

Conference of the Middle European Cooperation in Statistical Physics, MECO37,
March 18-22, 2012, Tatranské Matliare, Slovak Republik

Gauge/gravity duality: an overview

Z. Bajnok

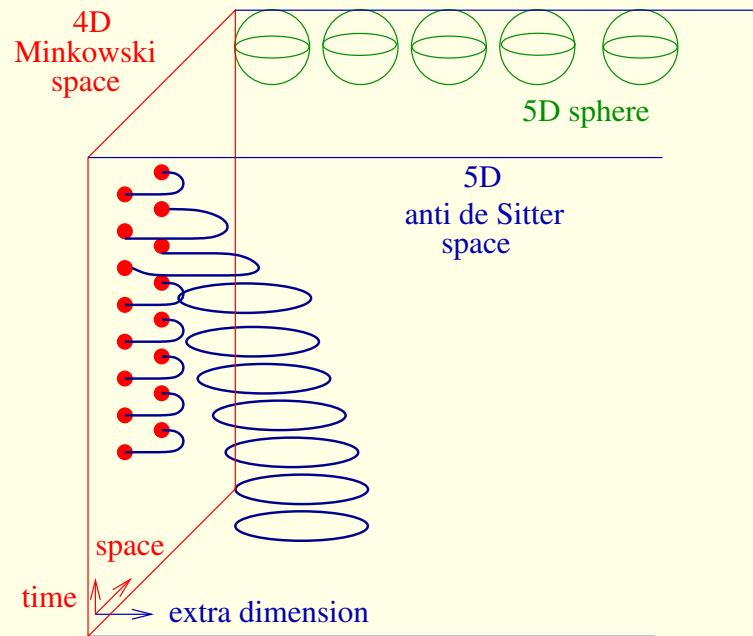
*Theoretical Physics Research Group of the
Hungarian Academy of Sciences, Eötvös University, Budapest*

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Gauge/gravity duality: an overview

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Alias: AdS/CFT correspondencce \subset gauge/gravity duality

Motivation: AdS=CFT

J. Polchinski: TASI lectures, arXiv:1010.6134: Physics World, reader poll:

What is the GREATEST EQUATION EVER?

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Leonhard Euler (1707-1783)

$$e^{i\pi} + 1 = 0$$

$i, \pi, e, 1, 0$ and $+, \cdot, ^$



James Clerk Maxwell (1831-1879)

$$d \star F = j \quad ; \quad dF = 0$$

unifies: electric+magnetic int.

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$$AdS = CFT$$

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J. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252: more than 8000 citations so far

Motivation: Organizing matter

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Periodic Table of the Elements © www.elementsdatabase.com

- hydrogen
- alkali metals
- alkali earth metals
- transition metals

- poor metals
- nonmetals
- noble gases
- rare earth metals

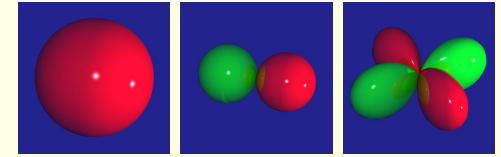
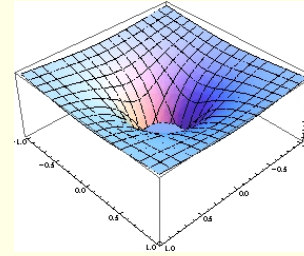
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								

58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Motivation: Organizing matter

Electric interaction (potential $\Phi(r) = k\frac{Zq}{r}$)

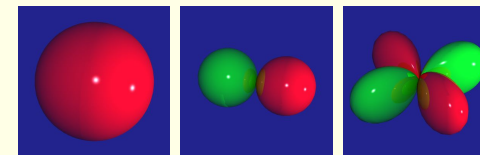
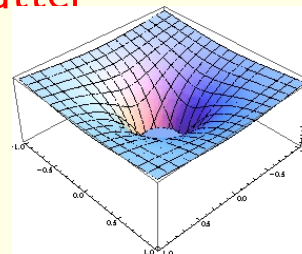
Quantum mechanics (Schrödinger eq.) $H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$



Motivation: Organizing matter

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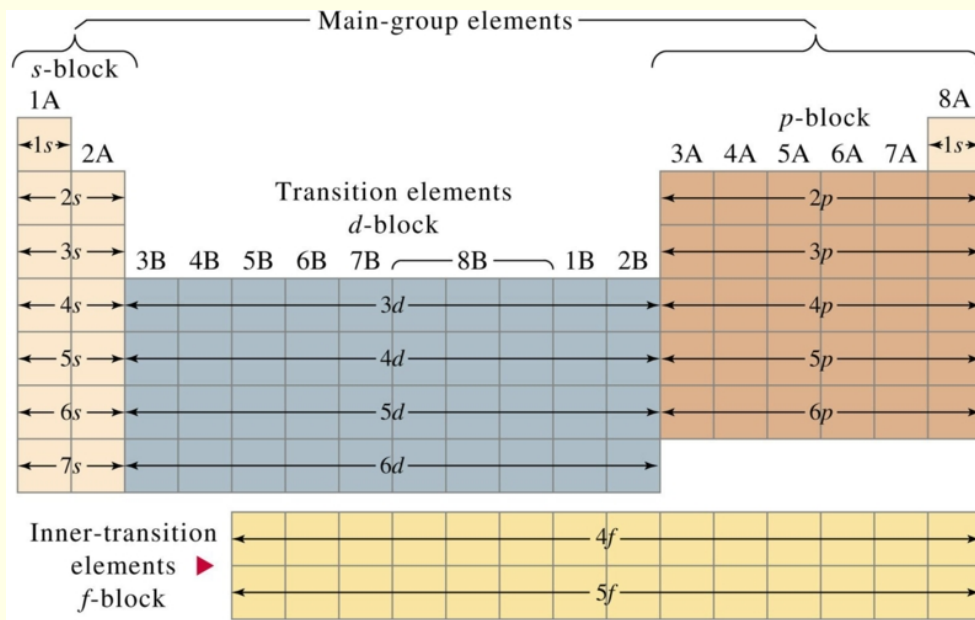
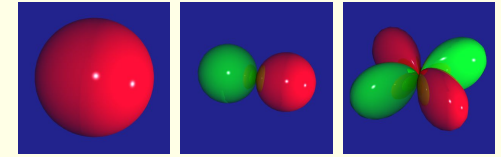
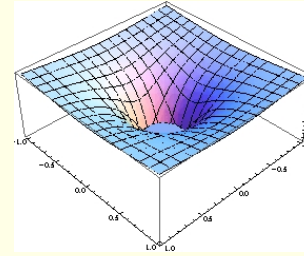


Main-group elements																			
s-block		Transition elements														p-block			
1A	2A	d-block										3A	4A	5A	6A	7A	8A		
←1s→																	←1s→		
←2s→																	←2p→		
←3s→		3B	4B	5B	6B	7B	8B	1B	2B								←3p→		
←4s→		←3d→																←4p→	
←5s→		←4d→																←5p→	
←6s→		←5d→																←6p→	
←7s→		←6d→																	
Inner-transition elements																			
f-block		←4f→																	
		←5f→																	

Motivation: Organizing matter

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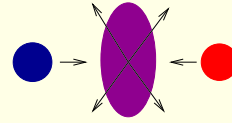
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Na	Mg											Al	Si	P	S	Cl	Ar
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
87	88	89	104	105	106	107	108	109	110								
Fr	Ra	Ac	Unq	Unp	Unh	Uns	Uno	Une	Uun								
		58	59	60	61	62	63	64	65	66	67	68	69	70	71		
		Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu		
		90	91	92	93	94	95	96	97	98	99	100	101	102	103		
		Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr		

