

Zimányi Winter School, Budapest, 3-7 December, 2012

# Integrable aspects of AdS/CFT

**Z. Bajnok**

*MTA-Lendület Holographic QFT Group*

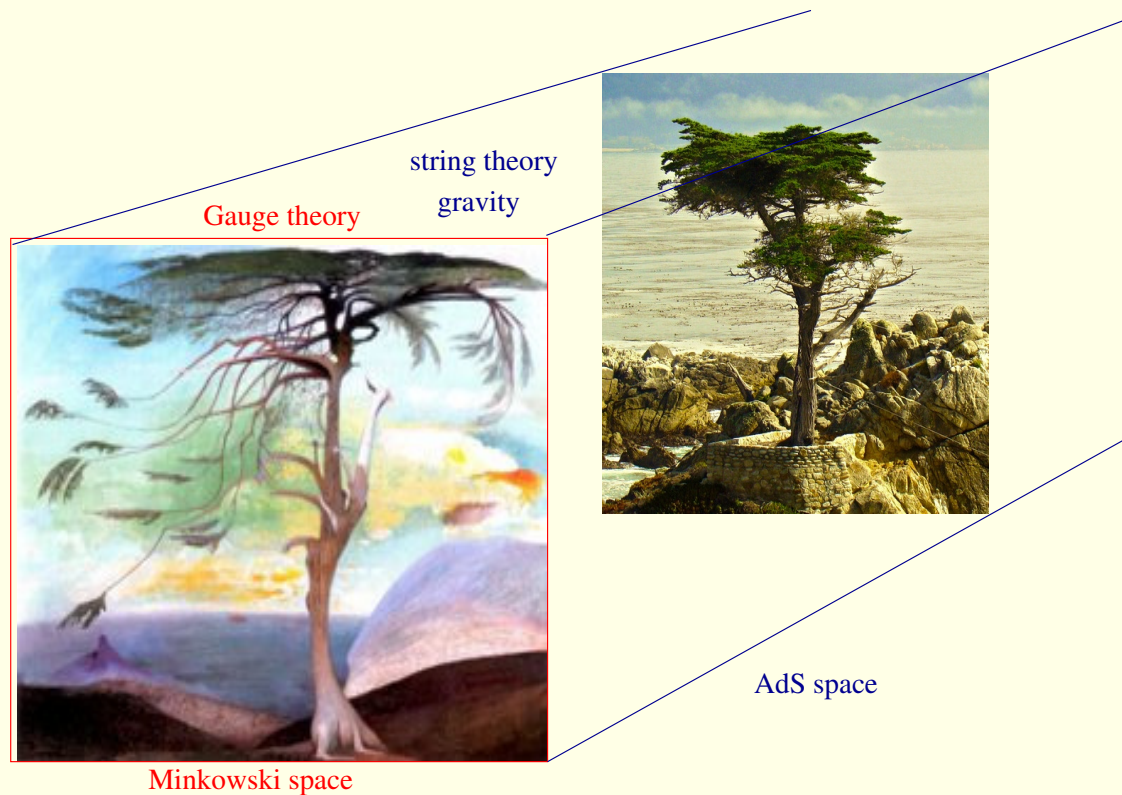
*Wigner Research Centre for Physics, Budapest*

# Integrable aspects of AdS/CFT

**Z. Bajnok**

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AdS/CFT correspondence  $\subset$  gauge/gravity duality

# Motivation: Organizing matter

Periodic Table of the Elements © www.elementsdatabase.com

- hydrogen
- poor metals
- alkali metals
- nonmetals
- alkali earth metals
- noble gases
- transition metals
- rare earth metals

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3	4											5	6	7	8	9	10
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		90	91	92	93	94	95	96	97	98	99	100	101	102	103		

Leptons		Quarks		Three Generations of Matter (Fermions)							
name	spin	charge	mass			name	spin				
electron	$\frac{1}{2}$	$-1$	0.511 MeV	electron neutrino	$\frac{1}{2}$	0	< 2.2 eV	up	$\frac{1}{2}$	$\frac{2}{3}$	2.4 MeV
muon	$\frac{1}{2}$	$-1$	105.7 MeV	muon neutrino	$\frac{1}{2}$	0	< 0.17 MeV	strange	$\frac{1}{2}$	$-\frac{1}{3}$	104 MeV
tau	$\frac{1}{2}$	$-1$	1.777 GeV	tau neutrino	$\frac{1}{2}$	0	< 15.5 MeV	bottom	$\frac{1}{2}$	$-\frac{1}{3}$	4.2 GeV
weak force	1	$\pm 1$	80.4 GeV	weak force	0	1	91.2 GeV	gluon	1	0	0
								photon	1	0	0

Bosons (Forces)

# Motivation: Organizing matter

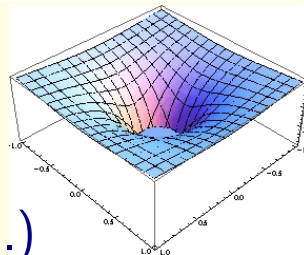
Periodic Table of the Elements © www.elementsdatabase.com

<ul style="list-style-type: none"> <li><span style="color: green;">■</span> hydrogen</li> <li><span style="color: yellow;">■</span> alkali metals</li> <li><span style="color: lightblue;">■</span> alkali earth metals</li> <li><span style="color: orange;">■</span> transition metals</li> <li><span style="color: blue;">■</span> poor metals</li> <li><span style="border: 1px solid black; display: inline-block; width: 10px; height: 10px;"></span> nonmetals</li> <li><span style="color: red;">■</span> noble gases</li> <li><span style="color: gray;">■</span> rare earth metals</li> </ul>										<table border="1" style="width: 100%; text-align: center;"> <tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>B</td><td>C</td><td>N</td><td>O</td><td>F</td><td>Ne</td></tr> <tr><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td></tr> <tr><td>Al</td><td>Si</td><td>P</td><td>S</td><td>Cl</td><td>Ar</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td></tr> <tr><td>Ga</td><td>Ge</td><td>As</td><td>Se</td><td>Br</td><td>Kr</td></tr> <tr><td>49</td><td>50</td><td>51</td><td>52</td><td>53</td><td>54</td></tr> <tr><td>In</td><td>Sn</td><td>Sb</td><td>Te</td><td>I</td><td>Xe</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td></tr> <tr><td>Tl</td><td>Pb</td><td>Bi</td><td>Po</td><td>At</td><td>Rn</td></tr> </table>										5	6	7	8	9	10	B	C	N	O	F	Ne	13	14	15	16	17	18	Al	Si	P	S	Cl	Ar	31	32	33	34	35	36	Ga	Ge	As	Se	Br	Kr	49	50	51	52	53	54	In	Sn	Sb	Te	I	Xe	81	82	83	84	85	86	Tl	Pb	Bi	Po	At	Rn																																																																	
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Leptons		Quarks		Bosons (Forces)	
electron $e$ 0.511 MeV $-\frac{1}{2}$	electron neutrino $\nu_e$ <2.2 eV $0$	down $d$ 4.8 MeV $-\frac{1}{3}$	up $u$ 2.4 MeV $\frac{2}{3}$	weak force $W$ 80.4 GeV $\pm 1$	weak force $Z$ 91.2 GeV $0$
muon $\mu$ 105.7 MeV $-\frac{1}{2}$	muon neutrino $\nu_\mu$ <0.17 MeV $0$	strange $s$ 104 MeV $-\frac{1}{3}$	charm $c$ 1.27 GeV $\frac{2}{3}$	gluon $g$ $1$	photon $\gamma$ $0$
tau $\tau$ 1.777 GeV $-\frac{1}{2}$	tau neutrino $\nu_\tau$ <15.5 MeV $0$	bottom $b$ 4.2 GeV $-\frac{1}{3}$	top $t$ 171.2 GeV $\frac{2}{3}$		
				I II III Three Generations of Matter (Fermions)	

