Institute for Theoretical Physics, Utrecht, 19th of January, 2011

Casimir effect, boundary quantum field theories and AdS/CFT

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in collaboration with L. Palla and G. Takács

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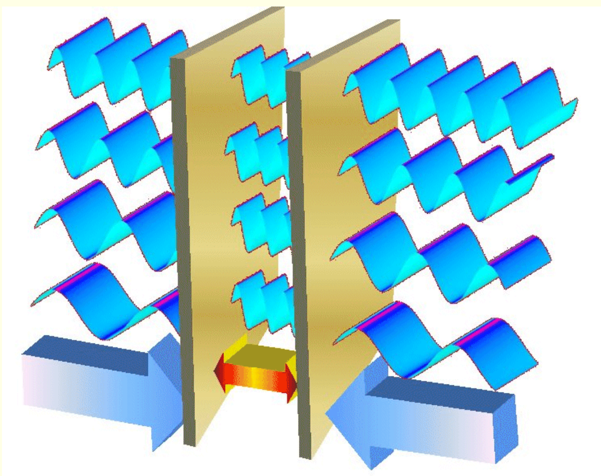


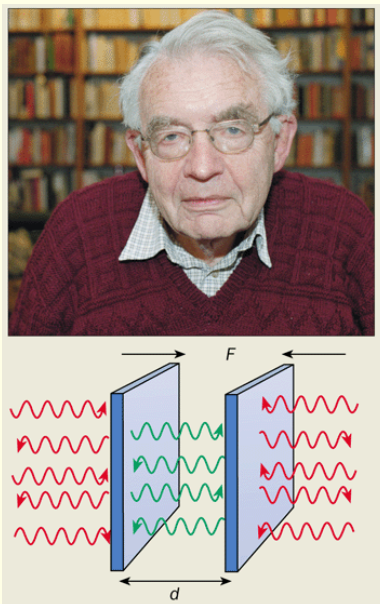
$$F(L) = \frac{dE_0(L)}{dL}$$

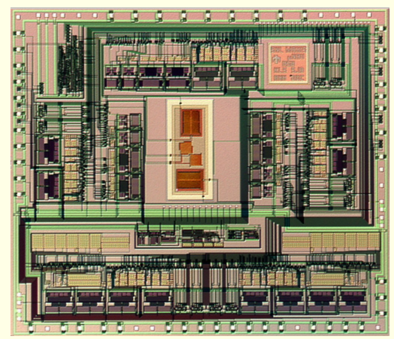
$\mathcal{N} = 4$ D=4 $SU(N)$ SYM
$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}$; $A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$
$\mathcal{O} = \text{Tr}(\Phi\dots\Phi)\ \ \det(\Phi\dots\Phi)$
$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{\vert x\vert^{2\Delta_n(\lambda)}}$

Planar Casimir energy $E_0(L) \equiv \Delta_n(\lambda)$ anomalous dimensions of determinant type operators:
from finite size effects in *(integrable)* boundary QFT

Motivation: Casimir effect

Hendrik Casimir (1909-2000) | Gecko legs | micro mechanical devices (Sandia)





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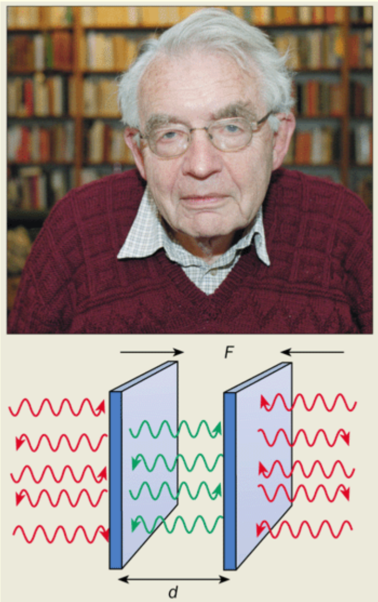
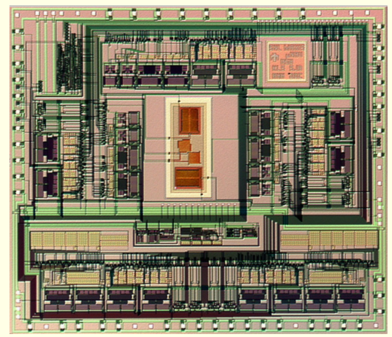
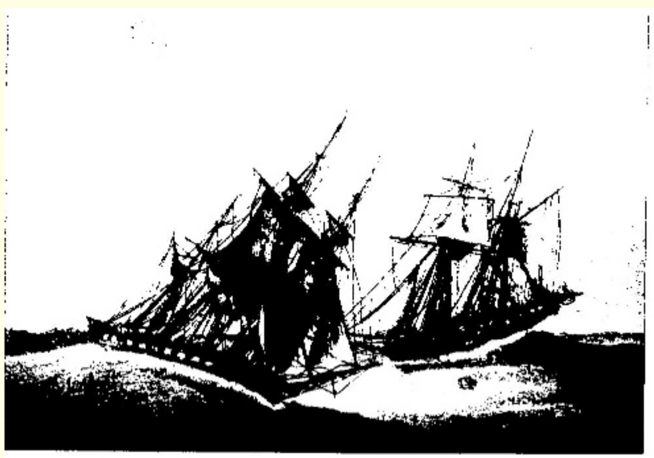
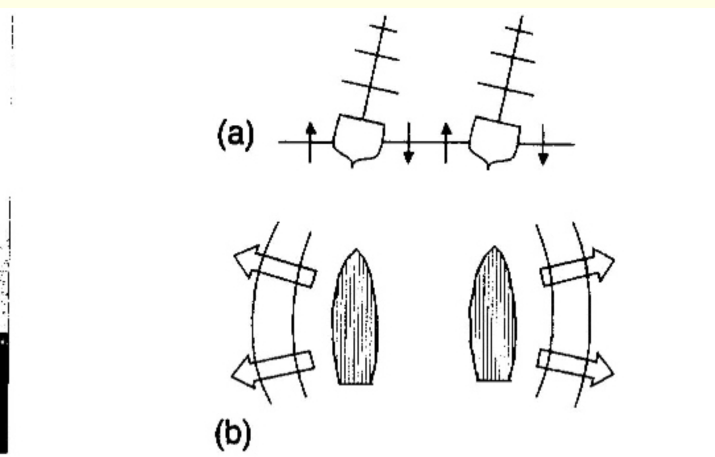




Maritime analogy:





Aim: understand/describe planar Casimir effect

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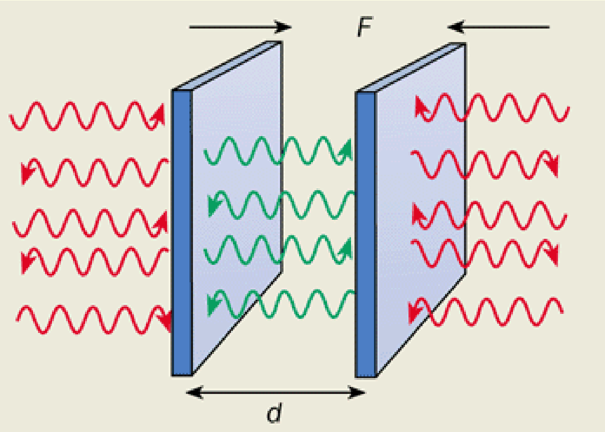
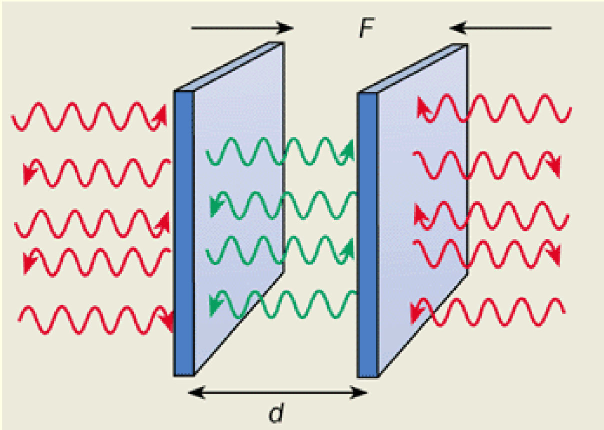


Usual explanation: energy of the vacuum: $E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$

Aim: understand/describe planar Casimir effect



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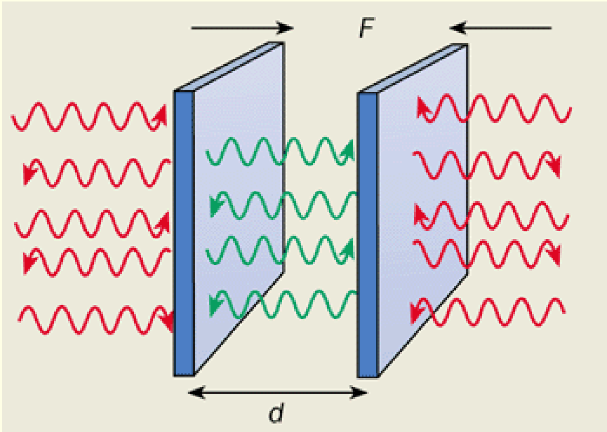
$$E_0(L) - E_0(\infty) - 2E_{plate} = \Delta E_0(L) \qquad ; \qquad \frac{\Delta E_0(L)}{A}$$

Aim: understand/describe planar Casimir effect



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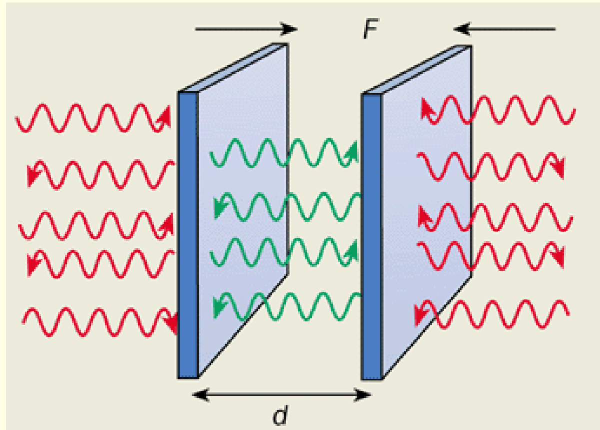
Lifshitz formula: QED, Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

$$\Delta E_0(L)/A = \sum_{i=\|,\perp} \int_0^\infty \frac{d^2q}{8\pi^2} d\zeta \log\left[1 - R_i^1(\zeta,q) R_i^2(\zeta,q) e^{-2L\sqrt{q^2+\zeta^2}}\right]$$

Aim: understand/describe planar Casimir effect



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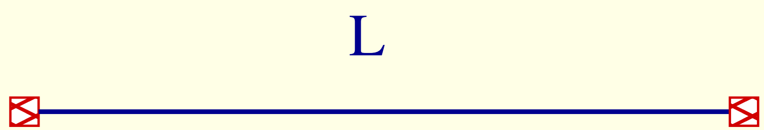
Physics can be understood in 1+1 D QFT



integrability helps to solve the problem even exactly $\rightarrow$ large volume expansion in any D

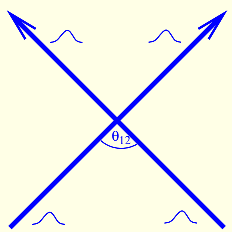
Plan of talk

Cylinder

Plan of talk

Cylinder

Infinite volume

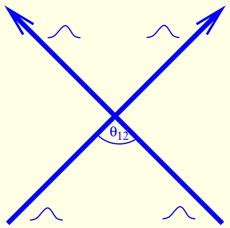


S-matrix

Plan of talk

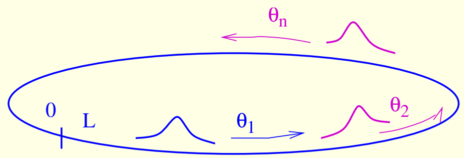
Cylinder

Infinite volume



S-matrix

Large volumes

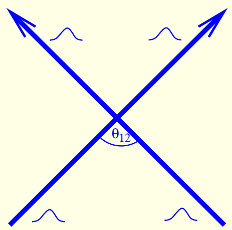


Bethe-Yang lines

Plan of talk

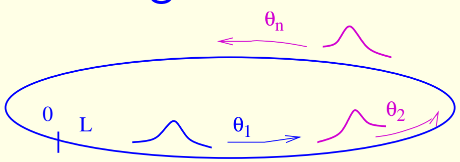
Cylinder

Infinite volume



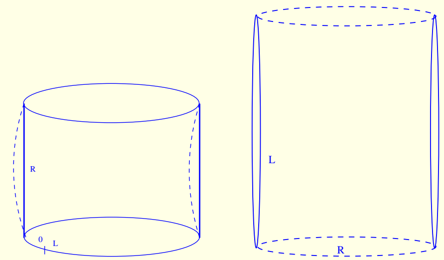
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction

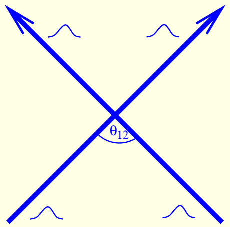


$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

Plan of talk

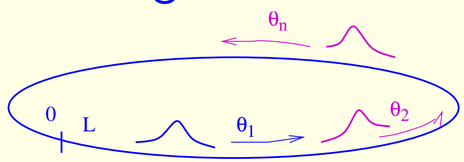
Cylinder

Infinite volume



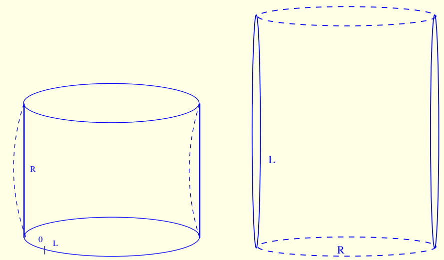
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



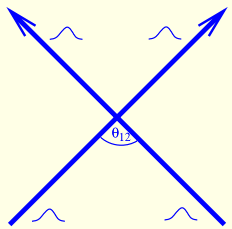
$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

Strip

Plan of talk

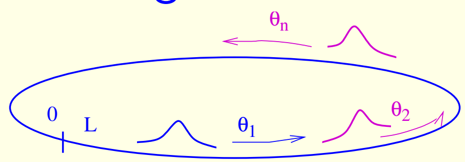
Cylinder

Infinite volume



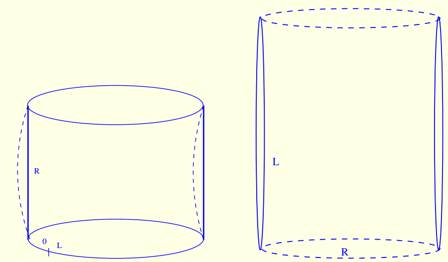
S-matrix

Large volumes



Bethe-Yang lines

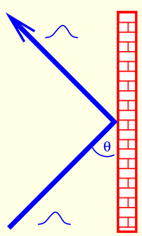
Lüscher correction



$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m \cosh(\theta) L}$$

Strip

Semiinfinite volume

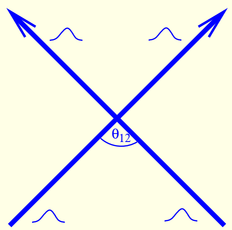


R-matrix

Plan of talk

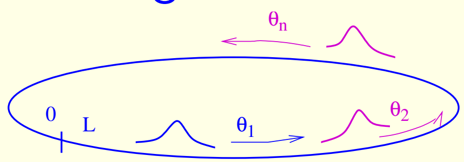
Cylinder

Infinite volume



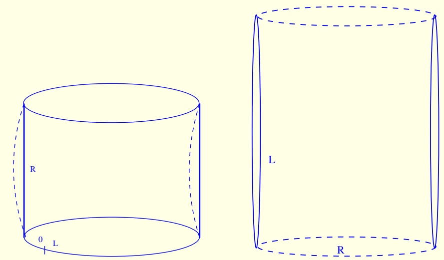
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



