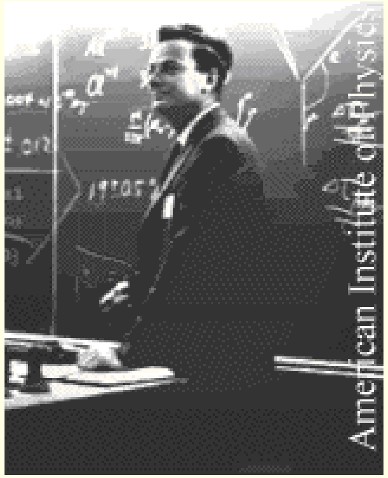
$\frac{g}{2} \;=\; 1 \;-1.3140\frac{\alpha}{2\pi}$

momentum-dependent coupling:

$\beta(\alpha) = \mu\frac{\partial\alpha}{\partial\mu} > 0$

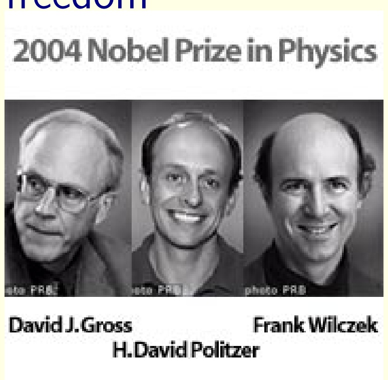


Quantum Chromodynamics



Quantum gauge theory

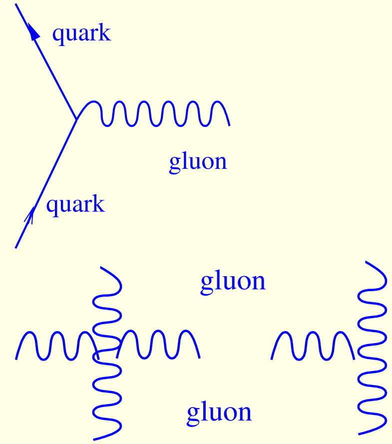
asymptotic freedom



photon $A_\mu \leftrightarrow G_\mu^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$

electron $\Psi_e \leftrightarrow \Psi_{kvark}$ quark

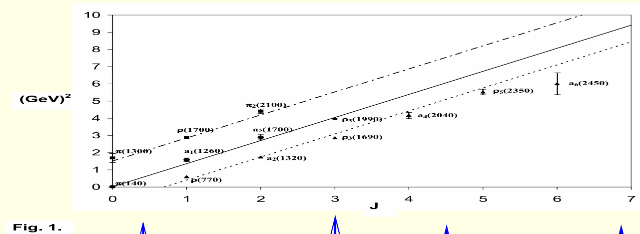
$SU(3)$ gauge theory: $G_\mu \rightarrow g^{-1} G_\mu g + g^{-1} \partial_\mu g$



$$\mathcal{L} = -\frac{1}{4} F^2 + \bar{\Psi}(i\partial\!\!\!/ - m)\Psi - g\bar{\Psi} G\!\!\!/ \Psi$$

experiments:

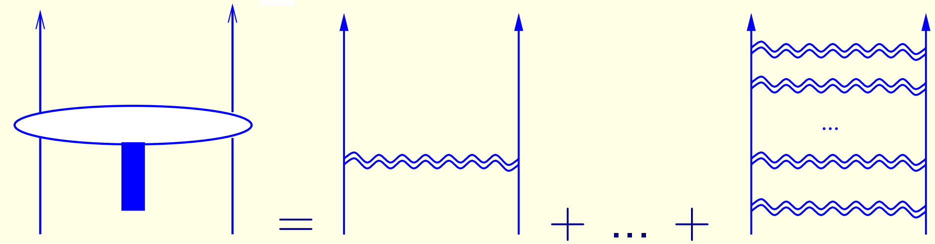
hadron spectrum



perturbation theory:

Feynman graphs

$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$

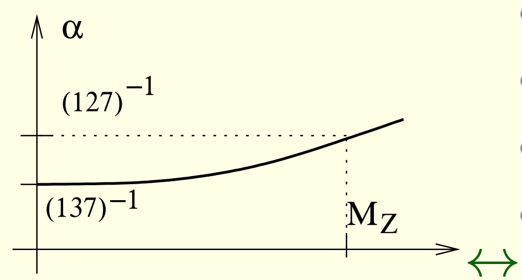


momentum-dependent coupling:

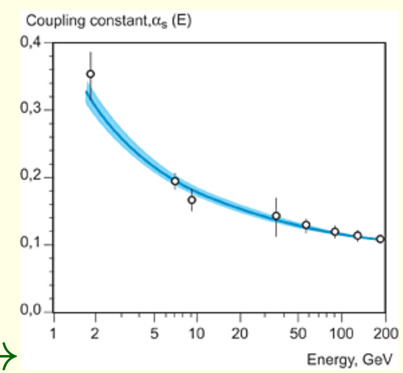
$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$

asymptotic freedom

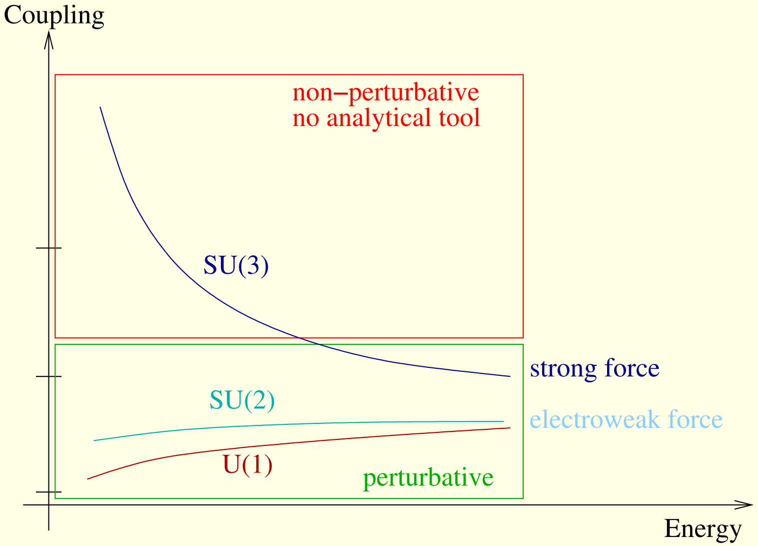
confinement



$\leftrightarrow$



CFT: maximally supersymmetric gauge theory



Fundamental interactions

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	$W^{\pm}, Z \quad \mu,\nu$+Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

only analytical tool: perturbation theory

maximally supersymmetric (N=4) gauge theory

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$

all fields $N^2 - 1$ component matrix

$$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}, \qquad A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$$

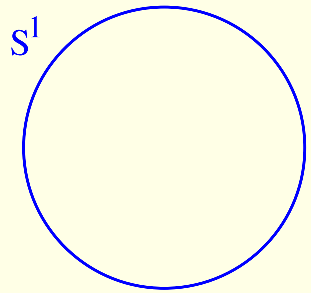
$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \,\mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

no running $\beta = 0 \rightarrow$ CFT

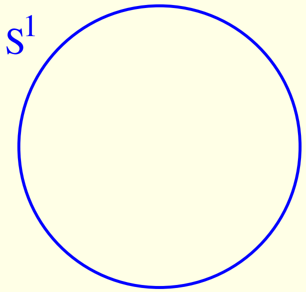
AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