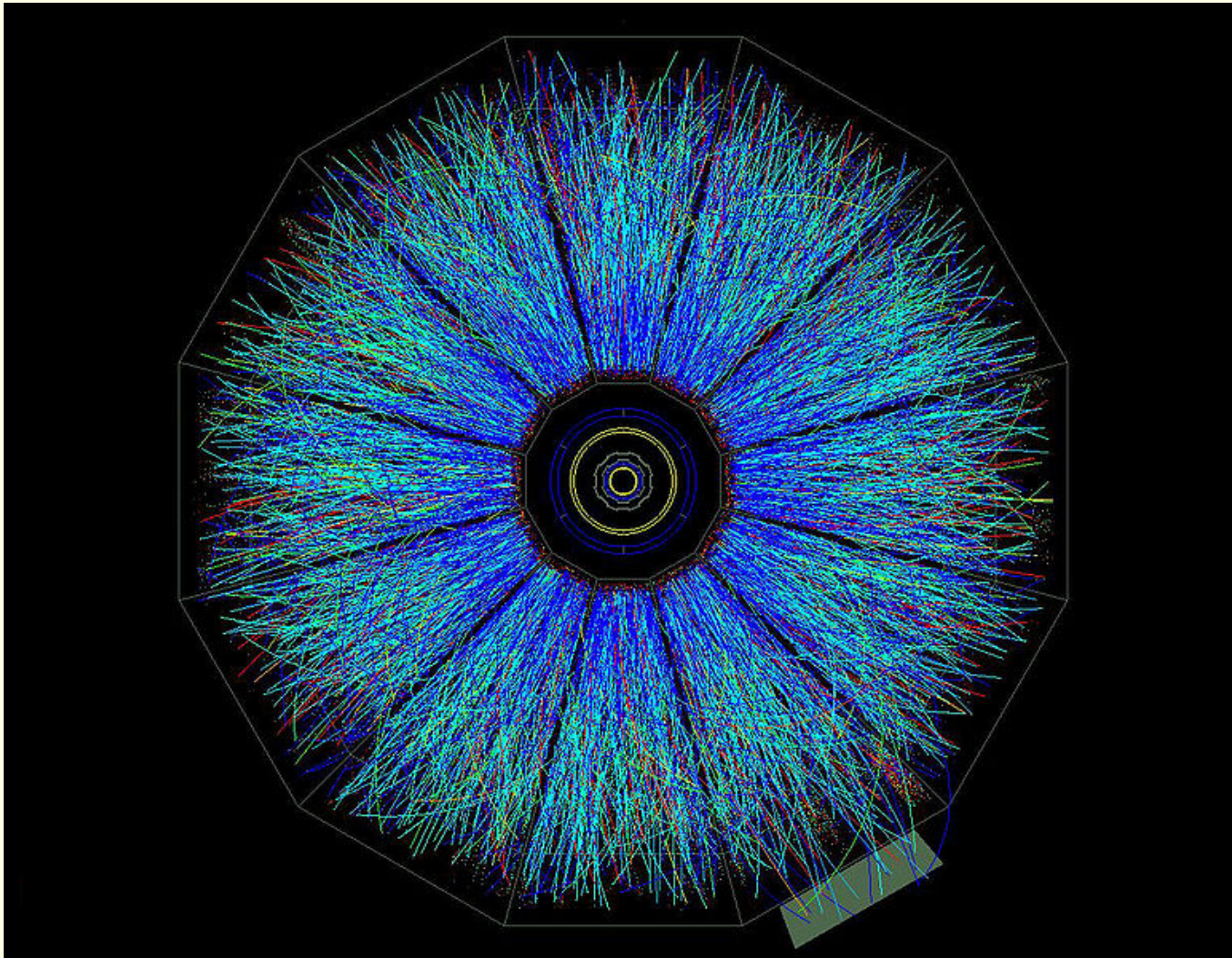
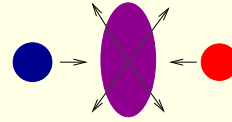
Organizing matter II

Brookhaven: Relativistic heavy ion collider (gold ion)



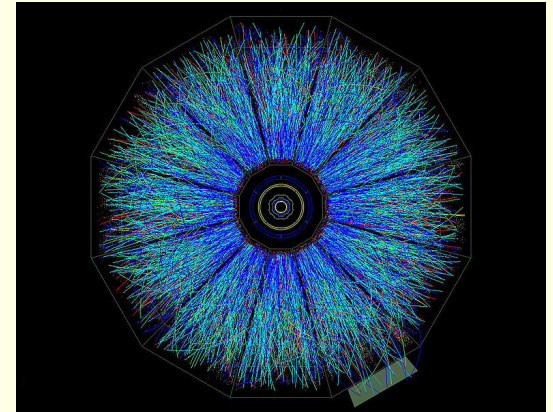
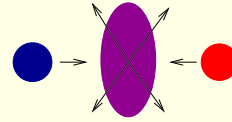
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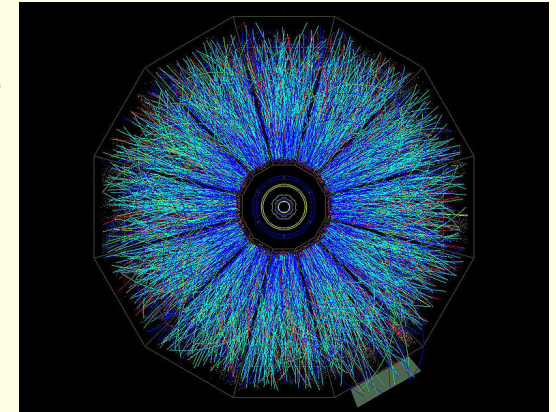
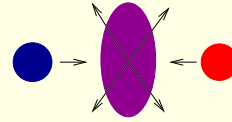
Brookhaven: Relativistic heavy ion collider (gold ion)



Organizing matter II

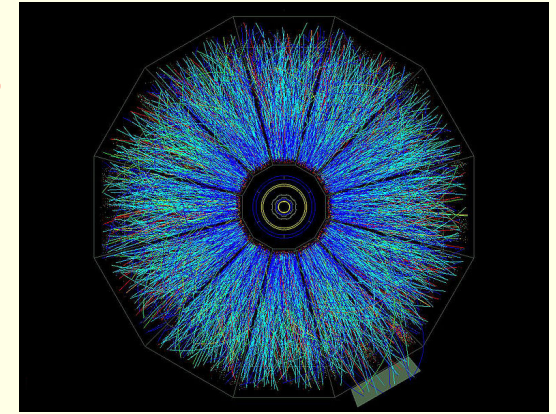
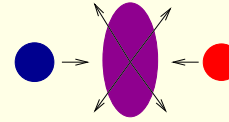
Brookhaven: Relativistic heavy ion collider (gold ion)