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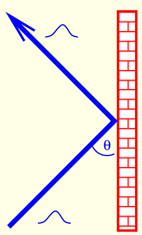
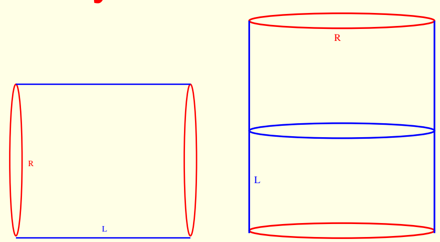
Strip

Semiinfinite volume



R-matrix

Boundary Lüscher correction



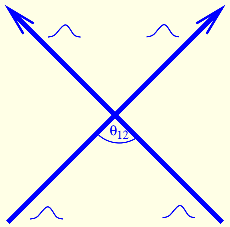
$$E_0(L) = -\int \frac{m d\theta}{4\pi} \cosh\theta K(\theta)\bar{K}(\theta) e^{-2mL\cosh(\theta)}$$

$$K(\theta) = R(\frac{i\pi}{2} - \theta)$$

Plan of talk

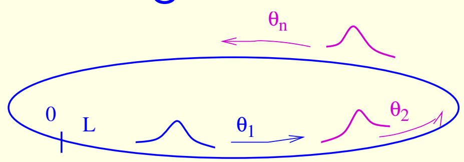
Cylinder

Infinite volume



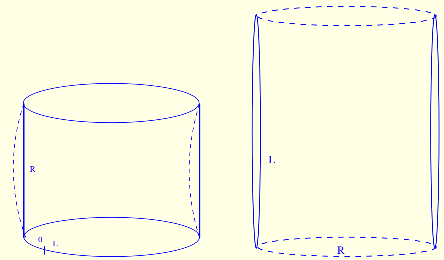
S-matrix

Large volumes



Bethe-Yang lines

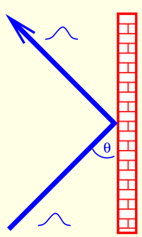
Lüscher correction



$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

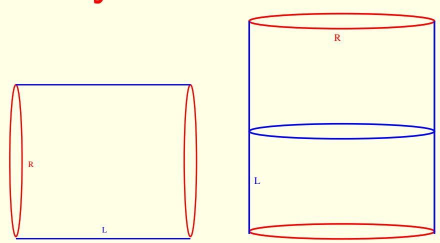
Strip

Semiinfinite volume



R-matrix

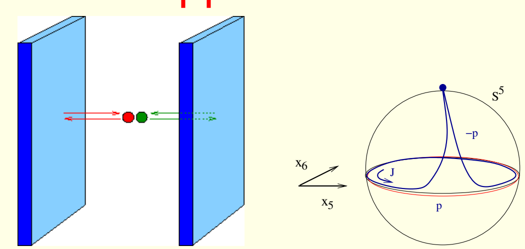
Boundary Lüscher correction



$$E_0(L) = -\int \frac{md\theta}{4\pi} \cosh\theta K(\theta)\bar{K}(\theta)e^{-2mL\cosh(\theta)}$$

$$K(\theta) = R(\frac{i\pi}{2} - \theta)$$

Application



Casimir effect, AdS/CFT

$$p_i = m \sinh \theta_i$$
$$E_i = m \cosh \theta_i$$

Integrable field theory: Bootstrap

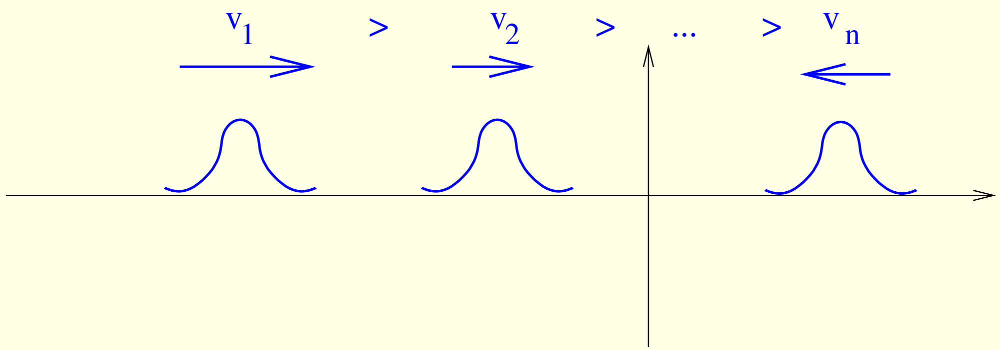
$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

Integrable field theory: Bootstrap

Bulk multiparticle state: with n particles

$$E(\theta_1, \theta_2, \ldots, \theta_n) = \sum_i m \cosh \theta_i$$



Integrable field theory: Bootstrap

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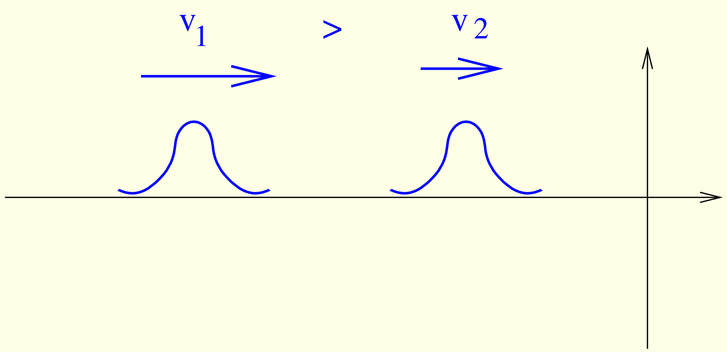
Bulk twoparticle state:



Integrable field theory: Bootstrap

$p_i = m \sinh \theta_i$

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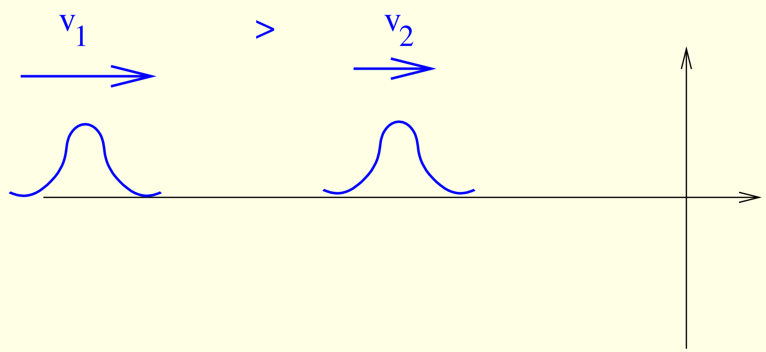
Bulk twoparticle in state: $t \to -\infty$



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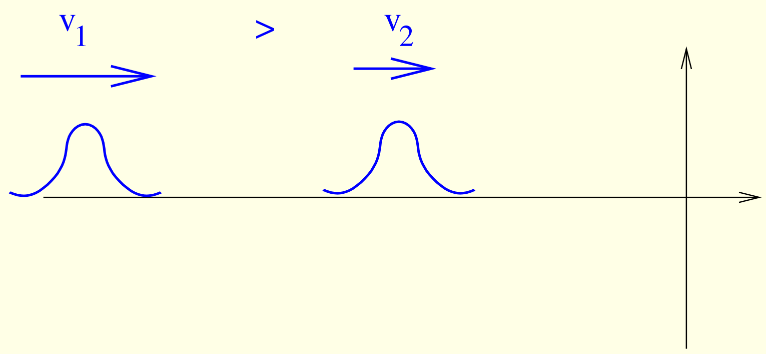
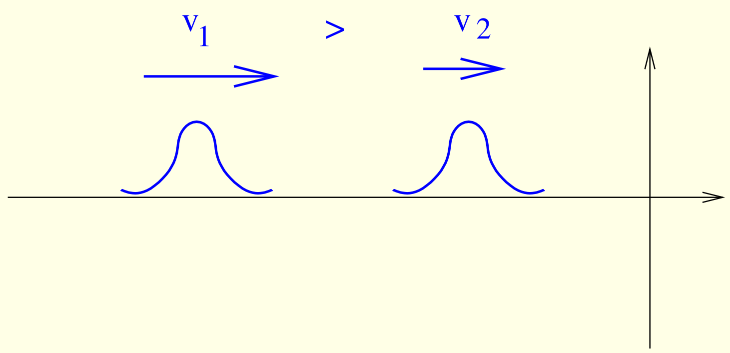


times develop



$p_i = m\,\sinh\theta_i$

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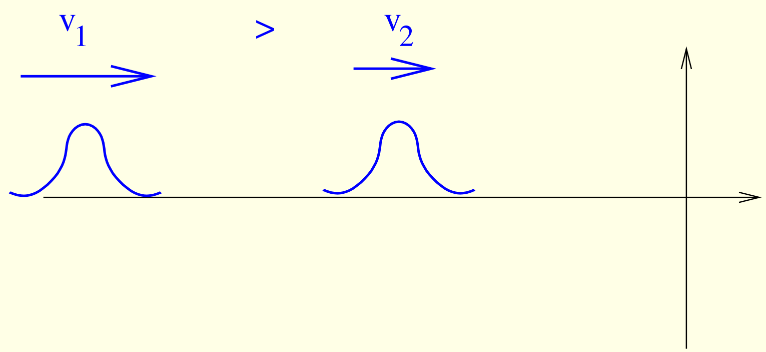
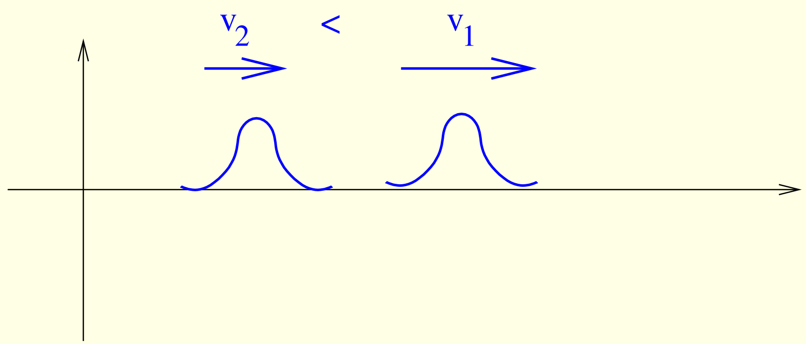
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$



times develop further



$p_i = m \sinh \theta_i$

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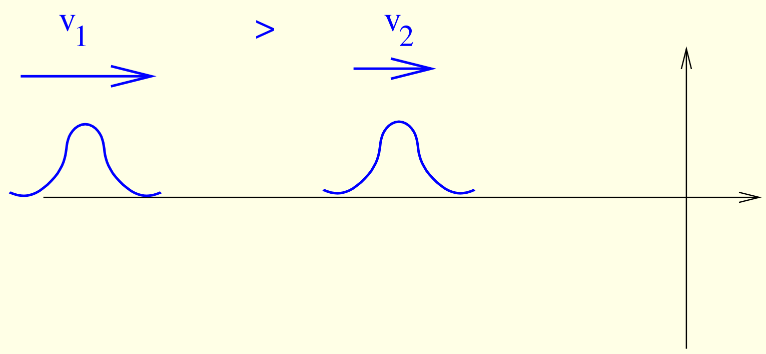
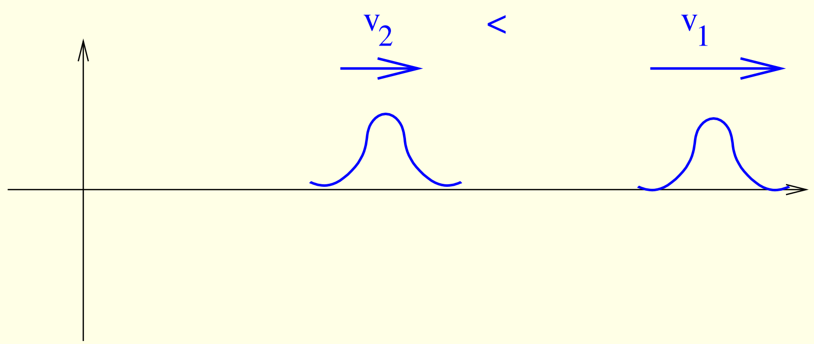
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$



Bulk twoparticle out state: $t \to \infty$



Integrable field theory: Bootstrap

$p_i = m \sinh \theta_i$

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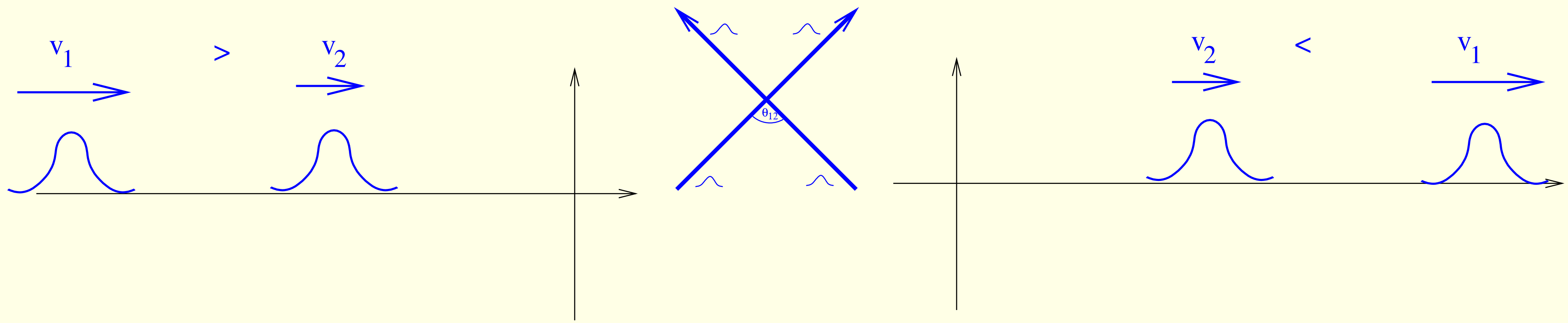
Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



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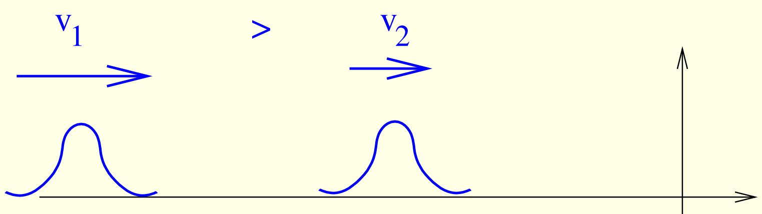
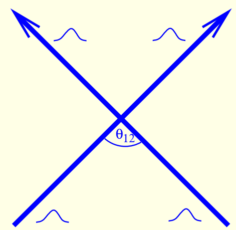
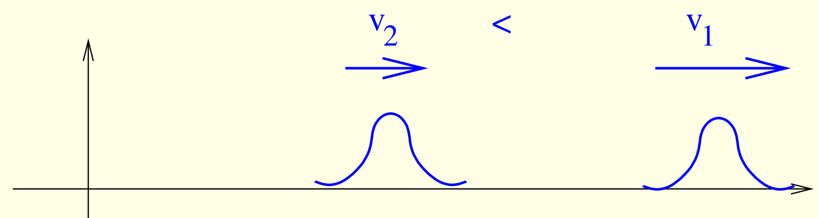