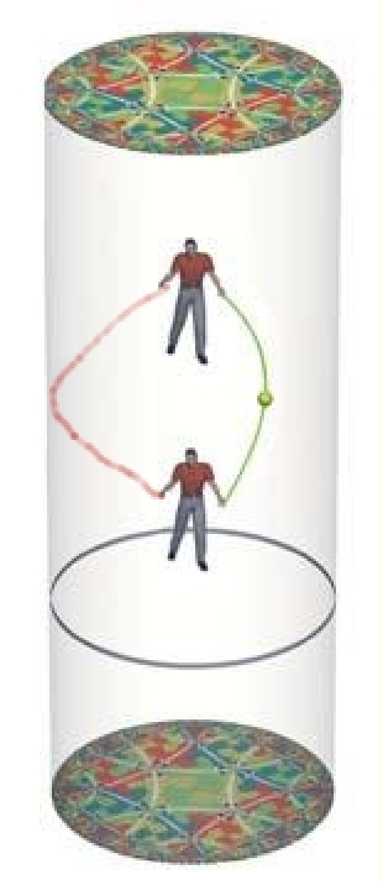


AdS: string theory on Anti de Sitter ⊃ gravitation

positively curved space

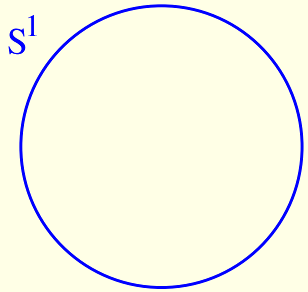


Anti de Sitter: negatively curved space

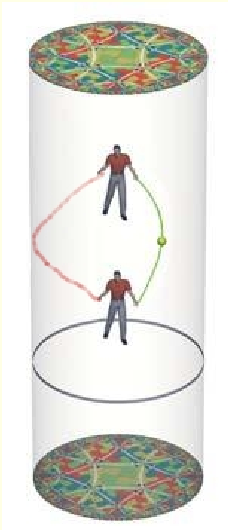


AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

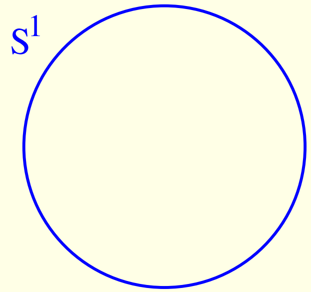


Anti de Sitter: negatively curved space

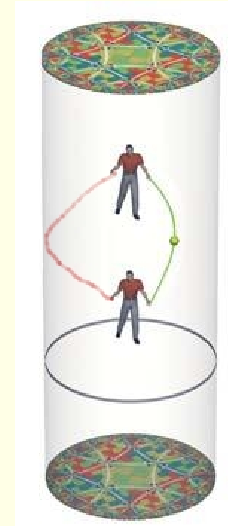


AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

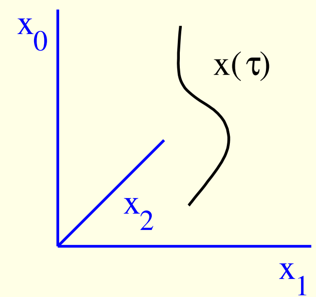


Anti de Sitter: negatively curved space



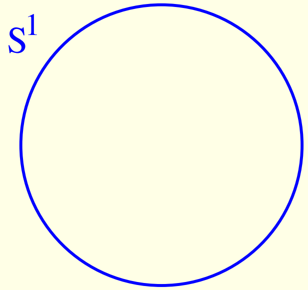
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$

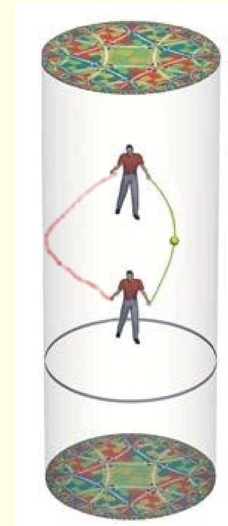


AdS: string theory on Anti de Sitter ⊃ gravitation

positively curved space

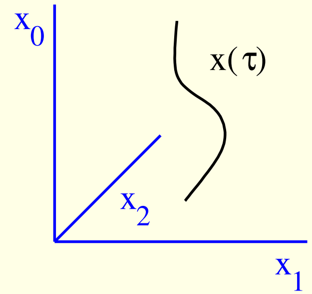


Anti de Sitter: negatively curved space



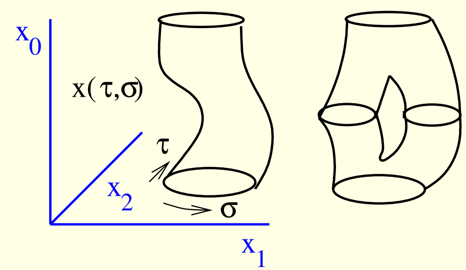
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



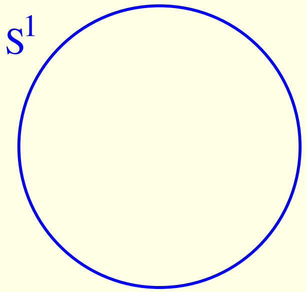
relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$

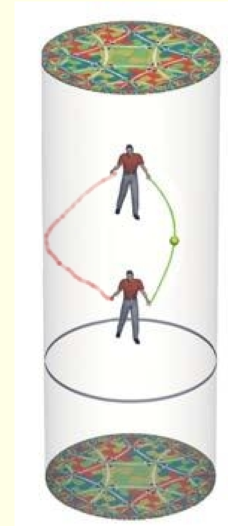


AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

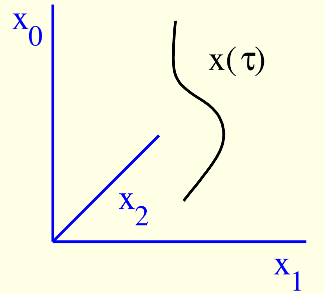


Anti de Sitter: negatively curved space



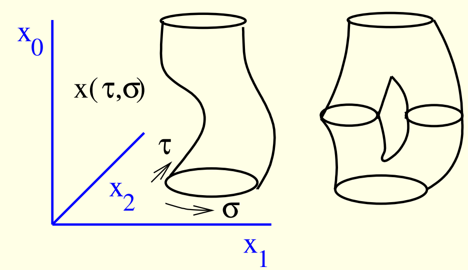
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

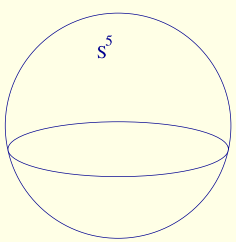
$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

