HoloGrav Helsinki 4-8 March 2013,

Review of progress in AdS/CFT integrability

Z. Bajnok

MTA-Lendület Holographic QFT Group,

Wigner Research Centre for Physics, Budapest

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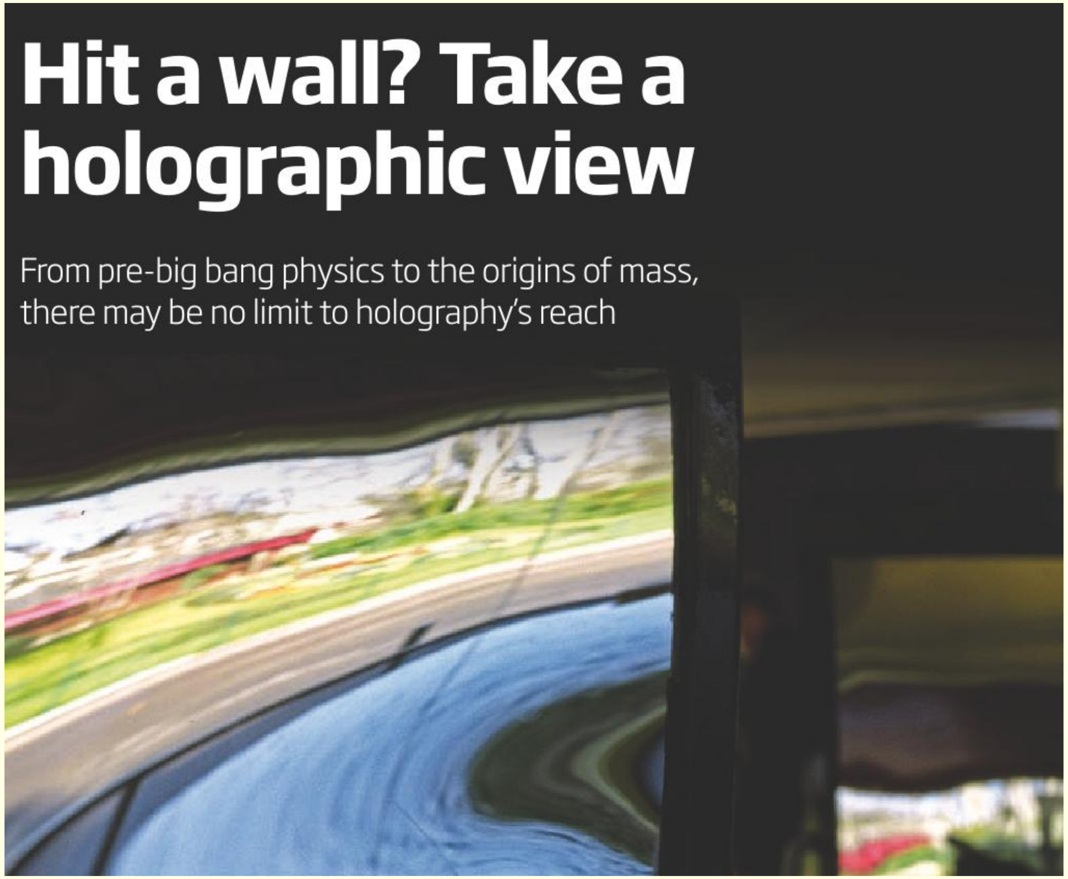

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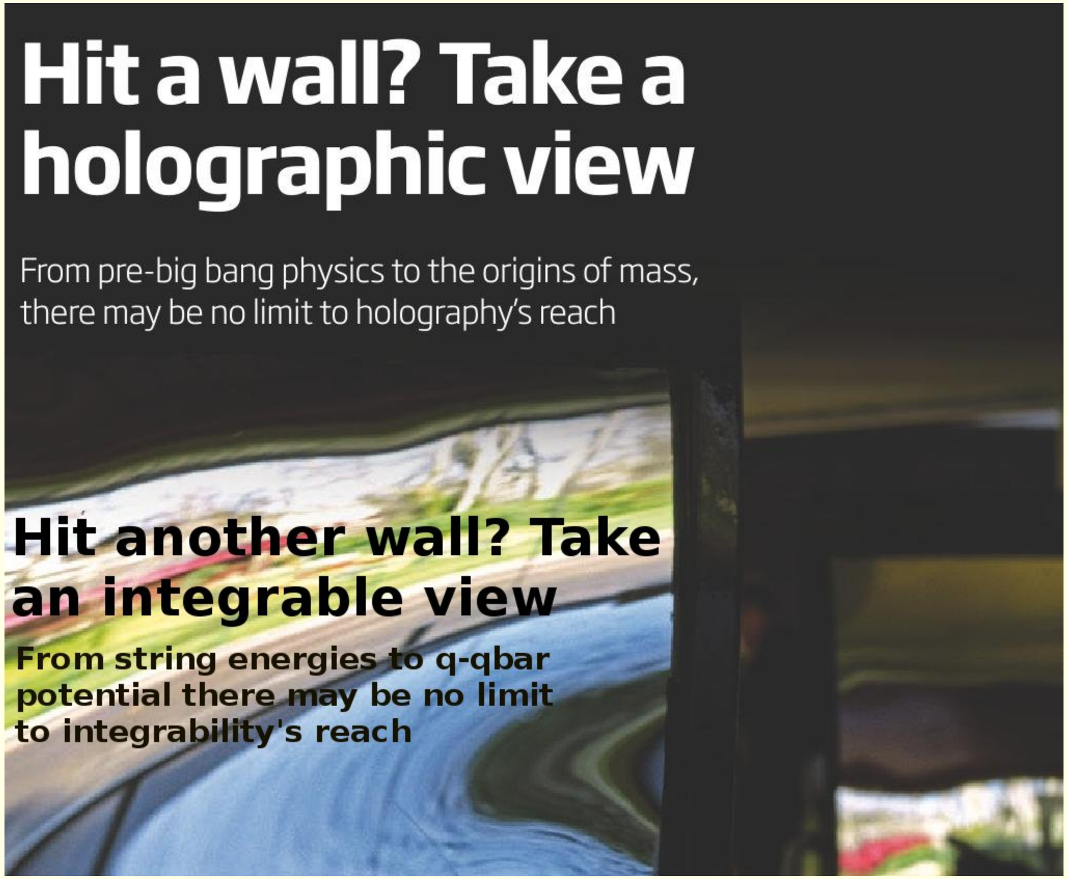
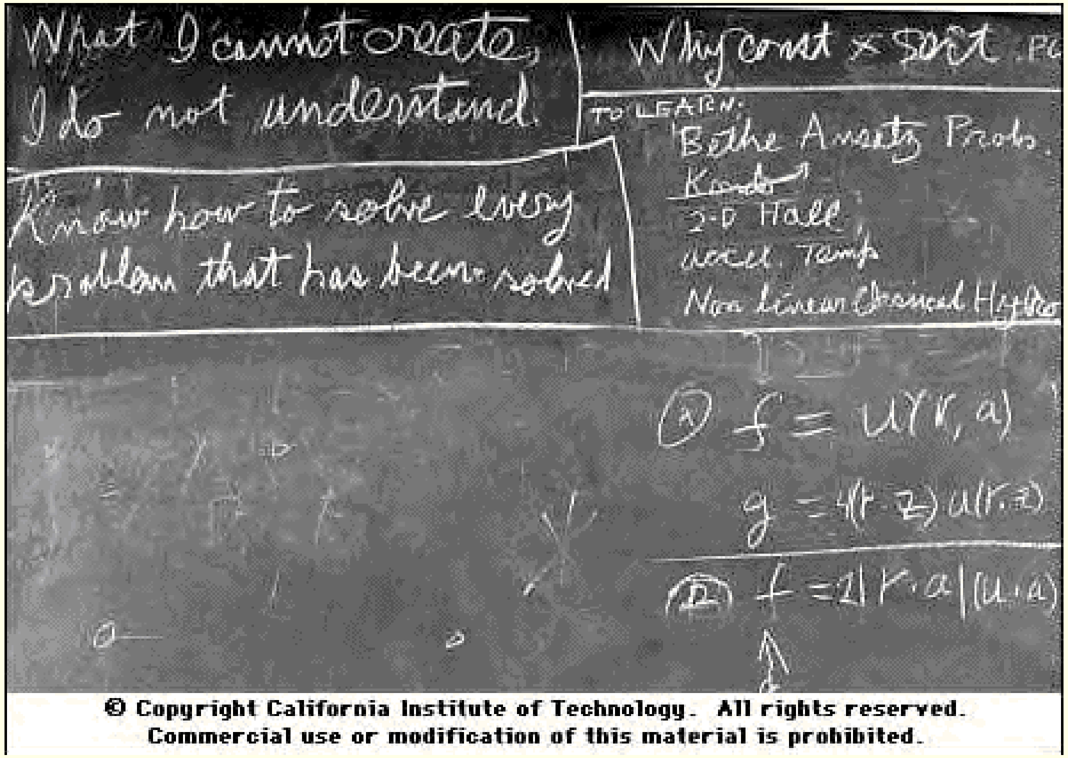


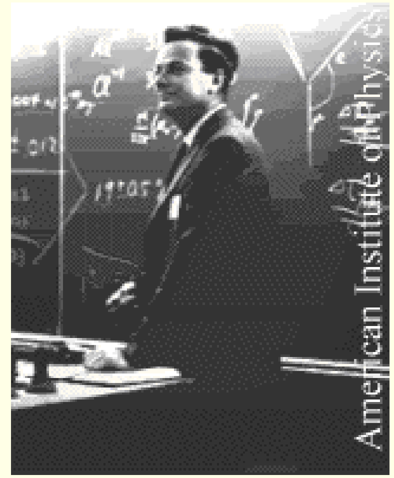
Gauge/gravity (string) duality: t' Hooft ⟷ Integrability in QCD: Feynman



What I cannot create I do not understand.

Know how to solve every problem that has been solved.

To learn: Bethe Ansatz Probs., ← Kondo, 2D Hall, accel temp, Non linear classical Hydro

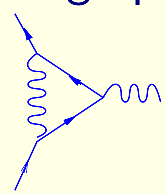


Richard P. Feynman (1918–1988)


1965

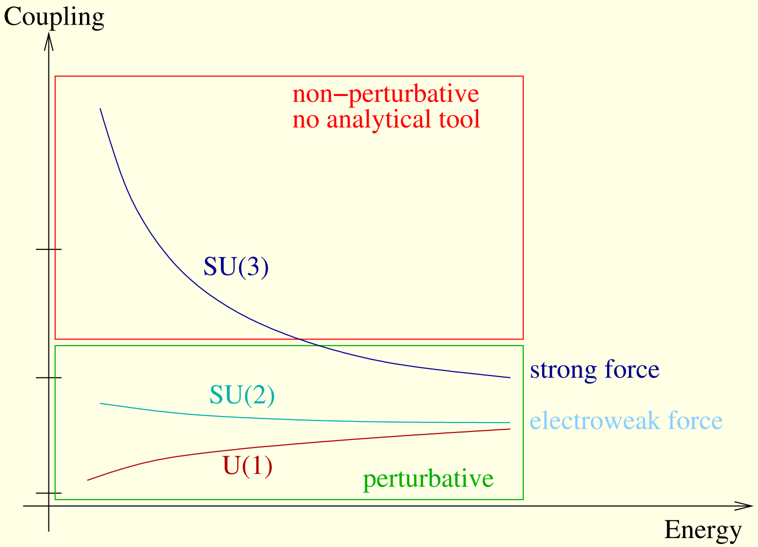
QED:

Feynman graphs



Strong interaction?

CFT: maximally supersymmetric gauge theory



Fundamental interactions (language of Nature: gauge theory)

interaction	particles	gauge theory
electromagnetic	photon+electron	$U(1)$
electroweak	$W^{\pm}, Z \quad \mu,\nu$+Higgs	$SU(2) \times U(1)$
strong	gluon+quarks	$SU(3)$

only analytical tool: perturbation theory

maximally supersymmetric gauge theory (harmonic oscillator)

interaction	particles	gauge theory
$\mathcal{N} = 4$ supersymm.	gluon+quarks+scalars	$SU(N)$

all fields $N^2 - 1$ component matrix

$$A_\mu \nearrow \Psi_{1,2,3,4} \searrow \Phi_{1,2,3,4,5,6}$$

$$A_\mu \searrow \overline{\Psi}_{1,2,3,4} \nearrow \Phi_{1,2,3,4,5,6}$$

$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$$

no running $\beta = 0 \to$ CFT
no confinement
supersymmetric

heavy ion collision:
finite T $\to$ SUSY is broken
quark-gluon plasma is not confined

CFT: Observables

maximally supersymmetric gauge theory

$A \quad \Psi_{1,2,3,4} \quad \overline{\Psi}_{1,2,3,4} \quad \Phi_{1,2,3,4,5,6}$ fields $SU(N)$ matrices

$$\mathcal{S} = \frac{2}{g_{YM}^2}\int d^4x \,\mathrm{Tr}\left[-\tfrac{1}{4}F^2 - \tfrac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \tfrac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

observables

parameters: g_{YM}, N

observables: partition function

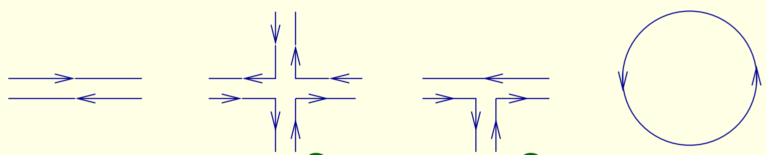
gauge-invariant operators

$\mathcal{O}(x) = \mathrm{Tr}(A^{L_1}\Psi^{L_2}\Phi^{L_3}..)$,det()

correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle$

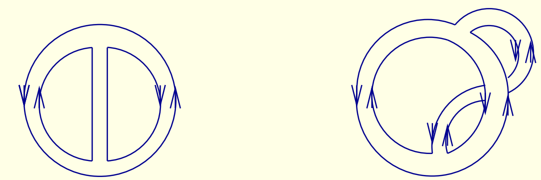
correlators: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \int [dA...] e^{-i\mathcal{S}}\mathcal{O}_1(x)\mathcal{O}_2(0) = \langle \mathcal{O}_1(x)\mathcal{O}_2(0)e^{-iV}\rangle_0$

perturbation:



$g_{YM}^2 \qquad g_{YM}^{-2} \qquad g_{YM}^{-2} \qquad N$

genus exp.



$g_{YM}^2 N^3 = N^2\lambda \qquad \lambda = g_{YM}^2 N$

partition func. $Z(\lambda, \frac{1}{N}) = N^2 \sum_g (\frac{1}{N})^{2g} \sum_n \alpha(g,n)\lambda^n$ string interactions? (t' Hooft)

conformal field theory: $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ scale dim.: Δ_i Konishi op. $\mathcal{O}_K = \mathrm{Tr}(\Phi_i^2)$

$$\Delta_K(\lambda) = 2 + 6\frac{\lambda}{4\pi^2} - 24\frac{\lambda^2}{(4\pi^2)^2} + 168\frac{\lambda^3}{(4\pi^2)^3} - (1410 + 144\zeta(3) + \tfrac{1}{2}(324 + 864\zeta(3) - 1440\zeta(5)))\frac{\lambda^4}{(4\pi^2)^4}$$

CFT: perturbative expansion

Observable: dimensions $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ Konishi op. $\mathcal{O}_K = \mathrm{Tr}(\Phi_i^2)$

$\Delta_K(\lambda) = 2 + 6g^2 - 24g^4 + 168g^6 + \quad ; \quad g^2 = \frac{\lambda}{4\pi^2}$

4loop:

[Fiamberti,A. Santambrogio,Sieg,Zanon '09] [Velizhanin]

$-(1410 + 144\zeta_3 + \frac{1}{2}(324 + 864\zeta_3 - 1440\zeta_5))g^8$

5loop:

[Eden, Heslop, Korchemsky, Smirnov, Sokatchev '12]

$+12(2209 + 360\zeta_3 + 240\zeta_5)g^{10}$
$-36(72\zeta_3(-1 + 2\zeta_3) + 5(63 + 64\zeta_5 - 168\zeta_7))g^{10}$

6loop, strong coupling?

CFT: perturbative expansion

Observable: dimensions $\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i}}$ Konishi op. $\mathcal{O}_K = \mathrm{Tr}(\Phi_i^2)$

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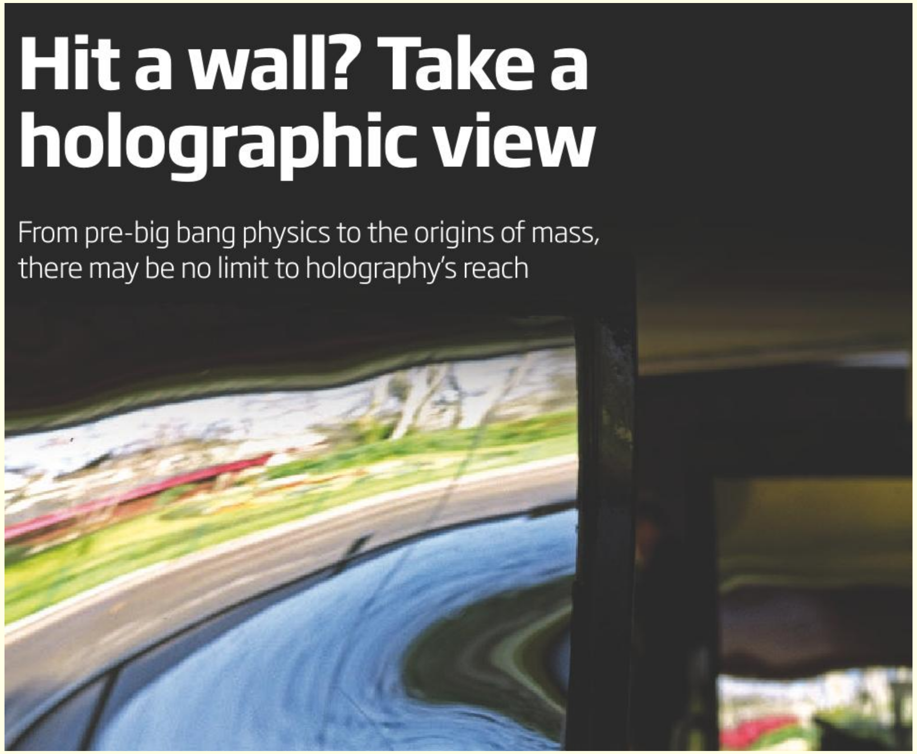

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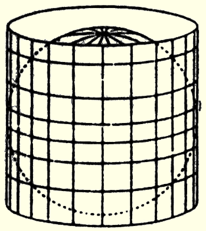
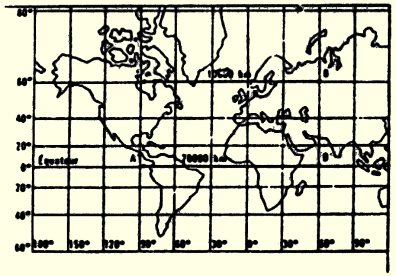
AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space





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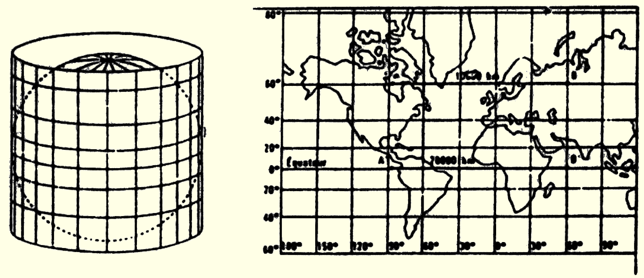
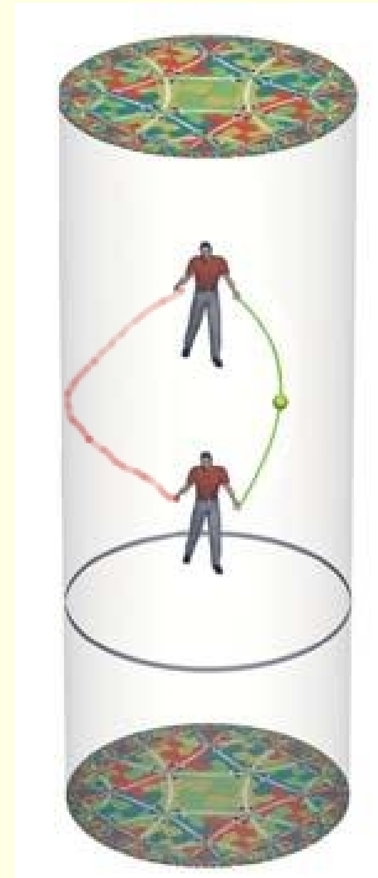


Anti de Sitter: negatively curved space



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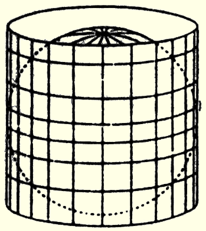

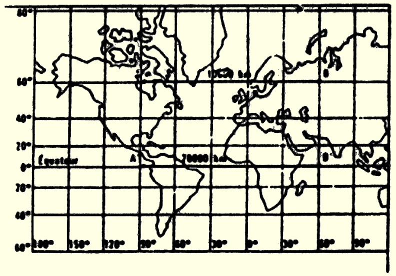

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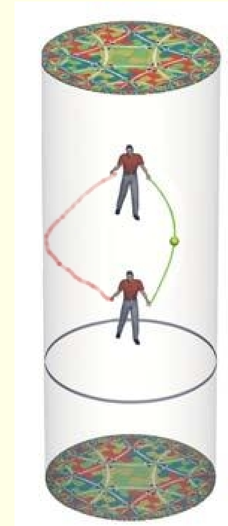