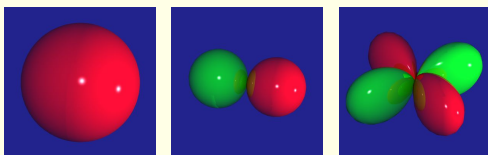
Electric interaction

(potential  $\Phi(r) = k \frac{Zq}{r}$ )



Quantum mechanics (Schrödinger eq.)

$$H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$$



# Motivation: Organizing matter

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- nonmetals
- alkali earth metals
- noble gases
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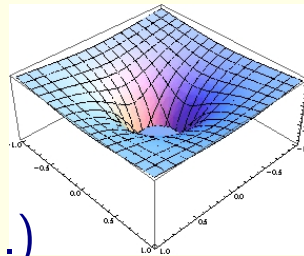
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Leptons		Quarks		Three Generations of Matter (Fermions)							
name	spin	charge	mass			name	spin				
electron $e$	$1/2$	$-1$	$0.511 \text{ MeV}$	down $d$	$1/2$	$-1/3$	$4.8 \text{ MeV}$	up $u$	$1/2$	$2/3$	$2.4 \text{ MeV}$
muon $\mu$	$1/2$	$-1$	$105.7 \text{ MeV}$	strange $s$	$1/2$	$-1/3$	$104 \text{ MeV}$	charm $c$	$1/2$	$2/3$	$1.27 \text{ GeV}$
tau $\tau$	$1/2$	$-1$	$1.777 \text{ GeV}$	bottom $b$	$1/2$	$-1/3$	$4.2 \text{ GeV}$	top $t$	$1/2$	$2/3$	$171.2 \text{ GeV}$
electron neutrino $\nu_e$	$1/2$	$0$	$< 2.2 \text{ eV}$	muon neutrino $\nu_\mu$	$1/2$	$0$	$< 0.17 \text{ MeV}$	tau neutrino $\nu_\tau$	$1/2$	$0$	$< 15.5 \text{ MeV}$
weak force $W$	$1$	$\pm 1$	$80.4 \text{ GeV}$	weak force $Z$	$0$	$0$	$91.2 \text{ GeV}$	gluon $g$	$1$	$0$	$0$
				photon $\gamma$	$0$	$0$	$0$				

Bosons (Forces)

Electric interaction

(potential  $\Phi(r) = k \frac{Zq}{r}$ )



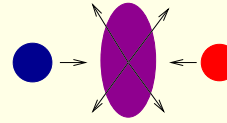
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$$H\Psi = \left(-\frac{(\hbar\nabla)^2}{2m} + \Phi(r)\right)\Psi = E\Psi$$



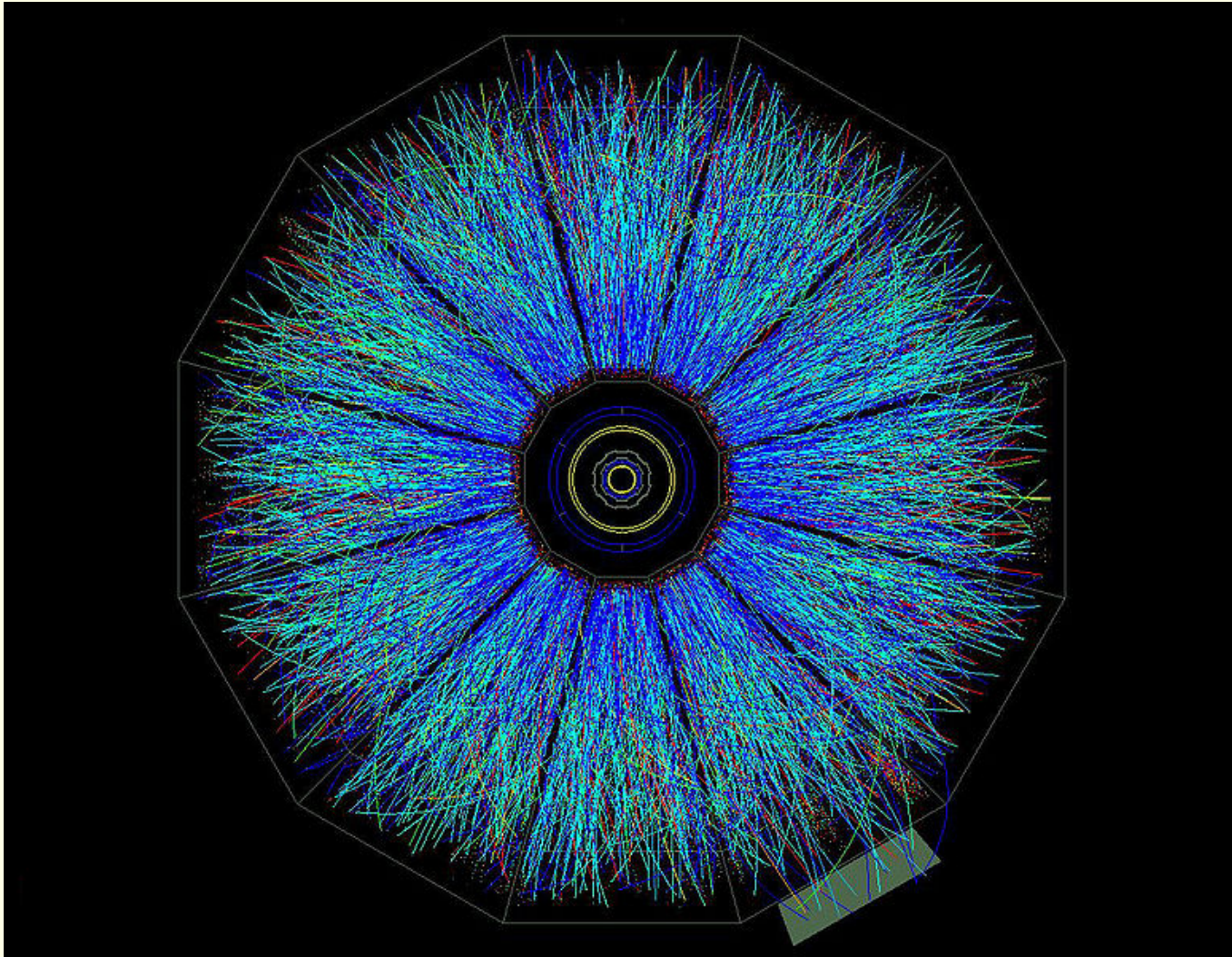
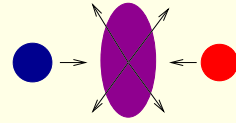
## Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



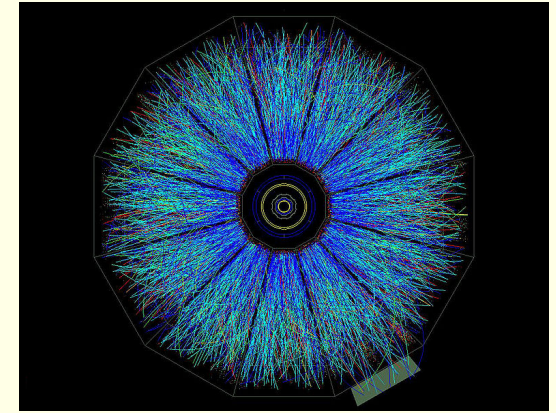
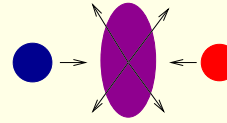
## Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