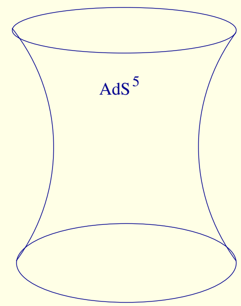
$S \propto$ worldsheet $\propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$





$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$



$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + fermionok$$

supercoset $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

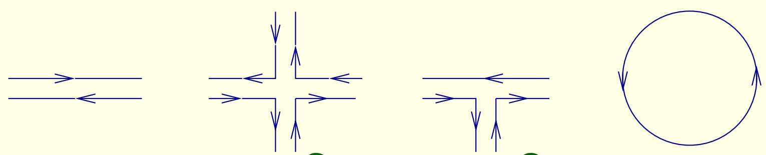
CFT: Observables

maximally supersymmetric gauge theory
A $\Psi_{1,2,3,4}$ $\overline{\Psi}_{1,2,3,4}$ $\Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices
$\mathcal{S} = \frac{2}{g_{YM}^2}\int d^4x \,\mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$
$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$

observables
parameters: g_{YM}, N
observables: partition function
gauge-invariant operators
$\mathcal{O}(x) = \mathrm{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$
correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle$

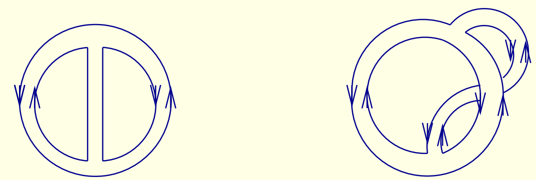
correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \int [dA...]e^{-i\mathcal{S}}\mathcal{O}_1(x)\mathcal{O}_2(0) = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV}\rangle_0$

perturbation:



g_{YM}^2 $\quad g_{YM}^{-2}$ $\quad g_{YM}^{-2}$ $\quad N$

genus exp.



$g_{YM}^2 N^3 = N^2\lambda \quad \lambda = g_{YM}^2 N$

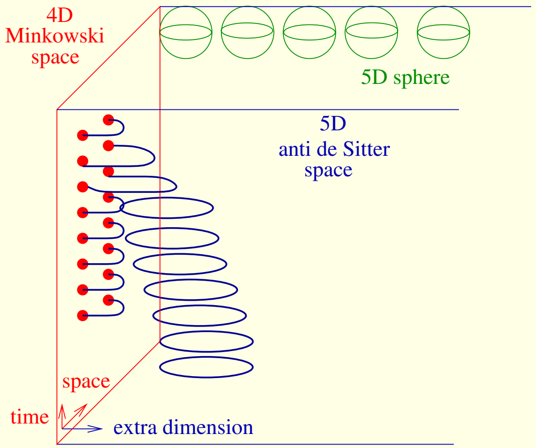
partition func. $Z(\lambda, \frac{1}{N}) = N^2 \sum_g (\frac{1}{N})^{2g} \sum_n \alpha(g,n)\lambda^n$ string interactions? (t' Hooft)

conformal field theory: $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ scale dim.: Δ_i Konishi op. $\mathcal{O}_K = \mathrm{Tr}(\Phi_i^2)$

$$\Delta_K(\lambda) = 2 + 6\frac{\lambda}{4\pi^2} - 24\frac{\lambda^2}{(4\pi^2)^2} + 168\frac{\lambda^3}{(4\pi^2)^3} - (1410 + 144\zeta(3) + \frac{1}{2}(324 + 864\zeta(3) - 1440\zeta(5)))\frac{\lambda^4}{(4\pi^2)^4}$$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \dots$$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\begin{array}{ccccc} & & \Psi_{1,2,3,4} & & \\ & \nearrow & & \searrow & \\ A_\mu & & & & \Phi_{1,2,3,4,5,6} \\ & \searrow & & \nearrow & \\ & & \overline{\Psi}_{1,2,3,4} & & \end{array}$$

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

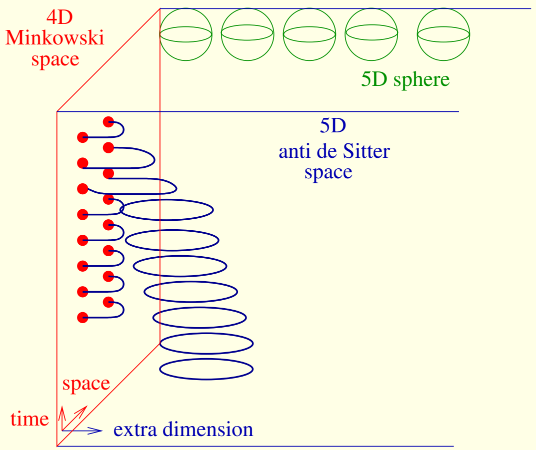
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \dots$$

$$\equiv$$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}, \quad A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