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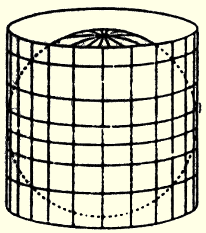

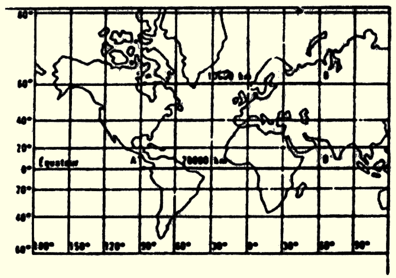

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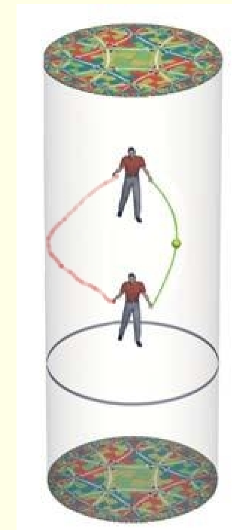
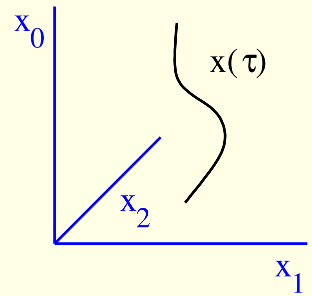


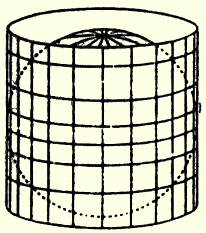


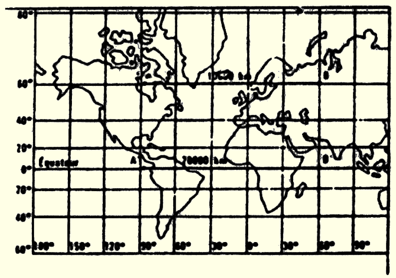
relativistic point particle: $ds^2 = -dx_0^2 + dx_1^2 + \ldots$

$S \propto$ worldline $\propto \int ds = \int \sqrt{\dot{x} \cdot \dot{x}} d\tau$

AdS: string theory on Anti de Sitter $\supset$ gravitation

positively curved space





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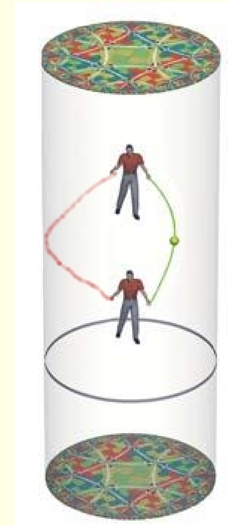

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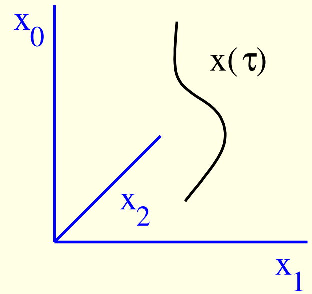
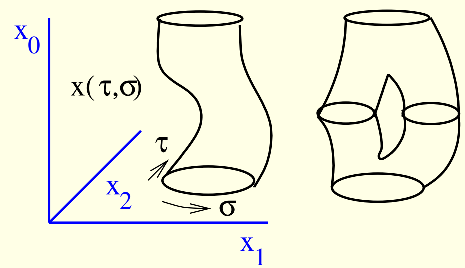


relativistic string: $ds^2 = -dx_0^2 + dx_1^2 + \dots$

$S \propto$ worldsheet $\propto \int dA = \int \sqrt{(\dot{x} \cdot x')^2 - \dot{x}^2 x'^2} d\tau d\sigma$



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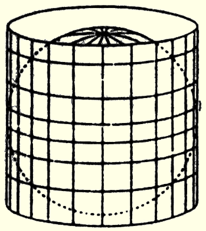

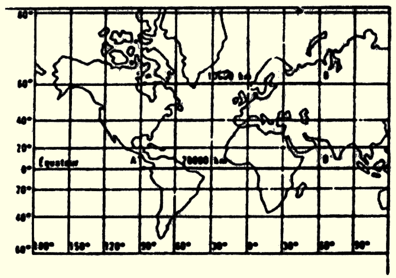

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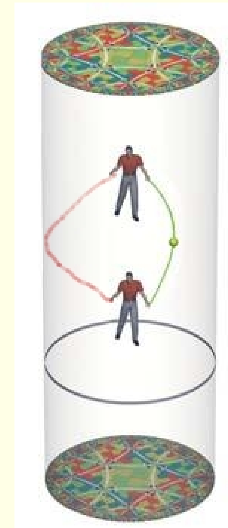

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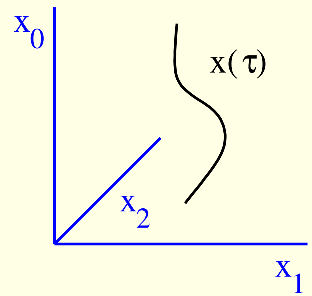

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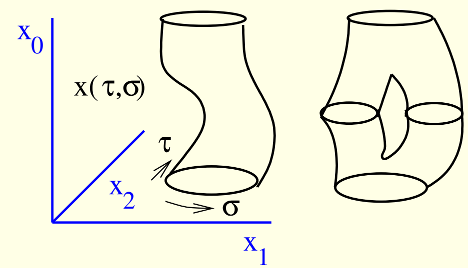
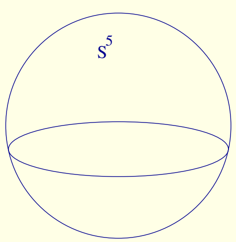
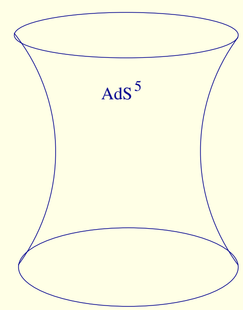




$S^5 : Y_0^2 + Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 + Y_5^2 = R^2$

$AdS_5 : -X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 - X_5^2 = -R^2$



$$S = \frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + fermionok$$

supercoset $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

AdS energies

AdS energies

Coset NLσ model: $h \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $\qquad J = h^{-1}dh = J_0 + J_1 + J_2 + J_3$

Z_4 graded structure: [Metsaev, Tseytlin 03]: $\mathcal{L} = \frac{g}{2}(\mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3))$

AdS energies

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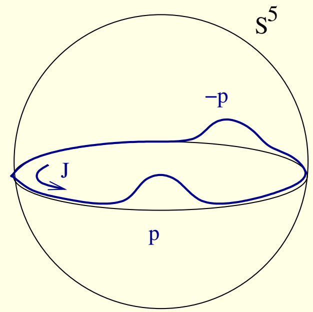
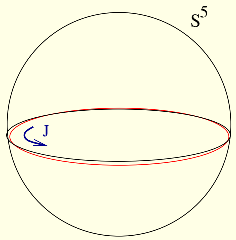
string configuration



BPS string configuration



$E_{BPS}(g) = J$

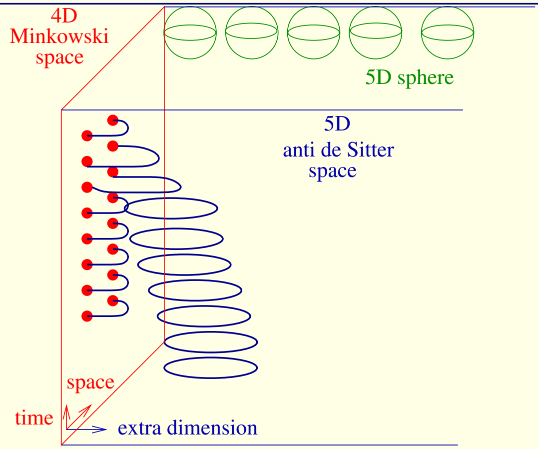
moving bumps [Hofman,Maldacena '07]

string action=saddle point+loop corr.

$E(\lambda) = E_0 + \frac{E_1}{g} + \frac{E_2}{g^2} + \dots$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \dots$$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

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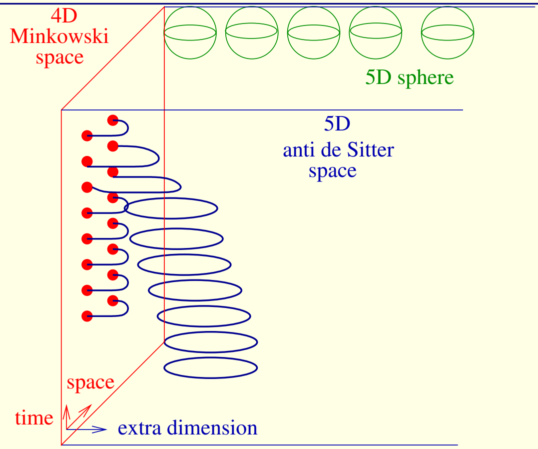

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$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i \overline{\Psi} \not{D} \Psi + V \right]$

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$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$

strong$\leftrightarrow$weak

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

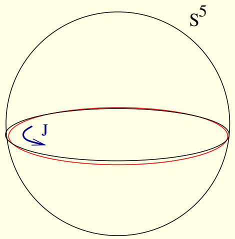
$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$

Anomalous dim $\Delta(\lambda)$

$\Delta(\lambda) = \Delta(0) + \lambda \Delta_1 + \lambda^2 \Delta_2 + \ldots$

AdS/CFT correspondence: how to match, charges?

BPS string configuration



$$E_{BPS}(\lambda) = J$$

strong↔weak

supersymmetric BPS operators

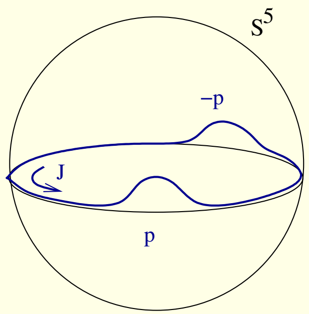
$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$$

$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \mathrm{Tr}(Z^J)$$

$$\Delta_{BPS} = J$$

string configuration



moving bumps (sine-Gordon) [Hofman .. '07]

string action=saddle point+loop corr.

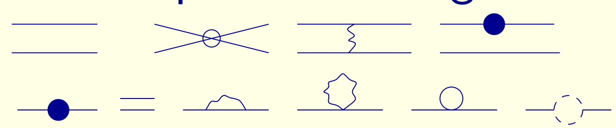
$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$$

$\equiv$

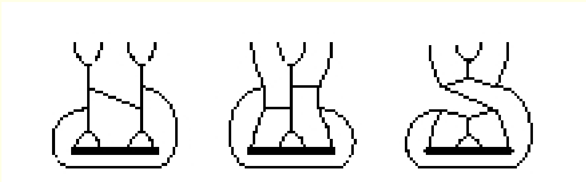
nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \mathrm{Tr}(ZXZX + \ldots)$$

operator mixing



$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \ldots + \lambda^4\Delta_4$$



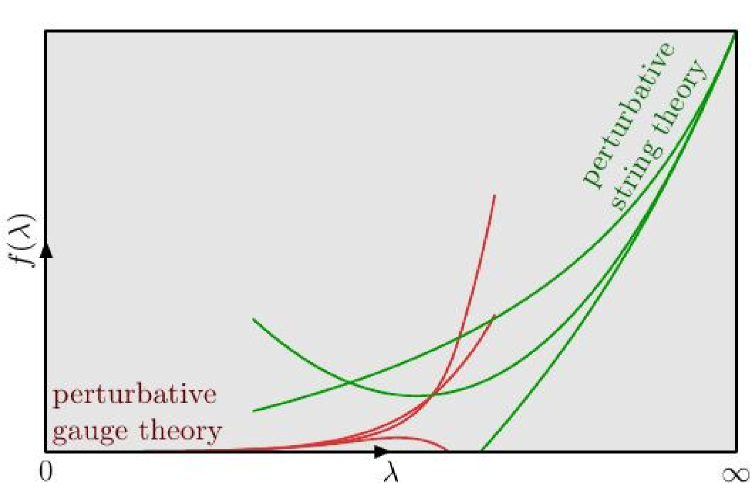
[Fiamberti ..'08]

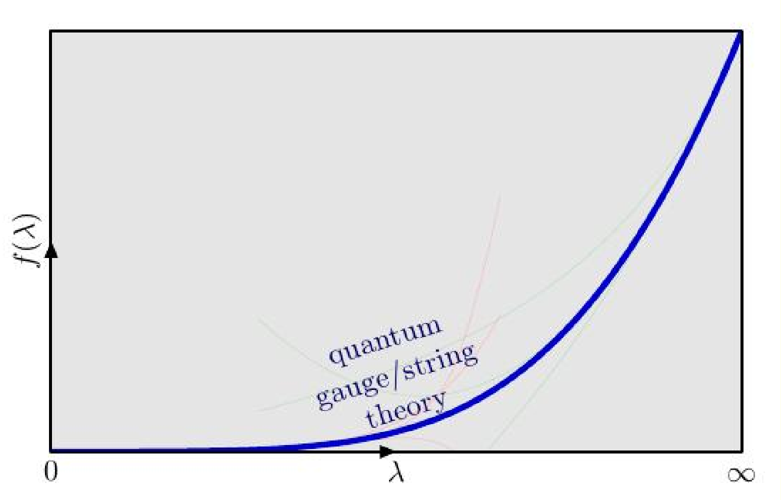
$$\Delta_4 = -2496 + 576\zeta_3 - 1440\zeta_5$$

AdS/CFT spectral problem

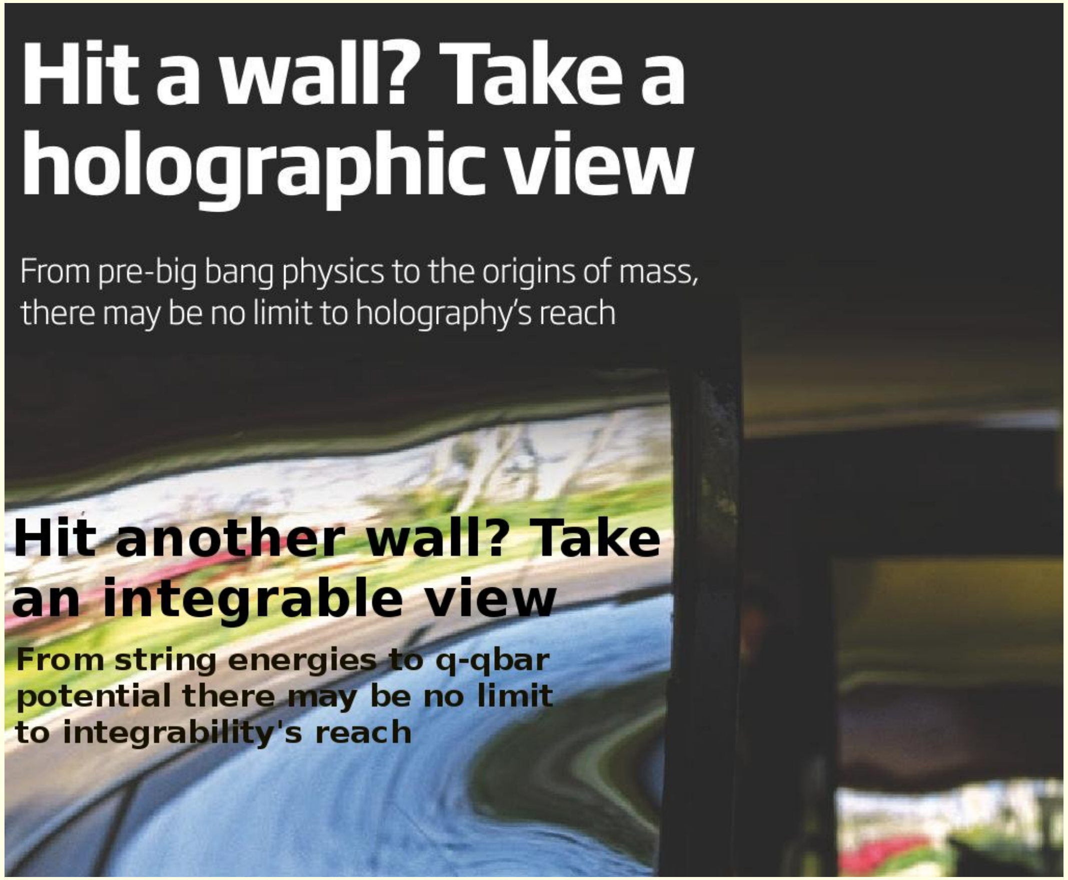
AdS/CFT spectral problem

Konishi dimension: $\mathrm{Tr}(ZXZX - ZZXX)$





AdS/CFT spectral problem



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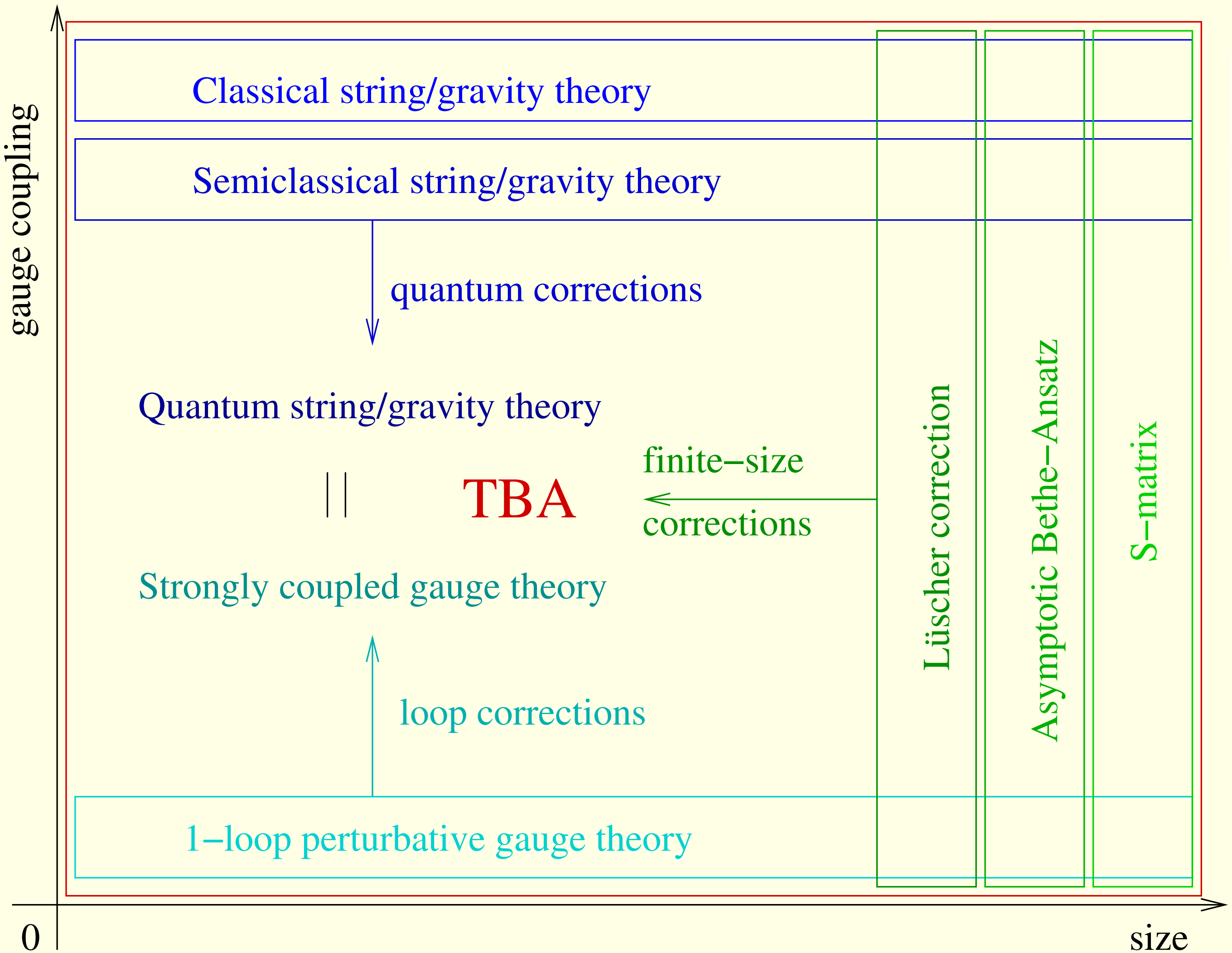
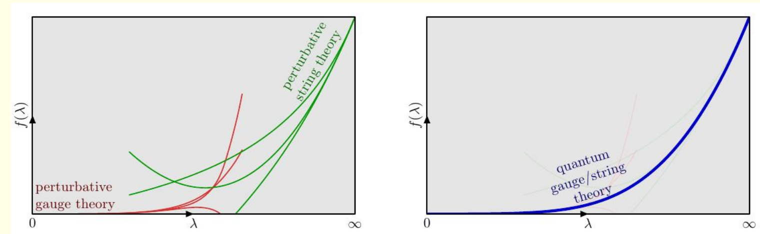