## Heavy ion collision

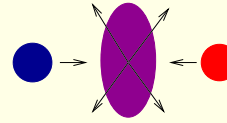
Brookhaven: Relativistic heavy ion collider (gold ion)





# Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)



Meson spectrum

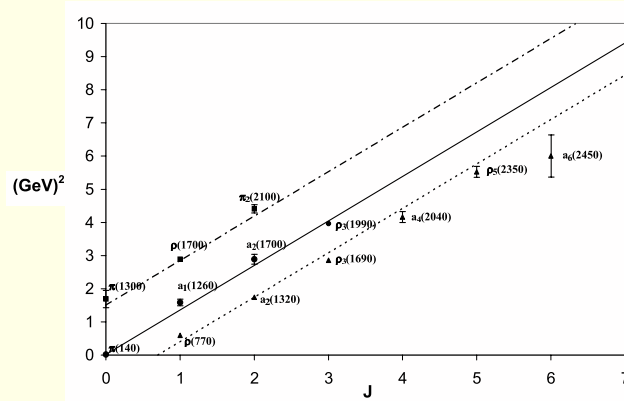
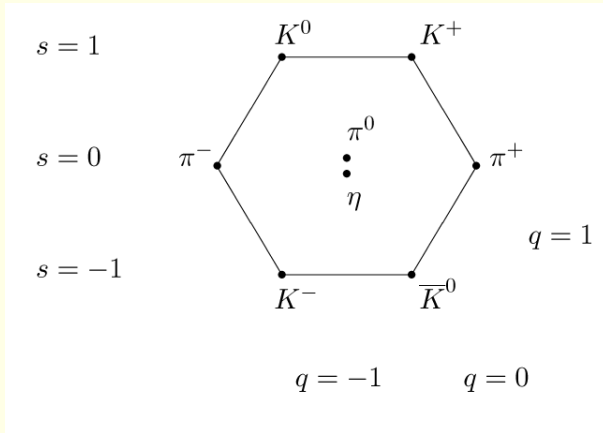
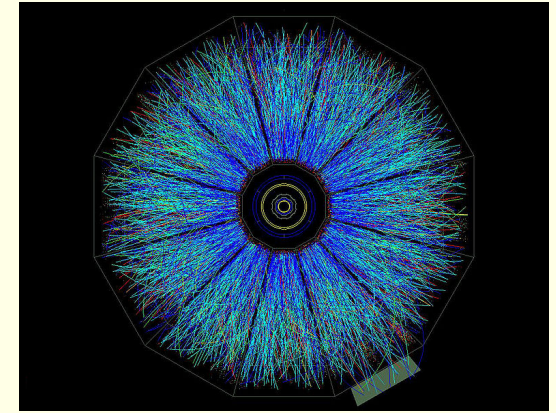


Fig. 1.



# Heavy ion collision

Brookhaven: Relativistic heavy ion collider (gold ion)

Meson spectrum

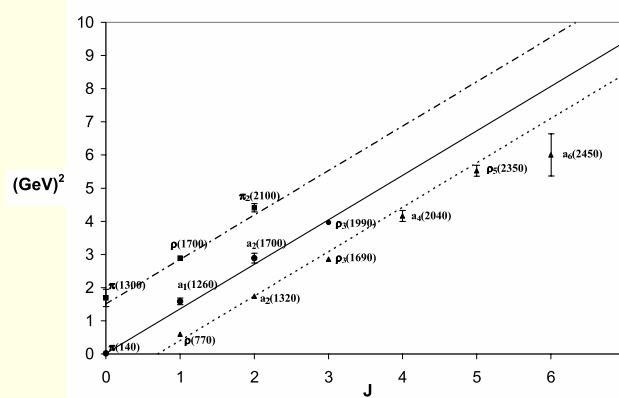
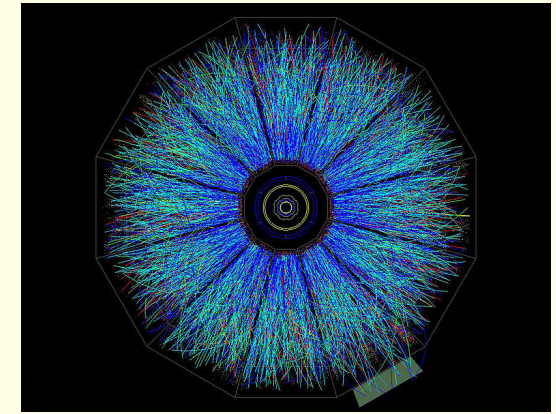
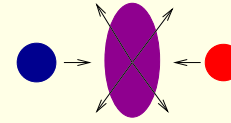
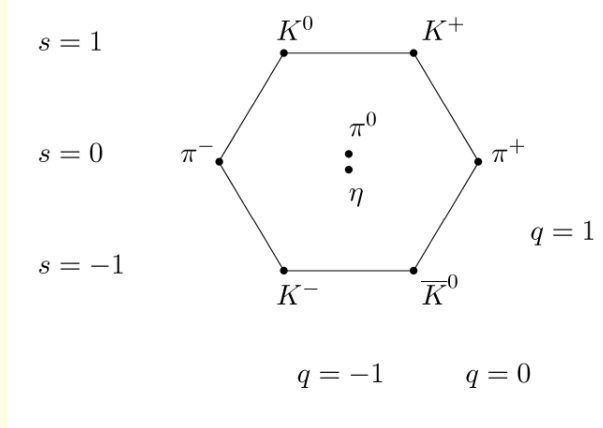


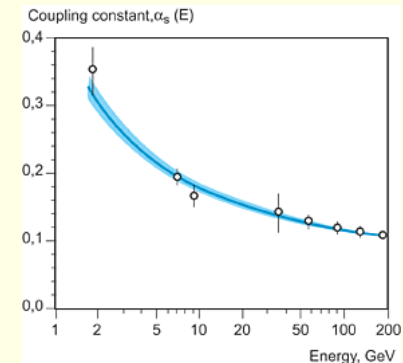
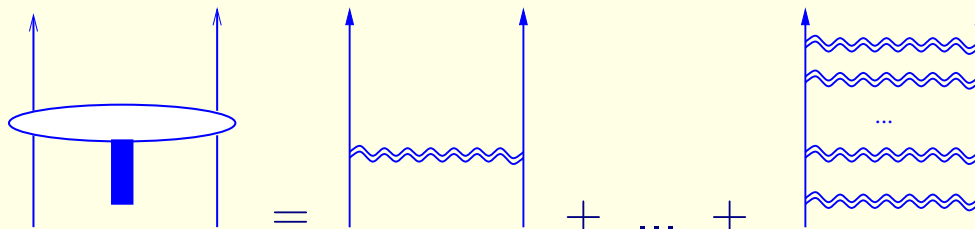
Fig. 1.