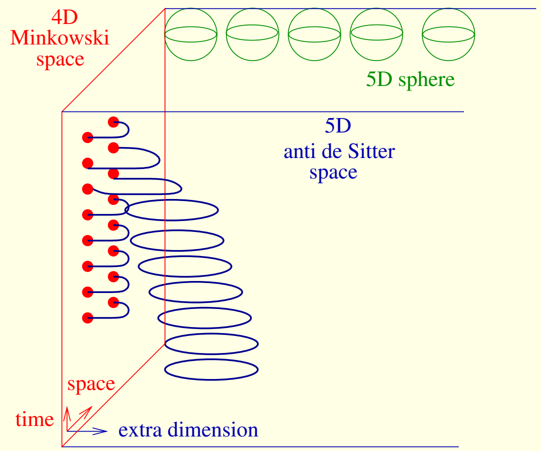
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$

$\frac{R^2}{\alpha'}\int \frac{d\tau d\sigma}{4\pi}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \ldots$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}, \quad A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$

Anomalous dim $\Delta(\lambda)$

$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \ldots$

2D integrable QFT

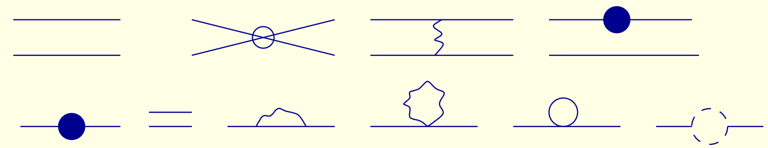
CFT: Integrablity

Perturbative correlator: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \langle \mathcal{O}_1(x)\mathcal{O}_2(0) e^{-i(\frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi])}\rangle_0$

Conformal (scale invariant) field theory: $= \frac{\delta_{ij}}{|x|^{2\Delta(\lambda)}} = \frac{1}{|x|^{2\Delta(0)}}\left[1 + \lambda\Delta_1 \log\frac{1}{|x|^2} + \ldots\right]$

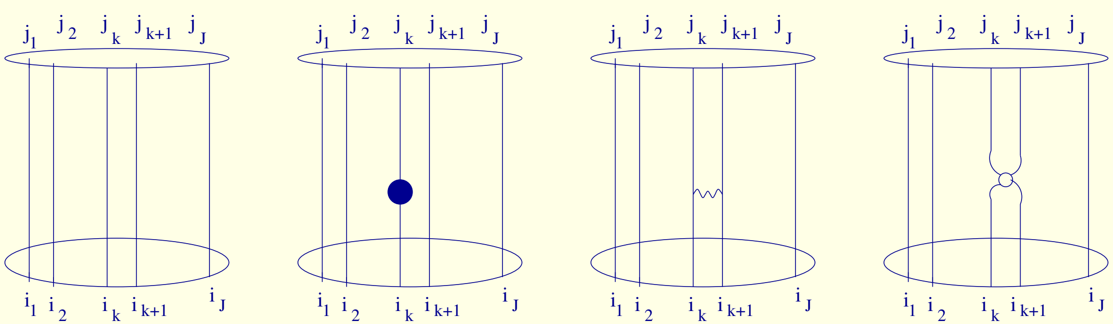
Scalar sector: $Z_1 = \Phi_1 + i\Phi_2$, $Z_2 = \Phi_3 + i\Phi_4$ SUSY st: $\mathcal{O} = \mathrm{Tr}\left[Z_i^J\right] \to \Delta_{\mathcal{O}}(\lambda) = J$

Operator mixing: $\mathcal{O}_1 = \mathrm{Tr}\,[Z_1 Z_1 Z_2 Z_2] \leftrightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$
$\mathcal{O}_2 = \mathrm{Tr}\,[Z_1 Z_2 Z_1 Z_2] \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle$



diagonalize the 1-loop mixing matrix: $\mathcal{O}_\pm = \mathcal{O}_1 \pm \mathcal{O}_2 \to$ $\Delta_{\mathcal{O}_+}(\lambda) = 4$, $\Delta_{\mathcal{O}_-}(\lambda) = 4 + 6\frac{\lambda}{4\pi^2}$

generic state at size J: $\mathcal{O}_{i_1\ldots i_J} = \mathrm{Tr}\left[Z_{i_1}\ldots Z_{i_J}\right] \leftrightarrow |i_1 \ldots i_J\rangle$



$$\Delta = J\,\mathbb{I} + \frac{\lambda}{8\pi^2}\sum_{k=1}^{J}(\mathbb{I} - \mathbb{P}_{k,k+1})$$

Heisenberg spin chain

CFT: Integrablity + Bethe Ansatz

Mixing matrix on the subspace Tr $\left[Z_{i_1} \ldots Z_{i_J}\right]$ of dim 2^J: Minahan-Zarembo 2002

$$\boxed{\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \ldots = J\,\mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \ldots}$$