Free, noninteracting in particles

Free, noninteracting out particles

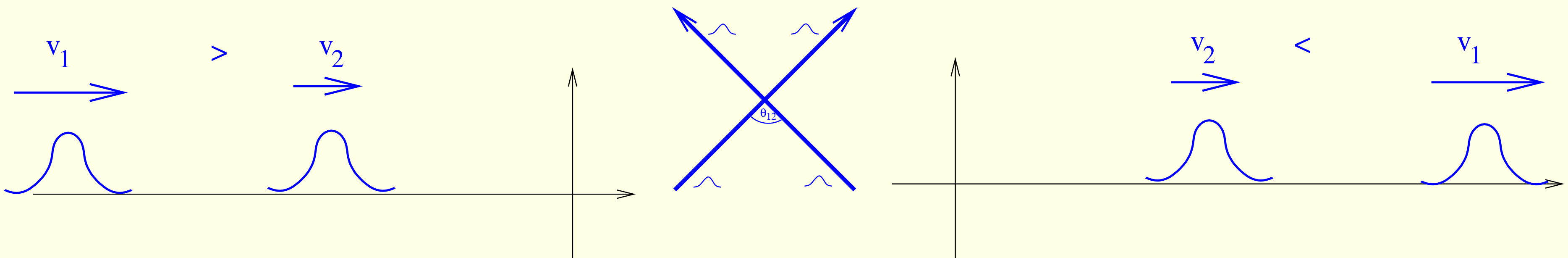
Integrable field theory: Bootstrap

$p_i = m \sinh \theta_i$

$E_i = m \cosh \theta_i$

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



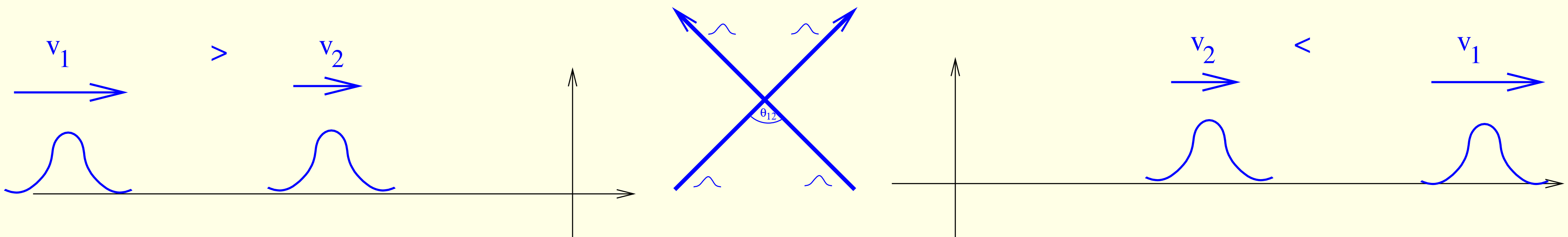
Free, noninteracting in particles $\longleftarrow$ **S-matrix** $\longrightarrow$ Free, noninteracting out particles

$$p_i = m \sinh \theta_i$$
$$E_i = m \cosh \theta_i$$

Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$



Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$

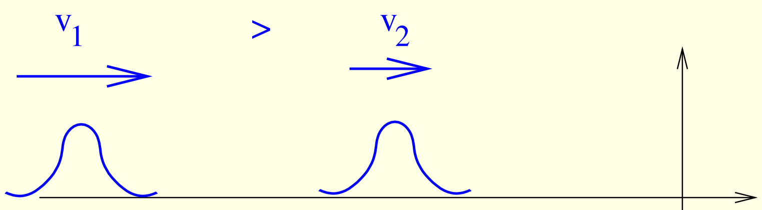
$p_i = m \sinh \theta_i$

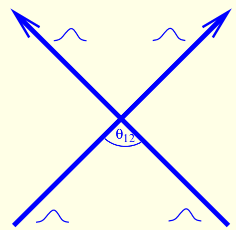
$E_i = m \cosh \theta_i$

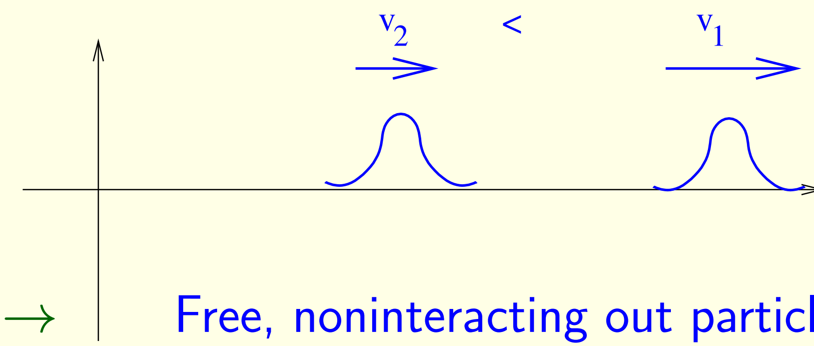
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$

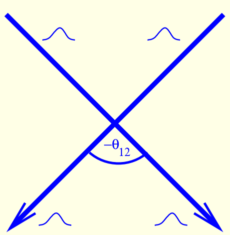




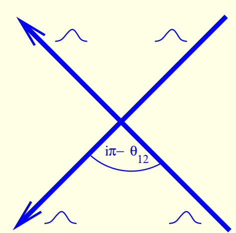


Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} \quad = \quad S(\theta_1 - \theta_2)|\theta_1, \theta_2\rangle^{out}$$



Unitarity $S^{-1}(\theta_{12}) = S(\theta_{21})$



Crossing $S(\theta_{12}) = S(i\pi - \theta_{12})$

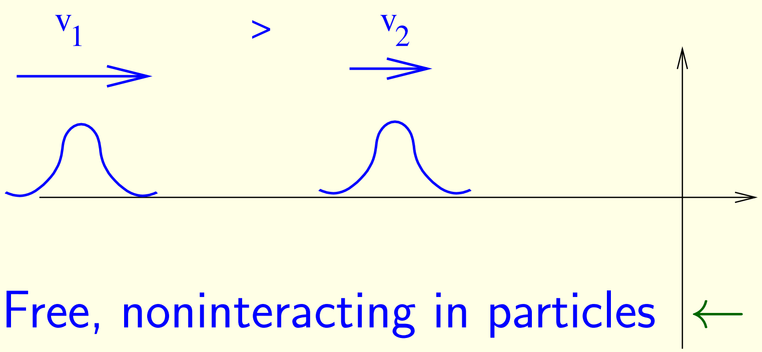
$p_i = m \sinh \theta_i$

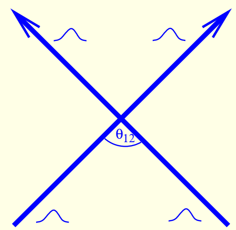
$E_i = m \cosh \theta_i$

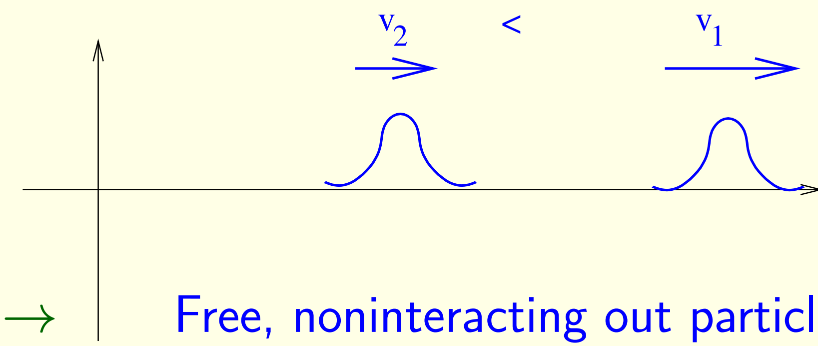
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$

Bulk twoparticle out state: $t \to \infty$







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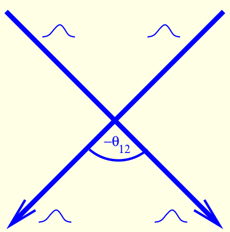

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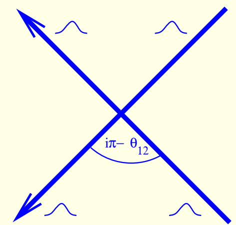

Crossing $S(\theta_{12}) = S(i\pi - \theta_{12})$

Integrability $\rightarrow$ factorizability: $|\theta_1, \theta_2, \ldots, \theta_n\rangle = \prod_{i<j} S(\theta_i - \theta_j)|\theta_n, \theta_{n-1}, \ldots, \theta_1\rangle$

$$p_i = m \sinh \theta_i$$
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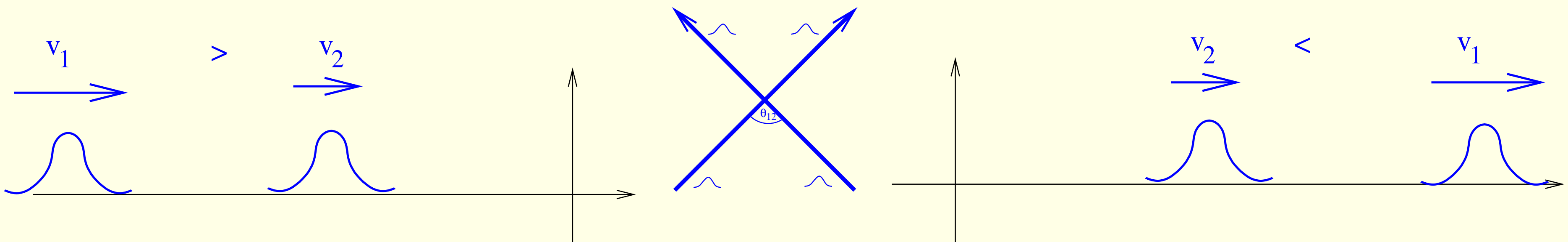

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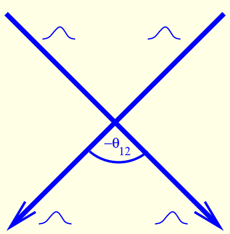

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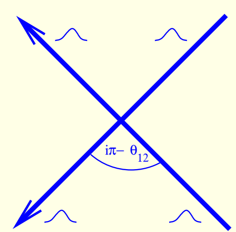

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Minimal solutions: free boson $S = 1$ sinh-Gordon $S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p}$, Lee-Yang $p = -\frac{1}{3}$

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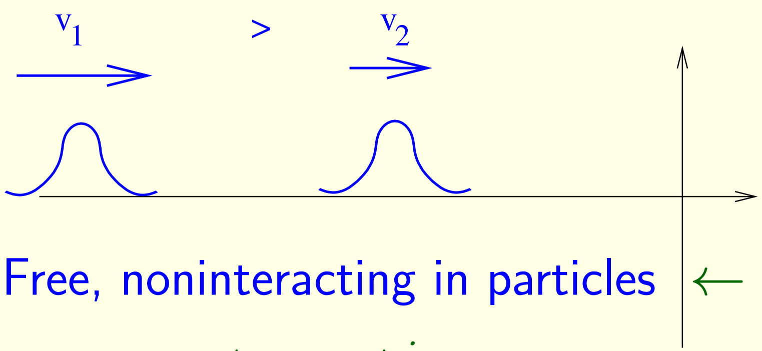
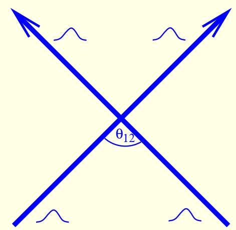
Integrable field theory: Bootstrap

Bulk twoparticle in state: $t \to -\infty$





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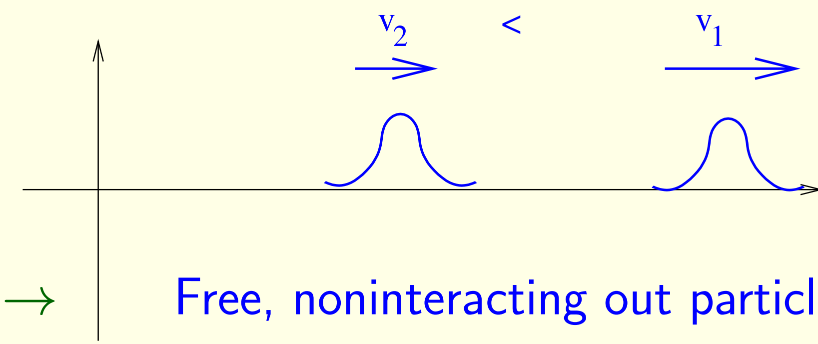

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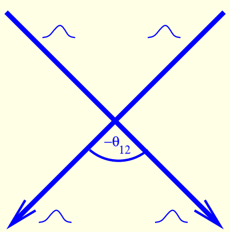

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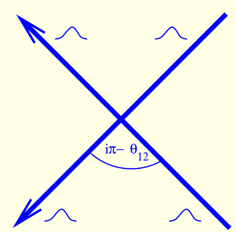

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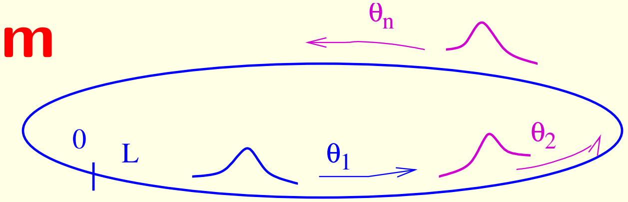
Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial \phi)^2 \qquad -\mu(\cosh b\phi - 1) \qquad p = \frac{b^2}{8\pi + b^2}$

Very large volume spectrum



$$\frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) \leftrightarrow S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p}$$

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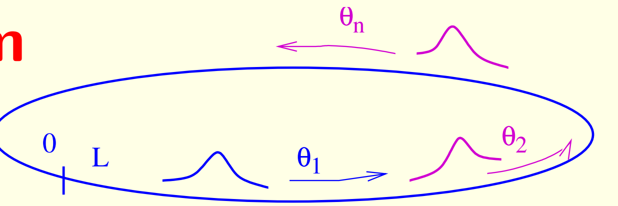

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Infinite volume

$$E(\theta) = m\cosh\theta$$

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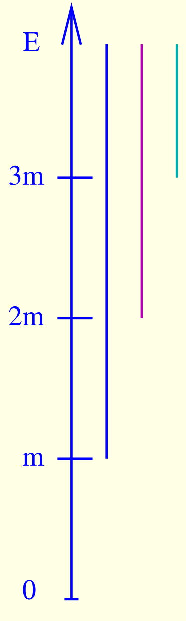
$$E(\theta_1, ..., \theta_n) = \sum_i E(\theta_i)$$



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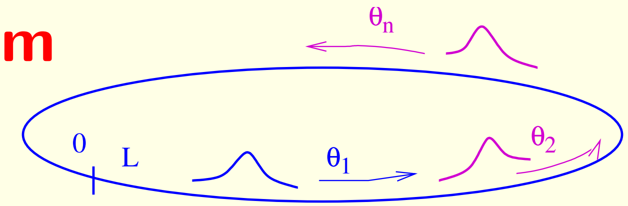

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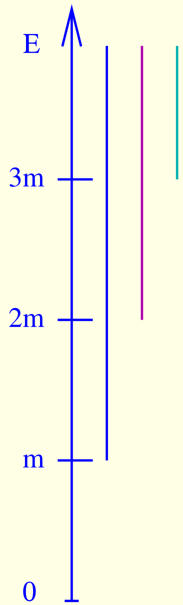
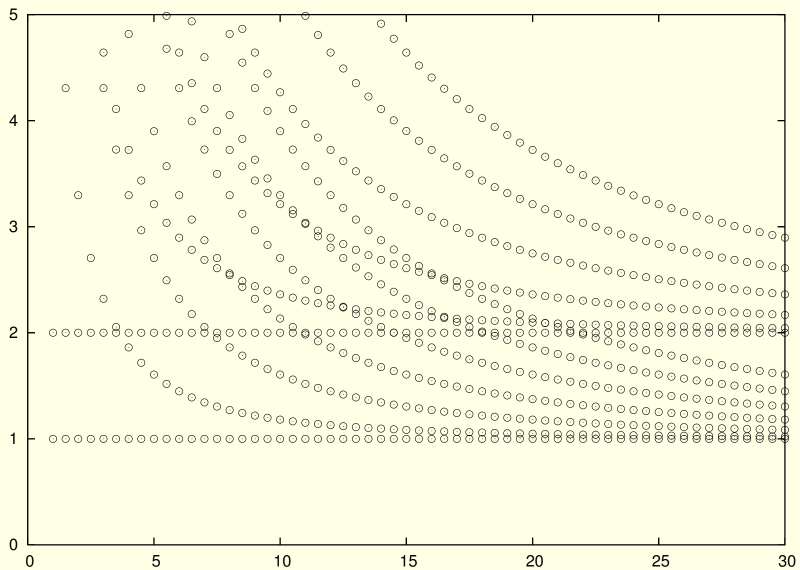