QCD =  $SU(3)$  gauge theory:  $G_{\mu}^{1..8}$  gluon  $\rightarrow F_{\mu\nu}^{1..8}$ ,  $\Psi_{kvarik}^{123}$

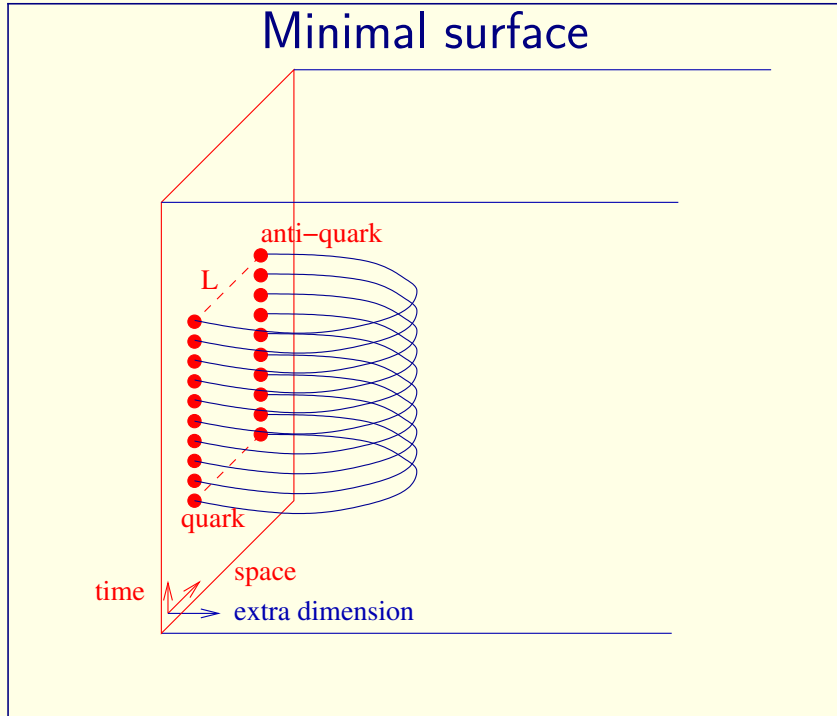
$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\Psi}(i\partial - m)\Psi - g\bar{\Psi}G\Psi$$

non-perturbative:  $\frac{\alpha_s}{4\pi} = O(1)$

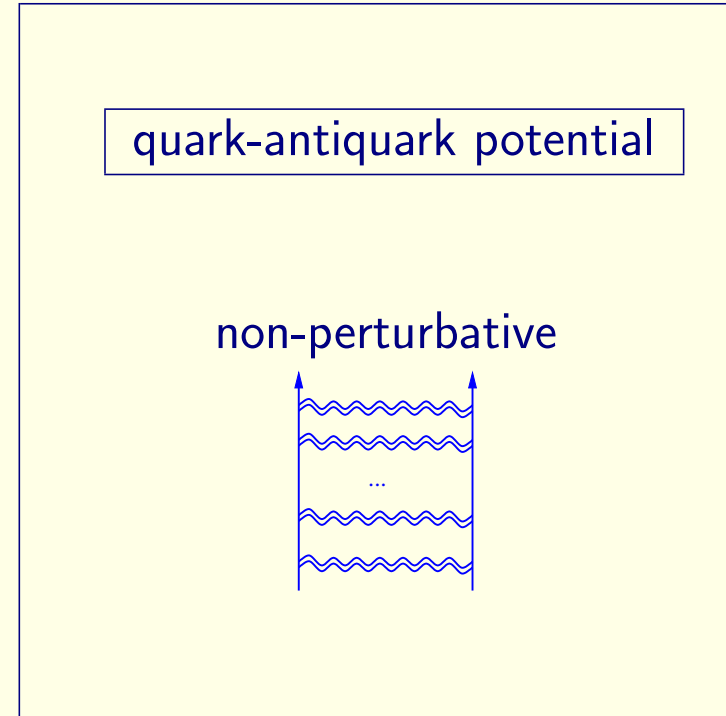
running coupl.:  $\beta(\alpha_s) = \mu \frac{\partial \alpha_s}{\partial \mu} < 0$   
asymptotic freedom, confinement



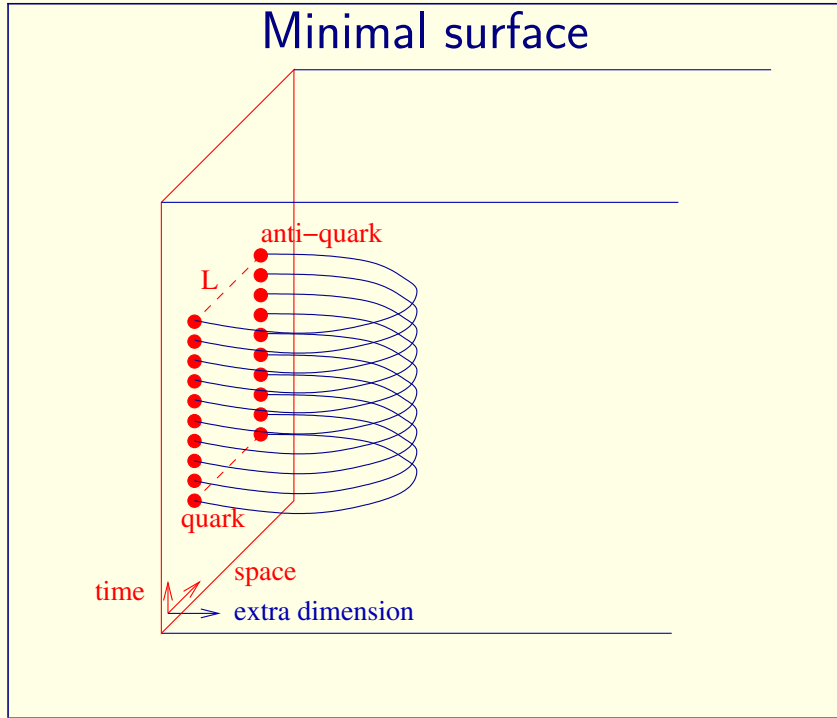
# AdS/CFT correspondence: other explanation