Number of elementary particles > number of atoms → classification

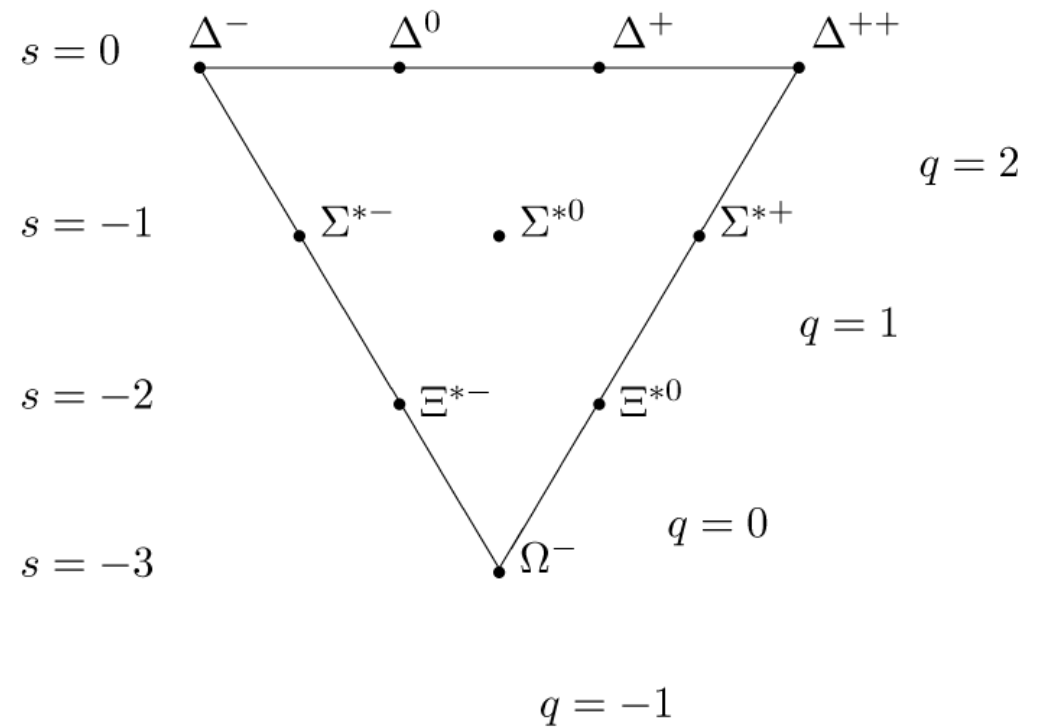
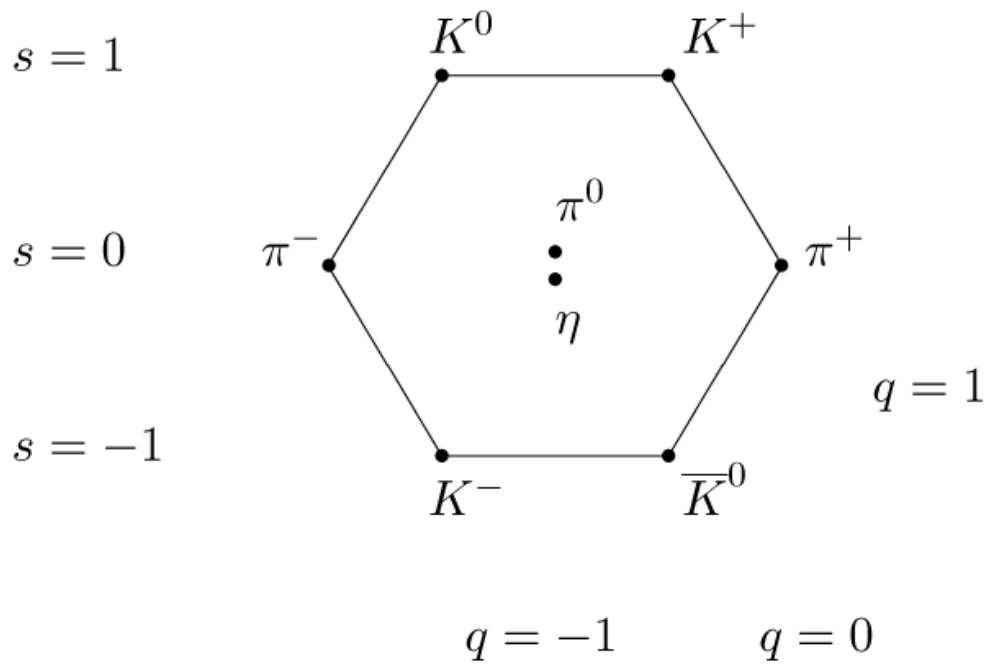


Organizing matter II

Brookhaven: Relativistic heavy ion collider (gold ion)

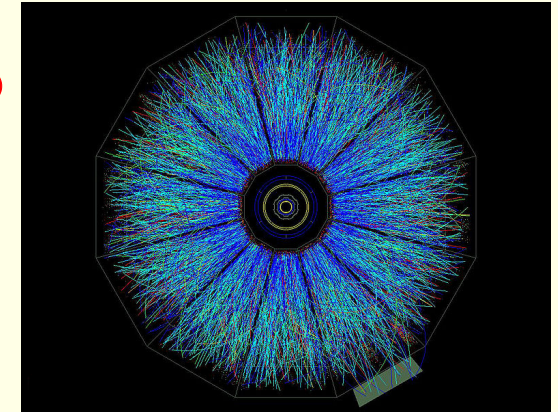
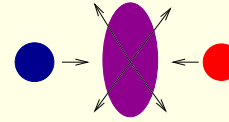


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Organizing matter II

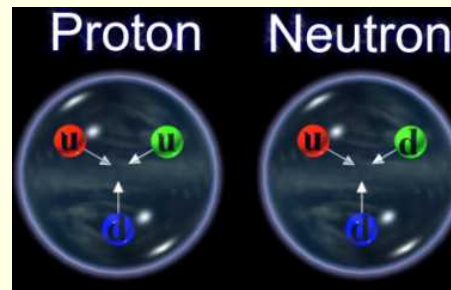
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Number of elementary particles \gg number of atoms \rightarrow classification

Decay \rightarrow strong, weak interaction \rightarrow Standard Model

Interaction			
γ	photon	electromagnetic	<i>I.</i>
W^{\pm}, Z	weak bosons	weak	
g	gluon	strong	<i>II.</i>
gr	graviton	gravitational	



Leptons		Quarks		Three Generations of Matter (Fermions)
name	charge	name	charge	
electron e	-1	up u	2/3	
electron neutrino ν_e	0	down d	-1/3	
muon μ	-1	charm c	2/3	
muon neutrino ν_μ	0	strange s	-1/3	
tau τ	-1	top t	2/3	
tau neutrino ν_τ	0	bottom b	-1/3	
weak force W^{\pm}	± 1	photon γ	0	
weak force Z	0	gluon g	0	

Bosons (Forces)

Quantum electrodynamics

Relativity theory: $A_\mu = (\Phi, \underline{A})$

electric + magnetic int: $F_{\mu\nu}$

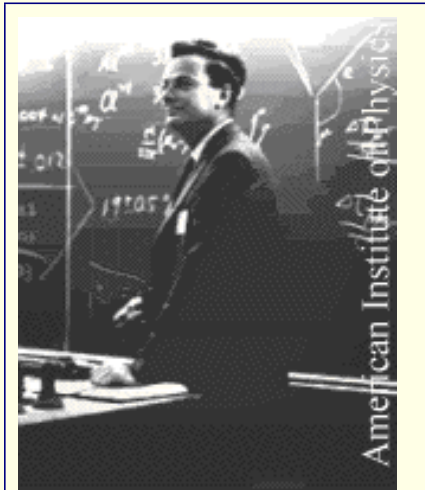
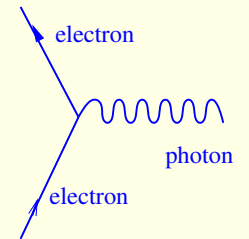
+ Quantum theory \rightarrow QED

$U(1)$ gauge theory: $A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x)$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - e\bar{\Psi}\not{A}\Psi$$

experiment: $\underline{\mu} = g \frac{e\hbar}{2mc} \underline{s}$ where $g = 2(1 + a)$