Finite volume: free particles

$e^{ip(\theta)L}=1$ Quantization

$$p(\theta)\rightarrow p(k)=\frac{2\pi k}{L}$$

$$|\theta_1,\ldots\theta_n\rangle\rightarrow|k_1,\ldots,k_n\rangle$$



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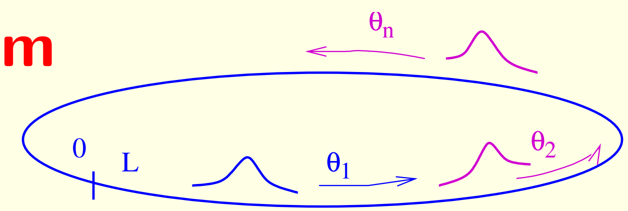

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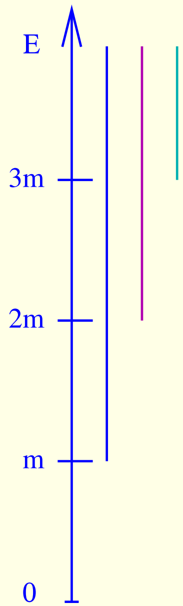

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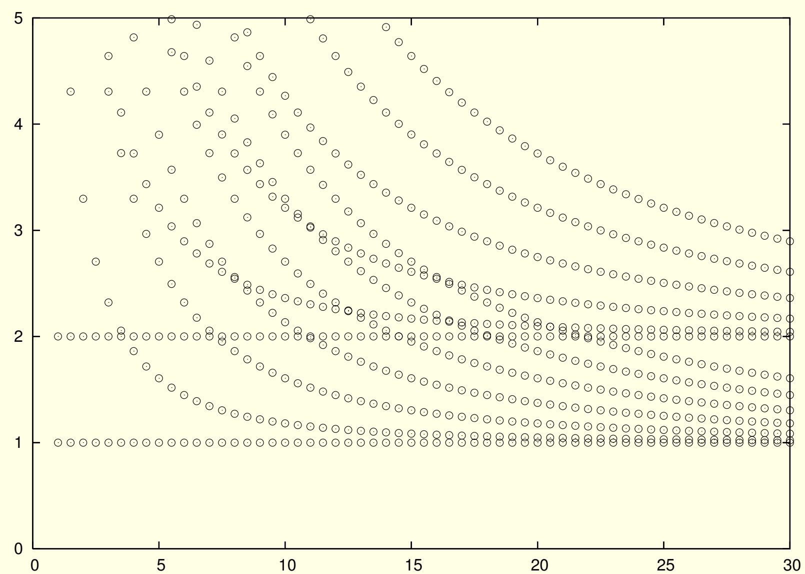


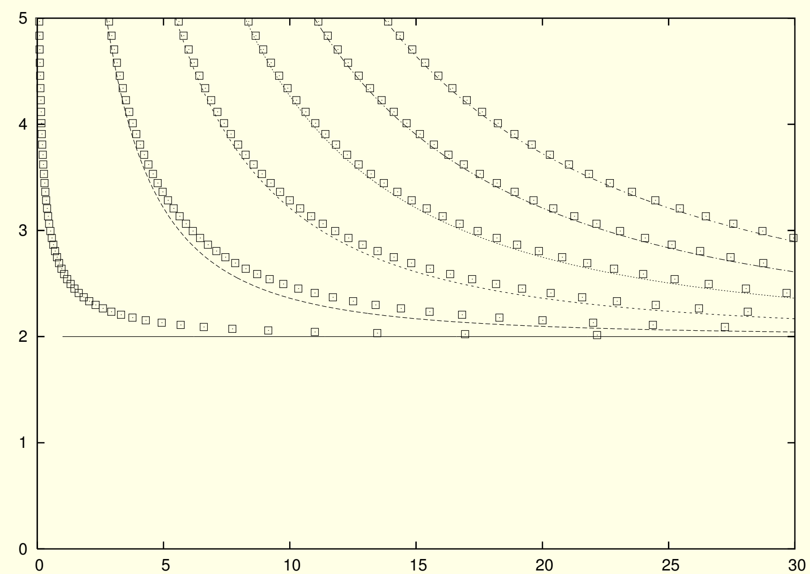
Very large volume, interacting particles

one particle $e^{ip(\theta)L}=1$;$\theta\to p(k)=\frac{2\pi k}{L}$

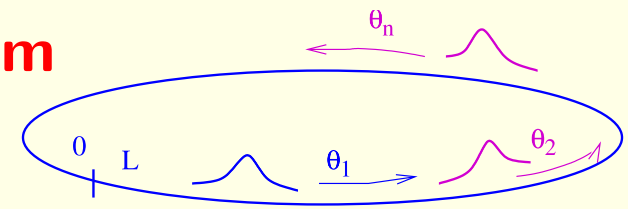
two particles $e^{ip(\theta_1)L}S(\theta_{12})=1$

$p(\theta_1)L+\varphi(\theta_{12})=2\pi n_1\ ;\quad S=e^{i\varphi}$

n particles $p(\theta_i)L+\sum_j\varphi(\theta_{ij})=2\pi n_i$



Very large volume spectrum



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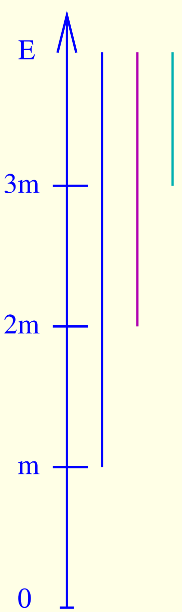

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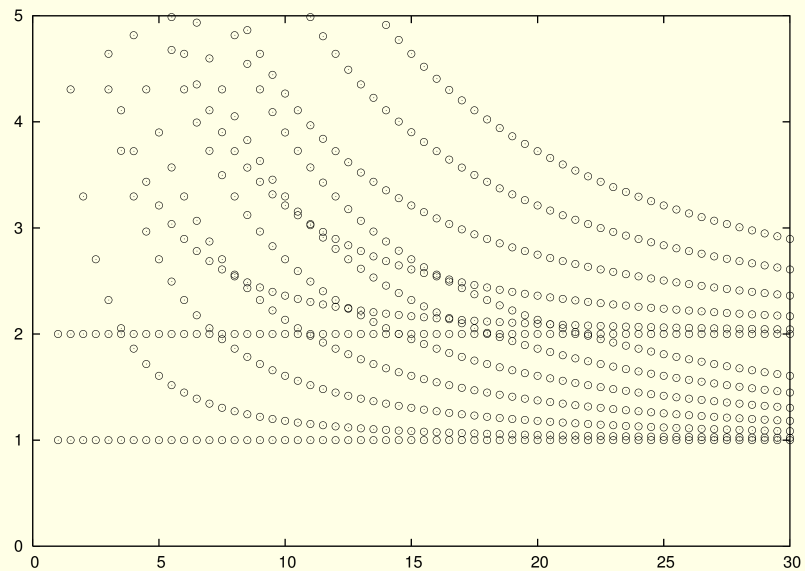

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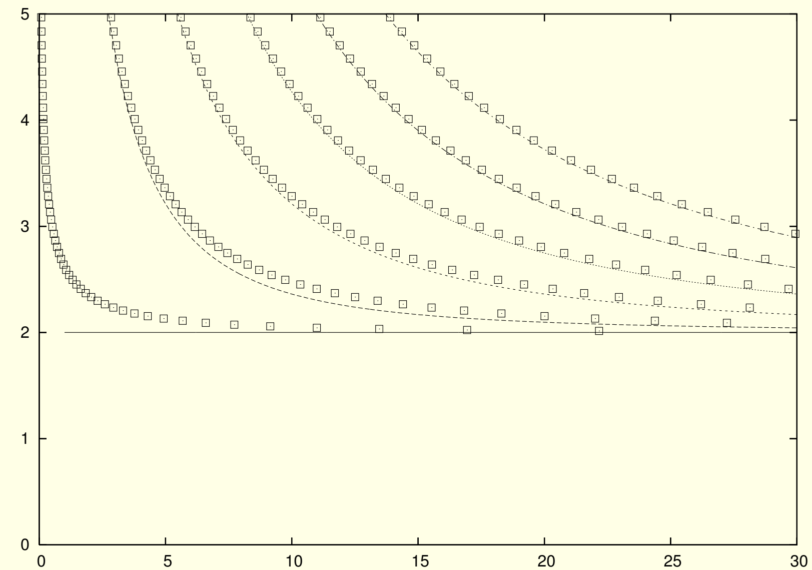
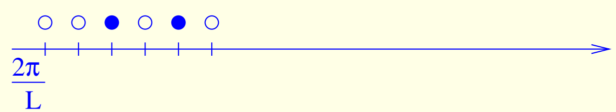
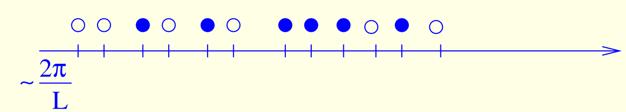


Momentum quantization $S(0) = -1$





Lüscher correction for the groundstate

Groundstate energy $E_0(L) =$

Lüscher correction for the groundstate

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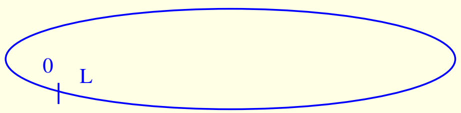

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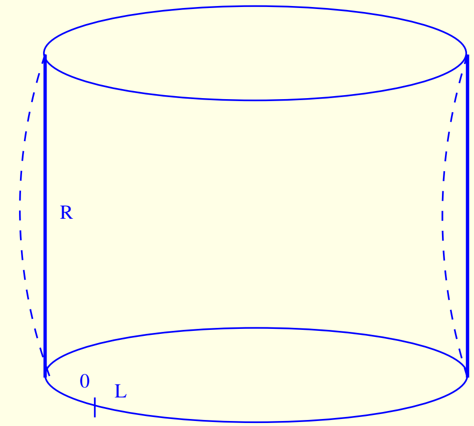
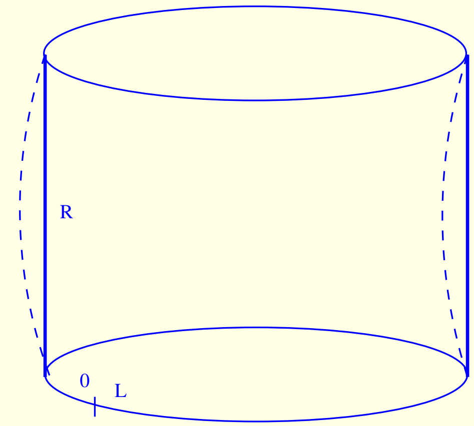


Groundstate energy $E_0(L) =$

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}\,(e^{-H(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log(e^{-E_0(L)R})$$

Lüscher correction for the groundstate



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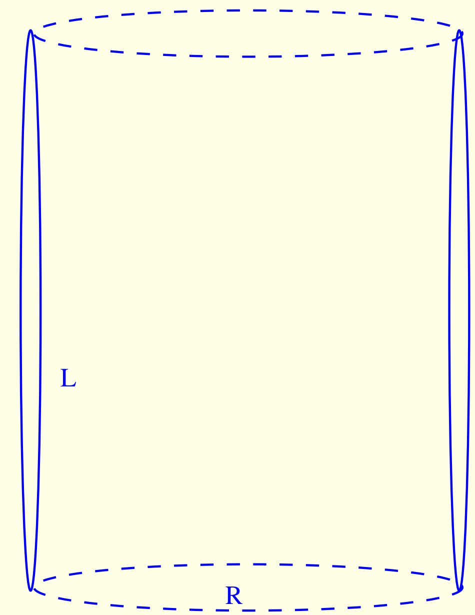

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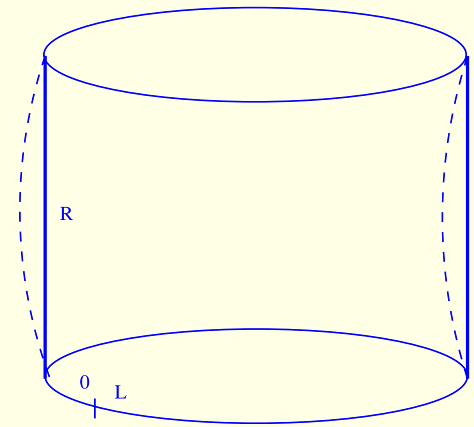

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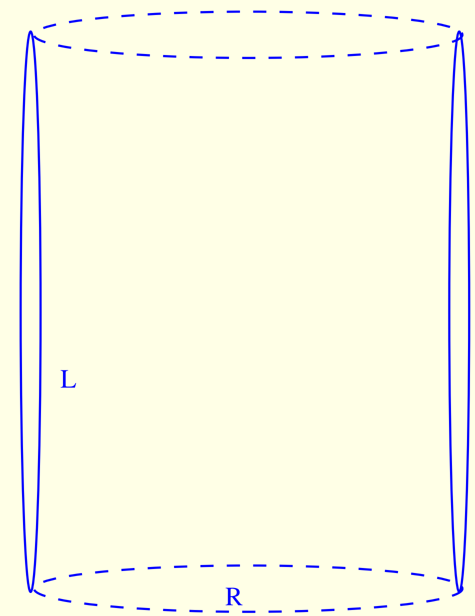
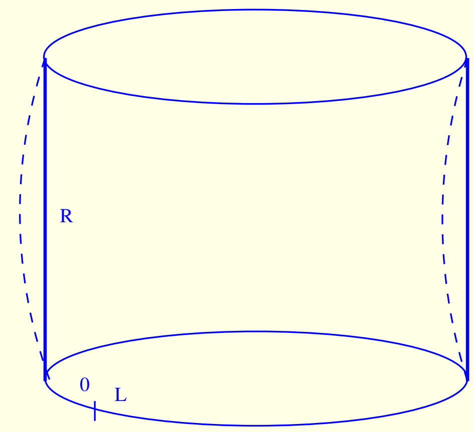


Dominant contribution for large L: one particle term

$$\mathrm{Tr}(e^{-H(R)L}) = 1 + \sum_k e^{-m\cosh\theta_k(R)L} + O(e^{-2mL})$$

Lüscher correction for the groundstate



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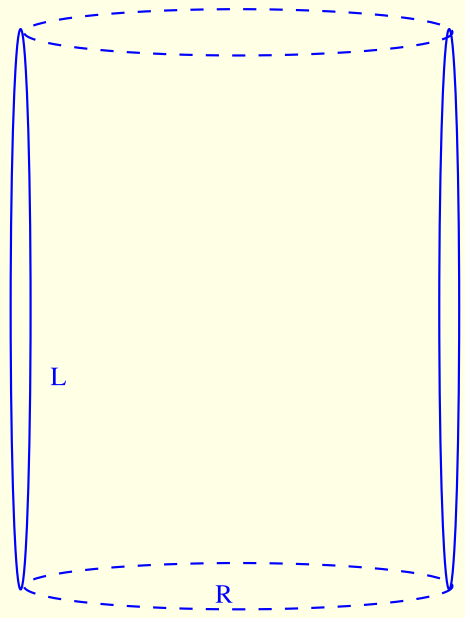