H_2: next-to-nearest neighbour integrable! $\rightarrow$ use Bethe ansatz

1. choose a groundstate: $Z = Z_1 \rightarrow \text{Tr}\left[Z^J\right] = \text{Tr}\left[ZZZZZ \ldots ZZZZ\right] \leftrightarrow |\uparrow \ldots \uparrow\rangle$
2. excitations $Z...ZXZ...X$ with SUSY multiplet $X = Z_2, Z_3, \Psi_a^{\alpha}, \Psi_a^{\dot{\alpha}}, D_\mu$
3. plane wave: $\sum_n e^{ipn} \text{Tr}(\overbrace{Z...Z}^{n} XZ \ldots ZZ)$
4. scattering states: $\sum_{n_1 n_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2} \text{Tr}(\overbrace{\underbrace{Z...Z}_{n_1} X_{a_1} Z...Z}^{n_2} X_{a_2} Z \ldots Z) + S(12)_{a_1 a_2}^{b_1 b_2} \sum$

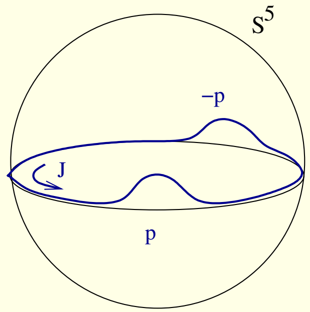
symmetry completely fixes the S-matrix for any λ (satisfies unitarity, crossing, Yang-Baxter)

Bethe ansatz follows from S-matrix: Shastry's Hubbard S-matrix

AdS/CFT correspondence: confirmation

two particle states

$$E_{BPS}(\lambda) = E_0$$



$$E_K(\lambda) = 2E(p,\lambda)$$

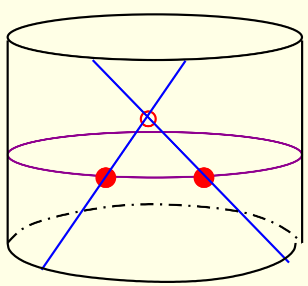
dispersion relation

$$E(p,\lambda) = \sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

elastic scattering $S(p,-p)$

Bethe Ansatz: $e^{ipJ}S(p,-p) = 1$

finite size corrections



$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

$\equiv$

Konishi anomalous dimension

$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

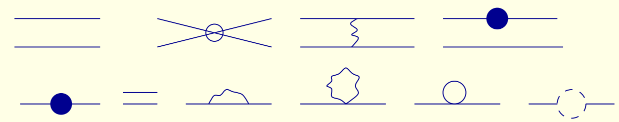
supersymmetric BPS operators

$$\mathcal{O}_{BPS} = \mathrm{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\ldots\uparrow\rangle$$

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \mathrm{Tr}(ZXZX + \ldots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$$

operator mixing

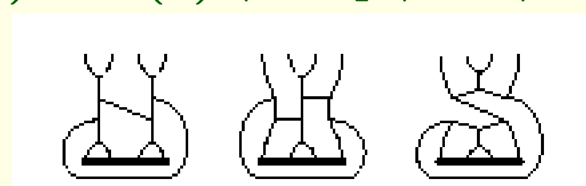


integrable spinchain

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$$

Bethe Ansatz + wrapping

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \ldots + \lambda^4\Delta_4 +$$