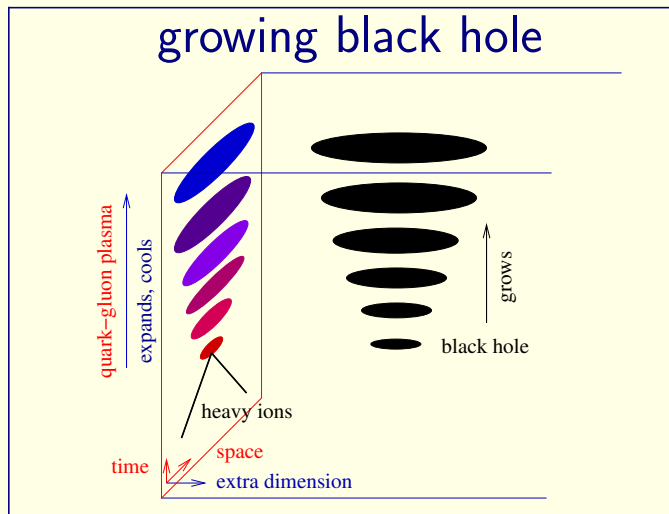
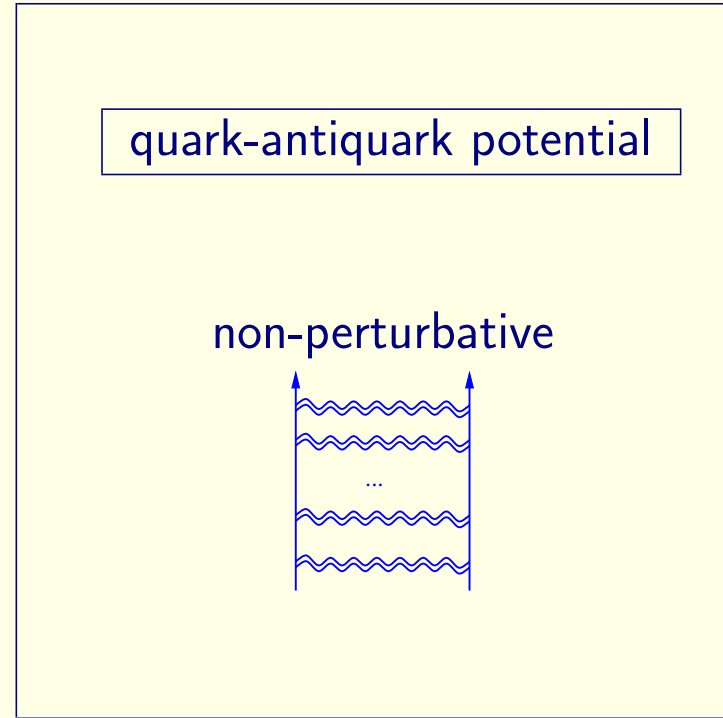
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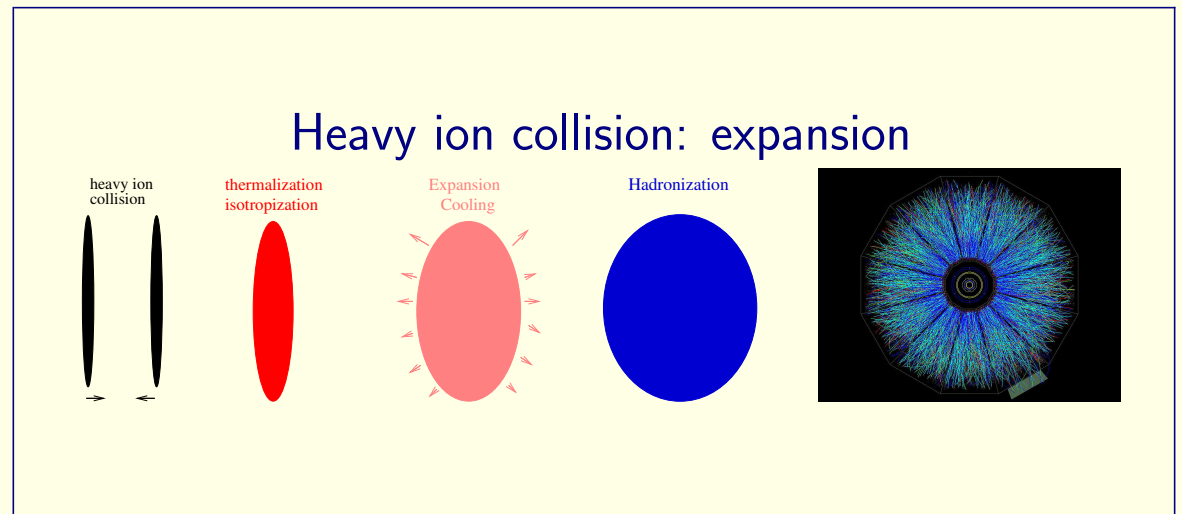
# AdS/CFT correspondence: other explanation



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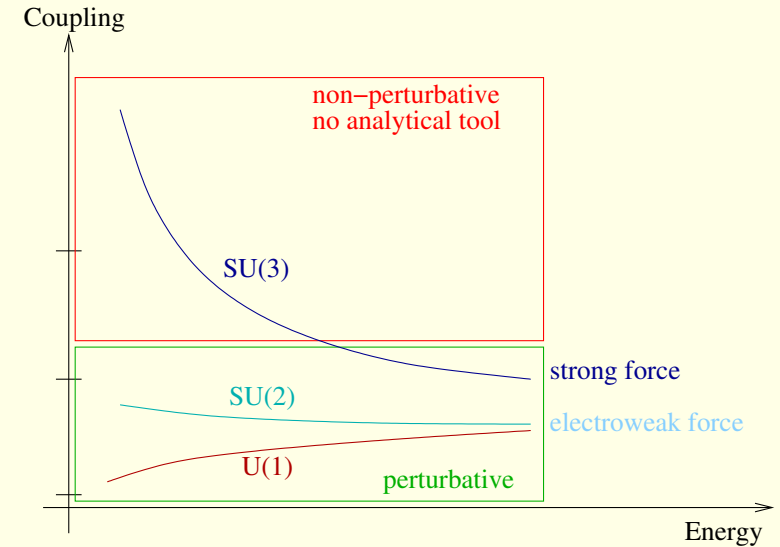


# QCD $\sim$ maximally supersymmetric gauge theory = CFT

## Fundamental interactions

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	$W^\pm, Z, \mu, \nu$ +Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

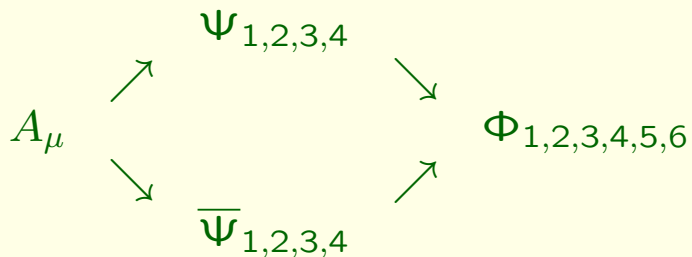
only analytical tool: perturbation theory