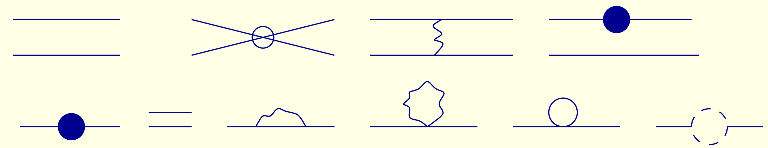
CFT: Integrablity

Perturbative correlator: $\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \langle \mathcal{O}_1(x)\mathcal{O}_2(0) e^{-i(\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi])}\rangle_0$

Conformal (scale invariant) field theory: $= \frac{\delta_{ij}}{|x|^{2\Delta(\lambda)}} = \frac{1}{|x|^{2\Delta(0)}}\left[1+\lambda\Delta_1 \log\frac{1}{|x|^2}+\ldots\right]$

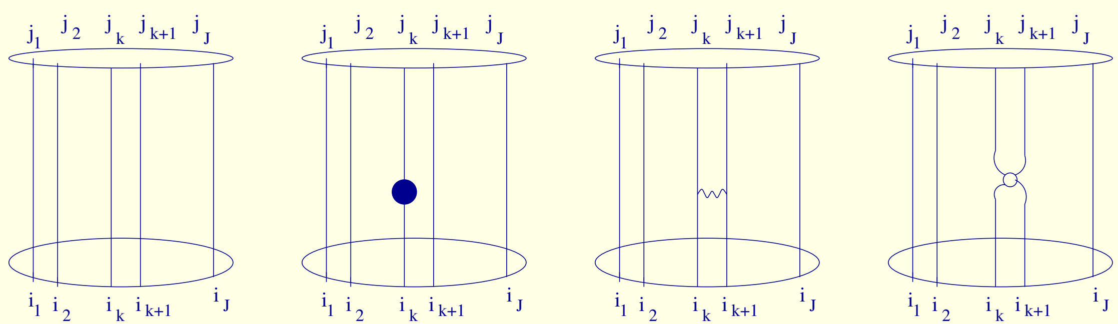
Scalar sector: $Z_1 = \Phi_1 + i\Phi_2$, $Z_2 = \Phi_3 + i\Phi_4$ SUSY st: $\mathcal{O} = \mathrm{Tr}\left[Z_i^J\right] \rightarrow \Delta_{\mathcal{O}}(\lambda) = J$

Operator mixing:
$\mathcal{O}_1 = \mathrm{Tr}\,[Z_1 Z_1 Z_2 Z_2] \leftrightarrow |\uparrow\uparrow\downarrow\downarrow\rangle$
$\mathcal{O}_2 = \mathrm{Tr}\,[Z_1 Z_2 Z_1 Z_2] \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle$



diagonalize the 1-loop mixing matrix: $\mathcal{O}_\pm = \mathcal{O}_1 \pm \mathcal{O}_2 \rightarrow$
$$\Delta_{\mathcal{O}_+}(\lambda) = 4$$
$$\Delta_{\mathcal{O}_-}(\lambda) = 4 + 6\frac{\lambda}{4\pi^2}$$

generic state at size J: $\mathcal{O}_{i_1\ldots i_J} = \mathrm{Tr}\left[Z_{i_1}\ldots Z_{i_J}\right] \leftrightarrow |i_1\ldots i_J\rangle$



$$\Delta = J\,\mathbb{I} + \frac{\lambda}{8\pi^2}\sum_{k=1}^{J}(\mathbb{I}-\mathbb{P}_{k,k+1})$$
Heisenberg spin chain

CFT: Integrablity + Bethe Ansatz

Mixing matrix on the subspace Tr $\left[Z_{i_1} \ldots Z_{i_J}\right]$ of dim 2^J: Minahan-Zarembo 2002

$$\boxed{\Delta = H_0 + \lambda H_1 + \lambda^2 H_2 + \ldots = J\,\mathbb{I} + \frac{\lambda}{8\pi^2} H_{XXX} + \lambda^2 H_2 + \ldots}$$

H_2: next-to-nearest neighbour integrable! $\rightarrow$ use Bethe ansatz: Feynman!

1. choose a groundstate: $Z = Z_1 \rightarrow \text{Tr}\left[Z^J\right] = \text{Tr}\left[ZZZZZ \ldots ZZZZ\right] \leftrightarrow |\uparrow \ldots \uparrow\rangle$
2. excitations $Z...ZXZ...X$ with SUSY multiplet $X = Z_2, Z_3, \Psi^{\alpha}_{a}, \Psi^{\dot\alpha}_{a}, D_\mu$
3. plane wave: $\sum_n e^{ipn} \text{Tr}(\overbrace{Z...Z}^{n} XZ \ldots ZZ)$
4. scattering states: $\sum_{n_1 n_2 a_1 a_2} e^{ip_1 n_1 + ip_2 n_2} \text{Tr}(\overbrace{\underbrace{Z...Z}_{n_1} X_{a_1} Z...Z}^{n_2} X_{a_2} Z \ldots Z) + S(12)^{b_1 b_2}_{a_1 a_2} \sum$

symmetry completely fixes the S-matrix for any λ (satisfies unitarity, crossing, Yang-Baxter)

Bethe ansatz follows from S-matrix: Shastry's Hubbard S-matrix

AdS: integrability

AdS: integrability

Coset NLσ model: $h \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $\qquad J = h^{-1}dh = J_0 + J_1 + J_2 + J_3$

Z_4 graded structure: [Metsaev, Tseytlin 03]: $\mathcal{L} = \frac{g}{2}(\mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3))$

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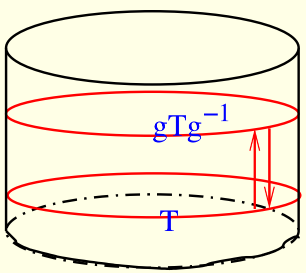
Z_4 graded structure: [Metsaev, Tseytlin 03]: $\mathcal{L} = \frac{g}{2}(\mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3))$

Integrability from flat connection: $dA - A \wedge A = 0$

$$A(\mu) = J_0 + \mu^{-1}J_1 + (\mu^2 + \mu^{-2})J_2/2 + (\mu^2 - \mu^{-2}) * J_2/2 + \mu J_3$$

Conserved charges from the trace of the monodromy matrix

$$T(\mu) = \mathrm{STr}(\mathcal{P}\exp \oint A(\mu)_\rho dx^\rho) = \sum \mu^n Q_n$$



AdS/CFT spectral problem: E(g,J) $g = \frac{\sqrt{\lambda}}{4\pi}$

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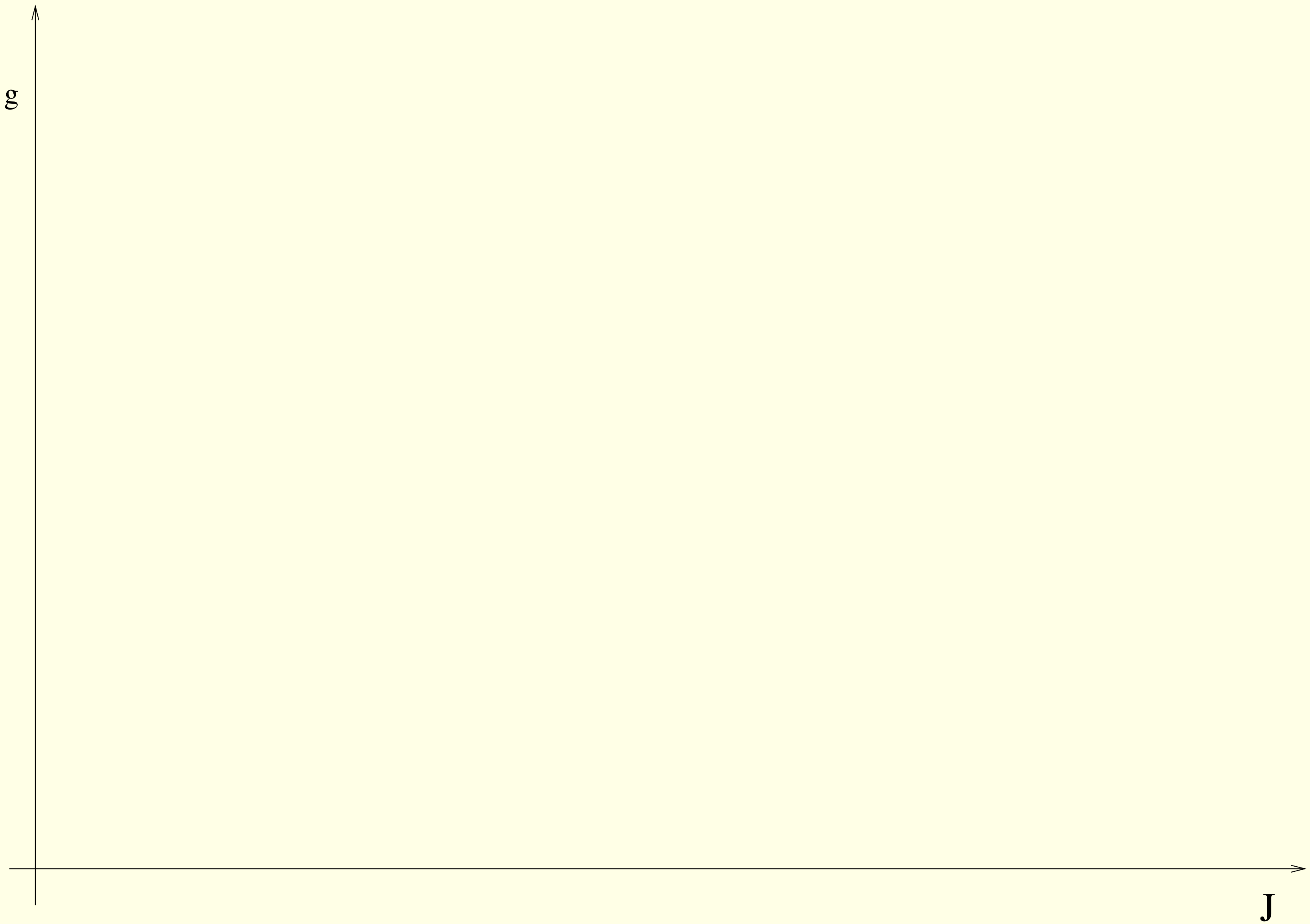

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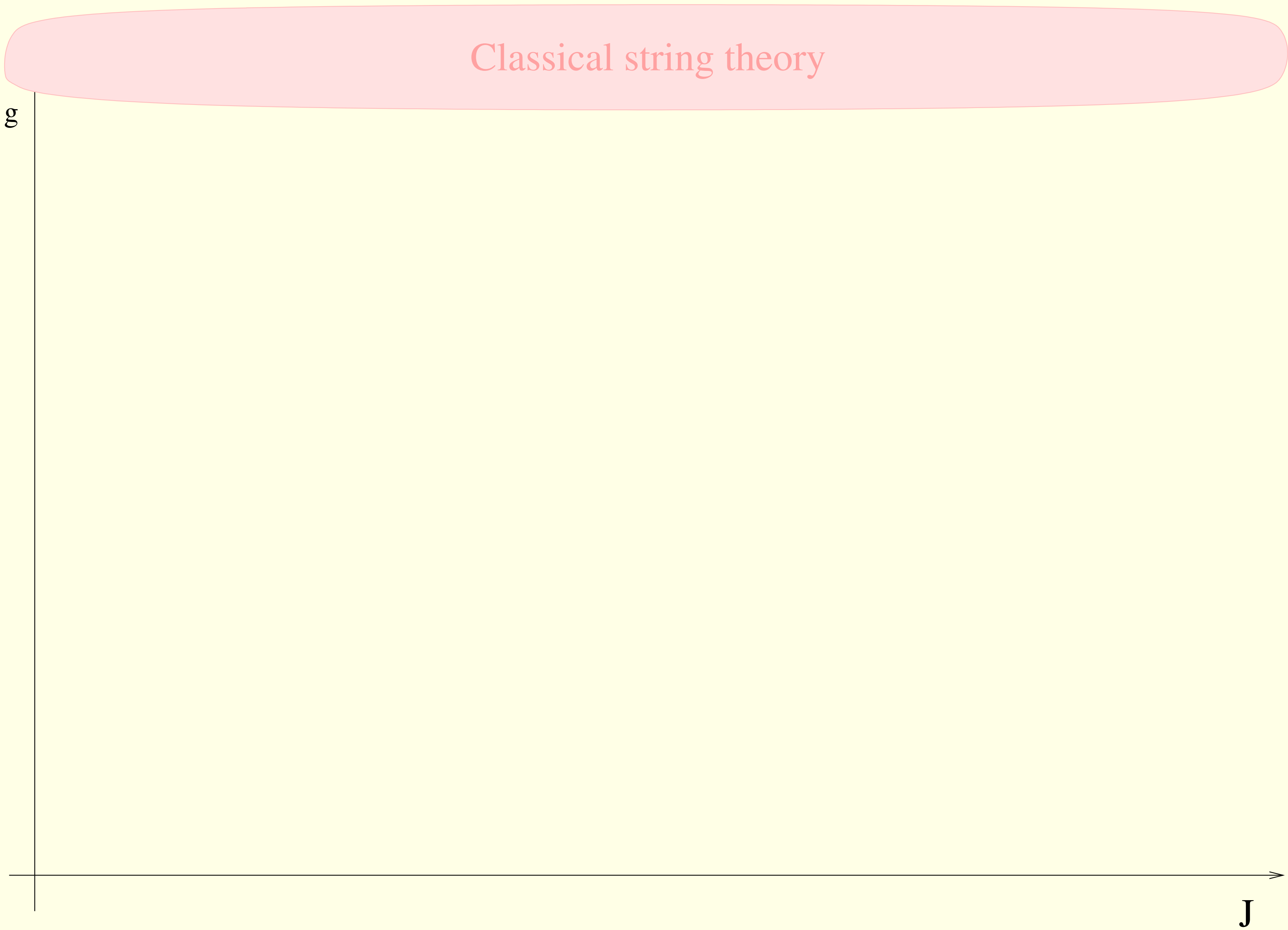

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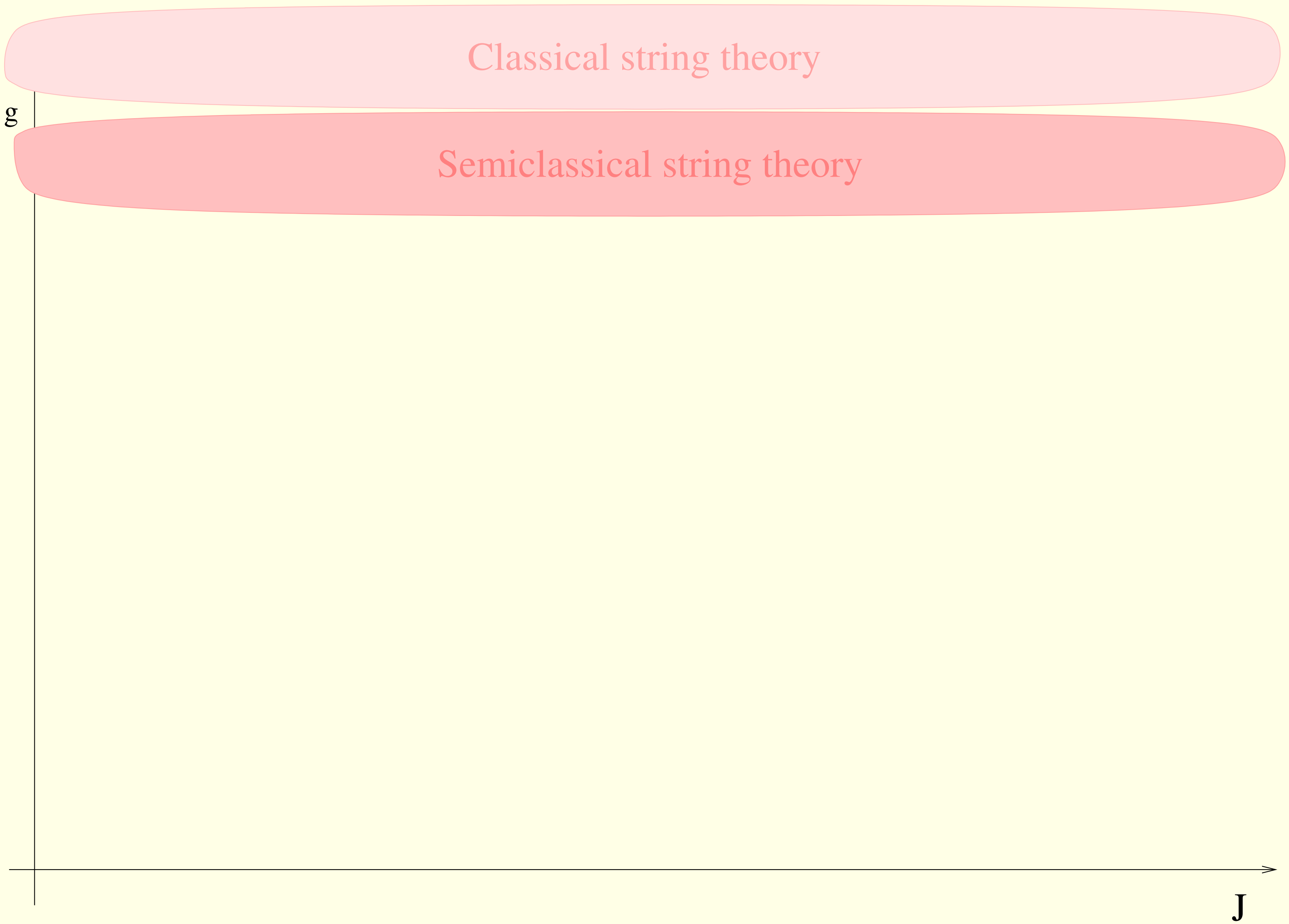

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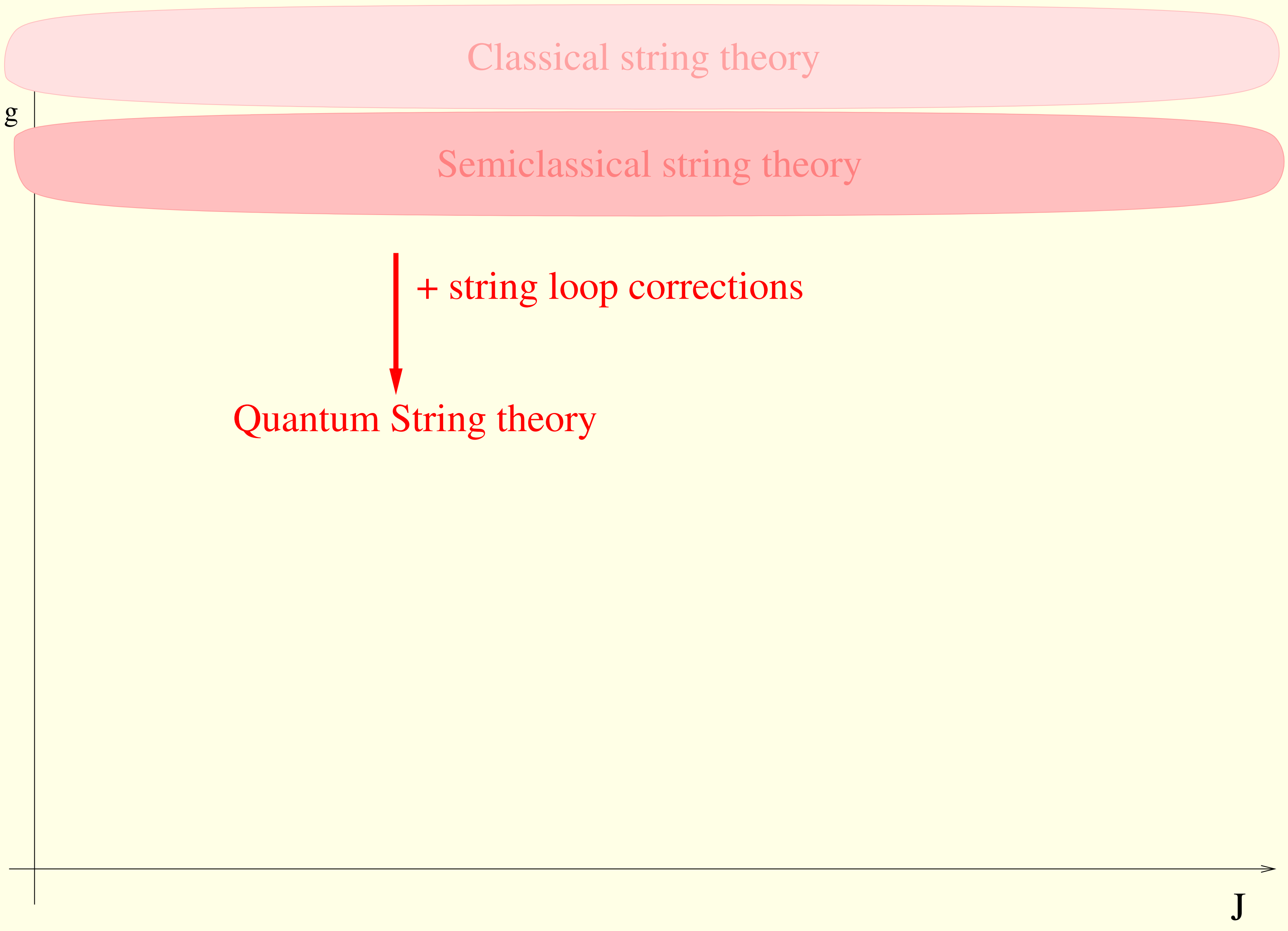