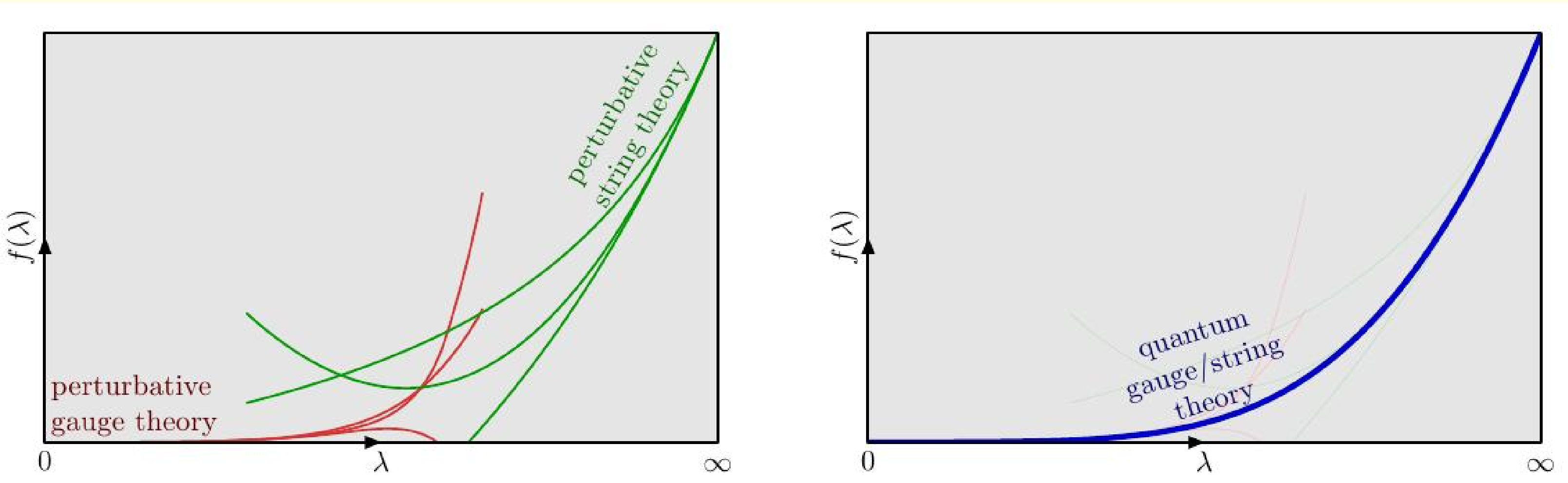
perturbatively $\propto 10^5$ diagrams

$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

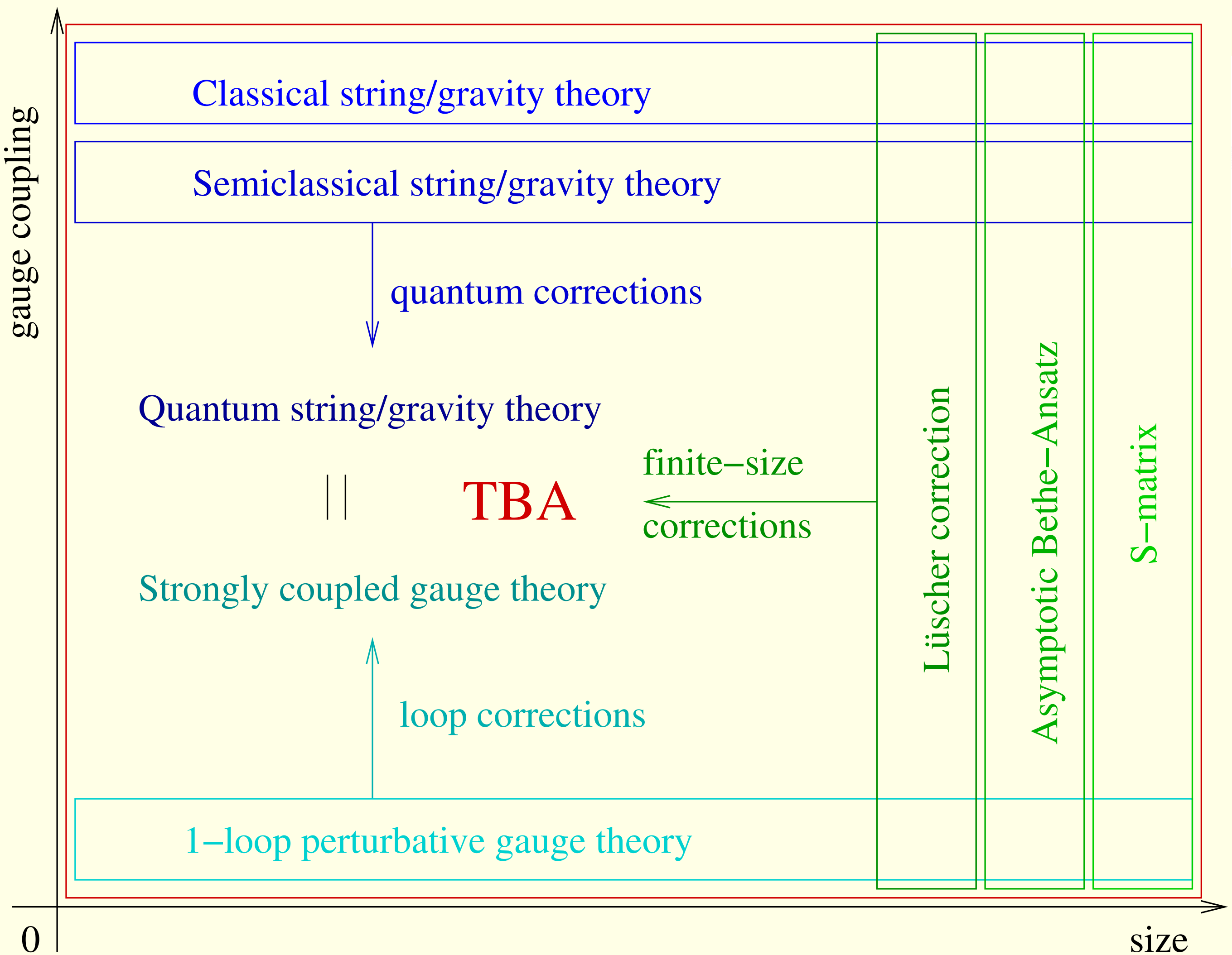
AdS/CFT spectral problem

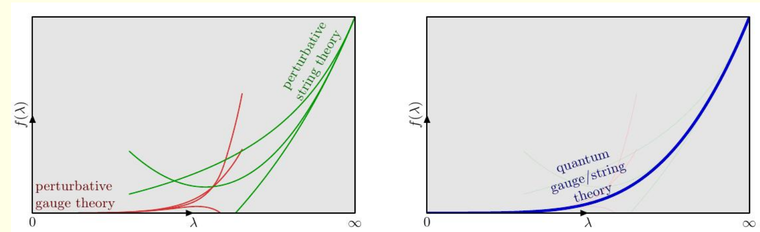
AdS/CFT spectral problem

Konishi dimension: $\mathrm{Tr}(ZXZX - ZZXX)$



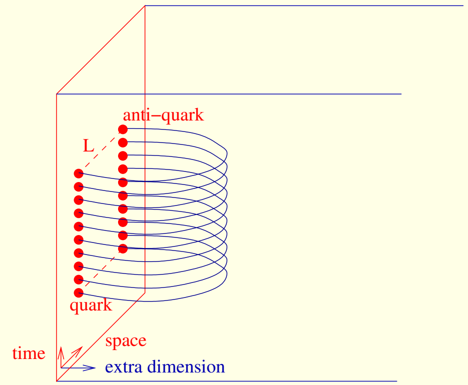
AdS/CFT spectral problem





AdS/CFT correspondence: applications

Minimal surface



exact for strong coupling $\lambda \to \infty$

$\equiv$

quark-antiquark potential

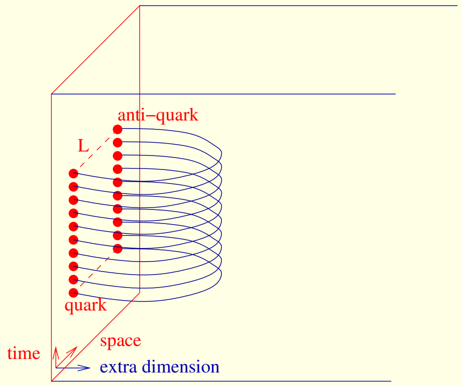
Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$

non-perturbative

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{r}$$

AdS/CFT correspondence: applications

Minimal surface



exact for strong coupling $\lambda \to \infty$

$\equiv$