## maximally supersymmetric ( $\mathcal{N}=4$ ) gauge theory

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$

all fields  $N^2 - 1$  component matrix



$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

no running  $\beta = 0 \rightarrow$  CFT

no confinement

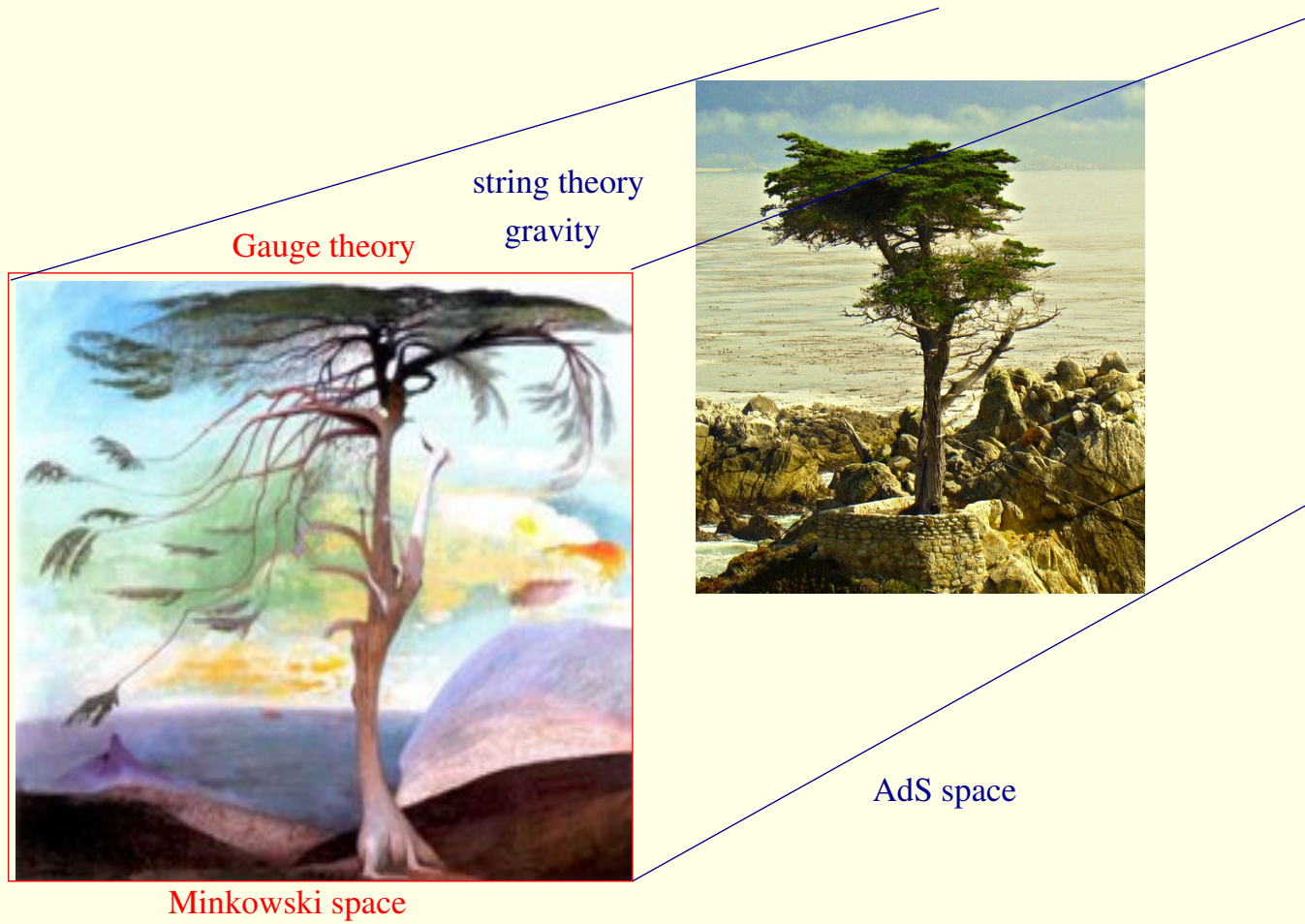
supersymmetric

heavy ion collision:

finite T  $\rightarrow$  SUSY is broken

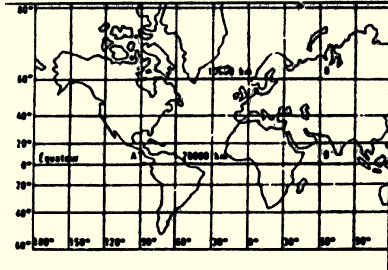
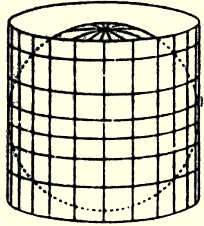
quark-gluon plasma is not confined

# AdS/CFT correspondence (Maldacena 1998)



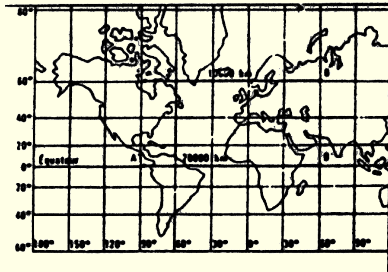
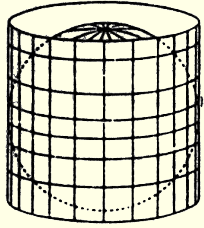
# AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space

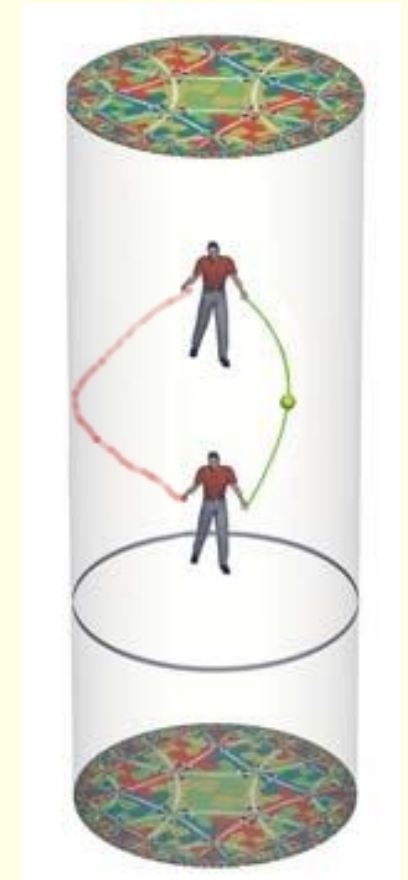


# AdS: string theory on Anti de Sitter $\supset$ gravitation

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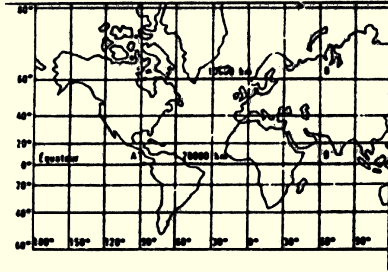
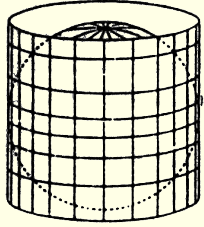
Anti de Sitter: negatively curved space



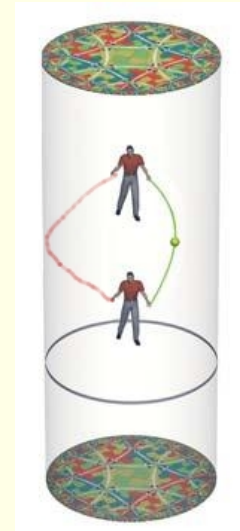


AdS: string theory on Anti de Sitter  $\supset$  gravitation

positively curved space

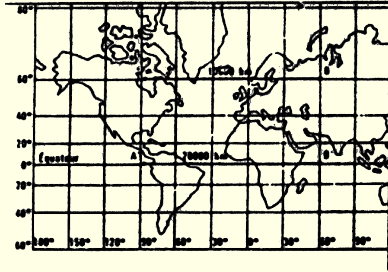
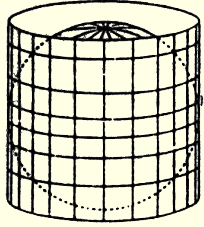


Anti de Sitter: negatively curved space



AdS: string theory on Anti de Sitter  $\supset$  gravitation

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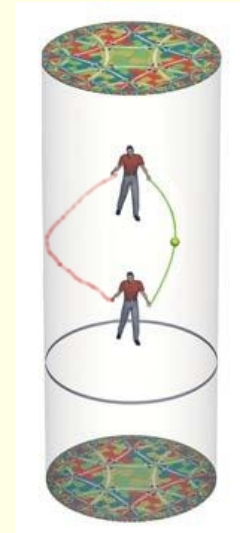
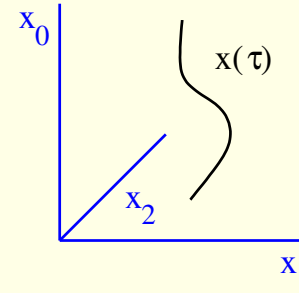