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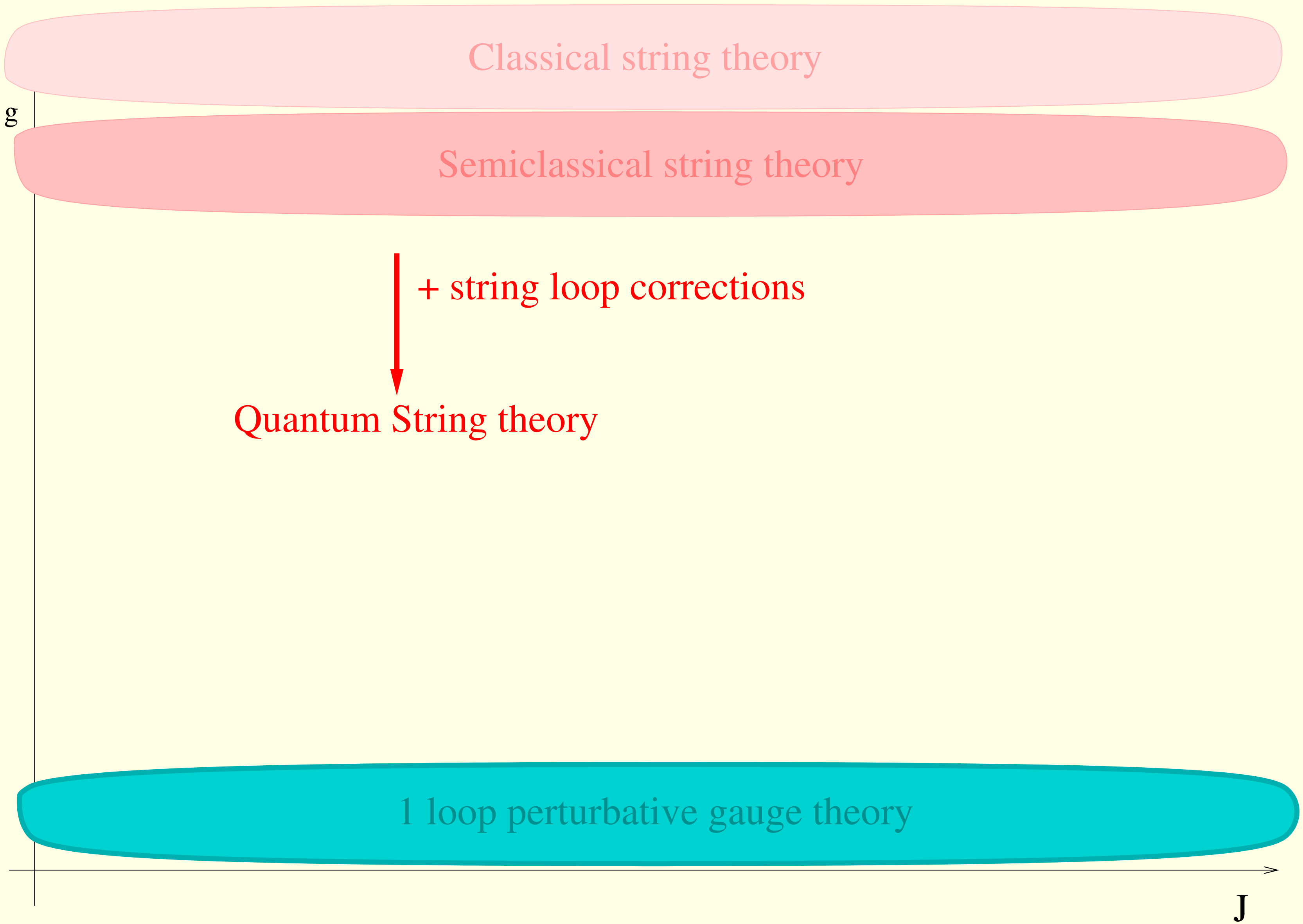

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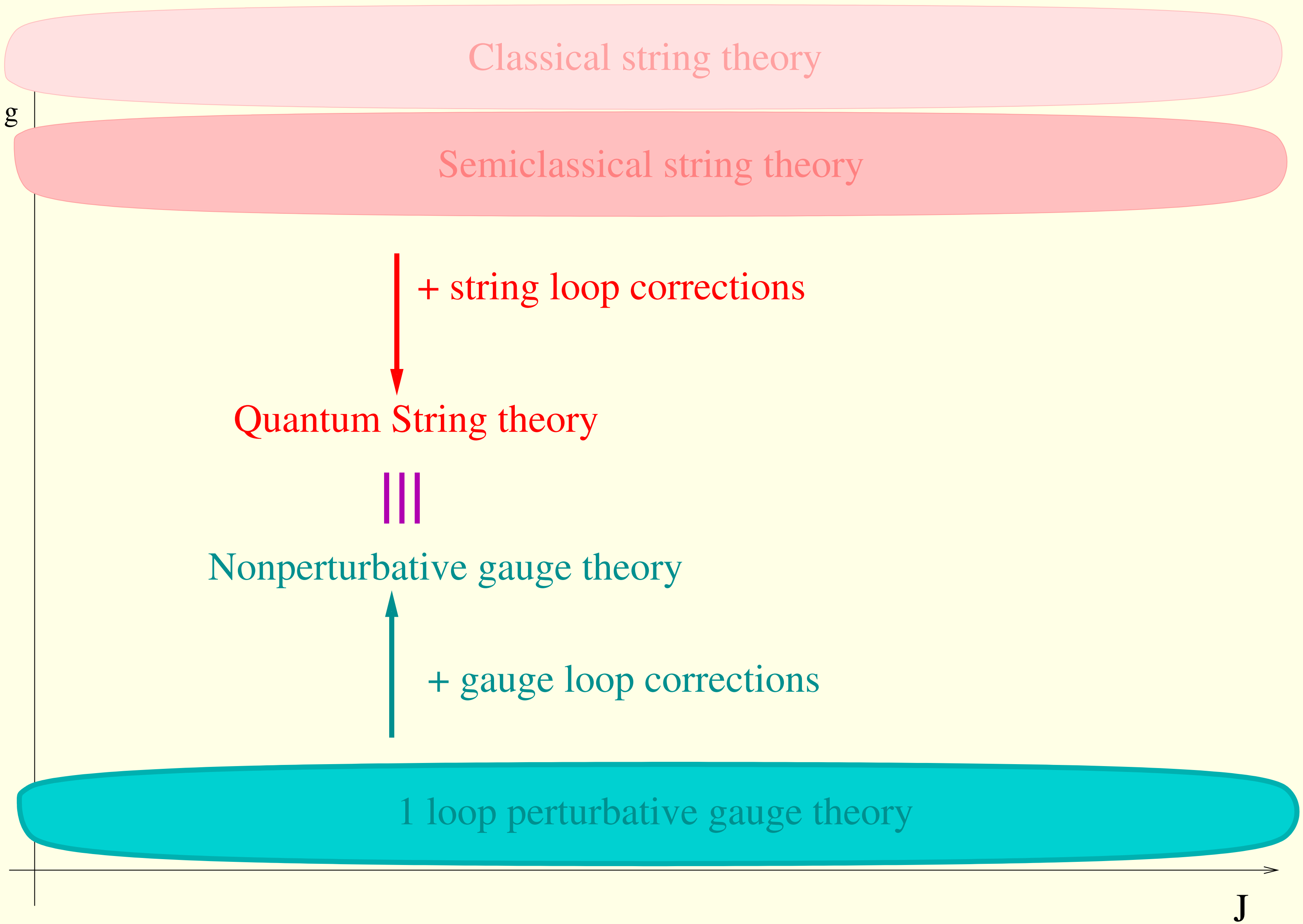

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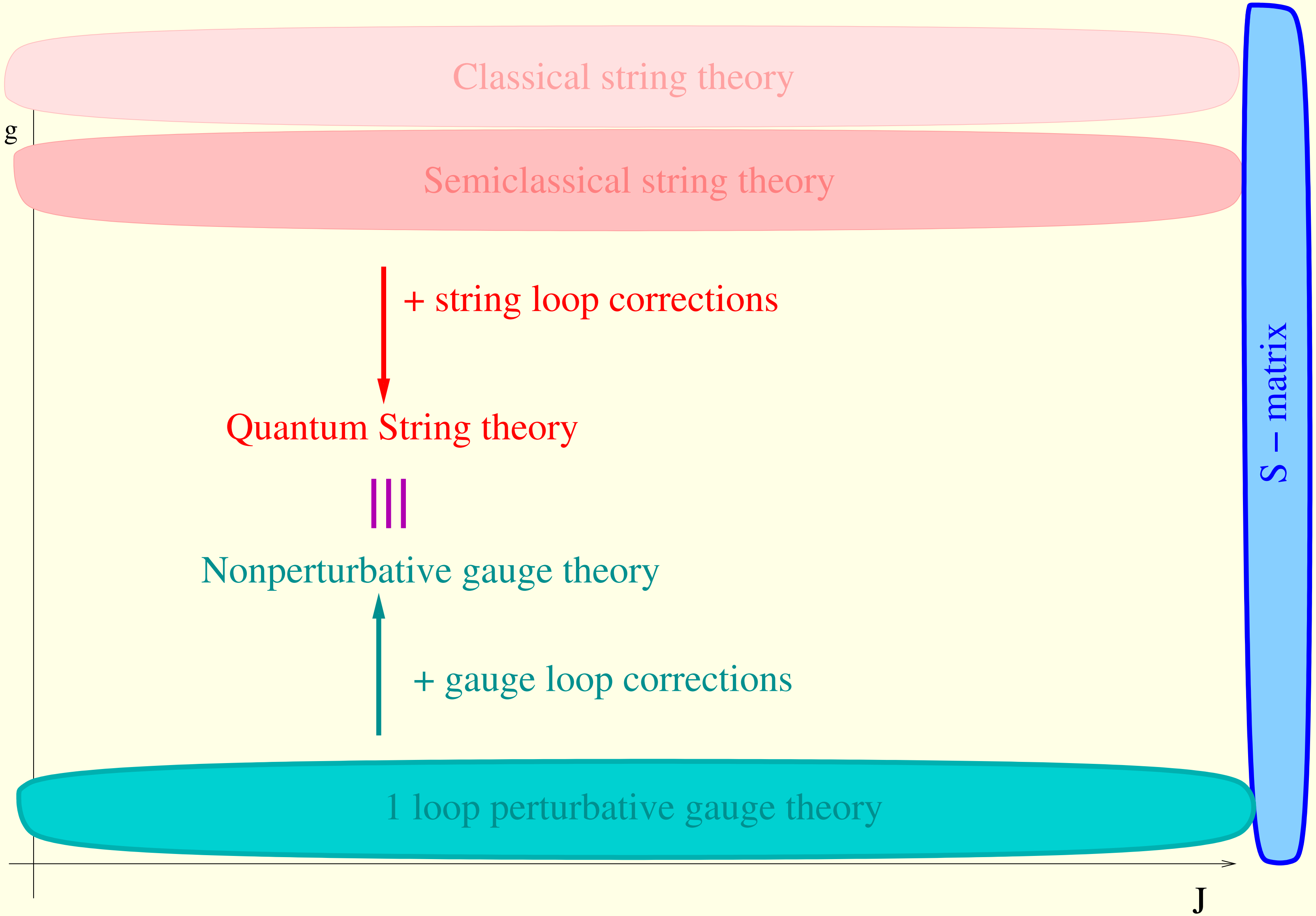

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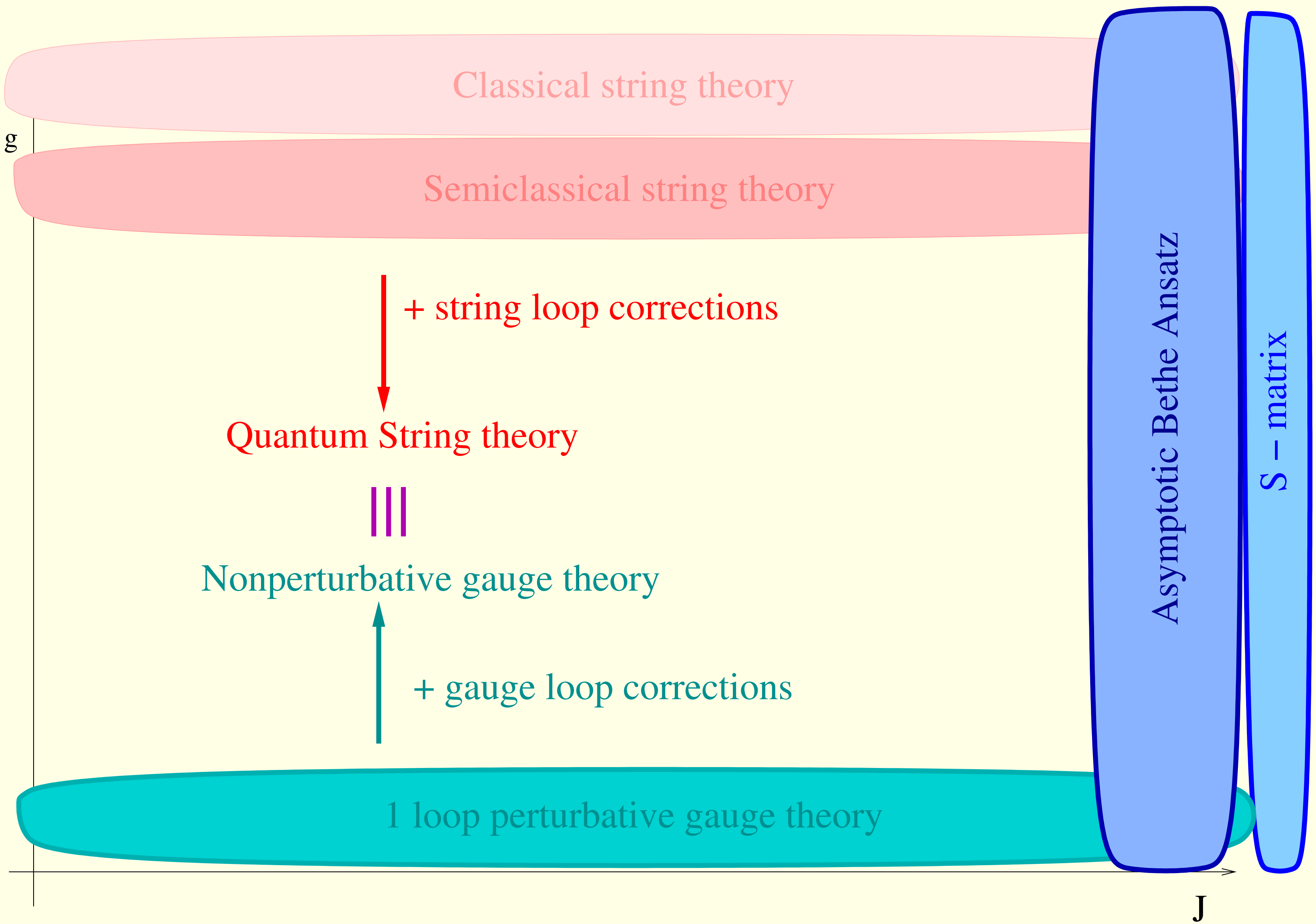

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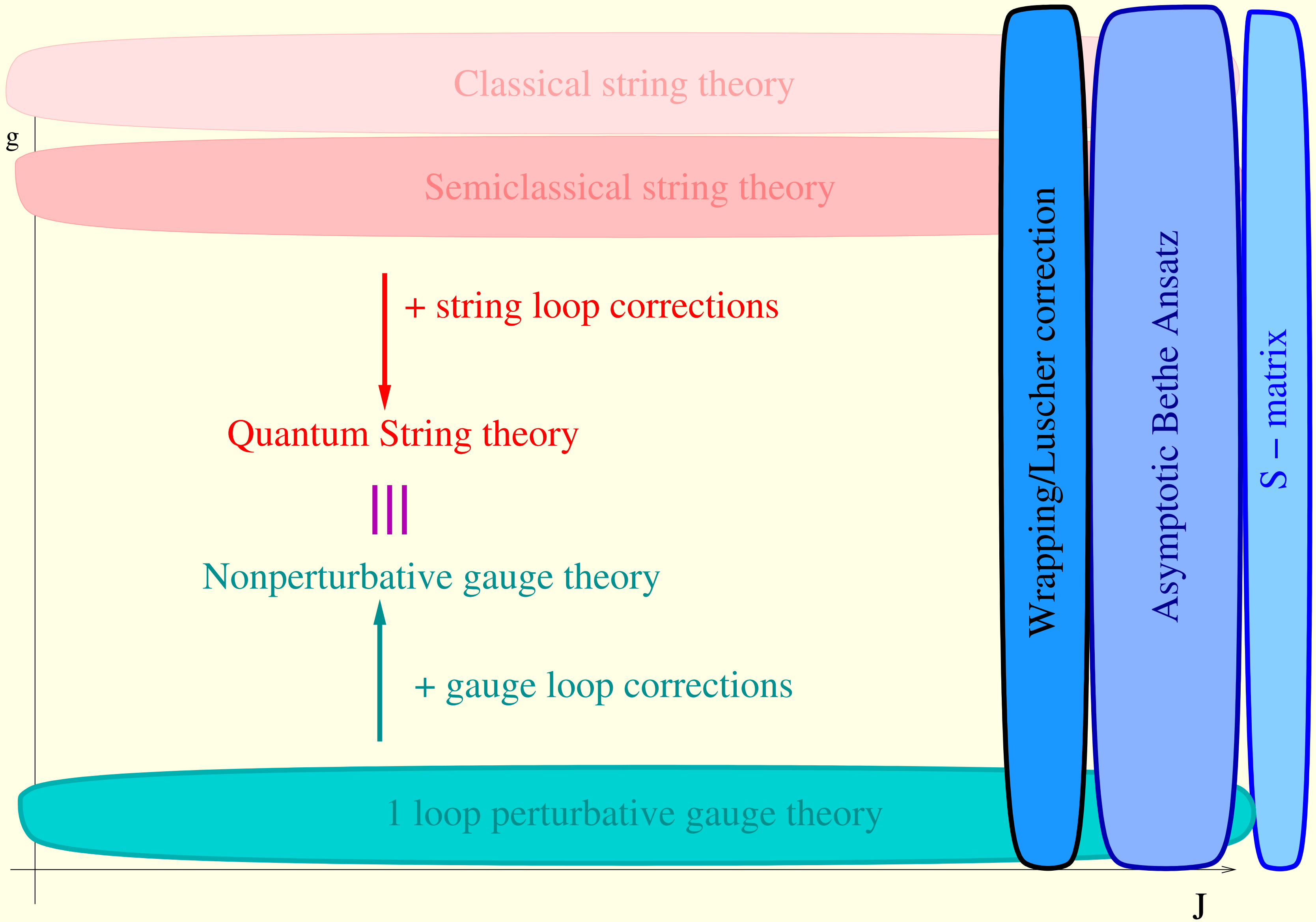
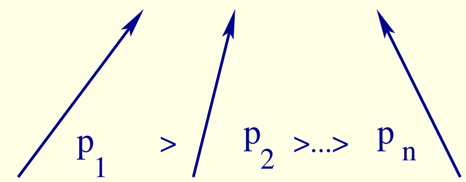


Need finite J (volume) solution of the spectral problem

S-matrix bootstrap program

Asymptotic states $|p_1, p_2, \dots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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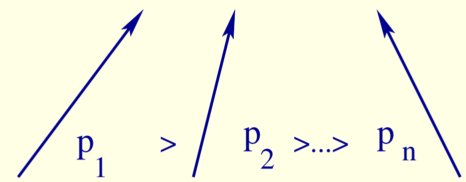

Lorentz: $P = \sum_i p_i \;\; E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$

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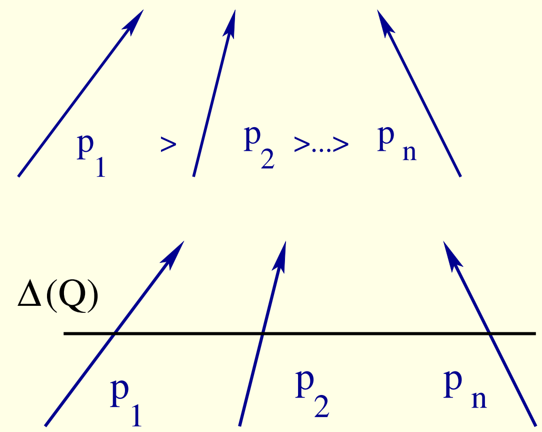

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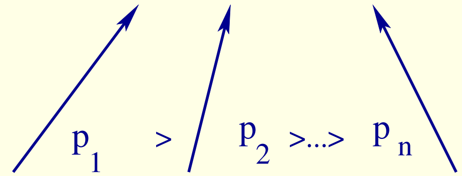

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Scattering matrix S: $|out\rangle \to |in\rangle$
commutes with symmetry $[S, \Delta(Q)] = 0$

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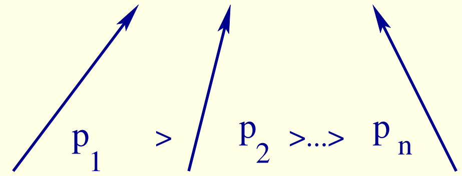