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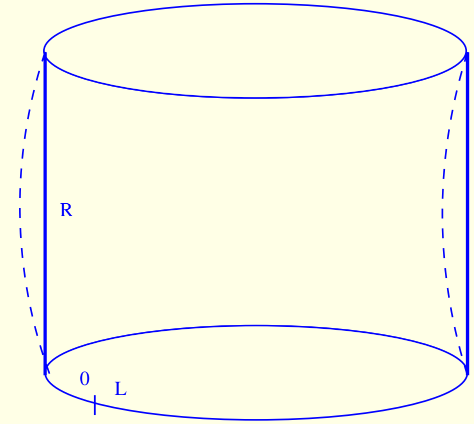
one particle quantization $m\sinh\theta = \frac{2\pi k}{R}$ $\quad \sum_k \to \frac{Rm}{2\pi}\int d\theta\cosh\theta$

$$E_0(L) = -m\int d\theta\cosh\theta\, e^{-mL\cosh\theta} + O(e^{-2mL})$$

Lüscher correction for the groundstate



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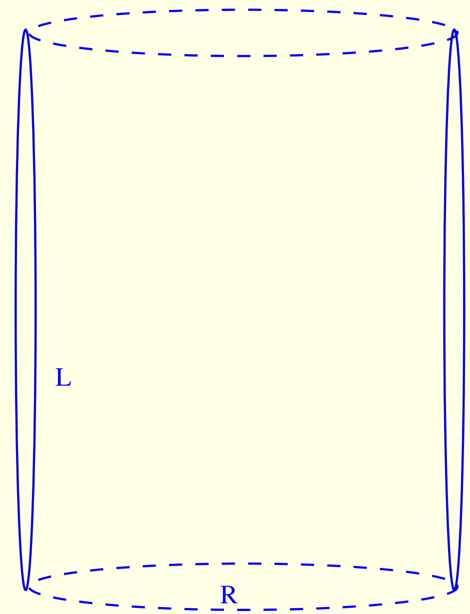

Dominant contribution for large L: one particle term

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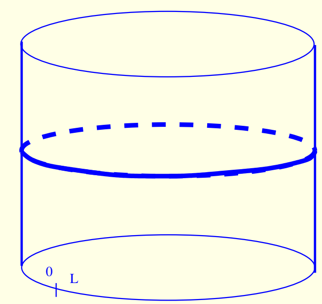
$$E_0(L) = -m\int d\theta\cosh\theta\, e^{-mL\cosh\theta} + O(e^{-2mL})$$

Ground state energy exactly: Al. Zamolodchikov '90

$$\boxed{E_0(L) = -m\int\frac{d\theta}{2\pi}\cosh(\theta)\log(1+e^{-\epsilon(\theta)})}$$

$$\epsilon(\theta) = mL\cosh\theta - \int\frac{d\theta'}{2\pi}\varphi'(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$



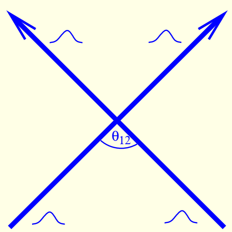
Plan of talk

Cylinder

Plan of talk

Cylinder

Infinite volume

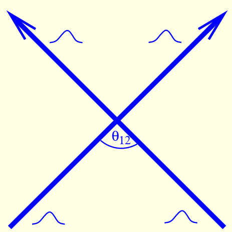


S-matrix

Plan of talk

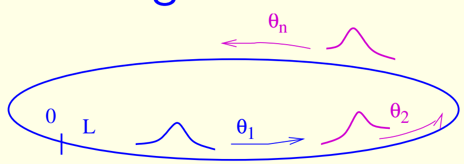
Cylinder

Infinite volume



S-matrix

Large volumes

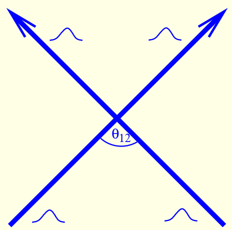


Bethe-Yang lines

Plan of talk

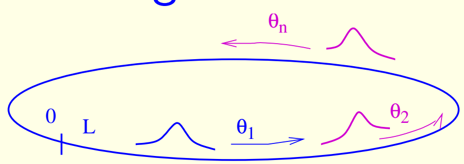
Cylinder

Infinite volume



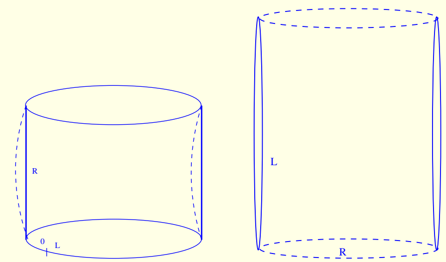
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction

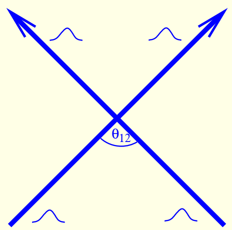


$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

Plan of talk

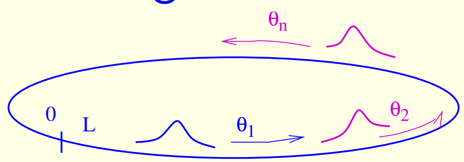
Cylinder

Infinite volume



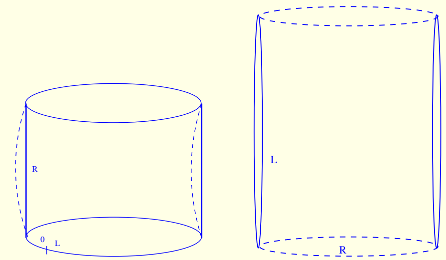
S-matrix

Large volumes



Bethe-Yang lines

Lüscher correction



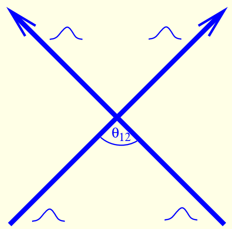
$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

Strip

Plan of talk

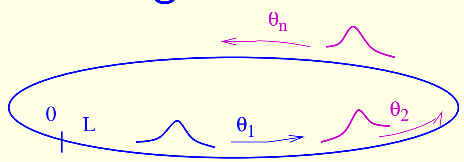
Cylinder

Infinite volume



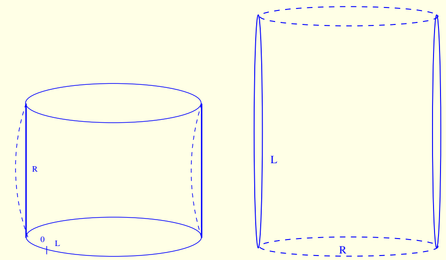
S-matrix

Large volumes



Bethe-Yang lines

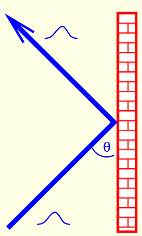
Lüscher correction



$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m \cosh(\theta) L}$$

Strip

Semiinfinite volume

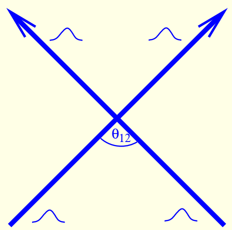


R-matrix

Plan of talk

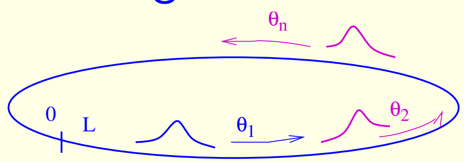
Cylinder

Infinite volume



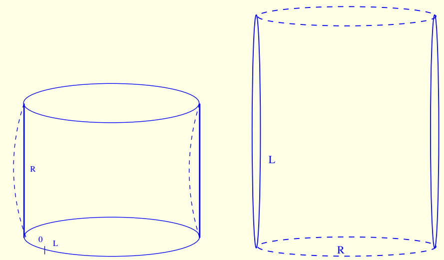
S-matrix

Large volumes



Bethe-Yang lines

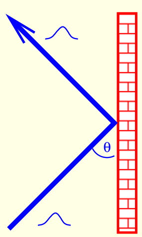
Lüscher correction



$$E_0(L) = -m\int \frac{d\theta}{2\pi}\cosh\theta\, e^{-m\cosh(\theta)L}$$

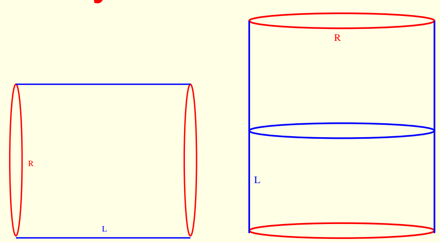
Strip

Semiinfinite volume



R-matrix

Boundary Lüscher correction



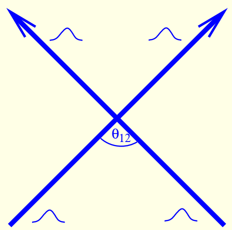
$$E_0(L) = -\int \frac{md\theta}{4\pi}\cosh\theta K(\theta)\bar{K}(\theta)e^{-2mL\cosh(\theta)}$$

$$K(\theta) = R(\frac{i\pi}{2}-\theta)$$

Plan of talk

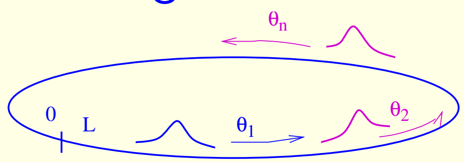
Cylinder

Infinite volume



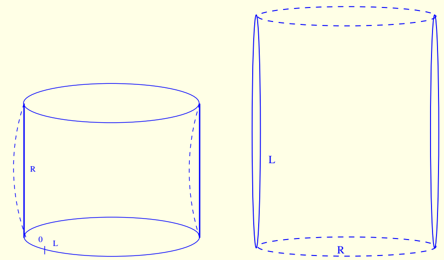
S-matrix

Large volumes



Bethe-Yang lines

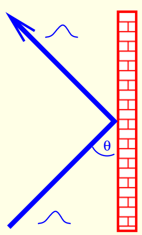
Lüscher correction



$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh\theta \, e^{-m\cosh(\theta)L}$$

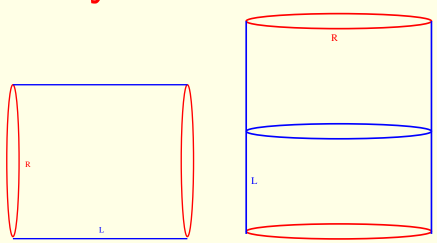
Strip

Semiinfinite volume



R-matrix

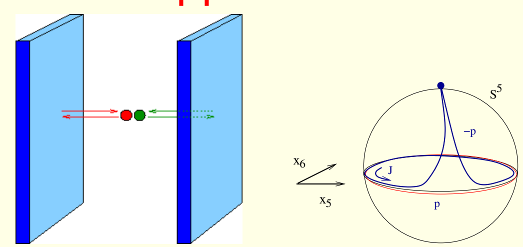
Boundary Lüscher correction



$$E_0(L) = -\int \frac{md\theta}{4\pi} \cosh\theta K(\theta)\bar{K}(\theta) e^{-2mL\cosh(\theta)}$$

$$K(\theta) = R(\frac{i\pi}{2} - \theta)$$

Application

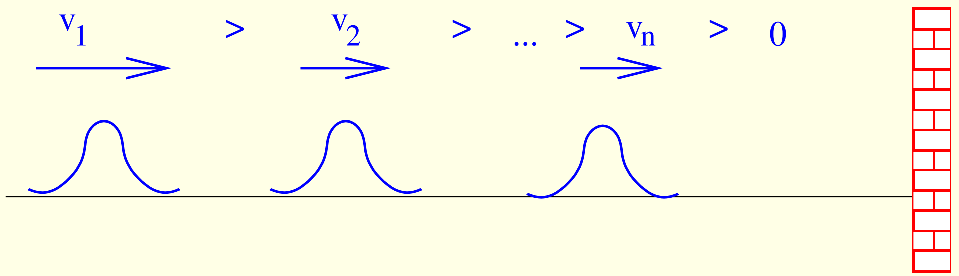


Casimir effect, AdS/CFT

Integrable boundary field theory: Bootstrap

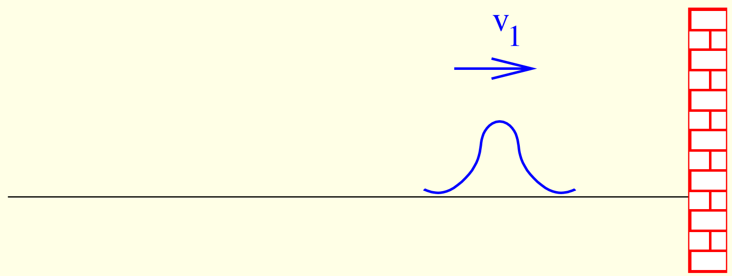
Integrable boundary field theory: Bootstrap

Boundary multiparticle state: with n particles



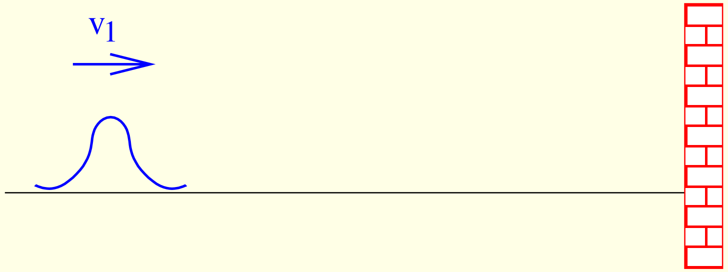
Integrable boundary field theory: Bootstrap

Boundary one particle state:



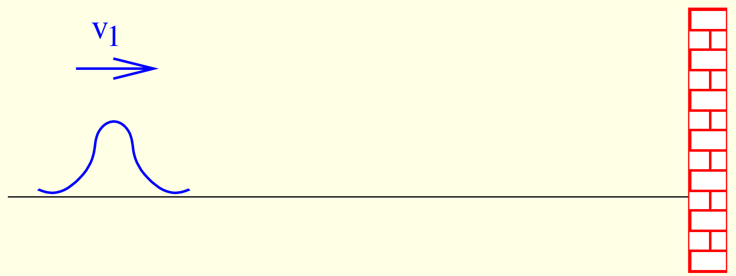
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