Anti de Sitter: negatively curved space



relativistic point particle:  $ds^2 = -dx_0^2 + dx_1^2 + \dots$

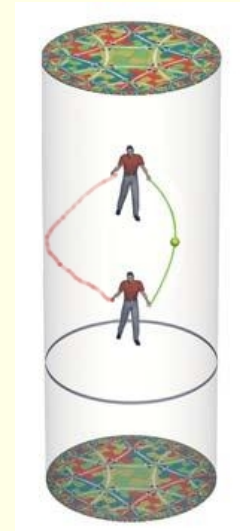
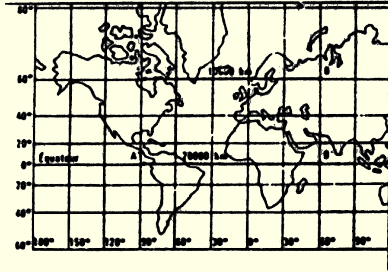
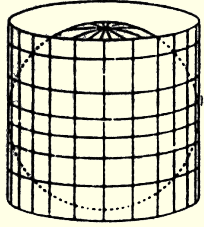
$S \propto \text{worldline} \propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$



# AdS: string theory on Anti de Sitter $\supset$ gravitation

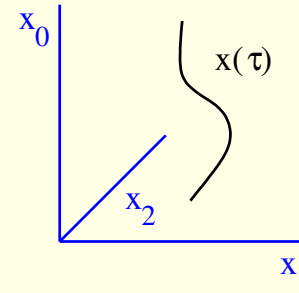
positively curved space

Anti de Sitter: negatively curved space



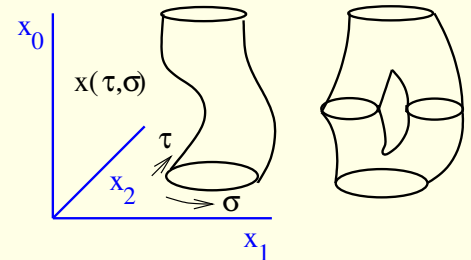
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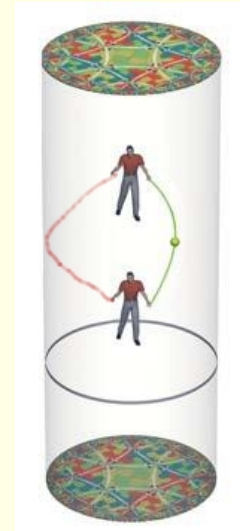
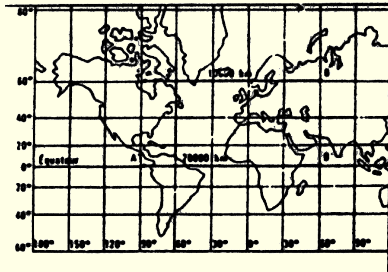
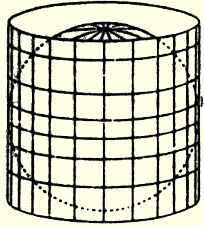
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# AdS: string theory on Anti de Sitter $\supset$ gravitation

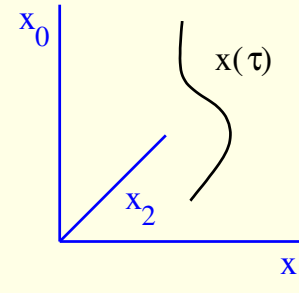
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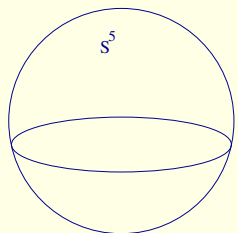
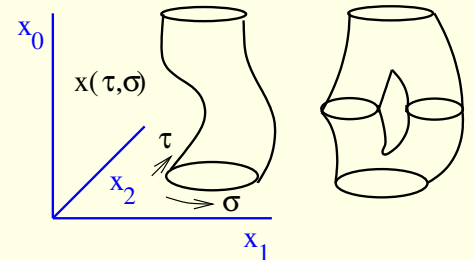
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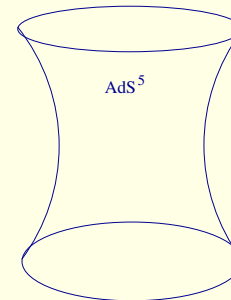
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$$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$$

$$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$$

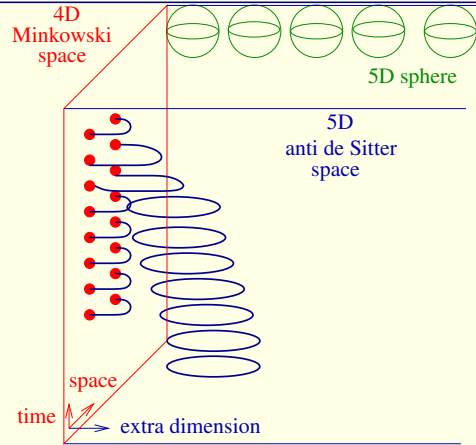


$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left( \partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \text{fermionok}$$

supercoset  $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

# AdS/CFT correspondence (Maldacena 1998)

## II<sub>B</sub> superstring on AdS<sub>5</sub> × S<sup>5</sup>



$$\sum_1^6 Y_i^2 = R^2 \quad - + + + + - = -R^2$$

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≡

## N = 4 D=4 SU(N) SYM

$$\frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[ -\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D}\Psi + V \right]$$

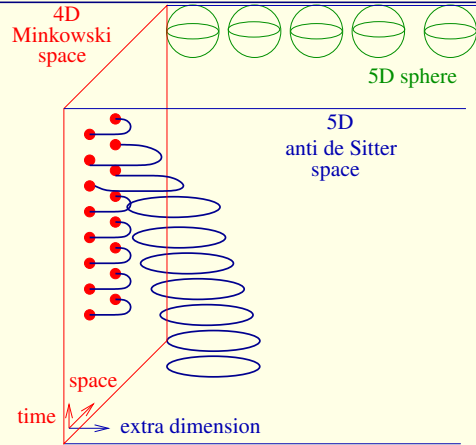
$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi} [\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$$

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# AdS/CFT correspondence (Maldacena 1998)

$II_B$  superstring on  $AdS_5 \times S^5$



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Couplings:  $\sqrt{\lambda} = \frac{R^2}{\alpha'}$ ,  $g_s = \frac{\lambda}{N} \rightarrow 0$

2D QFT

String energy levels:  $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

Dictionary

strong  $\leftrightarrow$  weak

$\Downarrow$

$\lambda = g_{YM}^2 N$ ,  $N \rightarrow \infty$  planar limit

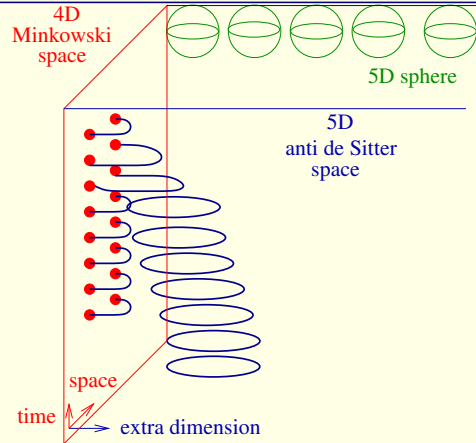
$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim  $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$$

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Anomalous dim  $\Delta(\lambda)$

$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \dots$

2D integrable QFT

spectrum:  $Q = 1, 2, \dots, \infty$  dispersion:  $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$   
Exact scattering matrix:  $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

## AdS/CFT correspondence: confirmation

supersymmetric **BPS** operators

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

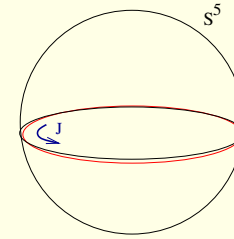
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow \dots \uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak  $\leftrightarrow$  strong