Gabrielse et.al.: $a = 1159652180.85(.76) \times 10^{-12}$



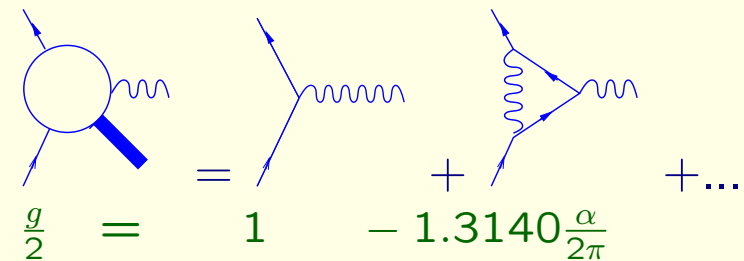
Feynman: *If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.*

Quantum gauge theory

perturbation theory:

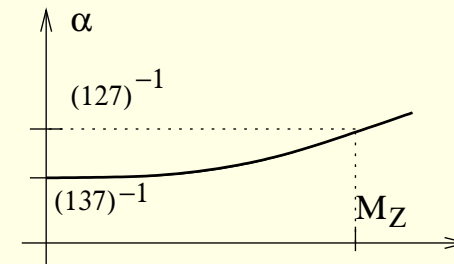
Feynman graphs

$$\frac{\alpha}{2\pi} = \frac{e^2}{2\pi\hbar c} = 0.001161$$



momentum-dependent coupling:

$$\beta(\alpha) = \mu \frac{\partial \alpha}{\partial \mu} > 0$$



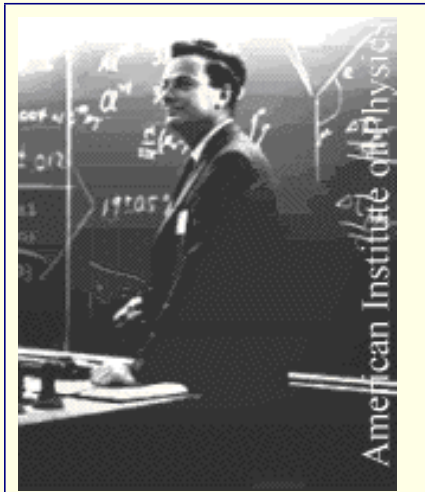
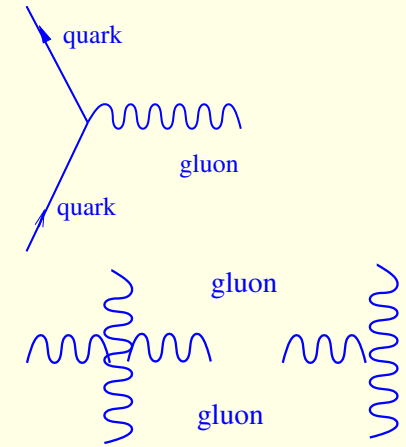
Quantum Chromodynamics

photon $A_\mu \leftrightarrow G_\mu^{1..8}$ gluon $\rightarrow F_{\mu\nu}^{1..8}$

electron $\Psi_e \leftrightarrow \Psi_{quark}$ quark

$SU(3)$ gauge theory: $G_\mu \rightarrow g^{-1}G_\mu g + g^{-1}\partial_\mu g$

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - g\bar{\Psi}Q\Psi$$



Quantum gauge theory

asymptotic freedom

2004 Nobel Prize in Physics



David J. Gross H. David Politzer Frank Wilczek

experiments:

hadron spectrum

perturbation theory:

Feynman graphs

$$0.001 = \frac{\alpha}{2\pi} \leftrightarrow \frac{\alpha_s}{4\pi} = O(1)$$

momentum-dependent coupling:

$$\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$$

asymptotic freedom

confinement

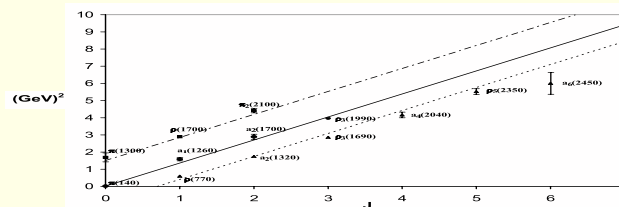
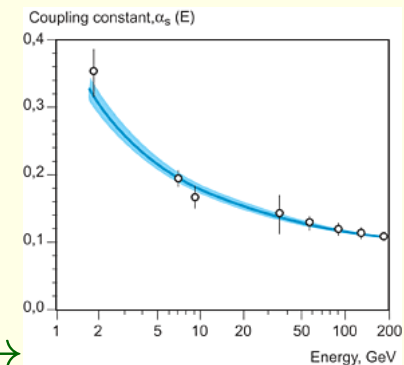
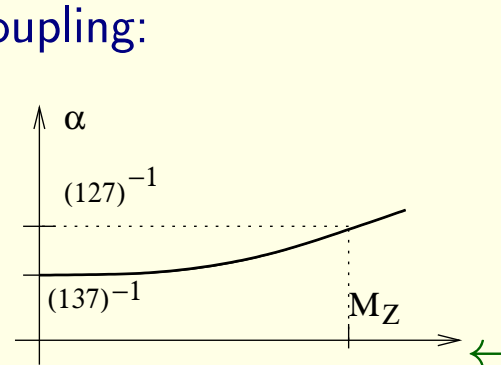
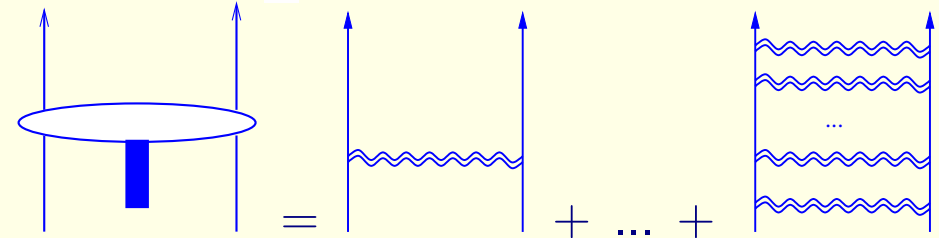


Fig. 1.



CFT: maximally supersymmetric gauge theory

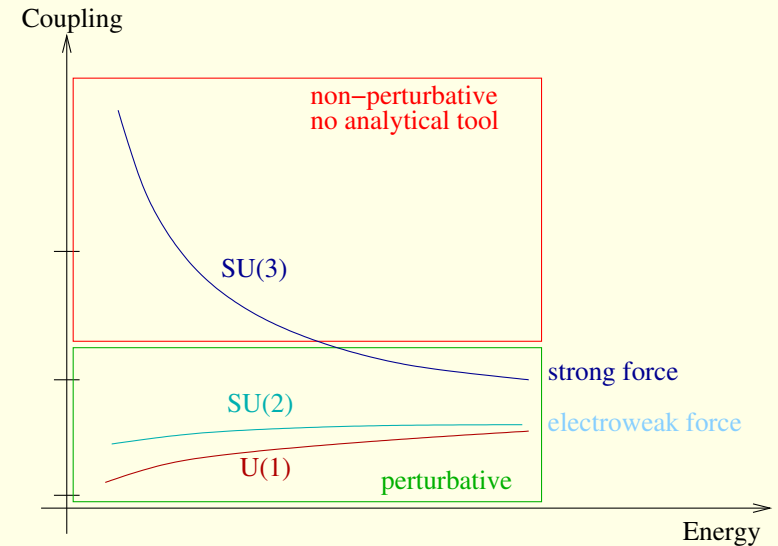
Fundamental interactions

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	W^\pm, Z, μ, ν +Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