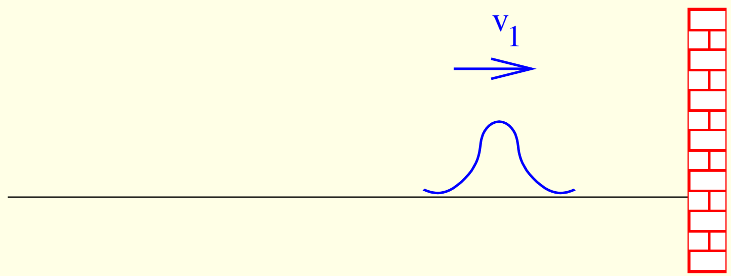


Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

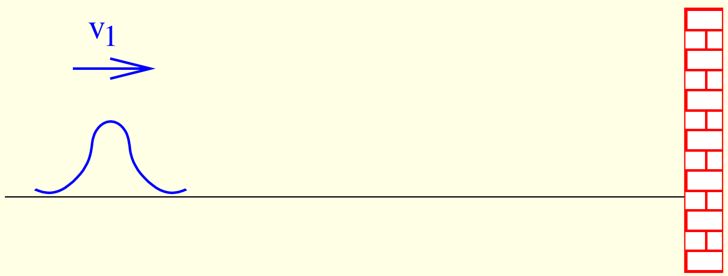


times develop

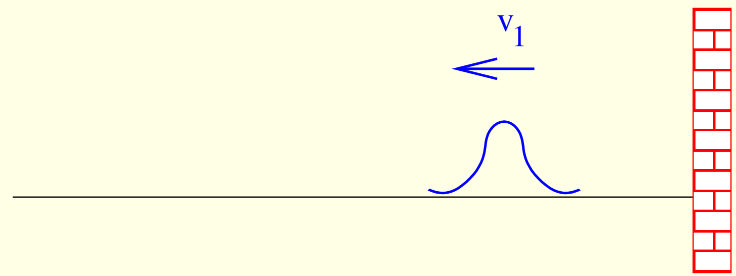


Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

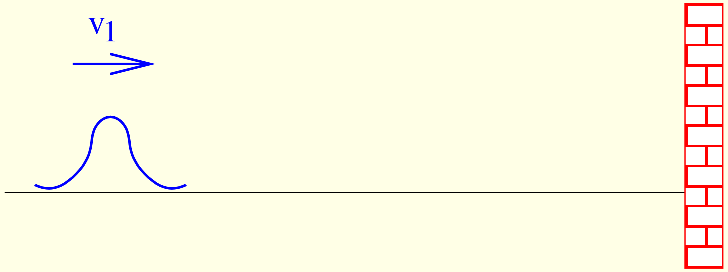


times develop further

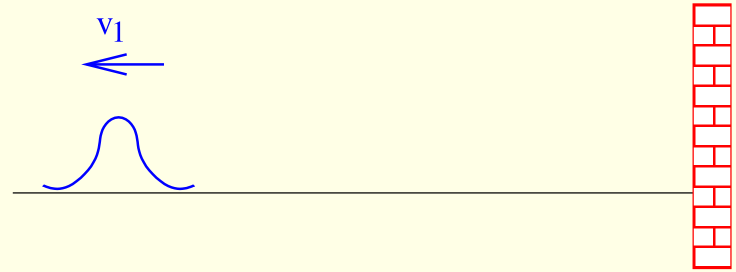


Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

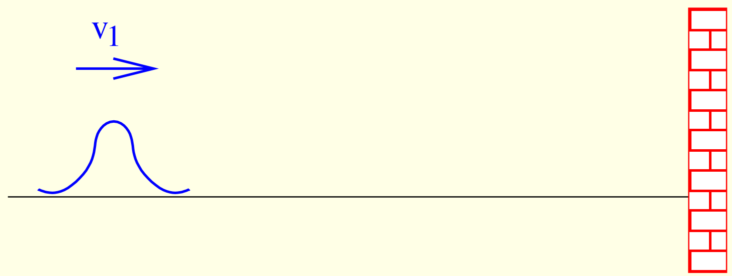


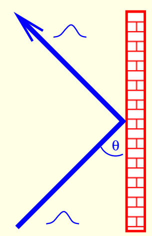
Boundary one pt out state: $t \to \infty$



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

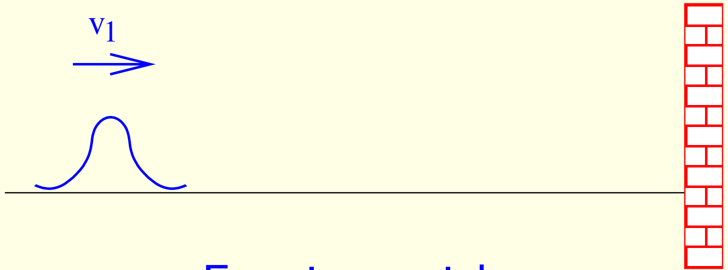




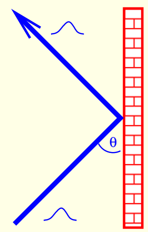
Boundary one pt out state: $t \to \infty$

Integrable boundary field theory: Bootstrap

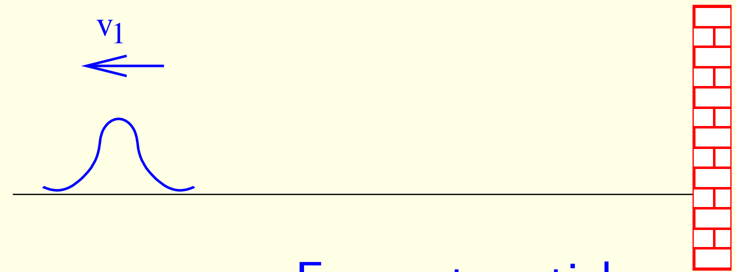
Boundary one particle in state: $t \to -\infty$



Free in particle



Boundary one pt out state: $t \to \infty$

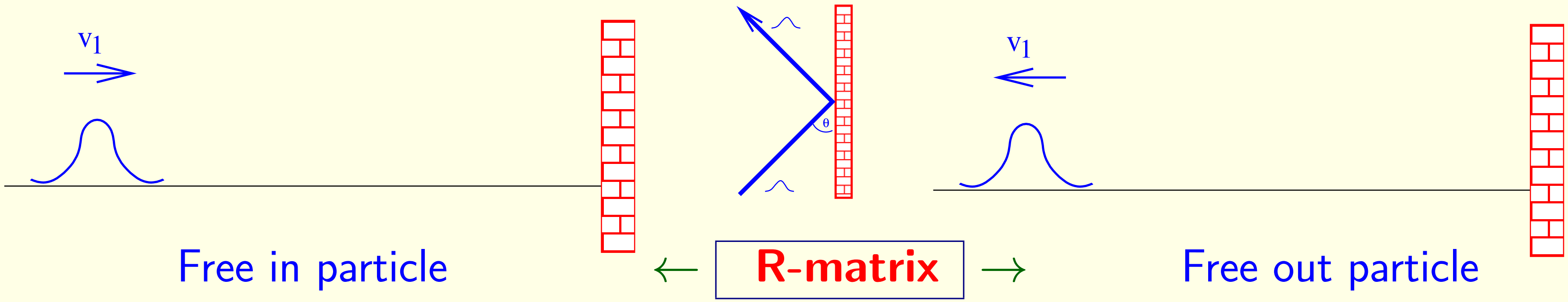


Free out particle

Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

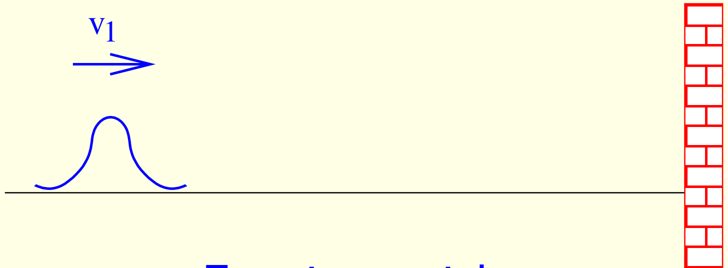
Boundary one pt out state: $t \to \infty$

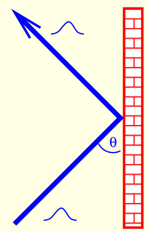


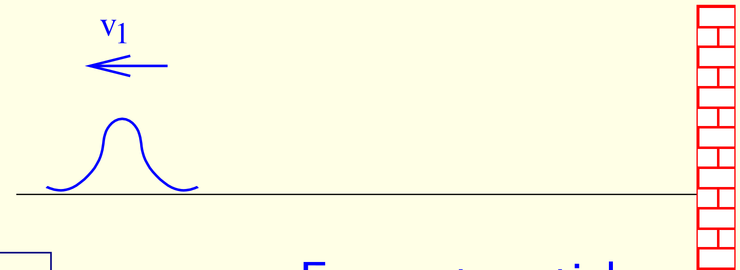
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

Boundary one pt out state: $t \to \infty$







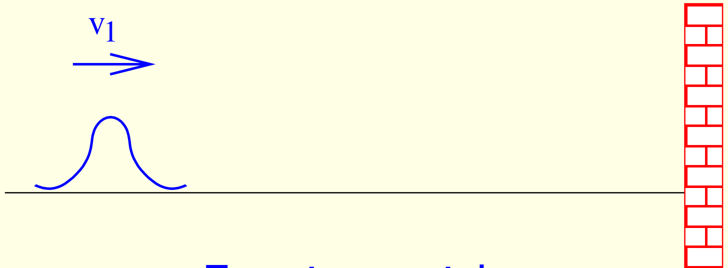
Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

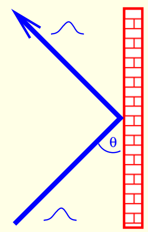
$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$

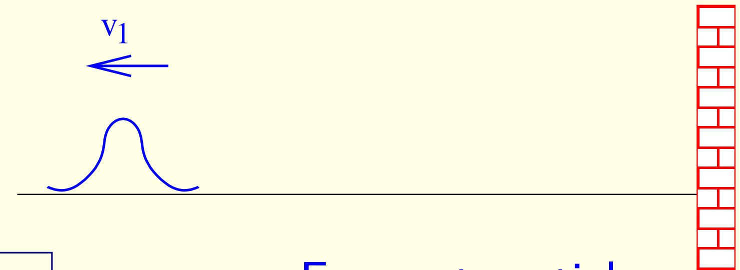
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

Boundary one pt out state: $t \to \infty$







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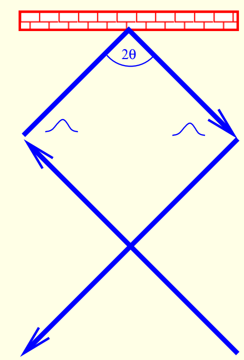
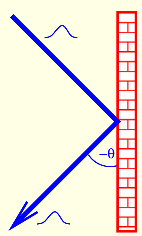


Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



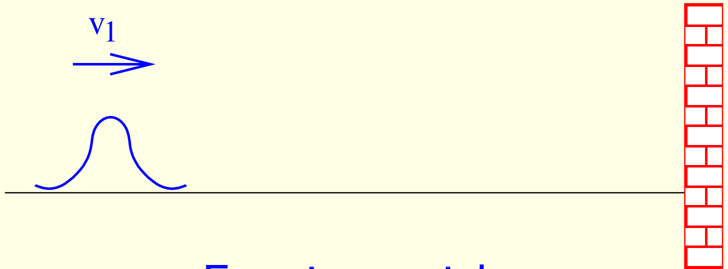
Boundary crossing unitarity

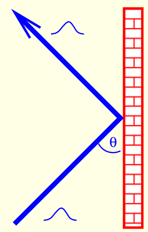
$$R(\tfrac{i\pi}{2} + \theta) = S(2\theta) R(\tfrac{i\pi}{2} - \theta)$$

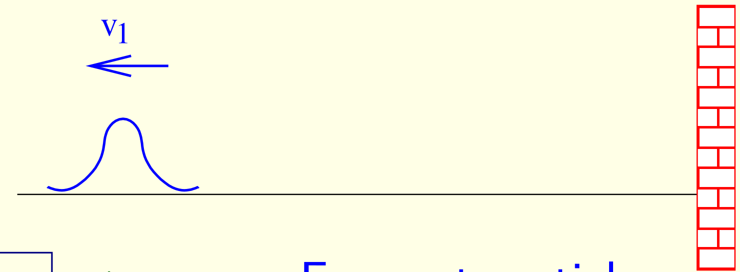
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

Boundary one pt out state: $t \to \infty$

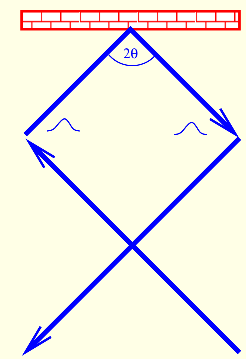






Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

$$|\theta\rangle_B = R(\theta)|-\theta\rangle_B$$



Unitarity

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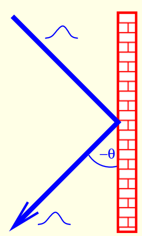


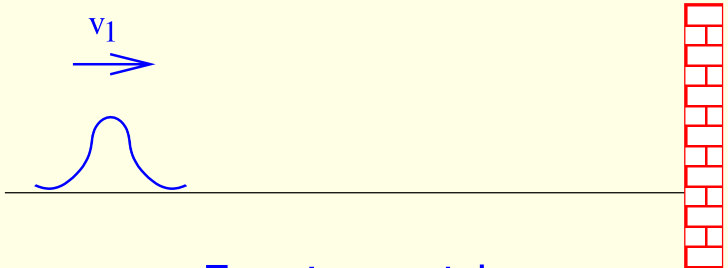
Boundary crossing unitarity

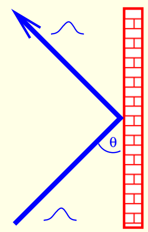
$$R(\tfrac{i\pi}{2}+\theta) = S(2\theta)R(\tfrac{i\pi}{2}-\theta)$$

sinh-Gordon $S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p} = [-p] = -(-p)(1+p)\,;\ (p) = \frac{\sinh\left(\frac{\theta}{2}+\frac{i\pi p}{2}\right)}{\sinh\left(\frac{\theta}{2}-\frac{i\pi p}{2}\right)}$

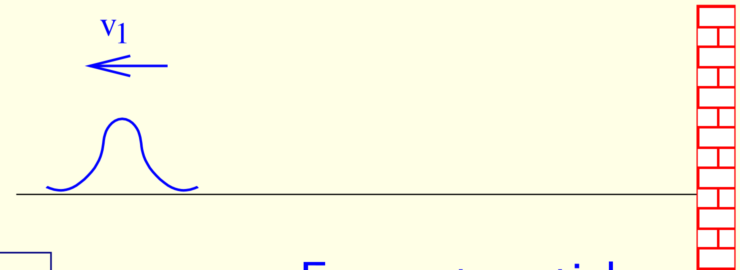
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$





Boundary one pt out state: $t \to \infty$



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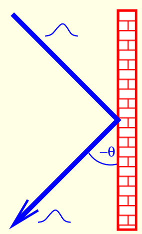

Unitarity

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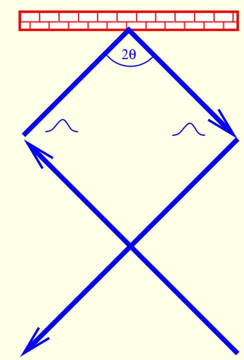

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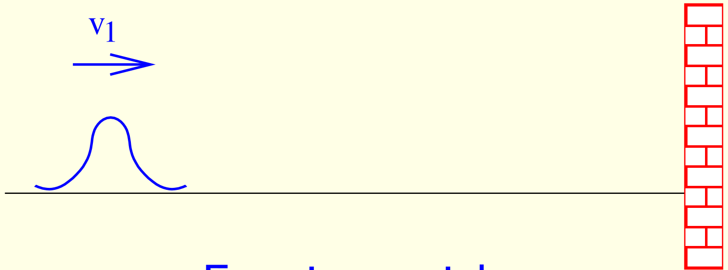
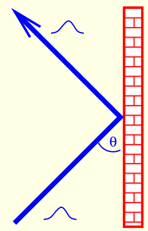
sinh-Gordon $S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p} = [-p] = -(-p)(1+p)\,;\ (p) = \frac{\sinh\left(\frac{\theta}{2}+\frac{i\pi p}{2}\right)}{\sinh\left(\frac{\theta}{2}-\frac{i\pi p}{2}\right)}$

reflection factor $R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1+p}{2}\right)\left(1-\frac{p}{2}\right)\left[\frac{3}{2}-\frac{\eta p}{\pi}\right]\left[\frac{3}{2}-\frac{\Theta p}{\pi}\right]$ Ghoshal-Zamolodchikov '94

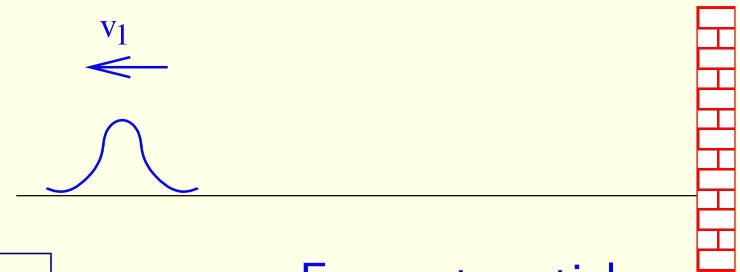
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$





Boundary one pt out state: $t \to \infty$



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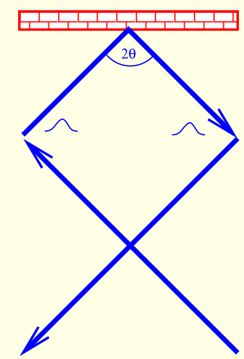