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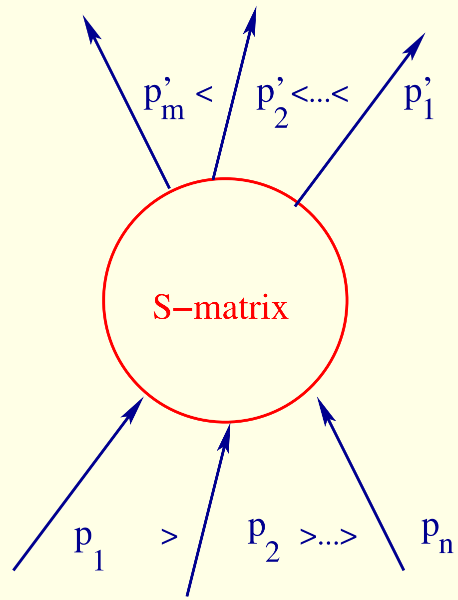

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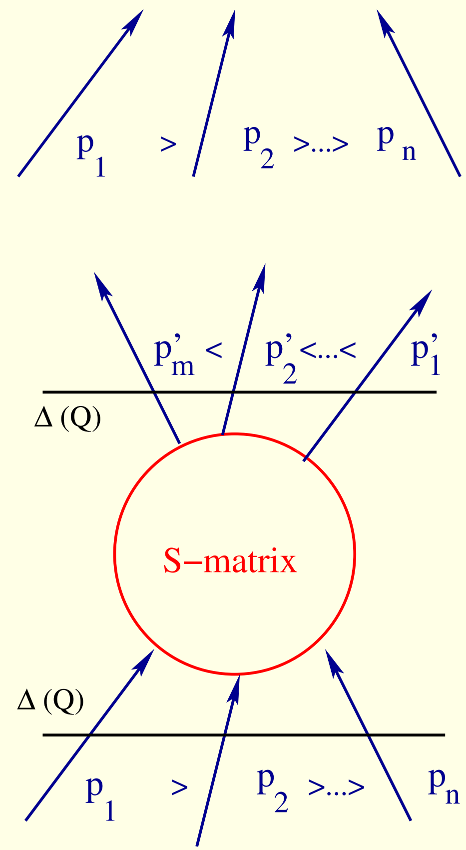

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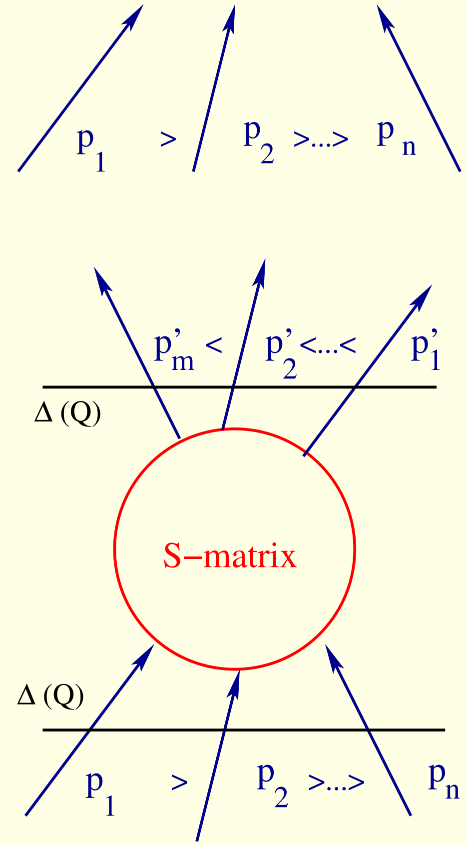

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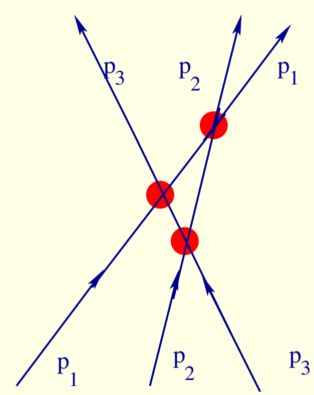
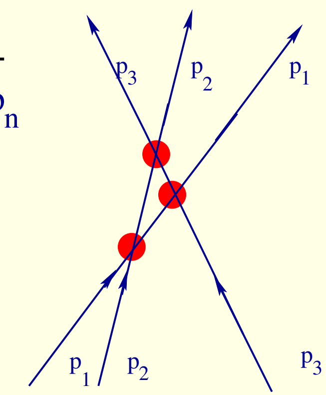
Higher spin concerved charge
factorization + Yang-Baxter equation
$S_{123} = S_{23}S_{13}S_{12} = S_{12}S_{13}S_{23}$





S-matrix = scalar . Matrix

S-matrix bootstrap program

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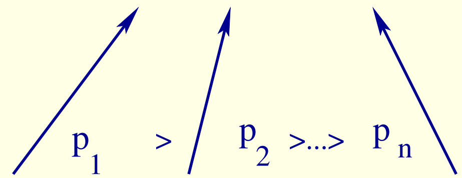

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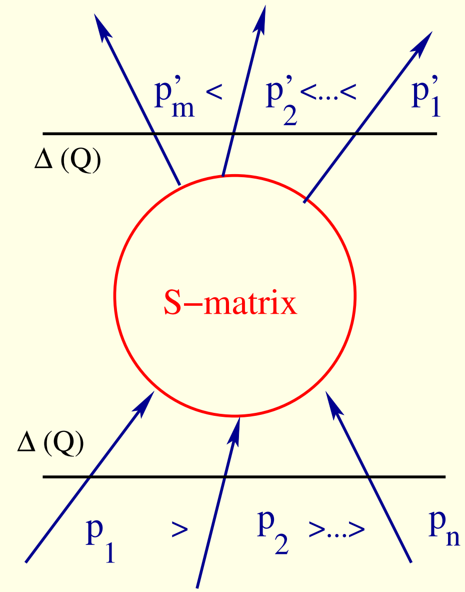

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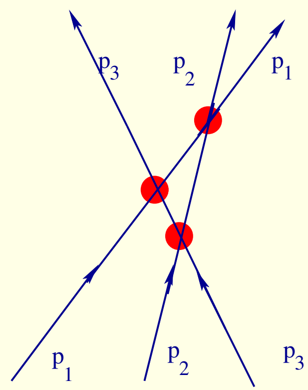

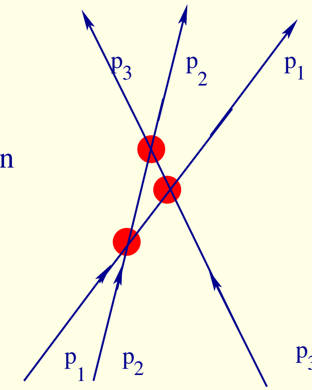

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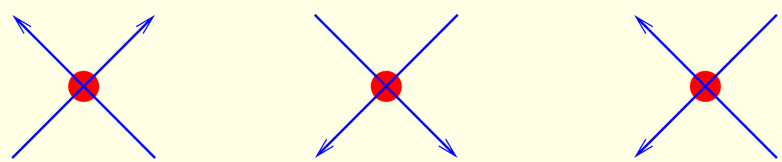
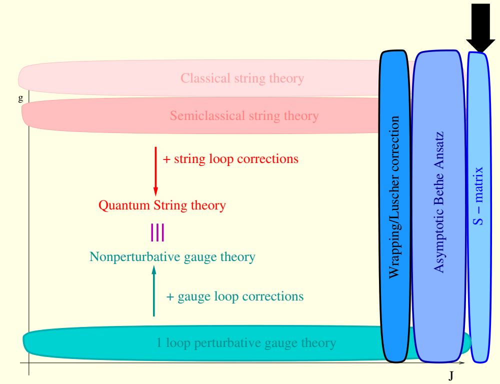
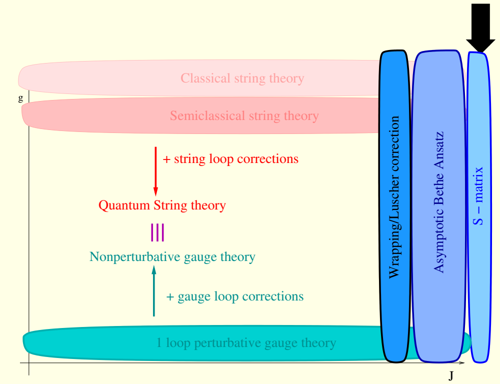


Unitarity $S_{12}S_{21} = Id$
Crossing symmetry $S_{12} = S_{2\bar{1}}$
Maximal analyticity: all poles have physical origin $\to$ boundstates, anomalous thresholds



S-matrix bootstrap program: AdS

Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar} \; . \; \text{Matrix}}$



S-matrix bootstrap program: AdS

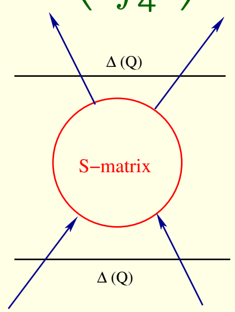
Nondiagonal scattering: S-matrix = scalar . Matrix

Matrix: [Beisert]

global symmetry $PSU(2|2)^2$

$$Q = 1 \text{ reps} \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_{\dot{1}} \\ b_{\dot{2}} \\ f_{\dot{3}} \\ f_{\dot{4}} \end{pmatrix}$$

$$[S, \Delta(Q)] = 0$$



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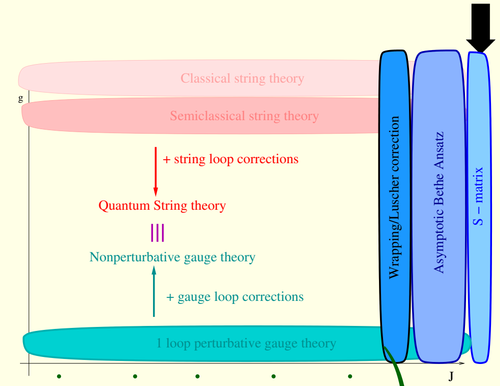

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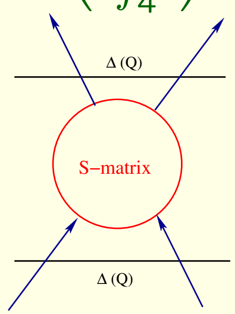
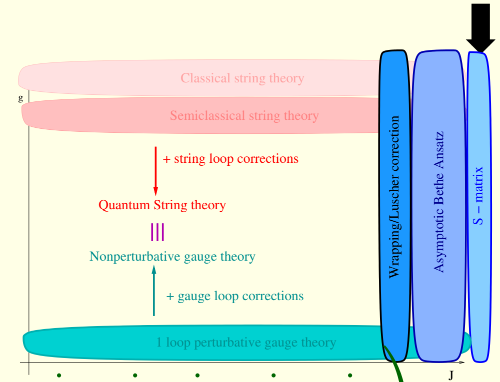


$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot \\
\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
\end{pmatrix}$$

S-matrix bootstrap program: AdS



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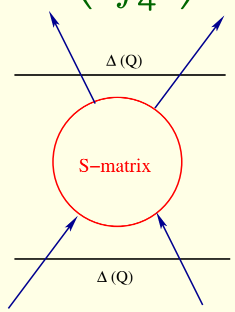
Matrix: [Beisert]

global symmetry $PSU(2|2)^2$

$Q=1$ reps $\begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_{\dot 1} \\ b_{\dot 2} \\ f_{\dot 3} \\ f_{\dot 4} \end{pmatrix}$

$[S,\Delta(Q)]=0$



$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot \\
\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
\end{pmatrix}$$

Unitarity

$\mathcal{S}(z_1,z_2)\mathcal{S}(z_2,z_1)=1$

Crossing symmetry [Janik] [Volin]

$\mathcal{S}(z_1,z_2)=\mathcal{S}^{c_1}(z_2,z_1+\omega_2)$

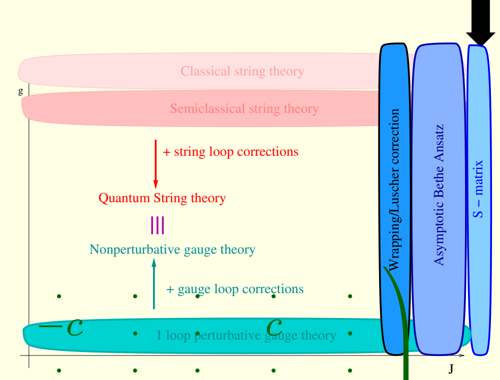
$$S^{11}_{11}=\frac{u_1-u_2-i}{u_1-u_2+i}e^{i2\theta(z_1,z_2)}$$

$$u=\tfrac{1}{2}\cot\tfrac{p}{2}E(p)$$

[Beisert,Eden,Staudacher]

$p=2\,\mathrm{am}(z)$

S-matrix bootstrap program: AdS



Nondiagonal scattering: **S-matrix = scalar . Matrix**

Matrix: [Beisert]

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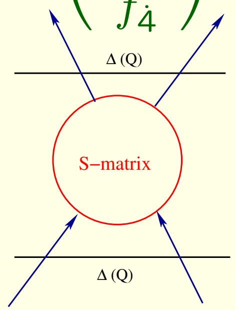

$$\begin{pmatrix}
1&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&A&\cdot&\cdot&-b&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&-c&\cdot&\cdot&c&\cdot\\
\cdot&\cdot&d&\cdot&\cdot&\cdot&\cdot&\cdot&e&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&-b&\cdot&d&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&e&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&A&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&c&\cdot&\cdot&-c&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&1&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&d&\cdot&e&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&d&\cdot&\cdot&\cdot&\cdot&\cdot&e&\cdot&\cdot\\
\cdot&\cdot&f&\cdot&\cdot&\cdot&\cdot&\cdot&g&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&f&\cdot&\cdot&g&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&a&\cdot&\cdot&\cdot&\cdot&\cdot\\
\cdot&-h&\cdot&\cdot&h&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&B&\cdot&\cdot&-i&\cdot\\
\cdot&\cdot&\cdot&f&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&g&\cdot&\cdot&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&f&\cdot&\cdot&\cdot&\cdot&\cdot&g&\cdot&\cdot\\
\cdot&h&\cdot&\cdot&-h&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&-i&\cdot&\cdot&B&\cdot\\
\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&\cdot&a
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[Beisert,Eden,Staudacher]

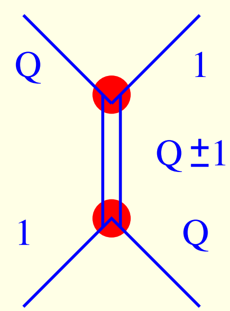
$$p=2\,\mathrm{am}(z)$$

Maximal analyticity:

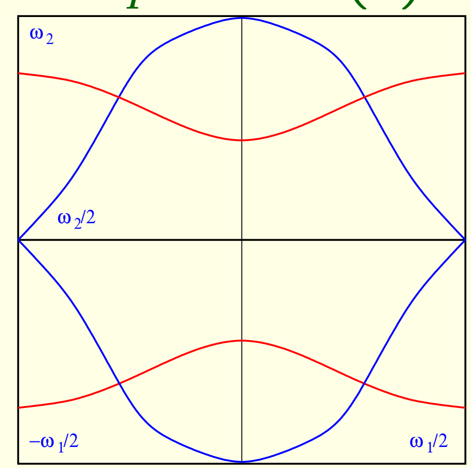
boundstates atyp symrep: $Q\in\mathbf{N}$

anomalous thresholds

[N.Dorey,Maldacena,Hofman,Okamura]

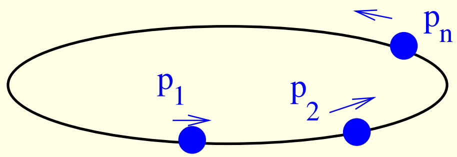


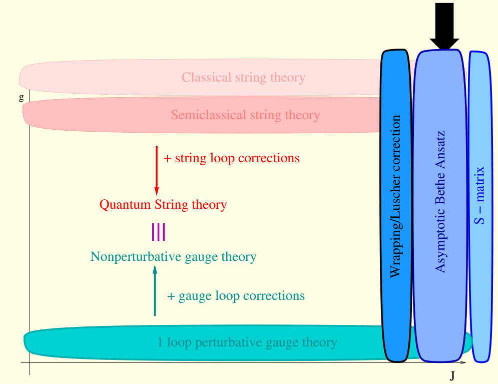
Physical domain



Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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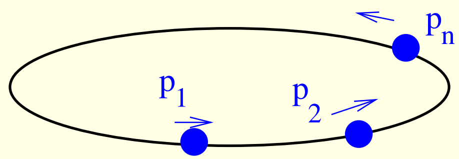

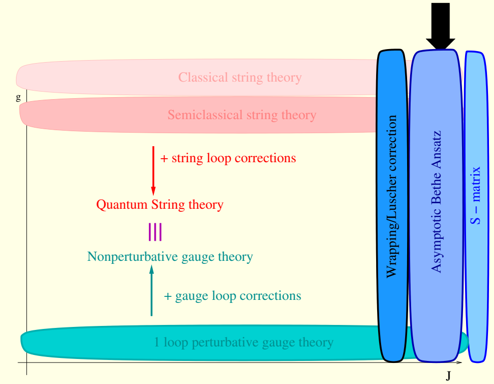

Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

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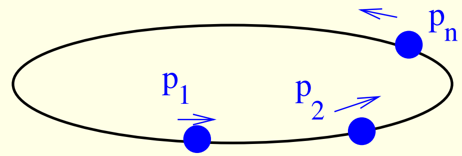

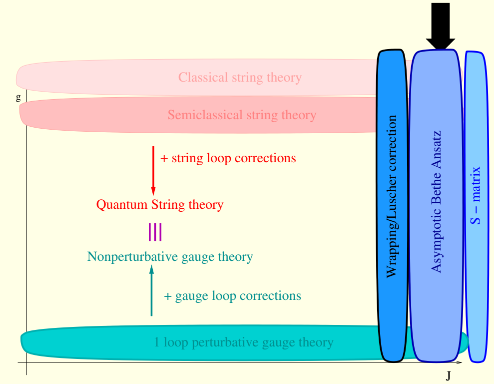

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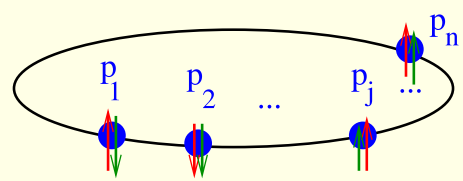
Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$



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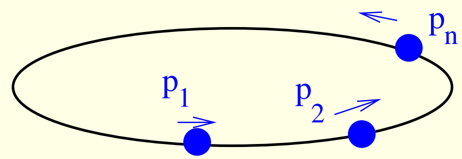

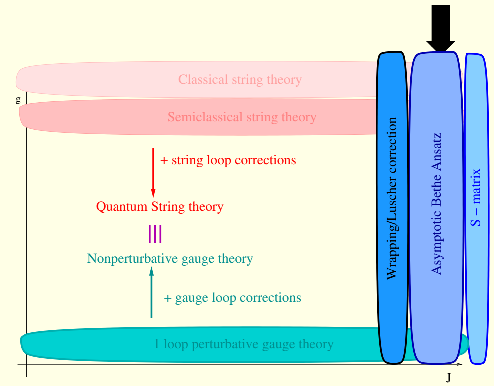

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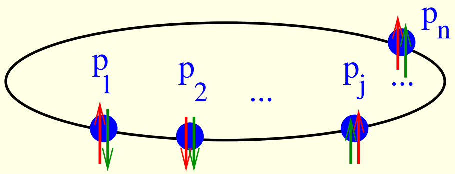
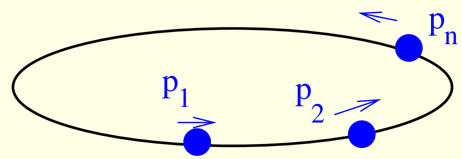
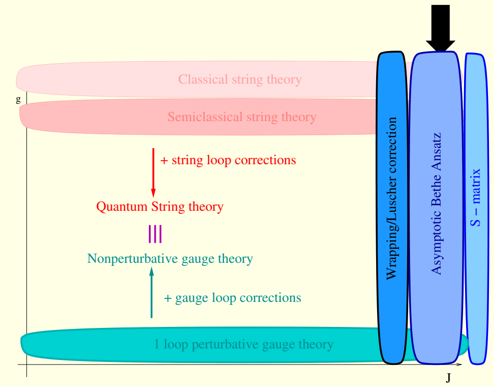


Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\frac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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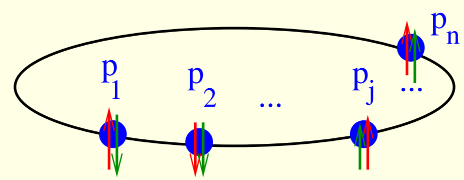

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$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$ [Beisert,Staudacher]

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}} \qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$

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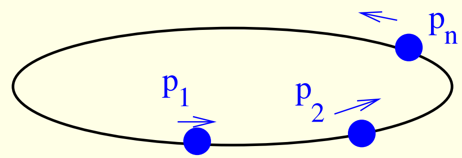

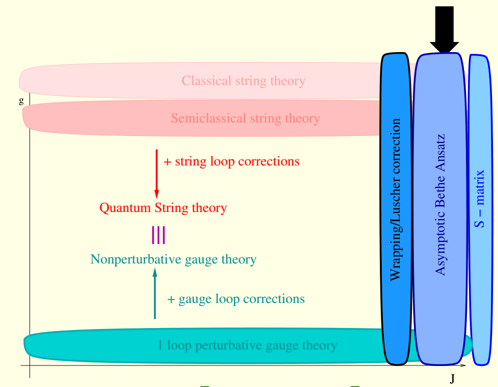

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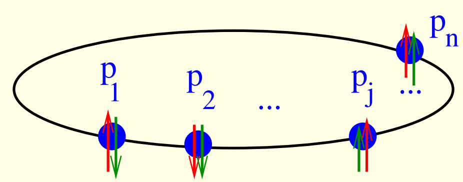