**BPS** string configuration



$$E_{BPS}(\lambda) = J$$



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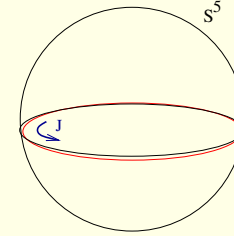
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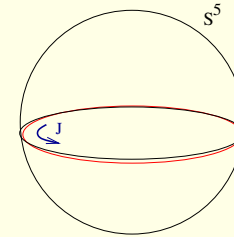
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$$E_{BPS}(\lambda) = J$$

2D integrable QFT

$$\text{supersymmetric groundstate } E_0(J) = \Delta(\lambda) - J = 0$$

Nontrivial anomalous dimension

supersymmetric theory: Excited state

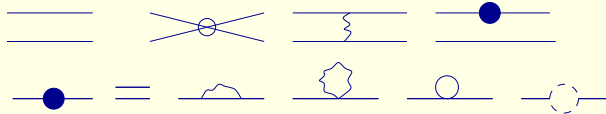
$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

## Confirmation: excited state - Konishi operator

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$$

operator mixing



$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \dots + \lambda^4\Delta_4 + \dots$$

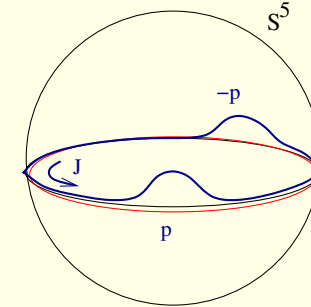


[Fiamberti ..'08]

$$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

≡

string configuration



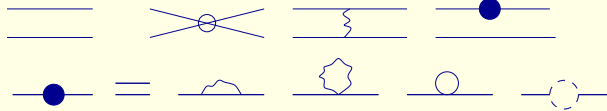
moving bumps (sine-Gordon) [Hofman .. '07]

string action = saddle point + loop corr.


$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

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 $\mathcal{O}_K = \text{Tr}(ZXZX + \dots) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + \dots$   
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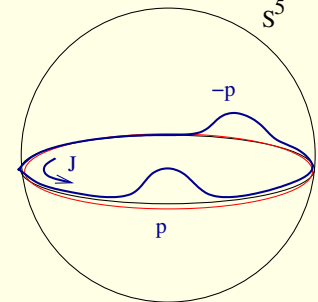
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[Fiamberti ..'08]  
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string configuration



moving bumps (sine-Gordon) [Hofman .. '07]  
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## two particle state

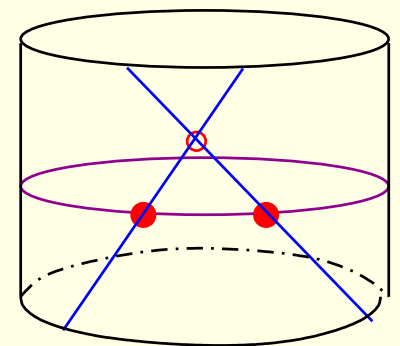
$$E = E_{BA} + E_{FSC}$$

Bethe Ansatz:  $e^{ipJ} S(p, -p) = 1$

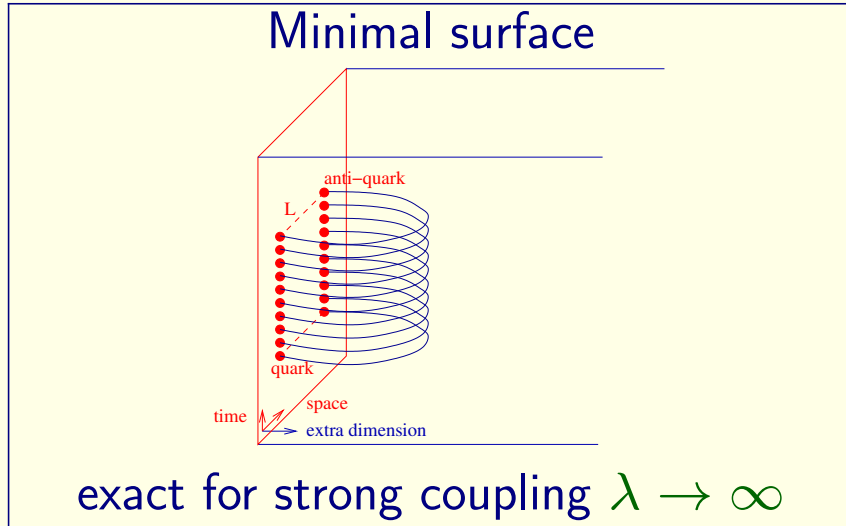
$$E_{BA} = 2E(p, \lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2} (\sin \frac{p}{2})^2}$$

$$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} S_{Q1}(q, p) S_{Q1}(q, -p) e^{-\epsilon_Q L}$$

$E_4 = \Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$  [Z.B., R. Janik '09]



## AdS/CFT correspondence: applications



$\equiv$

quark-antiquark potential

Wilson loop:  $\langle \oint_C A_\mu dx^\mu \rangle$   
non-perturbative

$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda} 1}{\Gamma(\frac{1}{4})^4 r}$$

# AdS/CFT correspondence: applications

### Minimal surface

exact for strong coupling  $\lambda \rightarrow \infty$

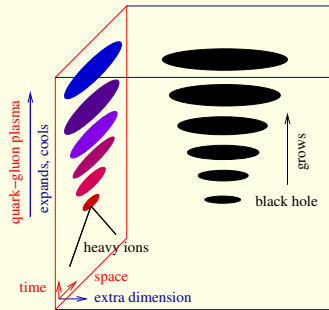
quark-antiquark potential

≡

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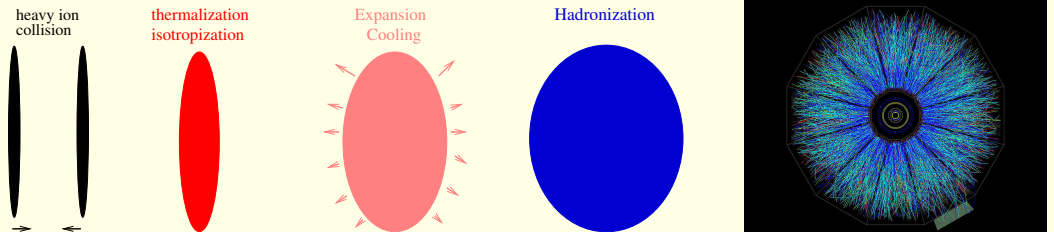
$$V(r) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{r}$$

### growing black hole



≡

### Heavy ion collision: expansion



metric  $\delta g(x, 0) \propto \langle T_{\mu\nu} \rangle$   
 $ds^2 = \frac{1}{z^2} (g(x, z)_{\mu\nu} dx^\mu dx^\nu + dz^2)$

Einstein equation

$$R_{ab} - \frac{1}{2} g_{ab} R - 6g_{ab} = 0$$

growing black hole

$$g_{tt} = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)^2}; \quad g_{xx} = 1 + \frac{z^4}{z_0^4}$$

$\langle T_{\mu\nu} \rangle$  matter distribution  
relativistic hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0 \text{ and } T^\mu_\mu = 0$$

viscous quark-gluon plasma

expansion in time: perfect fluid +  $\frac{\eta}{s} = \frac{1}{4\pi} + \dots$

Conclusion= Trust the AdS/CFT correspondence!