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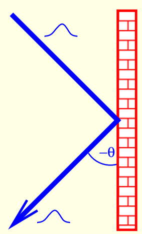


Boundary crossing unitarity

$$R(\tfrac{i\pi}{2} + \theta) = S(2\theta)R(\tfrac{i\pi}{2} - \theta)$$

sinh-Gordon $S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p} = [-p] = -(-p)(1+p)\,;\ (p) = \frac{\sinh\left(\frac{\theta}{2} + \frac{i\pi p}{2}\right)}{\sinh\left(\frac{\theta}{2} - \frac{i\pi p}{2}\right)}$

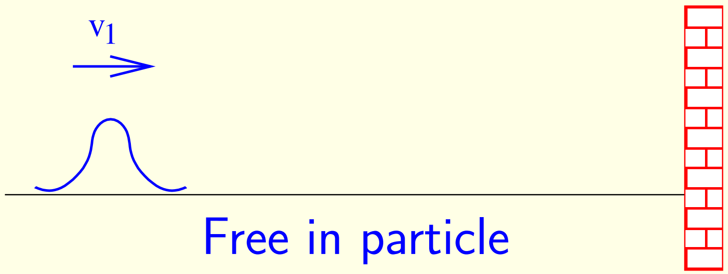
reflection factor $R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1+p}{2}\right)\left(1 - \frac{p}{2}\right)\left[\frac{3}{2} - \frac{\eta p}{\pi}\right]\left[\frac{3}{2} - \frac{\Theta p}{\pi}\right]$ Ghoshal-Zamolodchikov '94

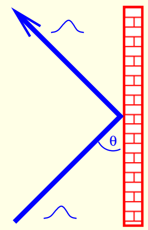
Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) - \delta\left[\mu_+^B e^{\frac{b}{2}\phi} + \mu_-^B e^{-\frac{b}{2}\phi}\right]$ $\sqrt{\frac{\mu}{\sin b^2}}\cosh b^2(\eta \pm \Theta) = \mu_\pm^B$

Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

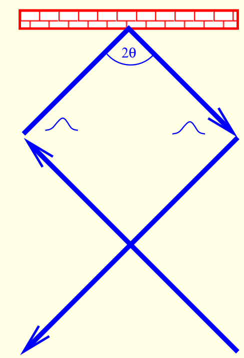
Boundary one pt out state: $t \to \infty$





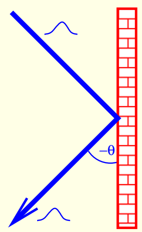
Free in particle ← **R-matrix** → Free out particle

$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$



Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



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Integrability →factorizability: $|\theta_1,\theta_2,\ldots,\theta_n\rangle_B = \prod_{i<j} S(\theta_i-\theta_j)S(\theta_i+\theta_j)\prod_i R(\theta_i)|-\theta_1,-\theta_2,\ldots,-\theta_n\rangle_B$

Boundary Lüscher correction

Groundstate energy for large L from IR reflection: $E_0(L) =$

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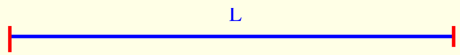

Boundary Lüscher correction

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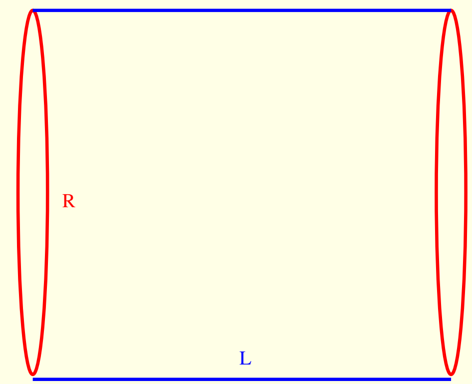
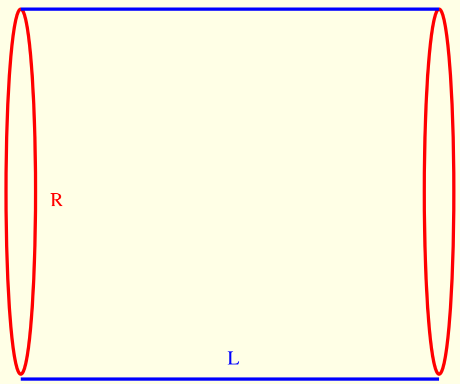
$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H^B(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log(e^{-E_0(L)R})$$



Boundary Lüscher correction



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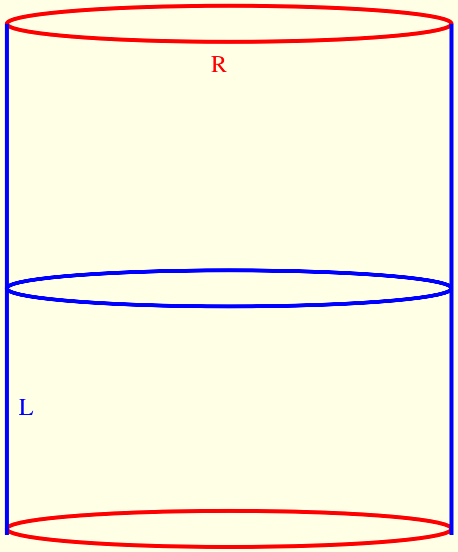

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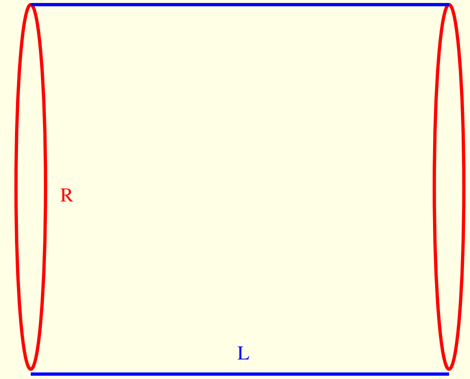

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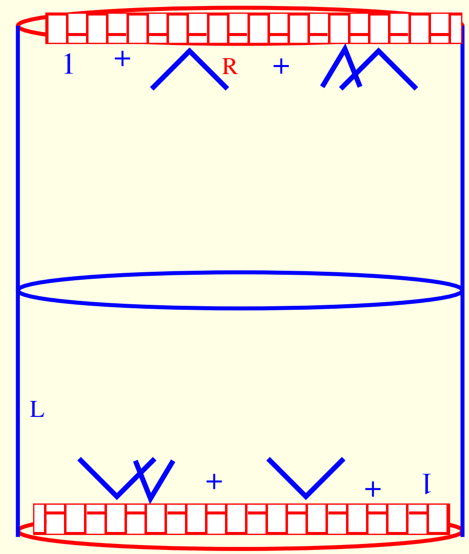

Boundary state $|B\rangle = \exp\left\{\int_{-\infty}^{\infty}\frac{d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)A^+(-\theta)A^+(\theta)\right\}|0\rangle$

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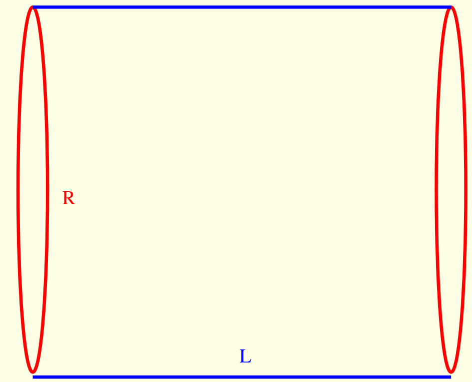

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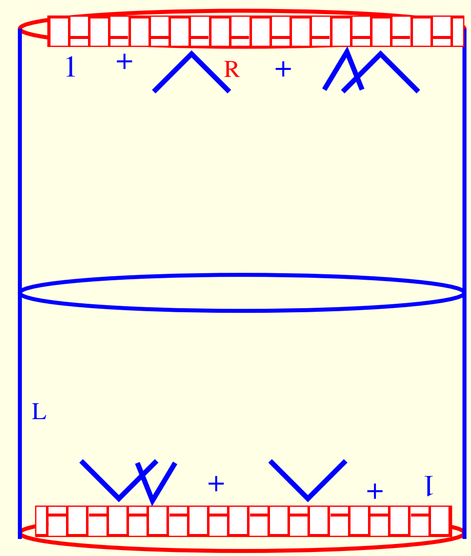
Dominant contribution for large L: two particle term

$$\langle B|e^{-H(R)L}|B\rangle = 1+\sum_k R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)e^{-2m\cosh\theta_k L}+\dots$$



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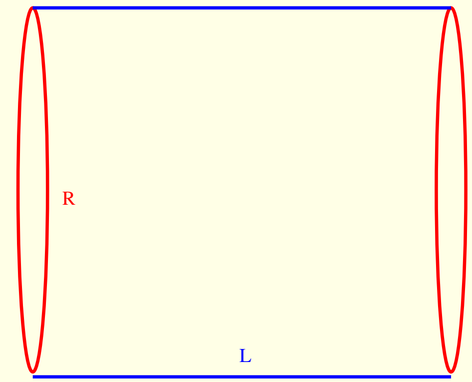

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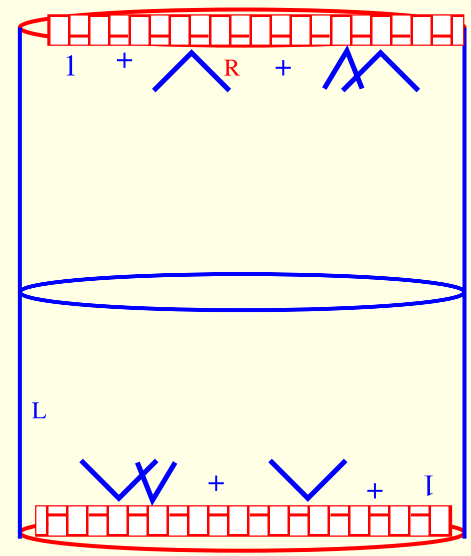

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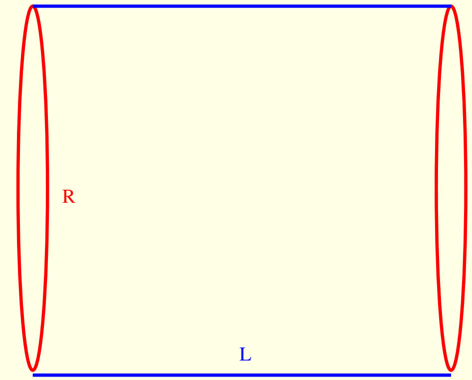
Boundary Lüscher correction



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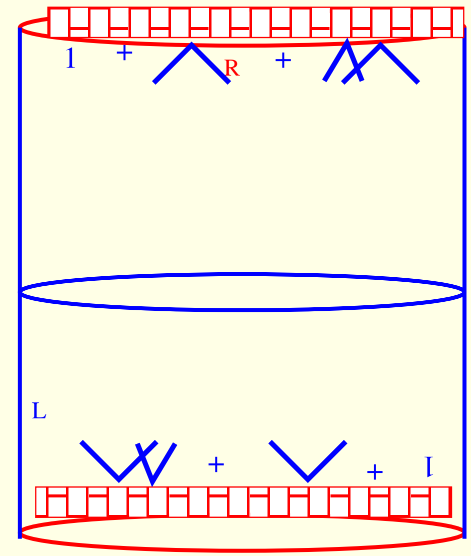

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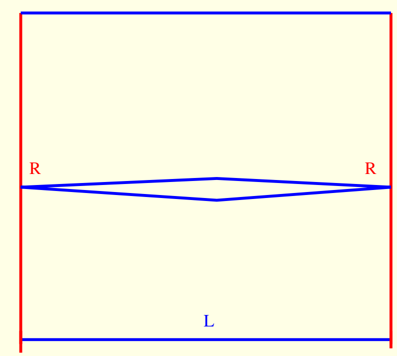
Ground state energy exactly: $\boxed{E_0(L) = -m\int\frac{d\theta}{4\pi}\cosh(\theta)\log(1+e^{-\epsilon(\theta)})}$



$$\epsilon(\theta) = 2mL\cosh\theta-\log(R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta))$$
$$-\int\frac{d\theta'}{2\pi}\varphi'(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$ LeClair, Mussardo, Saleur, Skorik

Casimir effect: Boundary finite size effect

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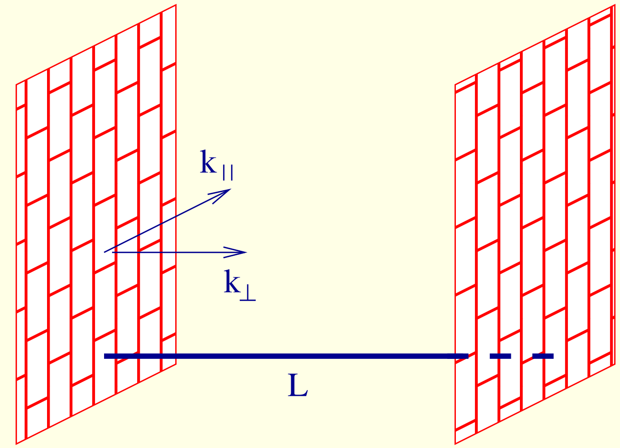
Extension to higher dimensions: $\vec{k}_{\|}$ label

Dispersion $\omega = \sqrt{m^2 + \vec{k}_{\|}^2 + k_{\perp}^2} = \sqrt{m_{\mathsf{eff}}^2 + k_{\perp}^2}$

rapidity $\omega = m_{\mathsf{eff}}(k_{\|}) \cosh\theta$, $k_{\perp} = m_{\mathsf{eff}}(k_{\|}) \sinh\theta$

Reflection $R(\theta, m_{\mathsf{eff}}(k_{\|}))$



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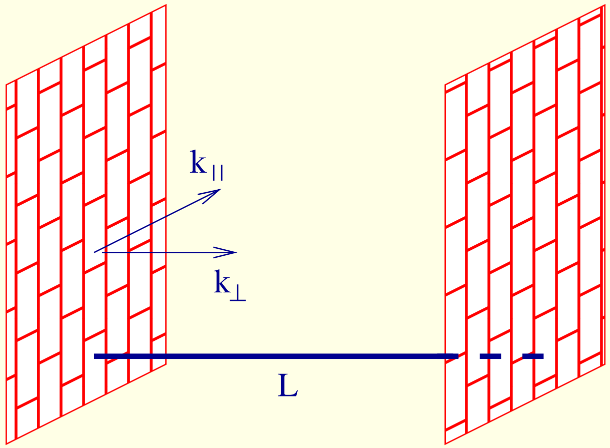

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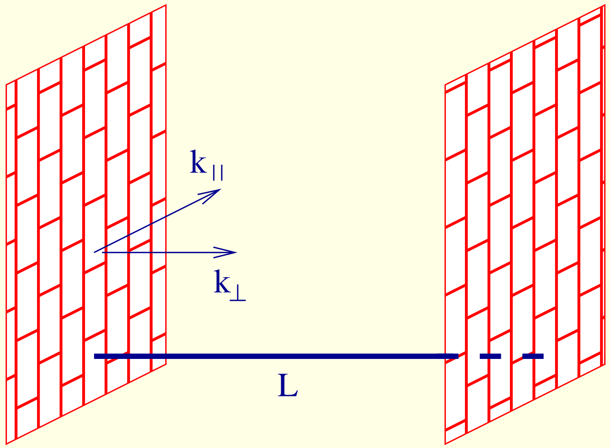