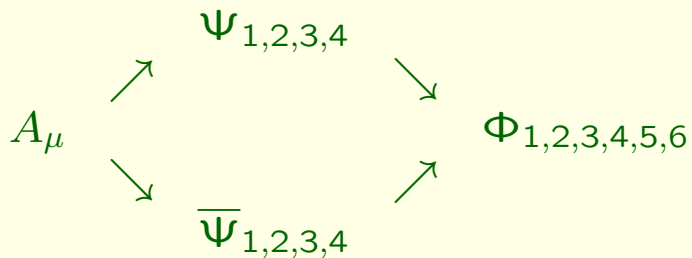
only analytical tool: perturbation theory

maximally supersymmetric (N=4) gauge theory

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$



all fields $N^2 - 1$ component matrix



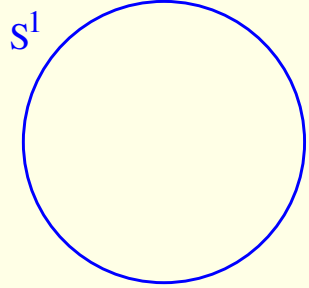
$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

no running $\beta = 0 \rightarrow$ CFT

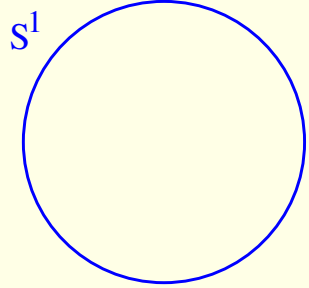
AdS: string theory on Anti de Sitter \supset gravitation

positively curved space

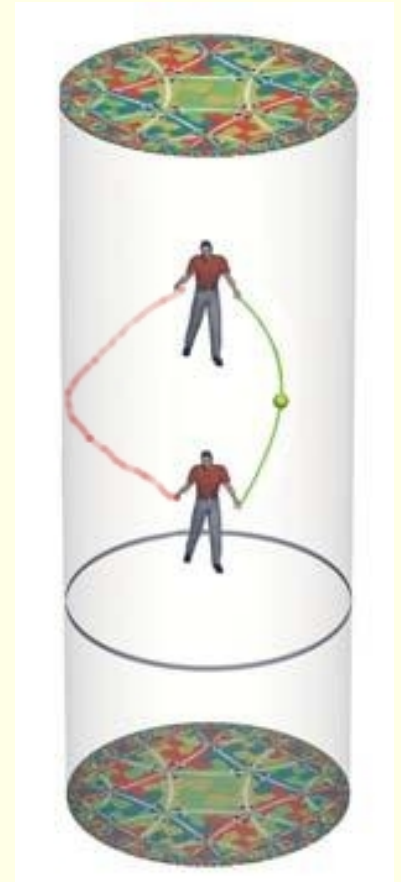


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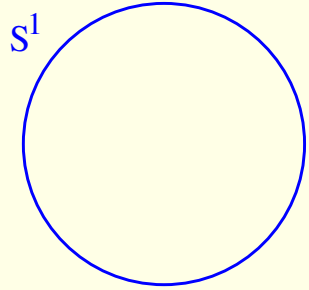


Anti de Sitter: negatively curved space

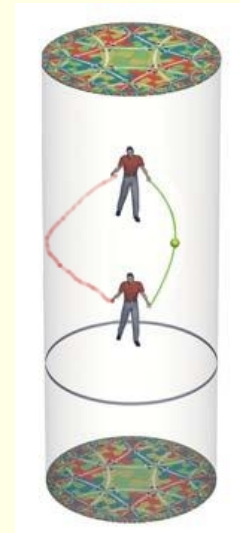


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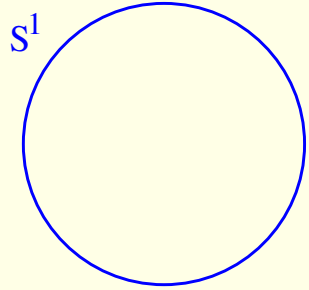


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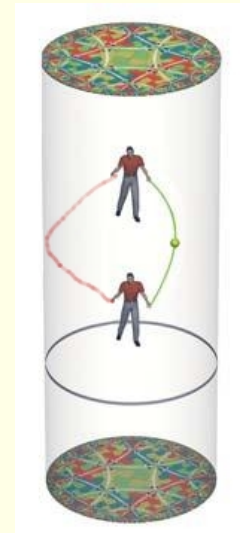
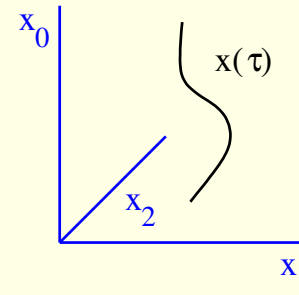


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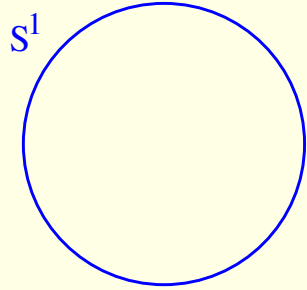
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



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Anti de Sitter: negatively curved space

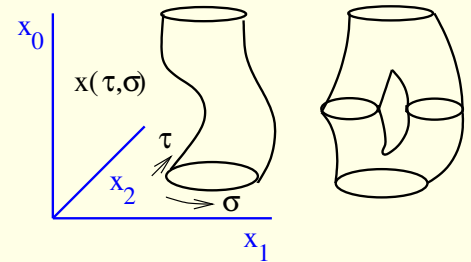
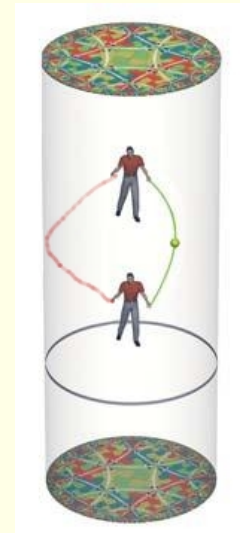
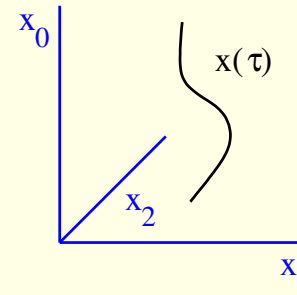


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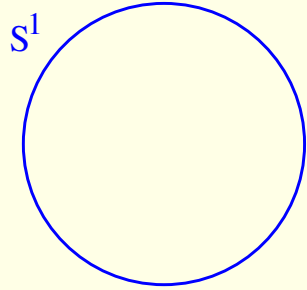
relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$$

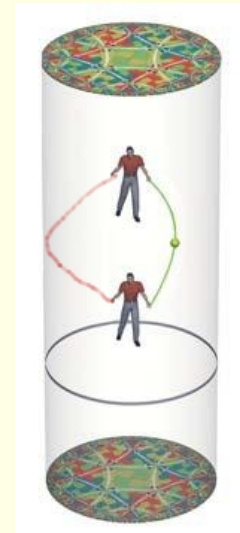


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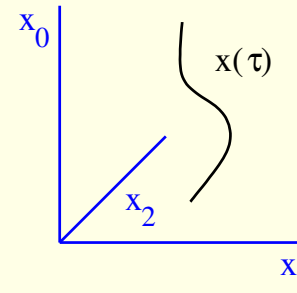


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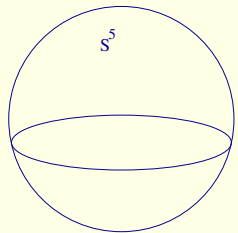
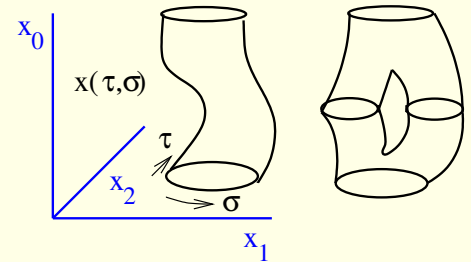
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$$