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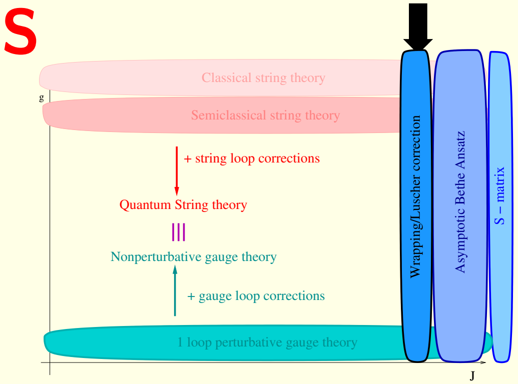
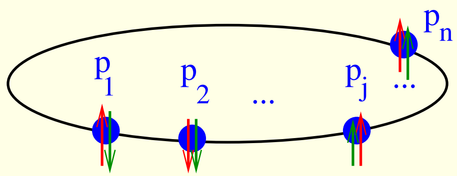
Bethe Ansatz: $\frac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1 \quad \frac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1 \quad \frac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$

Lüscher/wrapping correction in AdS



Finite volume spectrum



[Ambjorn,Janik,Kristjansen]

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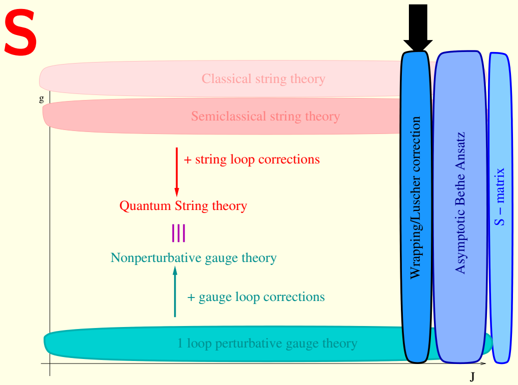

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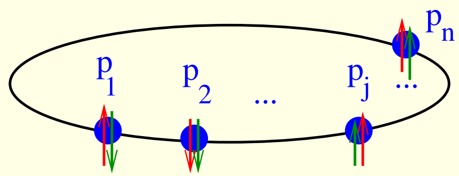
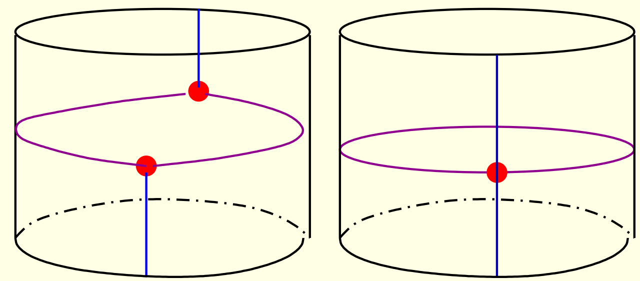


[Ambjorn,Janik,Kristjansen]

One particle correction: [Janik, Lukowski]

$$\Delta(L) = \left(1 - E^{'}(p)\tilde{E}^{'}(\tilde{p}_0)\right)\left(-i\mathrm{Res}_{\tilde{p}=\tilde{p}_0}\sum_b S\right)e^{-\tilde{E}(\tilde{p})L}$$

$$-\sum_b \int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}\left(1 - E^{'}(p)\tilde{E}^{'}(\tilde{p})\right)\left(S_{ba}^{ba}(\tilde{p},p) - 1\right)e^{-\tilde{E}(\tilde{p})L}$$



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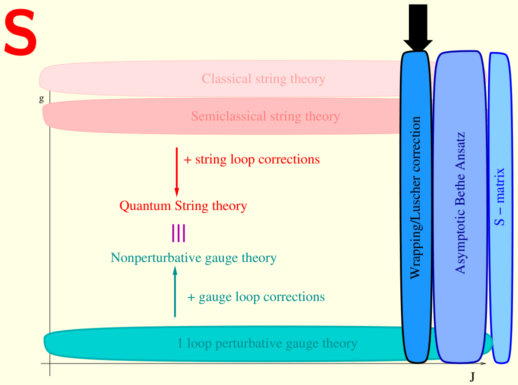
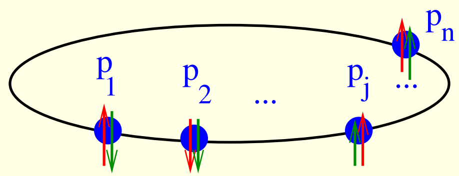


Finite volume spectrum

[Ambjorn,Janik,Kristjansen]



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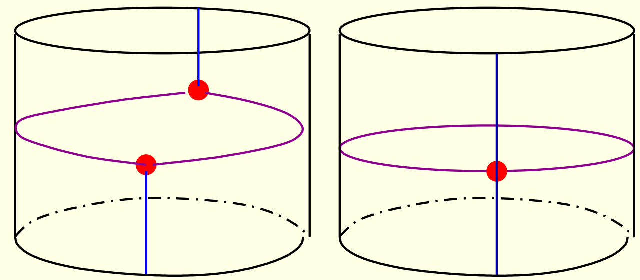
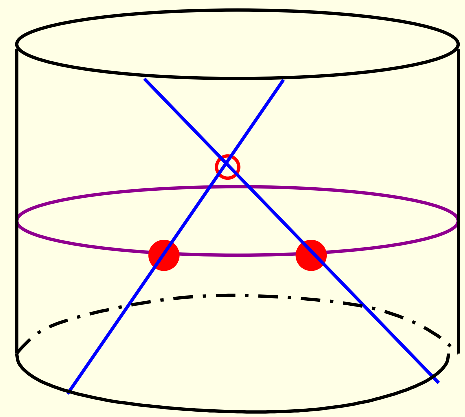


Two particle Lüscher correction (Konishi) [ZB,Janik]



BY: $j = 1,2 \quad \mathcal{S}(p_j,p_1)\mathcal{S}(p_j,p_2)\Psi = -e^{-ip_jL}\Psi$

$T(p,p_1,p_2)\Psi = t(p,p_1,p_2,\Psi)\Psi$

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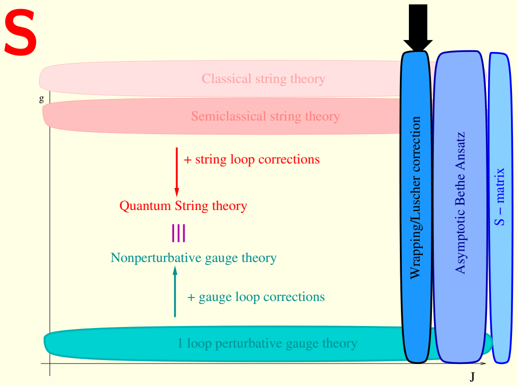
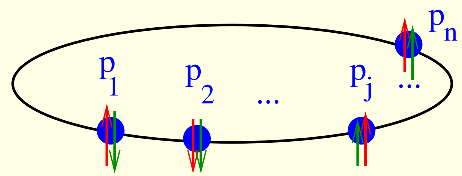


Finite volume spectrum

[Ambjorn,Janik,Kristjansen]



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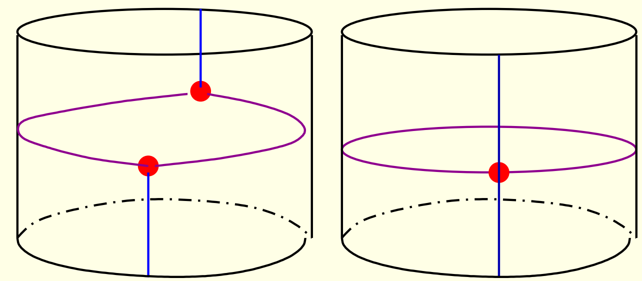

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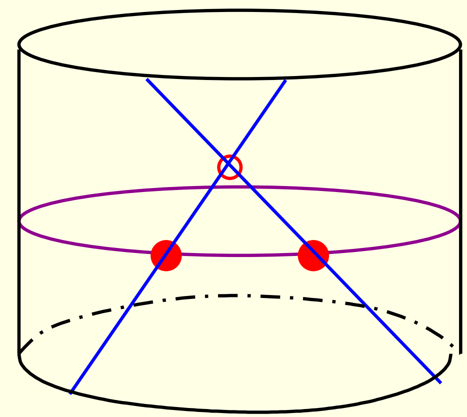

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$T(p,p_1,p_2)\Psi = t(p,p_1,p_2,\Psi)\Psi$

Modified momenta:

$$p_jL - i\log t(p_j,p_1,p_2,\Psi) = (2n+1)\pi + \Phi_j$$
$$\Phi_1 = -\int \frac{d\tilde{p}}{2\pi}\left(\frac{d}{d\tilde{p}}S(\tilde{p},p_1)\right)S(\tilde{p},p_2)e^{-L\tilde{E}(\tilde{p})}$$

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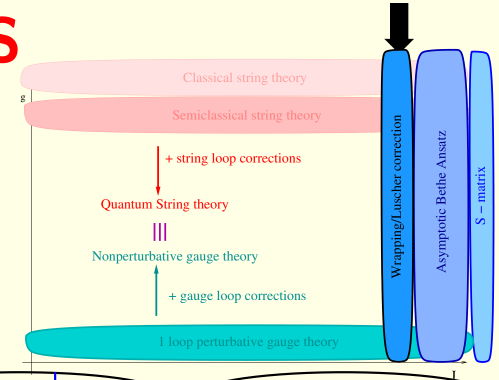
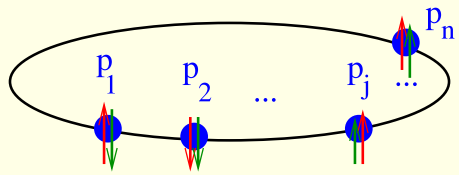


Finite volume spectrum

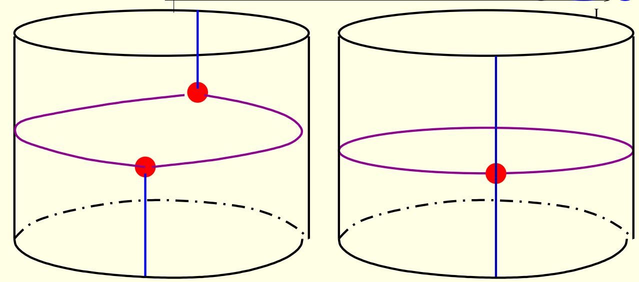
[Ambjorn,Janik,Kristjansen]



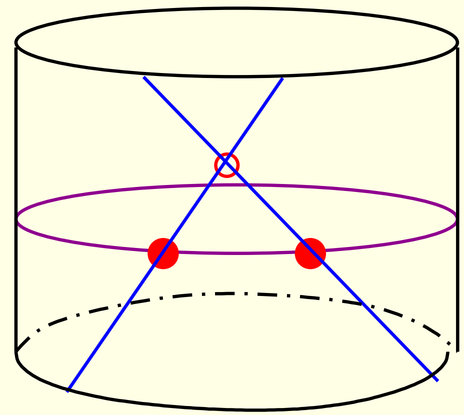
One particle correction: [Janik, Lukowski]

$$\Delta(L) = \Big(1 - E^{'}(p)\tilde{E}^{'}(\tilde{p}_0)\Big)\left(-i\mathrm{Res}_{\tilde{p}=\tilde{p}_0}\sum_b S\right)e^{-\tilde{E}(\tilde{p})L}$$

$$-\sum_b \int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}\Big(1 - E^{'}(p)\tilde{E}^{'}(\tilde{p})\Big)\left(S_{ba}^{ba}(\tilde{p},p) - 1\right)e^{-\tilde{E}(\tilde{p})L}$$



Two particle Lüscher correction (Konishi) [ZB,Janik]



BY: $j = 1,2 \quad \mathcal{S}(p_j,p_1)\mathcal{S}(p_j,p_2)\Psi = -e^{-ip_jL}\Psi$

$T(p,p_1,p_2)\Psi = t(p,p_1,p_2,\Psi)\Psi$

Modified momenta:

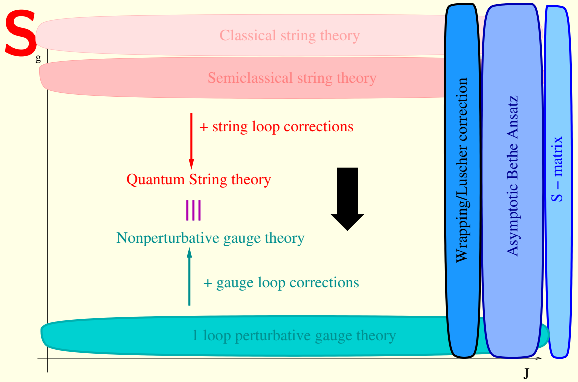
$$p_jL - i\log t(p_j,p_1,p_2,\Psi) = (2n+1)\pi + \Phi_j$$

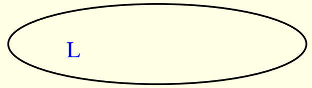
$$\Phi_1 = -\int \frac{d\tilde{p}}{2\pi}\left(\frac{d}{d\tilde{p}}S(\tilde{p},p_1)\right)S(\tilde{p},p_2)e^{-L\tilde{E}(\tilde{p})}$$

Modified energy:

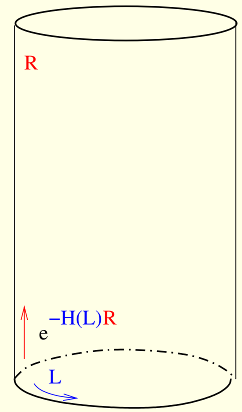
$$E(p_1,p_2) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi}t(q,p_1,p_2,\Psi)e^{-LE(q)}$$

Thermodynamic Bethe Ansatz: AdS





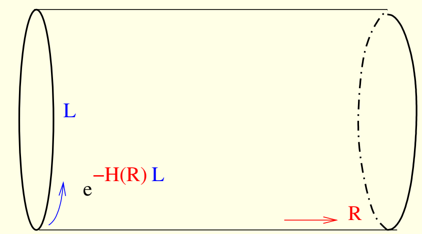
Ground-state energy exactly



[Bombardelli,Tateo,Fioravanti,Frolov,Arutyunov, Gromov,Kazakov,Viera,Kozak]

Euclidean $E^2 + (4g \sin \frac{p}{2})^2 = 1$ partition function:

$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1 + e^{-\Delta E\, R})$$



$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-\tilde{H}(R)L}) =_{R\to\infty} \sum_n e^{-\tilde{E}_n(L)R}$$

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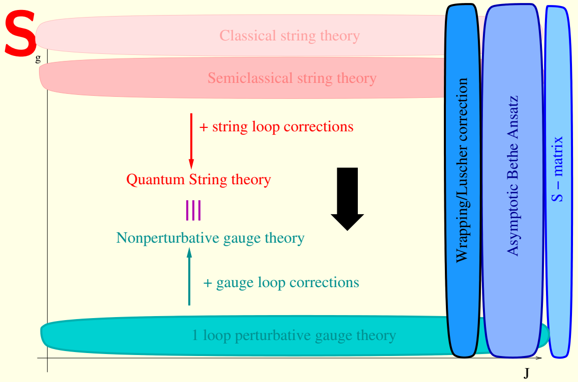

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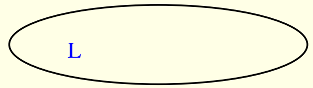

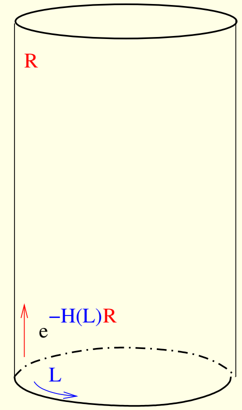

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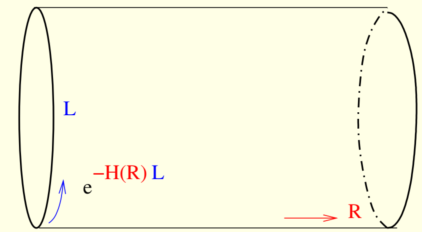
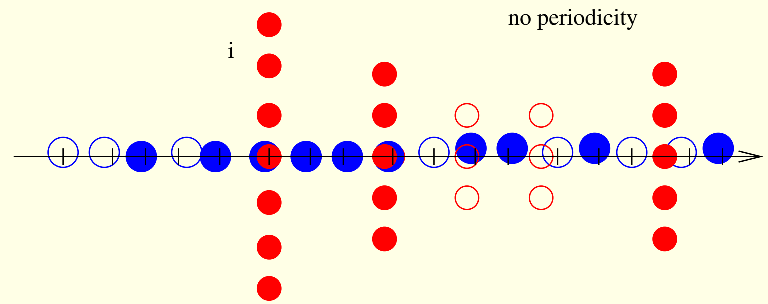


Finite particle/hole + Bethe root density $\rho^Q, \rho_h^Q, \rho^i, \rho_h^i$:



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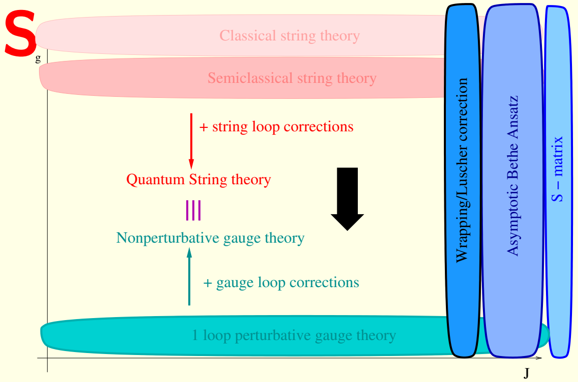

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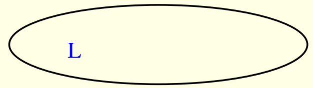

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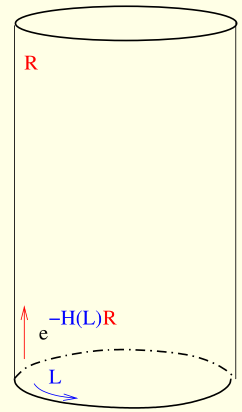

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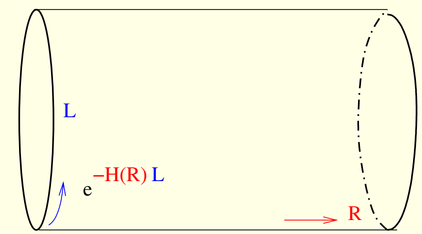

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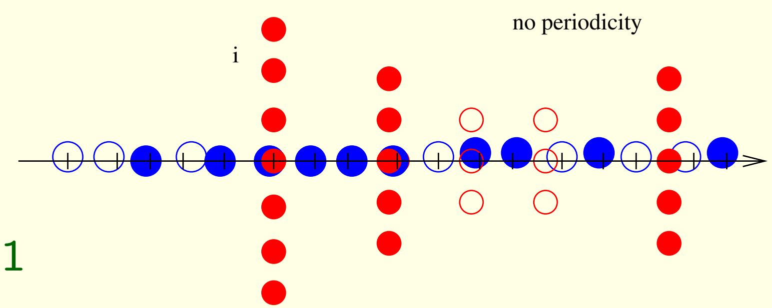


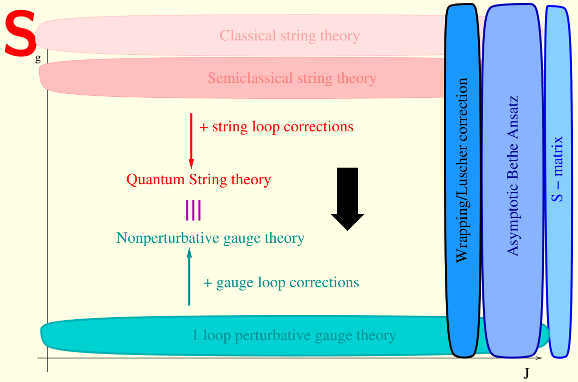
$$\tilde{E}_n(R) = \sum_{i,Q} \tilde{E}_Q(\tilde{p}_i) \to \sum_Q \int \tilde{E}_Q(\tilde{p})\rho^Q(\tilde{p})d\tilde{p}$$

$$e^{iLp}S_0^2\frac{Q_4^{++}}{Q_4^{--}}T\dot{T}|_4 = -1 \quad \frac{Q_2^{+}B_4^{(-)}}{Q_2^{-}B_4^{(+)}}|_1 = 1 \quad \frac{Q_2^{--}Q_1^{+}Q_3^{+}}{Q_2^{++}Q_1^{-}Q_3^{-}}|_2 = -1 \quad \frac{Q_2^{+}R_4^{(-)}}{Q_2^{-}R_4^{(+)}}|_3 = 1$$

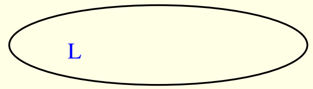
$$\int K_n^m(\tilde{p},\tilde{p}')\rho^n(\tilde{p}')d\tilde{p}' = 2\pi(\rho^m + \rho_h^m)$$

[Frolov,Kazakov,Gromov]

Thermodynamic Bethe Ansatz: AdS



Ground-state energy exactly



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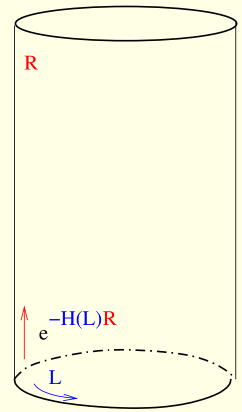