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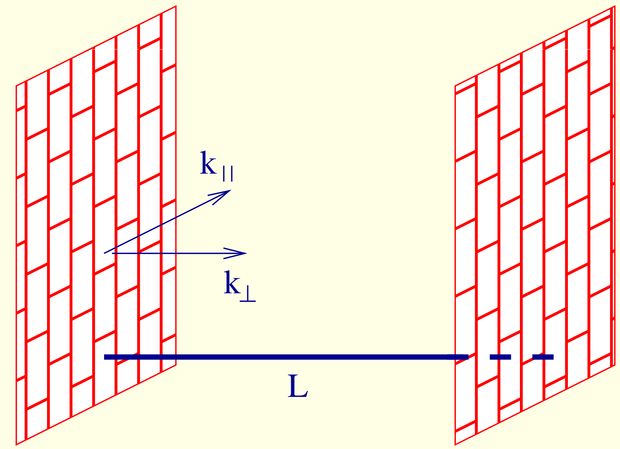

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QED: Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

reflections $E_{\|,\perp}, B_{\|,\perp} \longrightarrow R_{\|,\perp}$ look it up in Jackson:

$$R_\perp(\omega, k_{\|} = q) = \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}} \quad R_{\|}(\omega, k_{\|} = q) = \frac{\epsilon\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\epsilon\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}}$$

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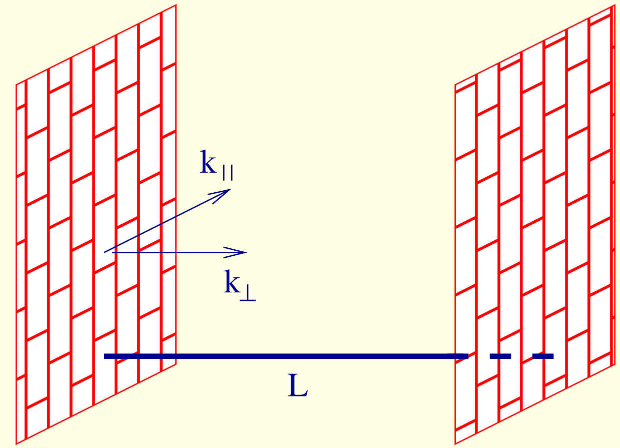

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Lifshitz formula

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Conclusion about Casimir

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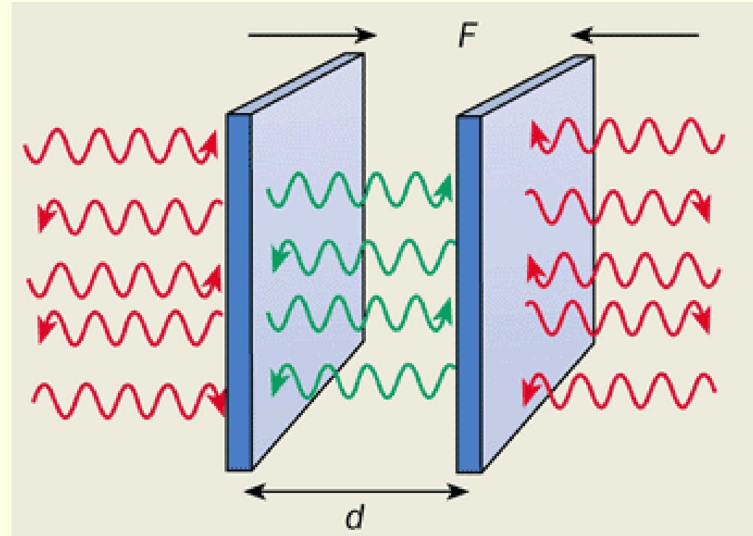


Usual derivation:

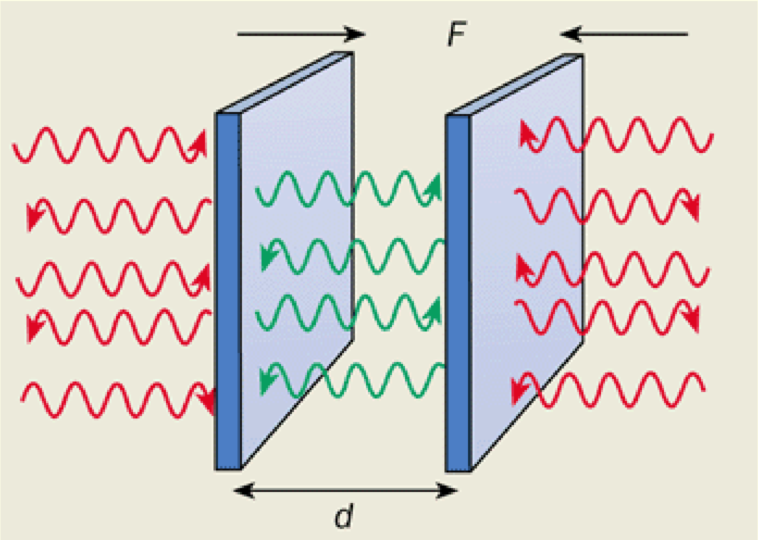
summing up zero freqencies

$E_0(L) = \frac{1}{2}\sum_{k(L)} \omega(k(L)) \propto \infty$

Complicated finite volume problem

$+$ divergencies

Conclusion about Casimir



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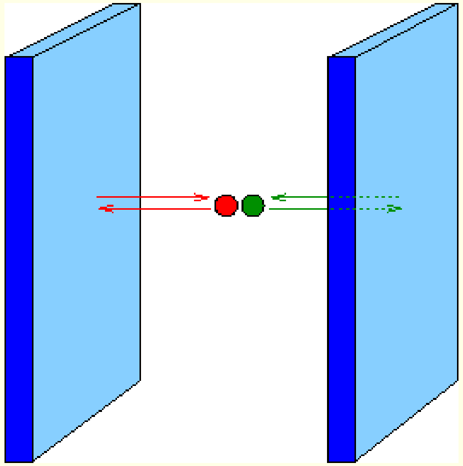
as a boundary finite size effect



$$E_0(L) = -\int \frac{d\tilde{p}}{2\pi} \log(1 + R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L})$$

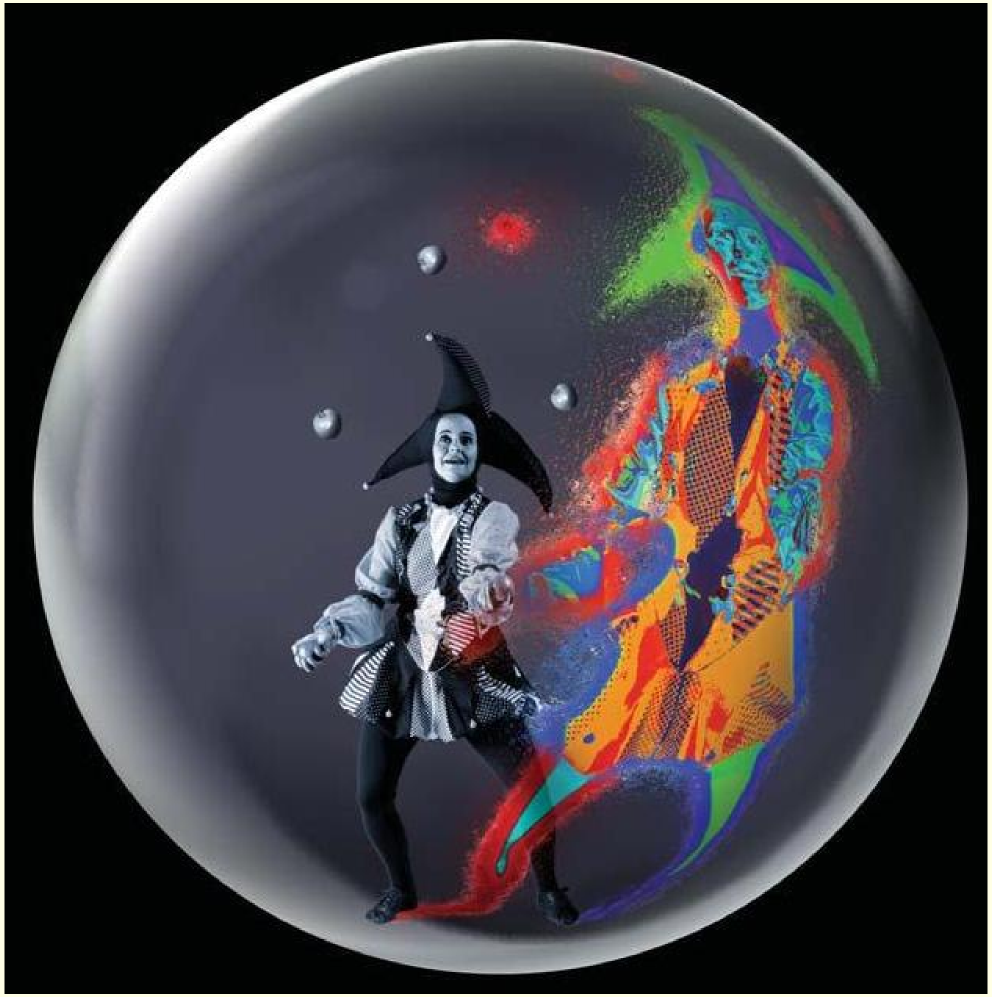
Reflection factor of the IR degrees of freedom:

semi infinite settings,

easier to calculate,

no divergences

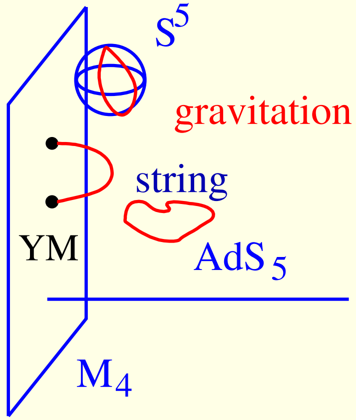
AdS/CFT correspondence (Maldacena 1997)



The Illusion of Gravity - Juan Maldacena, Scientific American (2005)

AdS/CFT correspondence (Maldacena 1997)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

$$\frac{R^2}{\alpha'}\int \frac{d\tau d\sigma}{4\pi}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \dots$$

$\equiv$

$\mathcal{N}=4$ D=4 $SU(N)$ SYM

$$\begin{array}{ccccc} & \nearrow & \Psi_{1,2,3,4} & \searrow & \\ A_\mu & & & & \Phi_{1,2,3,4,5,6} \\ & \searrow & \overline{\Psi}_{1,2,3,4} & \nearrow & \end{array}$$

$$\frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2)$, det( )

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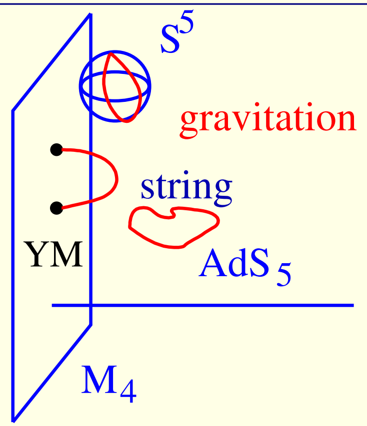

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$$\frac{2}{g_{YM}^2} \int d^4x \,\mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2)$, det( )

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

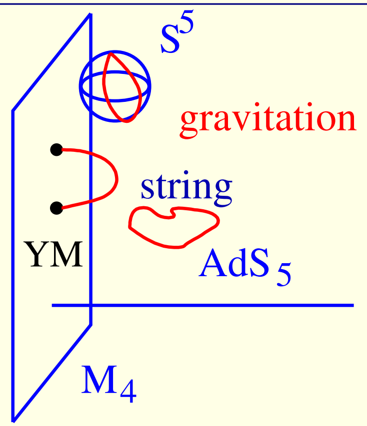
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

AdS/CFT correspondence (Maldacena 1997)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \dots$$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}, \qquad A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$$

$$\frac{2}{g_{YM}^2} \int d^4x \,\mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V\right]$$

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Anomalous dim $\Delta(\lambda)$

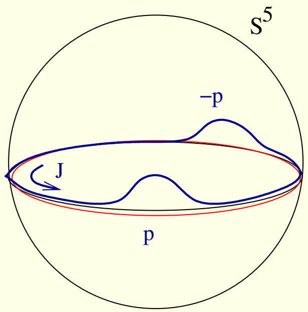
$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

2D integrable QFT

AdS/CFT correspondence: confirmation

two particle states

$$E_{BPS}(\lambda) = E_0$$



$$E_K(\lambda) = 2E(p,\lambda)$$

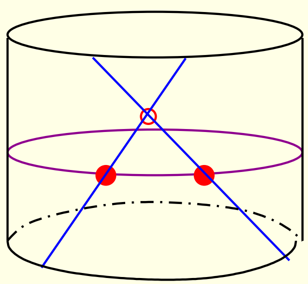
dispersion relation

$$E(p,\lambda) = \sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

elastic scattering $S(p,-p)$

Bethe Ansatz: $e^{ipJ} S(p,-p) = 1$

finite size corrections



$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

$\equiv$

Konishi anomalous dimension

$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

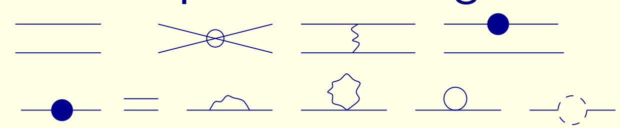
supersymmetric BPS operators

$$\mathcal{O}_{BPS} = \mathrm{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\ldots\uparrow\rangle$$

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \mathrm{Tr}(ZXZX + \ldots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$$

operator mixing

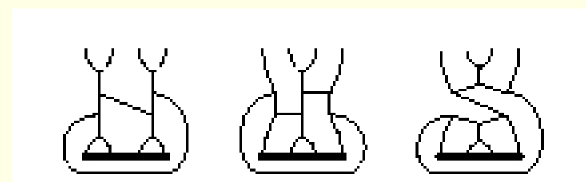


integrable spinchain

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$$

Bethe Ansatz + wrapping

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \ldots + \lambda^4\Delta_4 +$$

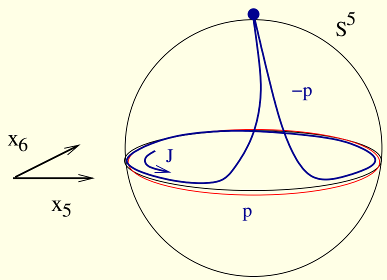


perturbatively $\propto 10^5$ diagrams

$$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

AdS/CFT correspondence: boundary

Z=0 brane: boundary vacuum



$$E_K(\lambda) = 2E_{Bdry}(\lambda)$$

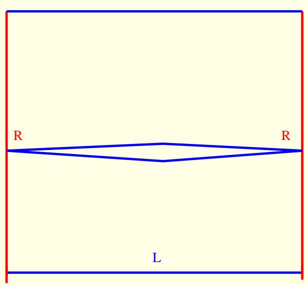
dispersion relation

$$E(p,\lambda) = \sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

elastic reflection $R(p)$

Bethe Ansatz: $e^{i2pJ}R(p)R(-p) = 1$

finite size corrections



$$\Delta E = -\int \frac{d\tilde{p}}{2\pi} R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L}$$

$\equiv$

det operator anomalous dimension

$$Z = \Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4$$

"Z=0 vacuum"

$$\mathcal{O} = \epsilon^{lm..nq}_{ij..kp} Z^i_l Z^j_m .. Z^k_n (Y Z^J Y)^p_q$$

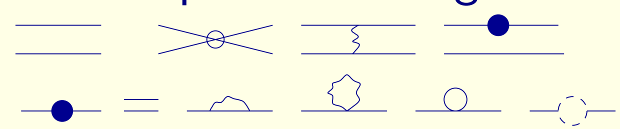
$$|\downarrow\uparrow\uparrow \dots \uparrow\uparrow\downarrow\rangle$$

"Y=0 vacuum"

$$\mathcal{O} = \epsilon^{lm..nq}_{ij..kp} Y^i_l Y^j_m .. Y^k_n (Z^J)^p_q$$

$$|\uparrow\uparrow \dots \uparrow\uparrow\rangle$$

operator mixing



integrable open spinchain

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$$

Z=0: Bethe Ansatz + wrapping

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 +$$

Y=0: Bethe Ansatz (Correa-Young '09)

direct test from BA!