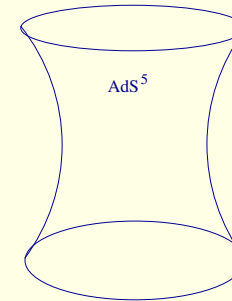
relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$$S \propto \text{worldsheet} \propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$$



$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$



$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \text{fermionok}$$

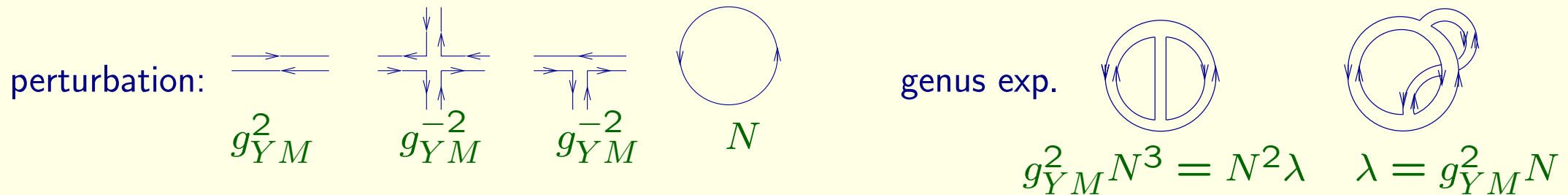
supercoset $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

CFT: Observables

maximally supersymmetric gauge theory	
A	$\Psi_{1,2,3,4}$ $\Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices
$\bar{\Psi}_{1,2,3,4}$	
$\mathcal{S} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\bar{\Psi}\not{D}\Psi + V \right]$	
$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$	

observables
parameters: g_{YM}, N
observables: partition function
gauge-invariant operators
$\mathcal{O}(x) = \text{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$
correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0) \rangle$

correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0) \rangle = \int [dA...] e^{-i\mathcal{S}} \mathcal{O}_1(x)\mathcal{O}_2(0) = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV} \rangle_0$



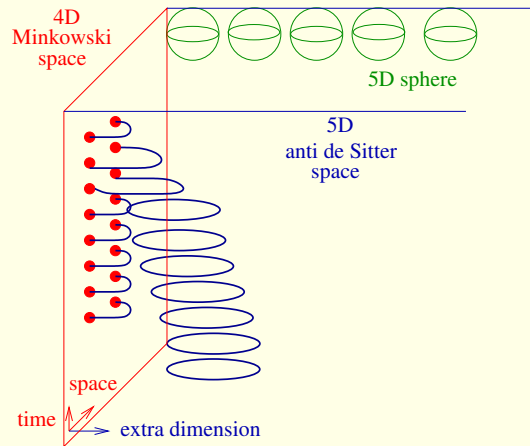
partition func. $Z(\lambda, \frac{1}{N}) = N^2 \sum_g (\frac{1}{N})^{2g} \sum_n \alpha(g, n) \lambda^n$ string interactions? (t' Hooft)

conformal field theory: $\langle \mathcal{O}_i(x)\mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ scale dim.: Δ_i Konishi op. $\mathcal{O}_K = \text{Tr}(\Phi_i^2)$

$$\Delta_K(\lambda) = 2 + 6\frac{\lambda}{4\pi^2} - 24\frac{\lambda^2}{(4\pi^2)^2} + 168\frac{\lambda^3}{(4\pi^2)^3} - (1410 + 144\zeta(3) + \frac{1}{2}(324 + 864\zeta(3) - 1440\zeta(5)))\frac{\lambda^4}{(4\pi^2)^4}$$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on AdS₅ × S⁵



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

≡

$\mathcal{N} = 4$ D=4 SU(N) SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

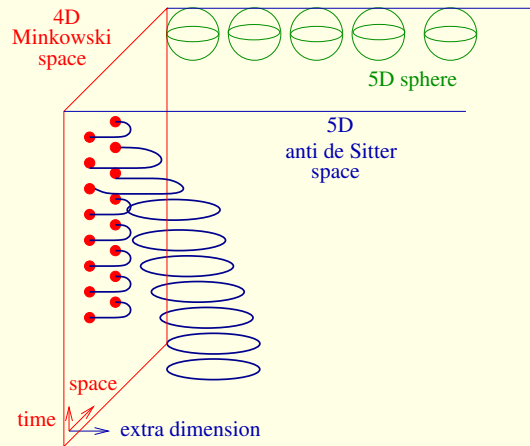
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \text{Tr}(\Phi^2), \det(\)$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

\equiv

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$A_\mu \begin{matrix} \nearrow \Psi_{1,2,3,4} \\ \searrow \bar{\Psi}_{1,2,3,4} \end{matrix} \quad \begin{matrix} \searrow \\ \nearrow \end{matrix} \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

$$\text{gaugeinvariants: } \mathcal{O} = \text{Tr}(\Phi^2), \det(\dots)$$

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha' l}, g_s = \frac{\lambda}{N} \rightarrow 0$
2D QFT

String energy levels: $E(\lambda)$
 $E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$

Dictionary

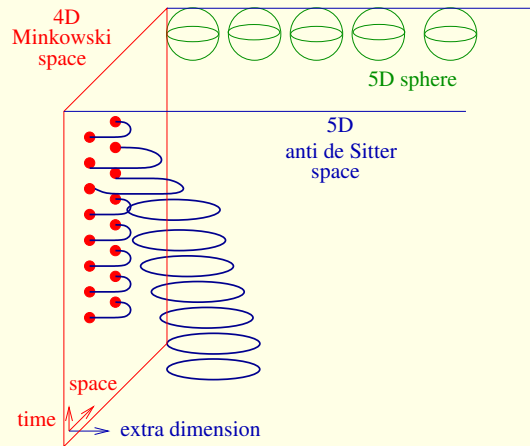
strong \leftrightarrow weak

\Downarrow

$\lambda = g_{YM}^2 N, N \rightarrow \infty$ planar limit
 $\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$
Anomalous dim $\Delta(\lambda)$
 $\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$

AdS/CFT correspondence (Maldacena 1998)

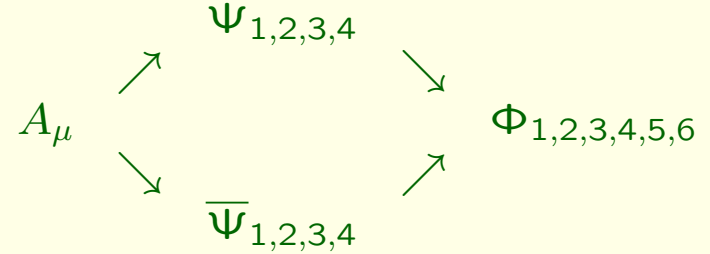
II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} (\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M) + \dots$$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM



$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

$$\text{gaugeinvariants: } \mathcal{O} = \text{Tr}(\Phi^2), \det(\)$$

\equiv

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \rightarrow 0$
2D QFT

String energy levels: $E(\lambda)$

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Dictionary

strong \leftrightarrow weak

\Downarrow

$\lambda = g_{YM}^2 N$, $N \rightarrow \infty$ planar limit

$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

2D integrable QFT

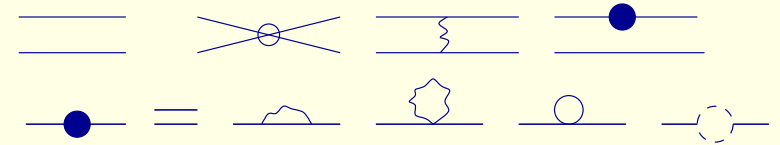
CFT: Integrability

Perturbative correlator: $\langle \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle = \langle \mathcal{O}_1(x) \mathcal{O}_2(0) e^{-i(\frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi])} \rangle_0$