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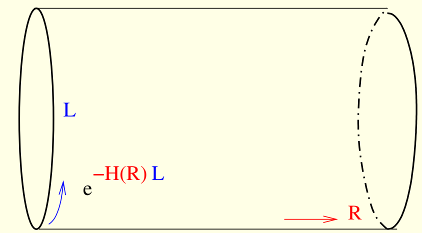

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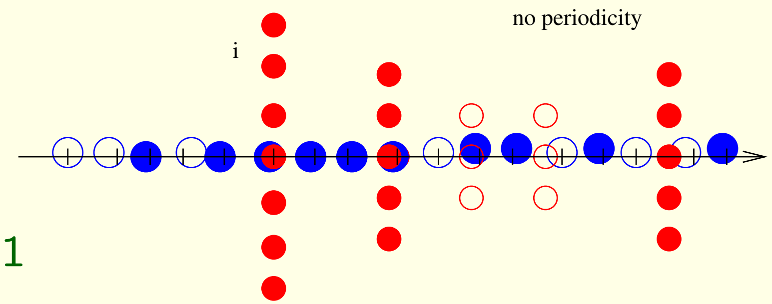


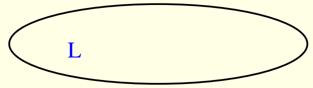
$$\tilde{E}_n(R)=\sum_{i,Q}\tilde{E}_Q(\tilde{p}_i)\to\sum_Q\int\tilde{E}_Q(\tilde{p})\rho^Q(\tilde{p})d\tilde{p}$$

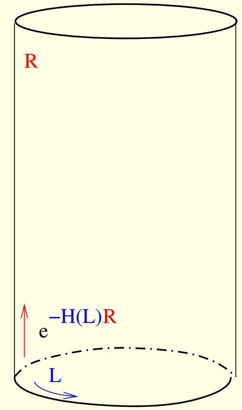
$$e^{iLp}S_0^2\frac{Q_4^{++}}{Q_4^{--}}T\dot{T}|_4=-1\;\;\frac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1\;\;\frac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1\;\;\frac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$$

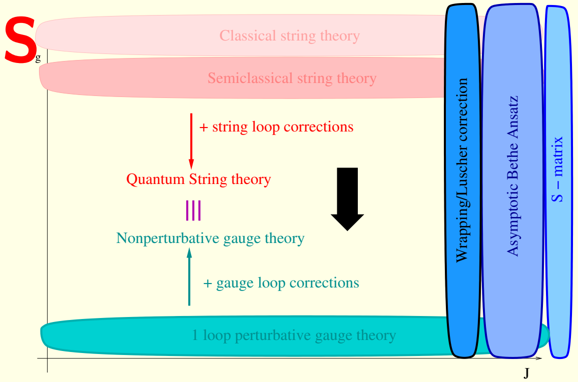
$$\int K_n^m(\tilde{p},\tilde{p}')\rho^n(\tilde{p}')d\tilde{p}'=2\pi(\rho^m+\rho_h^m)$$ [Frolov,Kazakov,Gromov]

$$Z(L,R)=\int d[\rho^i,\rho_h^i]e^{-LE(R)-\sum_i\int((\rho^i+\rho_h^i)\ln(\rho^i+\rho_h^i)-\rho^i\ln\rho^i-\rho_h^i\ln\rho_h^i)dp}$$

Thermodynamic Bethe Ansatz: AdS







Ground-state energy exactly

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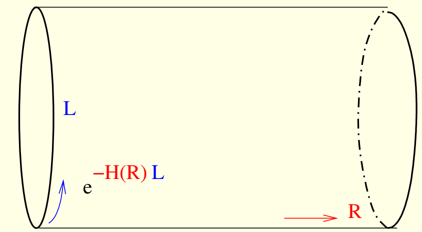

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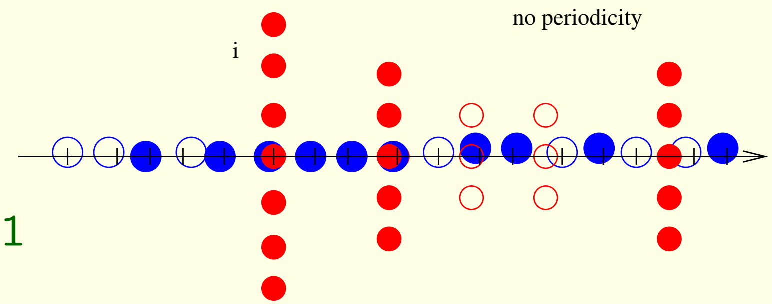


$$\tilde E_n(R)=\sum_{i,Q}\tilde E_Q(\tilde p_i)\to\sum_Q\int\tilde E_Q(\tilde p)\rho^Q(\tilde p)d\tilde p$$

$$e^{iLp}S_0^2\frac{Q_4^{++}}{Q_4^{--}}T\dot T|_4=-1\quad \frac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1\quad \frac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1\quad \frac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$$

$$\int K_n^m(\tilde p,\tilde p')\rho^n(\tilde p')d\tilde p'=2\pi(\rho^m+\rho^m_h)$$ [Frolov,Kazakov,Gromov]

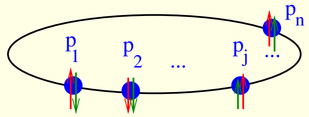
$$Z(L,R)=\int d[\rho^i,\rho^i_h]e^{-LE(R)-\sum_i\int((\rho^i+\rho^i_h)\ln(\rho^i+\rho^i_h)-\rho^i\ln\rho^i-\rho^i_h\ln\rho^i_h)dp}$$

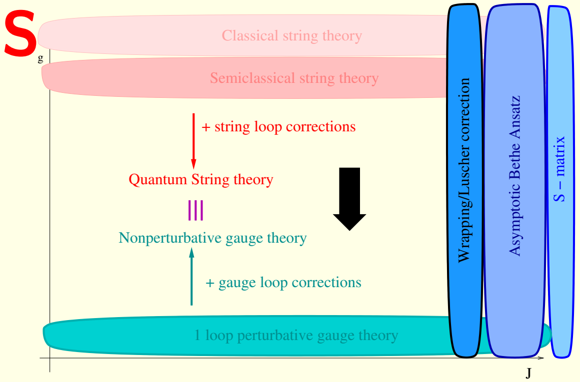
Saddle point : $\epsilon^i(\tilde p)=-\ln\frac{\rho^i(\tilde p)}{\rho^i_h(\tilde p)}$ $\boxed{\epsilon^j(\theta)=\delta^j_Q\tilde E_Q(\tilde p)L-\int K_i^j(\tilde p,\tilde p')\log(1+e^{-\epsilon^i(\tilde p')})d\tilde p'}$

Ground state energy exactly: $\boxed{E_0(L)=-\sum_Q\int\frac{d\tilde p}{2\pi}\log(1+e^{-\epsilon_Q(\tilde p)})d\tilde p}$

Excited states TBA, Y-system: AdS

Excited states exactly





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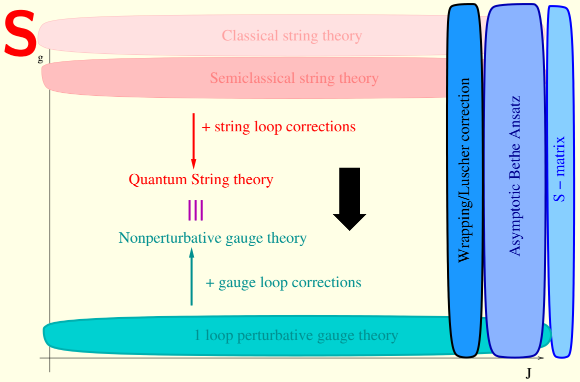

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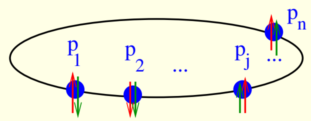
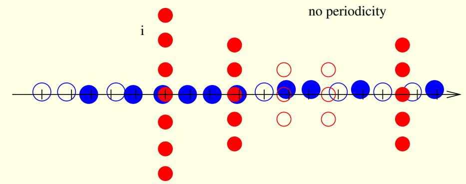
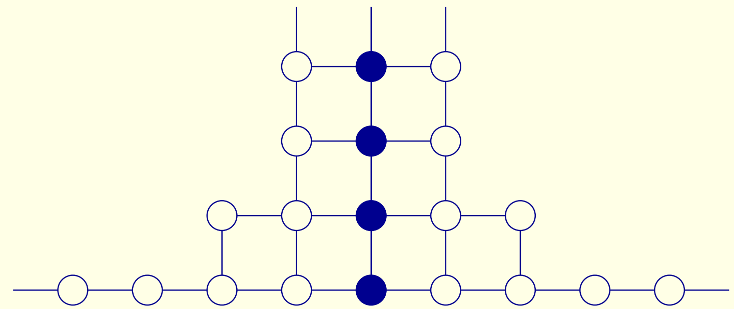


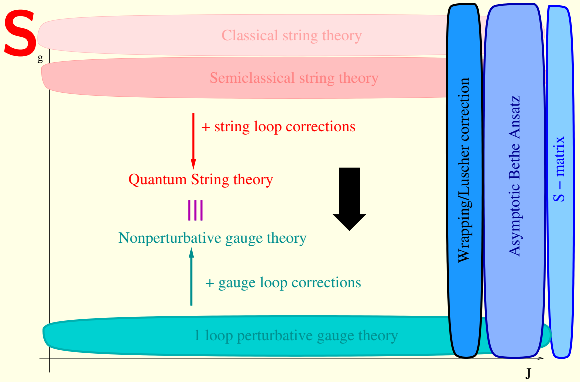


Y-system: AdS [Gromov,Kazakov,Viera]



$$\frac{Y_{a,s}(\theta+\frac{i}{2})Y_{a,s}(\theta-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

Excited states TBA, Y-system: AdS



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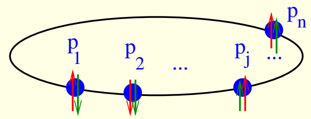

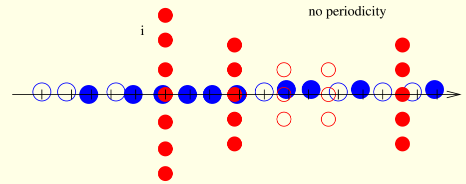

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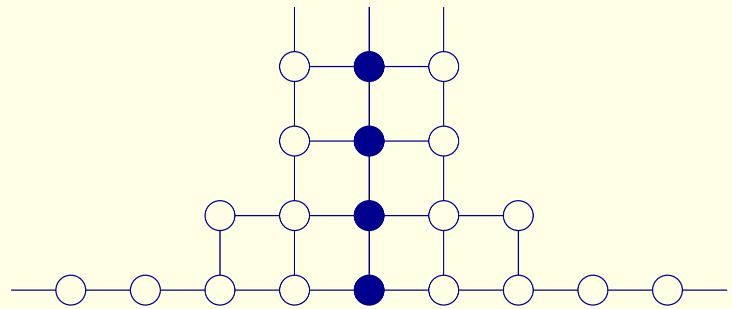


$$\frac{Y_{a,s}(\theta+\frac{i}{2})Y_{a,s}(\theta-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

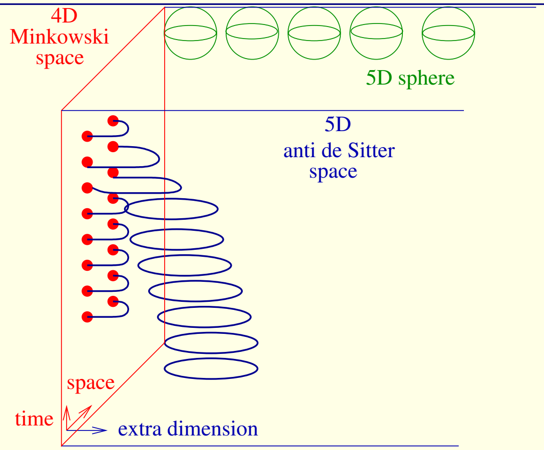
Excited states: analyticity from Lüscher

Assumption on analytical structure $\to$ excited state TBA [Gromov,Kazakov,Kozak,Viera], [Arutyunov, Frolov, Suzuki]

extra source terms: $\boxed{\epsilon^j(\theta)=\delta^j_Q\tilde{E}_Q(\tilde{p})L+sources-\int K^j_i(\tilde{p},\tilde{p}')\log(1+e^{-\epsilon^i(\tilde{p}')})d\tilde{p}'}$

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



$$\sum_1^6 Y_i^2 = R^2 \qquad - + + + + - = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M \right) + \ldots$$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\overline{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \overline{\Psi} [\Phi, \Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5) \times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2)$, det( )

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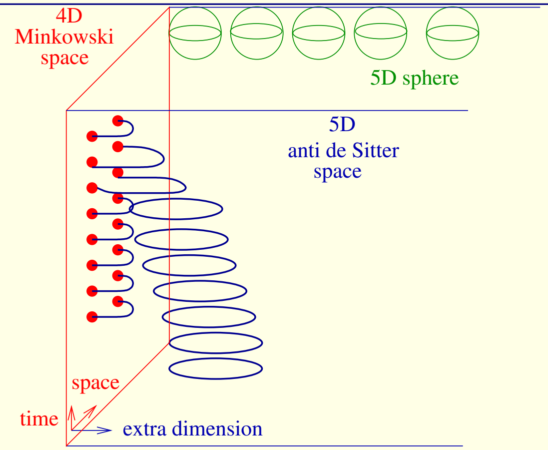

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Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong↔weak

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

$$\langle \mathcal{O}_n(x) \mathcal{O}_m(0) \rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \dots$$

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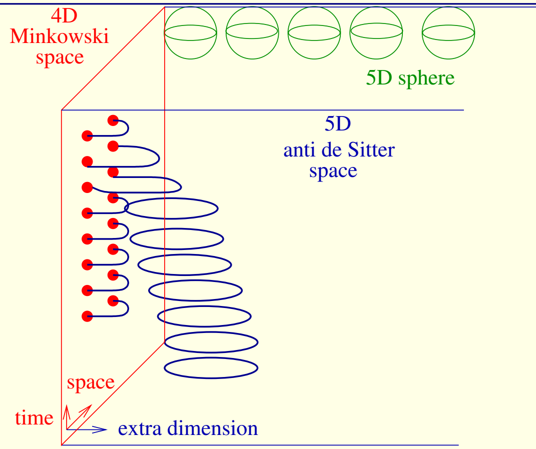

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$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

$$\frac{2}{g_{YM}^2}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

$\beta = 0$ superconformal $\frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^2), \det(\ )$

Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$$

strong$\leftrightarrow$weak

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

$$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{|x|^{2\Delta_n(\lambda)}}$$

Anomalous dim $\Delta(\lambda)$

$$\Delta(\lambda) = \Delta(0) + \lambda\Delta_1 + \lambda^2\Delta_2 + \ldots$$

2D integrable QFT

spectrum: $Q = 1, 2, \ldots, \infty$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2}\sin^2\frac{p}{2}}$

Exact scattering matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$

finite volume spectrum: $E(\lambda, J)$

AdS/CFT correspondence

supersymmetric BPS operators

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

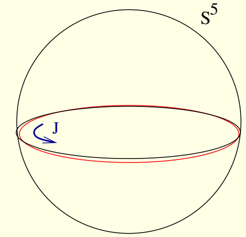
$$Z = \Phi_1 + i\Phi_2, X = \Phi_3 + i\Phi_4$$

$$\mathcal{O}_{BPS} = \mathrm{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\ldots\uparrow\rangle$$

$$\Delta_{BPS} = J$$

weak↔strong

BPS string configuration



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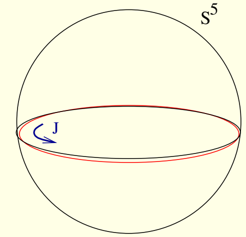

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supersymmetric groundstate $E_0(J)=\Delta(\lambda)-J=0$

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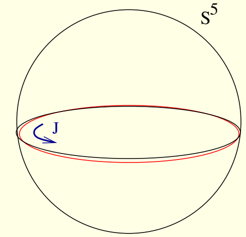
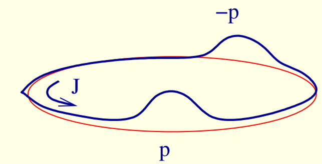


$$E_{BPS}(\lambda) = J$$

2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

Konishi anomalous dimension at weak coupling

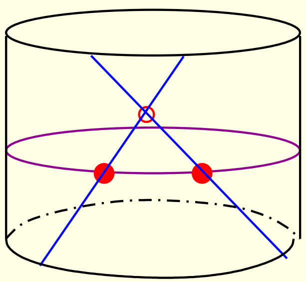


Konishi: two particles in finite volume:

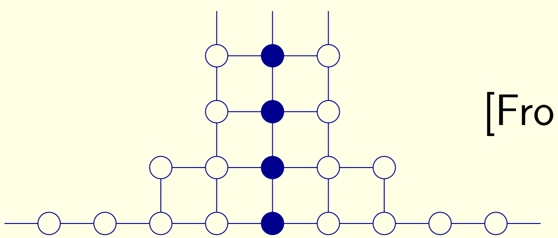
Bethe Ansatz: $e^{ipJ}S(p,-p)=1$ + dispersion relation $E(p,\lambda)=\sqrt{1+16g^2(\sin\frac{p}{2})^2}$

$$\Delta_{BA}(\lambda)=2+6g^2-24g^4+168g^6-(1410+144\zeta_3)g^8-12(22429+4608\zeta_3+3672\zeta_5+2520\zeta_7)g^{12}+\dots$$

Lüscher correction $\Delta_{Luscher}(\lambda)=-\int\frac{d\tilde{p}}{2\pi}S(\tilde{p},p_1)S(\tilde{p},p_2)e^{-L\tilde{E}(\tilde{p})}$



4loop: $(324+864\zeta_3-1440\zeta_5)g^8$	[ZB, Janik '09]
5loop: $-36(72\zeta_3(-1+2\zeta_3)+$ $+5(63+64\zeta_5-168\zeta_7))g^{10}$	[ZB, Hegedus, Janik,Lukowski '10]
6loop: $-108(-2421+96\zeta_3(20+2\zeta_3-15\zeta_5)$ $-1448\zeta_5-980\zeta_7+4536\zeta_9)g^{12}$	[ZB, Janik '12]
	7loop



TBA
[Frolov,Arutyunov,Suzuki]

Finite equations
[Balog, Hegedus '12]

FiNLIE
[Gromov, Kazakov, Leurent, Volin '12]

FiNLIE weak coupling: 8loop; $\zeta_{1,2,8}$
[Leurent, Serban, Volin '12] [Leurent, Volin '13]

Konishi at strong coupling (only numerical)