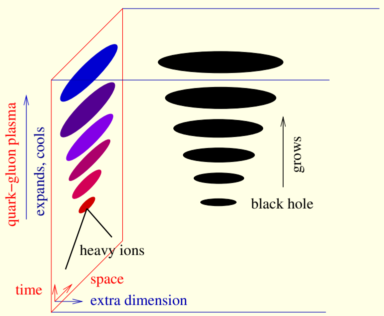
quark-antiquark potential

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$

non-perturbative

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{r}$$

growing black hole



metric

$$ds^2 = \frac{1}{z^2}(g(x,z)_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

Einstein equation

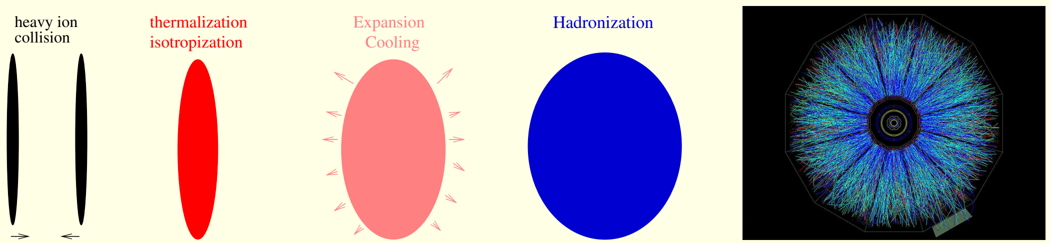
$$R_{ab} - \frac{1}{2}g_{ab}R - 6g_{ab} = 0$$

growing black hole

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}\ ;\ g_{xx} = 1 + \frac{z^4}{z_0^4}$$

$\equiv$

Heavy ion collision: expansion



$\langle T_{\mu\nu} \rangle$ matter distribution

relativistic hydrodynamics

$\partial_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$

viscous quark-gluon plasma

expansion in time: perfect fluid + $\frac{\eta}{s} = \frac{1}{4\pi}$ +...