Conformal (scale invariant) field theory: $= \frac{\delta_{ij}}{|x|^{2\Delta(\lambda)}} = \frac{1}{|x|^{2\Delta(0)}} \left[1 + \lambda \Delta_1 \log \frac{1}{|x|^2} + \dots \right]$

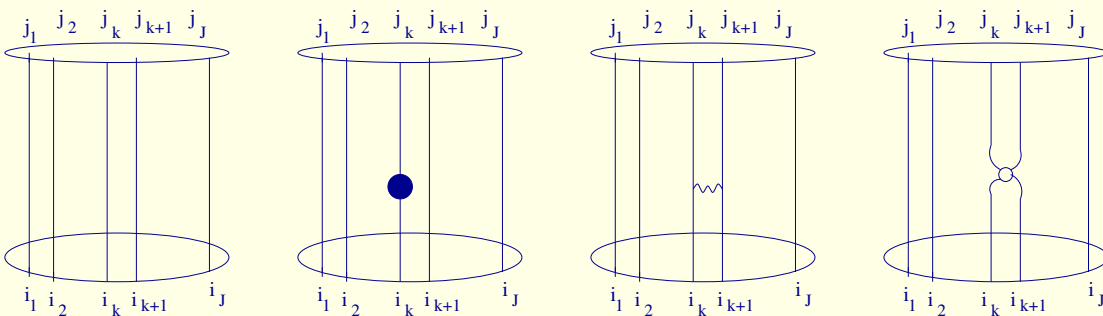
Scalar sector: $Z_1 = \Phi_1 + i\Phi_2, Z_2 = \Phi_3 + i\Phi_4$ SUSY st: $\mathcal{O} = \text{Tr} [Z_i^J] \rightarrow \Delta_{\mathcal{O}}(\lambda) = J$

Operator mixing: $\mathcal{O}_1 = \text{Tr} [Z_1 Z_1 Z_2 Z_2] \leftrightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$
 $\mathcal{O}_2 = \text{Tr} [Z_1 Z_2 Z_1 Z_2] \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle$



diagonalize the 1-loop mixing matrix: $\mathcal{O}_{\pm} = \mathcal{O}_1 \pm \mathcal{O}_2 \rightarrow \Delta_{\mathcal{O}_+}(\lambda) = 4$
 $\Delta_{\mathcal{O}_-}(\lambda) = 4 + 6 \frac{\lambda}{4\pi^2}$

generic state at size J : $\mathcal{O}_{i_1 \dots i_J} = \text{Tr} [Z_{i_1} \dots Z_{i_J}] \leftrightarrow |i_1 \dots i_J\rangle$



$$\Delta = J\mathbb{I} + \frac{\lambda}{8\pi^2} \sum_{k=1}^J (\mathbb{I} - \mathbb{P}_{k,k+1})$$

Heisenberg spin chain

CFT: Integrability + Bethe Ansatz

Mixing matrix on the subspace $\text{Tr} [Z_{i_1} \dots Z_{i_J}]$ of dim 2^J : Minahan-Zarembo 2002

$$\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \dots = J \mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \dots$$

H_2 : next-to-nearest neighbour integrable! \rightarrow use Bethe ansatz

1. choose a groundstate: $Z = Z_1 \rightarrow \text{Tr} [Z^J] = \text{Tr} [ZZZZZ \dots ZZZZ] \leftrightarrow |\uparrow \dots \uparrow\rangle$

2. excitations $Z \dots Z X Z \dots X$ with SUSY multiplet $X = Z_2, Z_3, \Psi_a^\alpha, \Psi_a^{\dot{\alpha}}, D_\mu$

3. plane wave: $\sum_n e^{ipn} \text{Tr} (\overbrace{Z \dots Z}^n X Z \dots Z)$

4. scattering states: $\sum_{n_1 n_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2} \text{Tr} (\underbrace{Z \dots Z}_{n_1} \overbrace{X_{a_1} Z \dots Z}^{n_2} X_{a_2} Z \dots Z) + S(12)_{a_1 a_2}^{b_1 b_2} \Sigma$

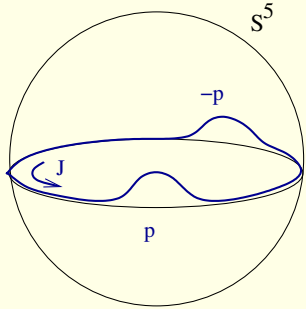
symmetry completely fixes the S-matrix for any λ (satisfies unitarity, crossing, Yang-Baxter)

Bethe ansatz follows from S-matrix: Shastry's Hubbard S-matrix

AdS/CFT correspondence: confirmation

two particle states

$$E_{BPS}(\lambda) = E_0$$



$$E_K(\lambda) = 2E(p, \lambda)$$

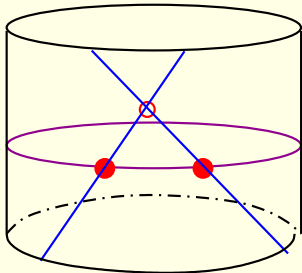
dispersion relation

$$E(p, \lambda) = \sqrt{1 + \frac{\lambda}{\pi^2} (\sin \frac{p}{2})^2}$$

elastic scattering $S(p, -p)$

Bethe Ansatz: $e^{ipJ} S(p, -p) = 1$

finite size corrections



$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

Konishi anomalous dimension

$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

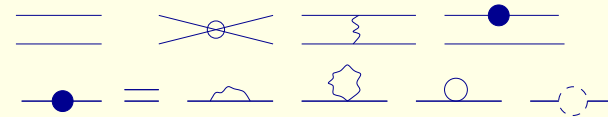
supersymmetric **BPS** operators

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

operator mixing



integrable spinchain

$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$$

Bethe Ansatz + **wrapping**

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 + \dots$$



perturbatively $\propto 10^5$ diagrams

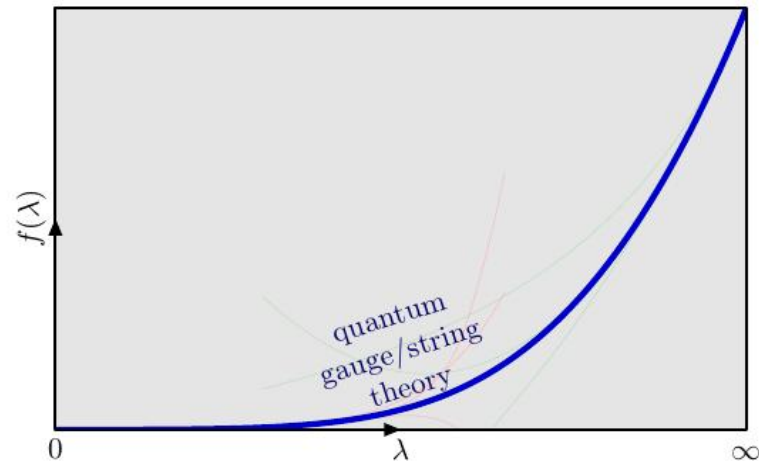
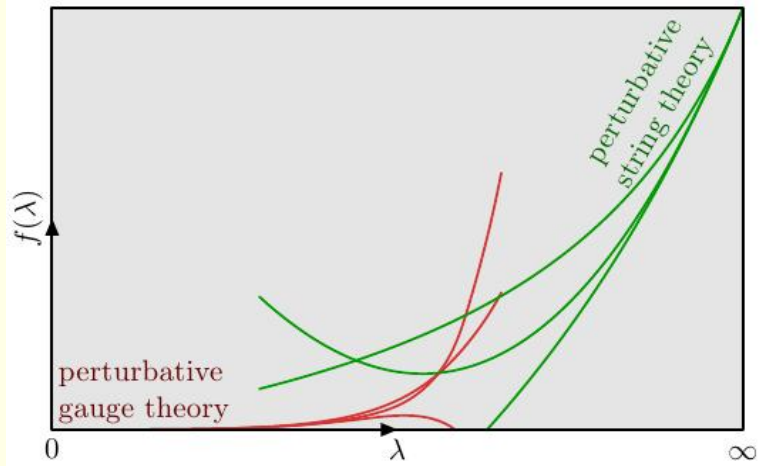
$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