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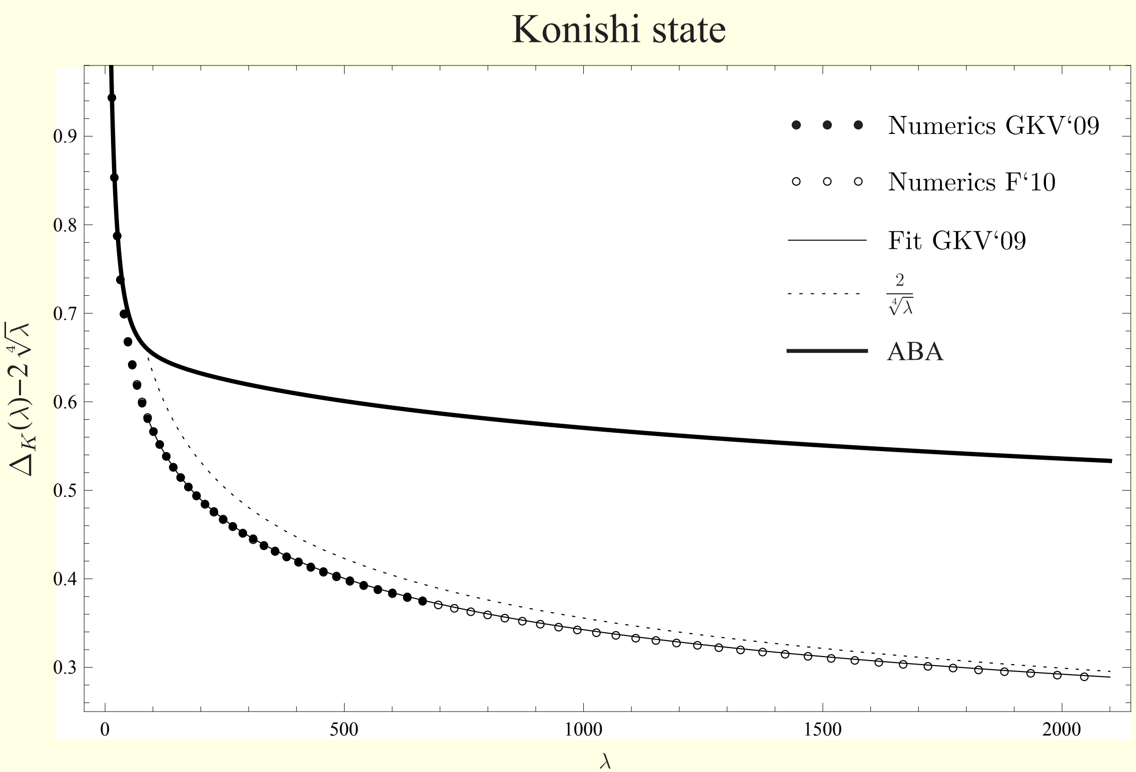
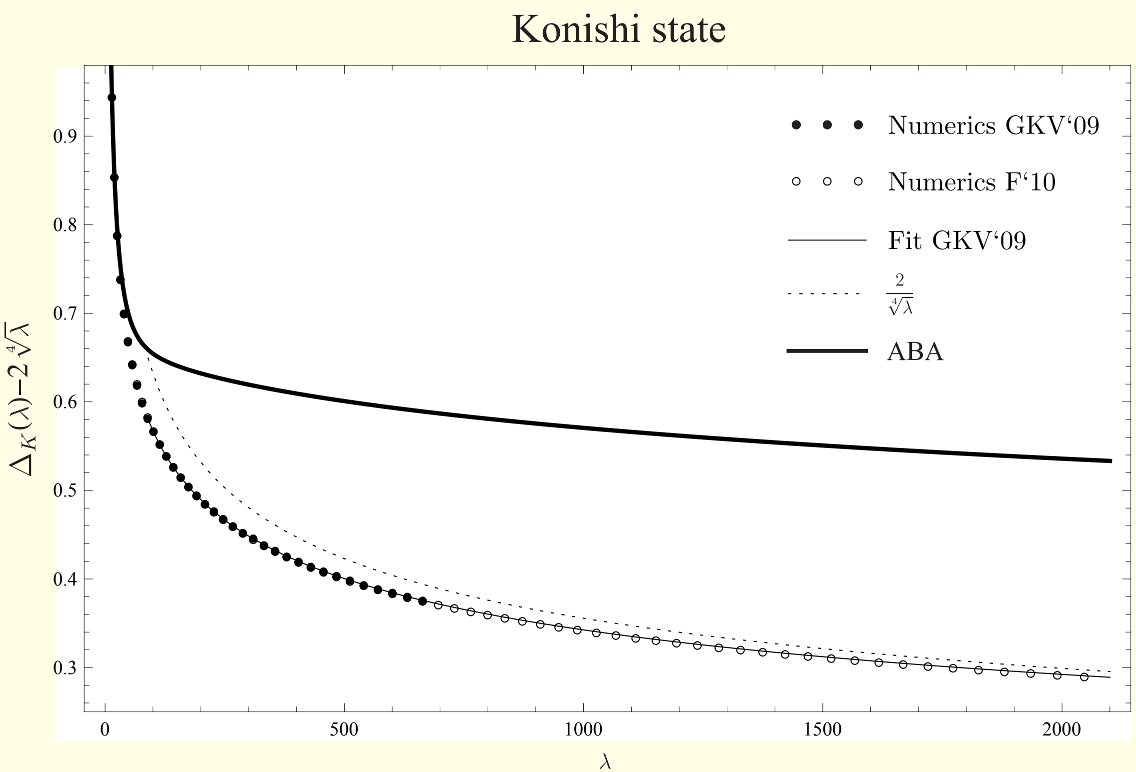


Konishi at strong coupling (only numerical)

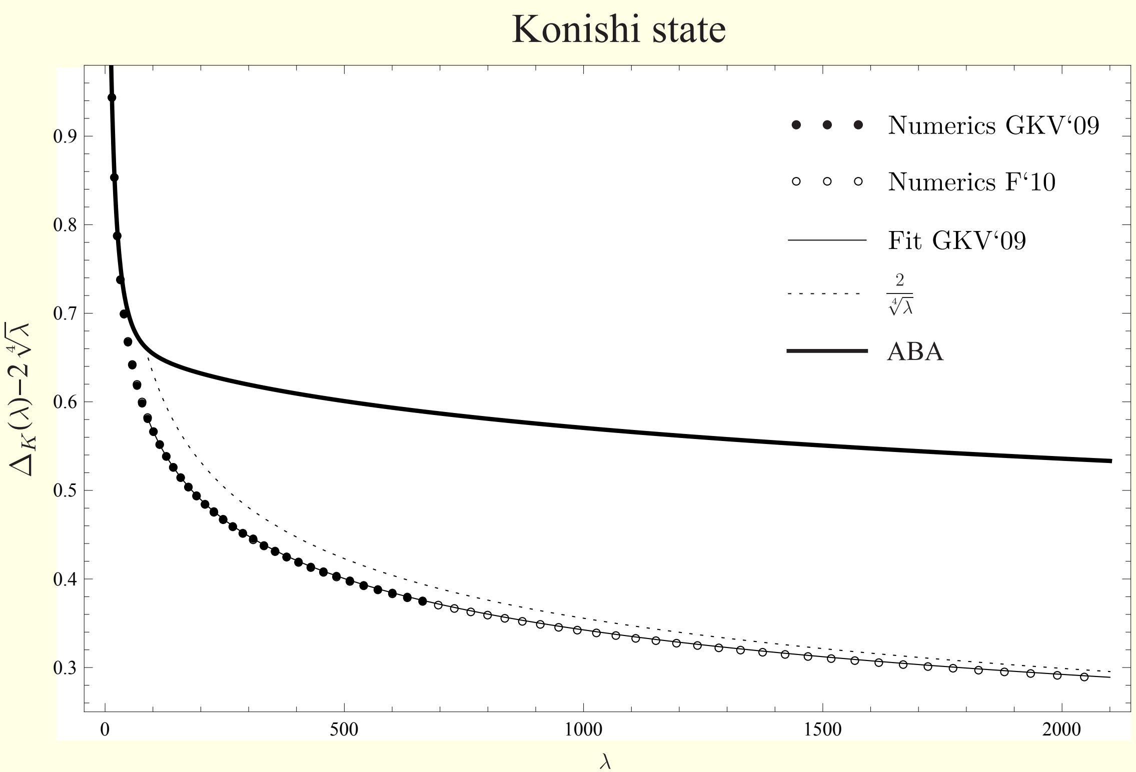


Numerical TBA for Konishi operator: $\mathrm{Tr}(\Phi^2)$

Strong coupling (numerically) agrees with string theory calculations

$$E(g) = 2\lambda^{\frac{1}{4}} - 4 + \frac{2}{\lambda^{\frac{1}{4}}} + \ldots$$

Konishi at strong coupling (only numerical)



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$$E(g) = 2\lambda^{\frac{1}{4}} - 4 + \frac{2}{\lambda^{\frac{1}{4}}} + \dots$$

AdS/CFT: spectral problem for twisted theory

non supersymmetric theory [Frolov .. '05]

$$V(\Phi,\Psi)=\frac{1}{4}[\Phi,\Phi]^2_{\gamma_i}+\overline{\Psi}[\Phi,\Psi]_{\gamma_i}$$

$$[\Phi_i,\Phi_j]_{\gamma_k}=e^{i\epsilon_{ijk}\gamma_k}\Phi_i\Phi_j-e^{-i\epsilon_{ijk}\gamma_k}\Phi_j\Phi_i$$

$$Z=\Phi_1+i\Phi_2, X=\Phi_3+i\Phi_4$$

$$\mathcal{O}=\mathrm{Tr}(Z^J)\leftrightarrow|\uparrow\uparrow\ldots\uparrow\rangle$$

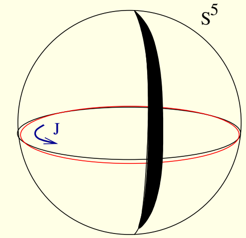
$$\Delta_{\mathcal{O}}=J+\Delta_{\text{wrap.}}$$

$$=J+\lambda^J\Delta_J+\ldots+\lambda^{2J}\Delta_{2J}$$

$\leftrightarrow$

TST deformed AdS

[Frolov '05][Alday .. '06]

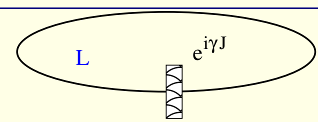


$\equiv$AdS with twisted BC.

$E(\lambda)=J+$finite size corr.

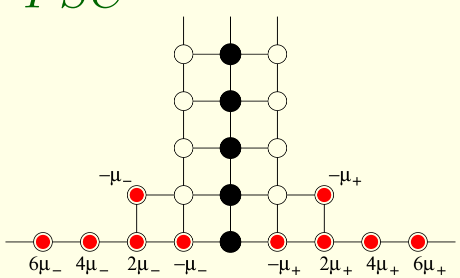
[Arutyunov et al '11]

twisted groundstate



$$E-J=E_{FSC}$$

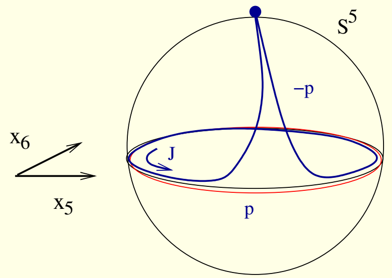
E_{FSC} =finite size correction! , [ZB et al '11] twisted TBA



untwisted Y-system [Gromov .. '11]

AdS/CFT correspondence: boundary

Z=0 brane: boundary vacuum



$$E_K(\lambda) = 2E_{Bdry}(\lambda)$$

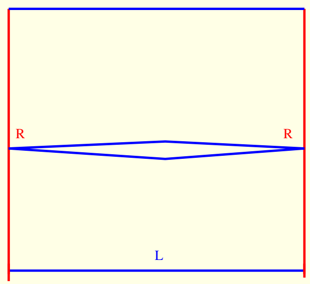
dispersion relation

$$E(p,\lambda) = \sqrt{1+\frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

elastic reflection $R(p)$

Bethe Ansatz: $e^{i2pJ}R(p)R(-p)=1$

finite size corrections



$$\Delta E = -\int \frac{d\tilde{p}}{2\pi} R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L}$$

$\equiv$

det operator anomalous dimension

$$Z=\Phi_5+i\Phi_6, Y=\Phi_3+i\Phi_4$$

"Z=0 vacuum"

$$\mathcal{O}=\epsilon^{lm..nq}_{ij..kp}Z^i_l Z^j_m..Z^k_n(YZ^JY)^p_q$$

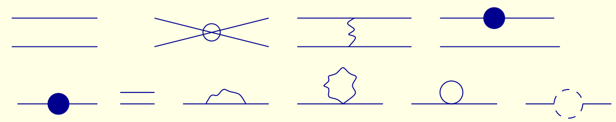
$$|\downarrow\uparrow\uparrow\dots\uparrow\uparrow\downarrow\rangle$$

"Y=0 vacuum"

$$\mathcal{O}=\epsilon^{lm..nq}_{ij..kp}Y^i_l Y^j_m..Y^k_n(Z^J)^p_q$$

$$|\uparrow\uparrow\dots\uparrow\uparrow\rangle$$

operator mixing



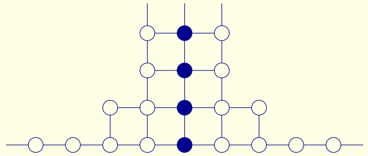
integrable open spinchain

$$\langle\mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_i(\lambda)}}$$

Z=0: Bethe Ansatz + wrapping

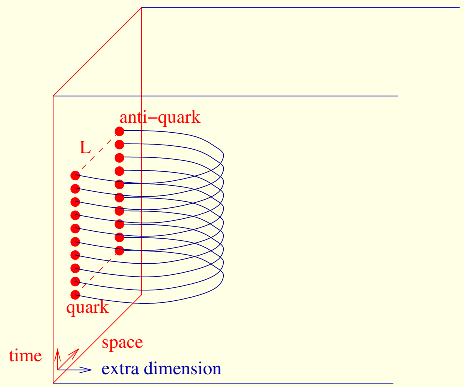
Y=0: Bethe Ansatz + wrapping

Y-system [ZB et al '12]



AdS/CFT integrability: other observables

quark-antiquark potential



$$V(L) = \frac{-\lambda}{4\pi L} + \frac{\lambda^2}{8\pi^2 L}\log\frac{T}{L} + \dots$$

$\equiv$

Minimal surface

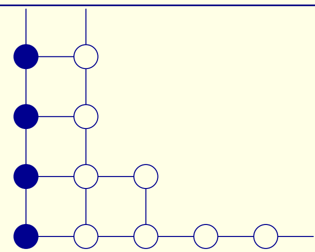
Wilson loop: $\langle \oint_C A_\mu dx^\mu \rangle \propto e^{-TV(L,\lambda)}$

strong coupling

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}\left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimal surface+fluctuations

Integrable system on the strip



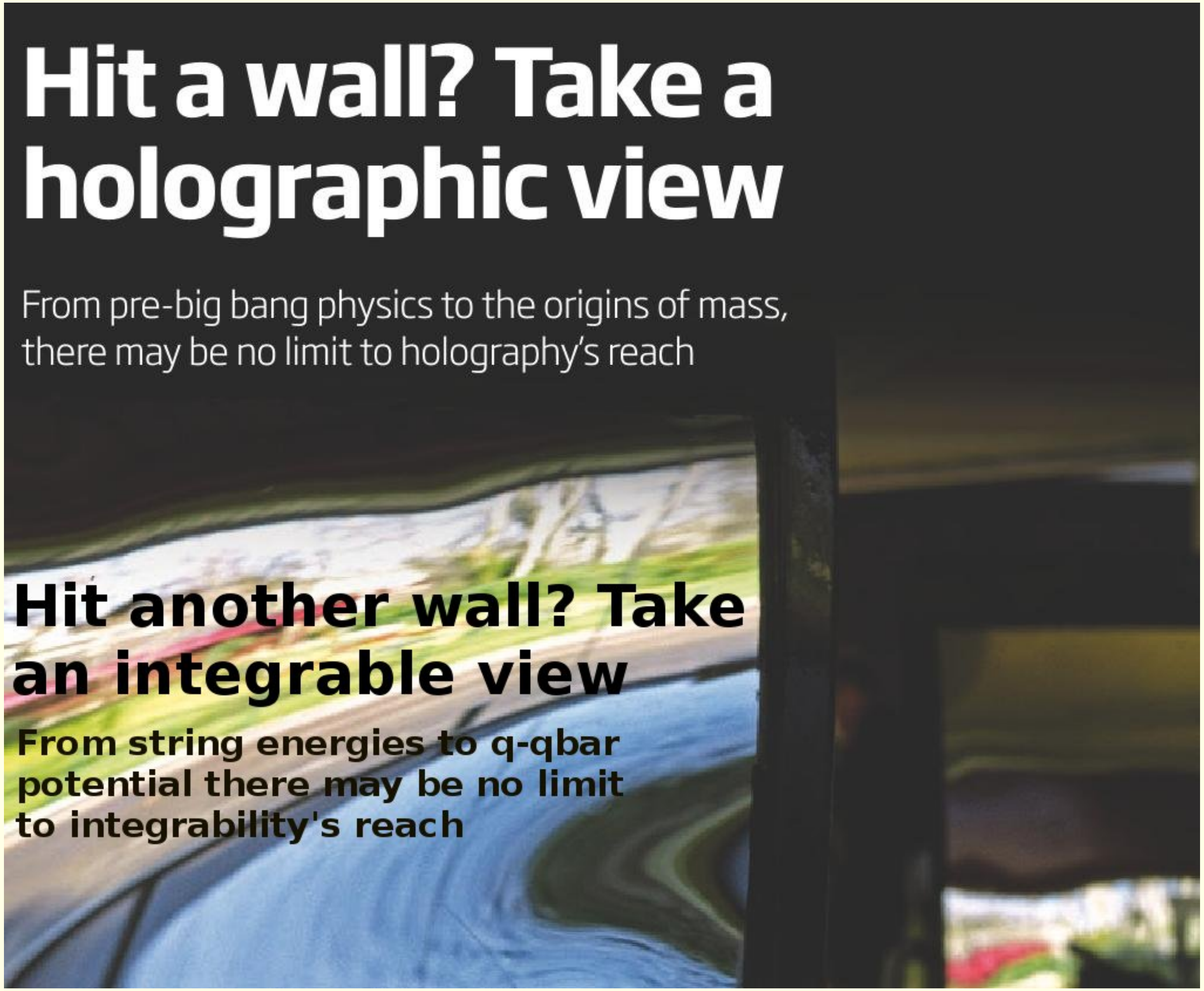
[Correa,Maldacena,Sever '12][Drukker '12]

Boundary problem: open spin chain with reflections

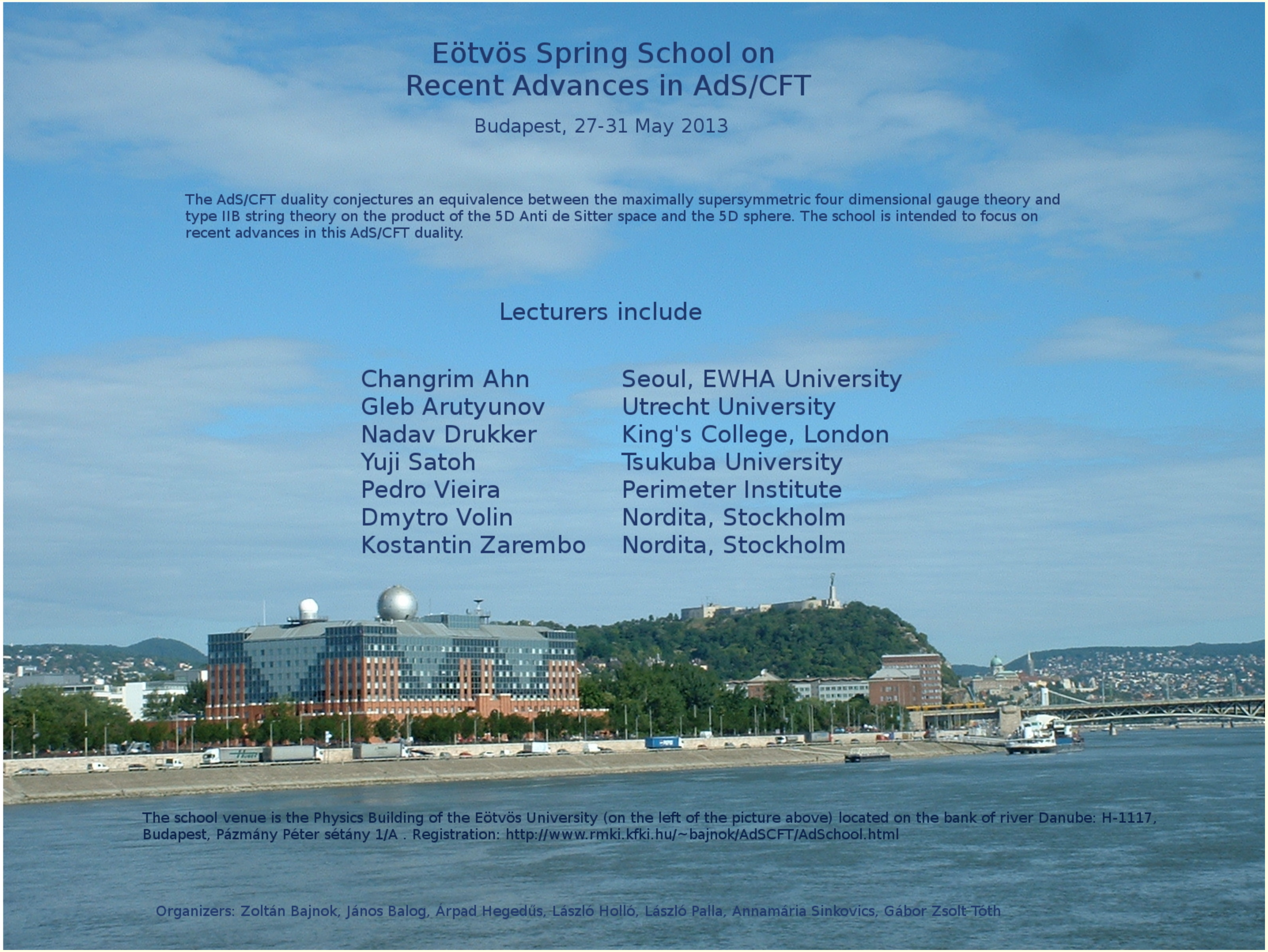
$E_0(L)$, Casimir energy

$$\epsilon^j(\theta) = \delta^j_Q(\sigma_Q(\tilde{p}) + \tilde{E}_Q(\tilde{p})L - \int K^j_i(\tilde{p},\tilde{p}')\log(1+e^{-\epsilon^i(\tilde{p}')})d\tilde{p}'$$

Conclusion



Hit a wall? Take a holographic view
From pre-big bang physics to the origins of mass, there may be no limit to holography's reach
Hit another wall? Take an integrable view
From string energies to q-qbar potential there may be no limit to integrability's reach


Eötvös Spring School on
Recent Advances in AdS/CFT
Budapest, 27-31 May 2013
The AdS/CFT duality conjectures an equivalence between the maximally supersymmetric four dimensional gauge theory and type IIB string theory on the product of the 5D Anti de Sitter space and the 5D sphere. The school is intended to focus on recent advances in this AdS/CFT duality.
Lecturers include
Changrim Ahn Seoul, EWHA University
Gleb Arutyunov Utrecht University
Nadav Drukker King's College, London
Yuji Satoh Tsukuba University
Pedro Vieira Perimeter Institute
Dmytro Volin Nordita, Stockholm
Kostantin Zarembo Nordita, Stockholm
The school venue is the Physics Building of the Eötvös University (on the left of the picture above) located on the bank of river Danube: H-1117, Budapest, Pázmány Péter sétány 1/A . Registration: http://www.rmki.kfki.hu/~bajnok/AdSCFT/AdSchool.html
Organizers: Zoltán Bajnok, János Balog, Árpad Hegedűs, László Holló, László Palla, Annamária Sinkovics, Gábor Zsolt Tóth