\equiv

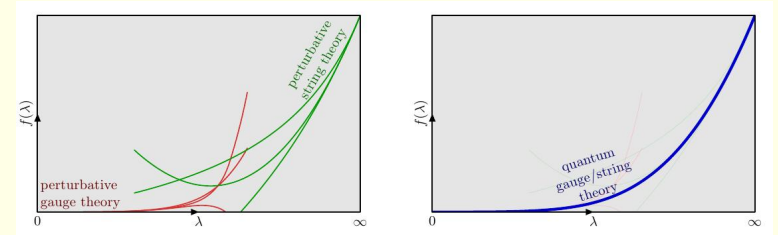
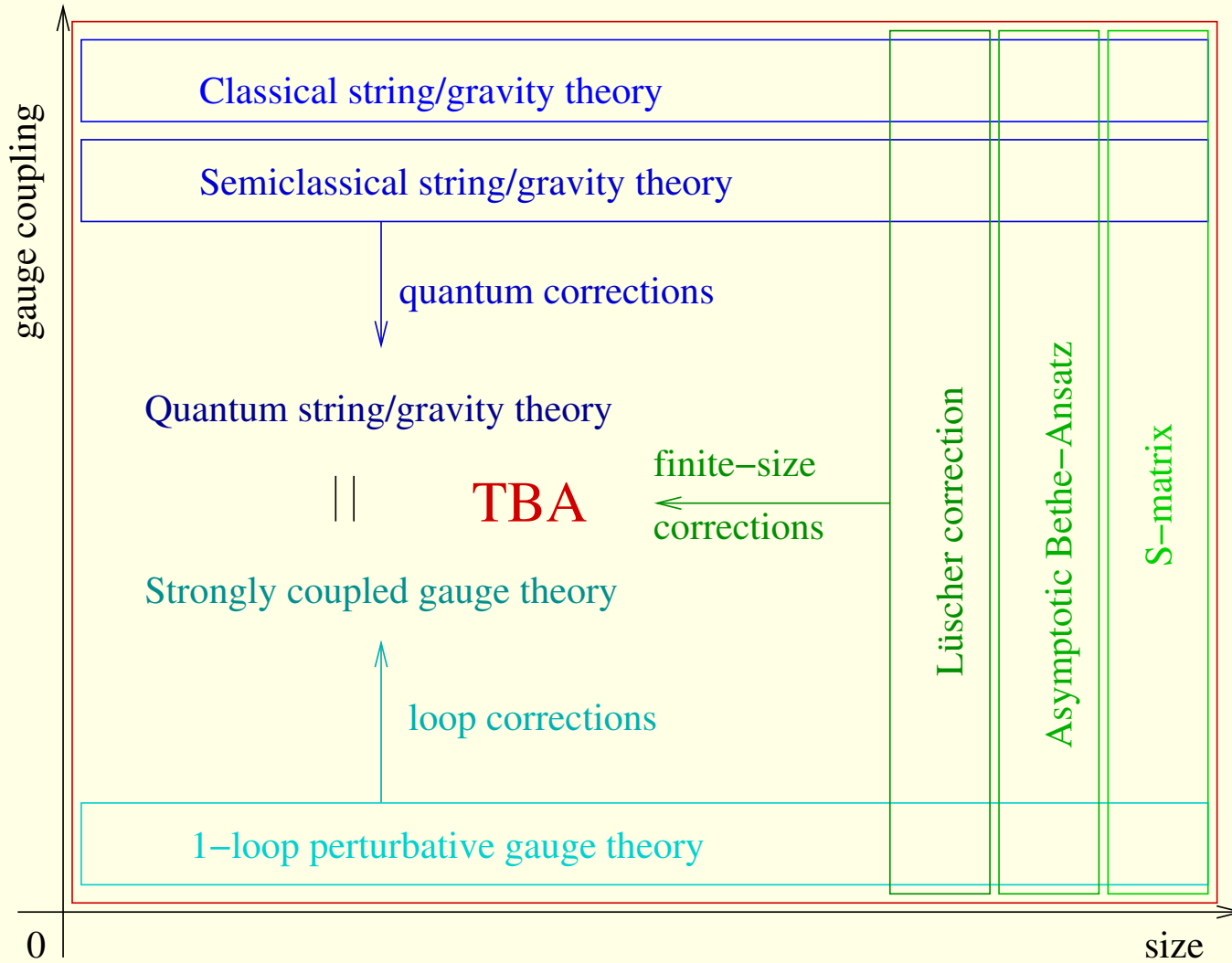
AdS/CFT spectral problem

AdS/CFT spectral problem

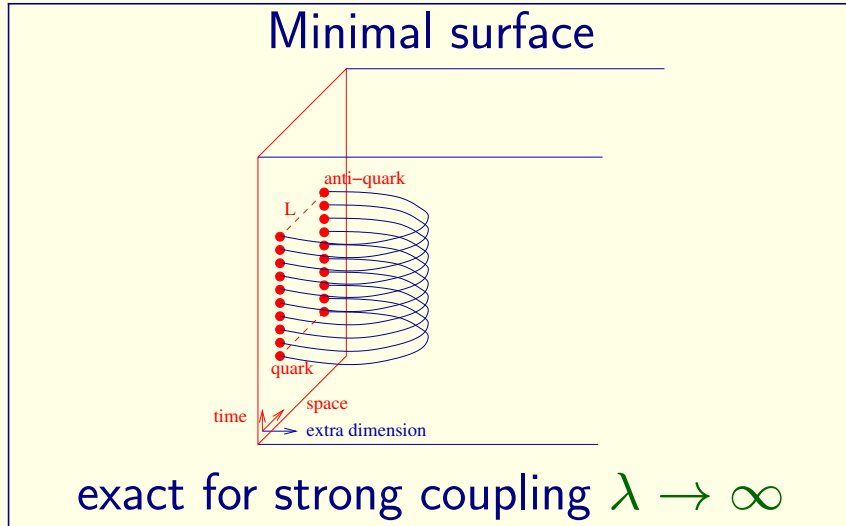
Konishi dimension: $\text{Tr}(ZXZX - ZZXX)$



AdS/CFT spectral problem



AdS/CFT correspondence: applications



\equiv

quark-antiquark potential

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$
non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

AdS/CFT correspondence: applications

Minimal surface

exact for strong coupling $\lambda \rightarrow \infty$

quark-antiquark potential

Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle$
 non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

≡

growing black hole

metric

$$ds^2 = \frac{1}{z^2} (g(x, z)_{\mu\nu} dx^\mu dx^\nu + dz^2)$$

Einstein equation

$$R_{ab} - \frac{1}{2} g_{ab} R - 6g_{ab} = 0$$

growing black hole

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}; \quad g_{xx} = 1 + \frac{z^4}{z_0^4}$$

≡

Heavy ion collision: expansion

$\langle T_{\mu\nu} \rangle$ matter distribution
 relativistic hydrodynamics
 $\partial_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$
 viscous quark-gluon plasma

expansion in time: perfect fluid + $\frac{\eta}{s} = \frac{1}{4\pi} + \dots$