Integrable Models and Applications: from Strings to Condensed Matter

Santiago de Compostela, Spain 12-16 September 2005

On the boundary form factor program

Z. Bajnok, L. Palla, and G. Takács

Institute for Theoretical Physics, Eötvös University, Budapest

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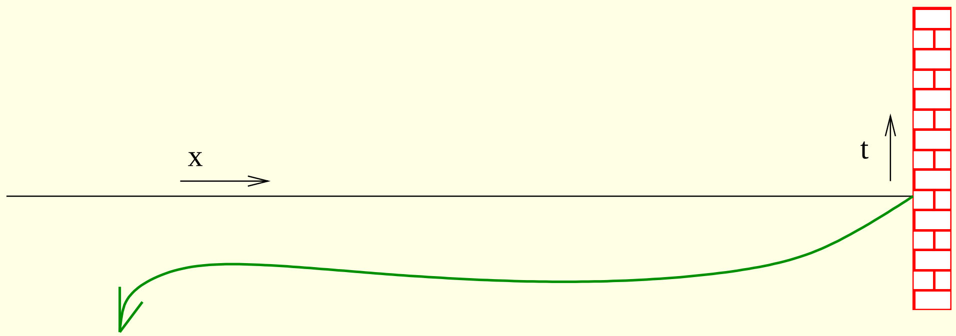
Massive integrable boundary QFT in 1+1 D (diagonal)



$$ {}_B\langle 0|\varphi_B(t)|\theta_1,\theta_2,\dots,\theta_n\rangle_B^{in} $$

Plan of talk

$$\boxed{{}_B\langle 0|\varphi_B(t)|\theta_1,\theta_2,\ldots,\theta_n\rangle_B^{in}}$$

Boundary form factor

Plan of talk

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Boundary form factor

↙

$\varphi_B(t)$: local boundary operator

Plan of talk

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Boundary form factor

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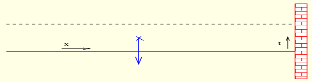
Bulk $< >$ 

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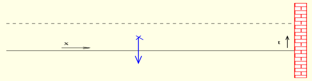
Bulk $< \; >$ 

Bulk form factors$+|B\rangle$

Plan of talk

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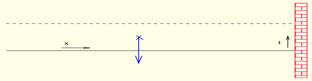
Boundary form factor

$\varphi_B(t)$: local boundary operator

Bulk $< >$ 

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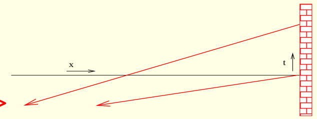
Boundary $< >$ 

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Boundary form factor

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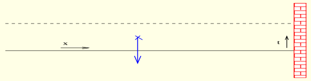
Bulk $< >$ 

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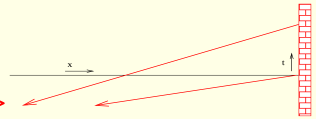

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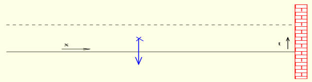

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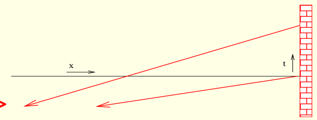

Boundary $<\ >$

Boundary form factors!

$|\theta_1,\theta_2,\ldots,\theta_n\rangle_B^{in}$

Asymptotic states $>$

Plan of talk

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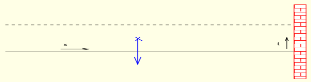
Boundary form factor

$\varphi_B(t)$: local boundary operator

Bulk $< >$ 

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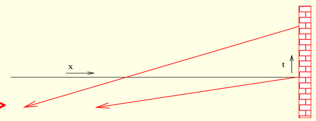
Boundary $< >$ 

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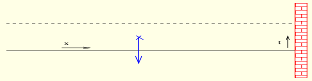
Bulk $<\ >$ 

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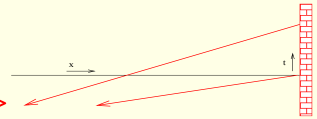

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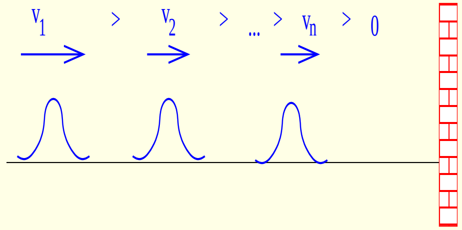

Plan of talk

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Boundary form factor

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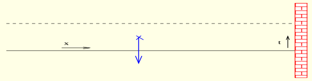

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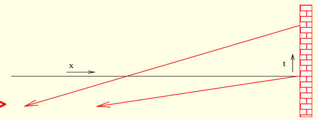

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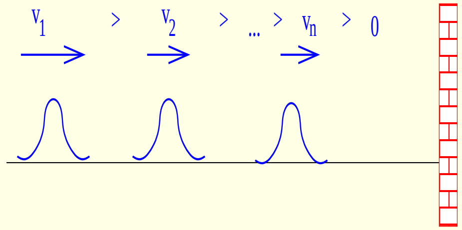
Asymptotic states $>$



Boundary reduction formula $>$

Plan of talk

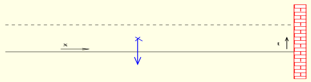
$$_B\langle 0|\varphi_B(t)|\theta_1,\theta_2,\dots,\theta_n\rangle_B^{in}$$

Boundary form factor

$\swarrow$

$\varphi_B(t)$: local boundary operator

Bulk $< >$



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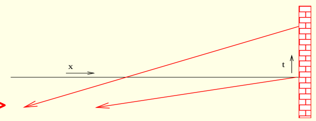
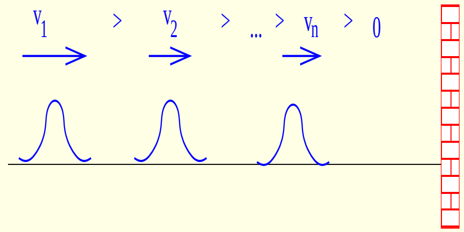


Boundary form factors!

$\searrow$

$|\theta_1,\theta_2,\dots,\theta_n\rangle_B^{in}$

Asymptotic states $>$



Boundary reduction formula $>$

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Plan of talk

$${}_B\langle 0|\varphi_B(t)|\theta_1,\theta_2,\dots,\theta_n\rangle_B^{in}$$

Boundary form factor

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$\varphi_B(t)$: local boundary operator

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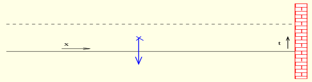

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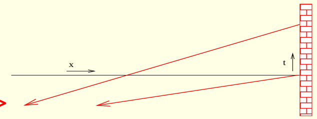

Boundary form factors!

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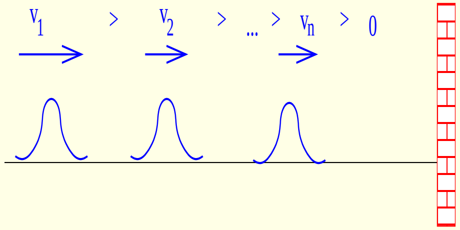
Asymptotic states $>$



Boundary reduction formula $>$

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Consistency equations !

Plan of talk

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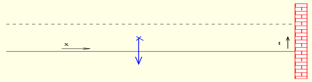

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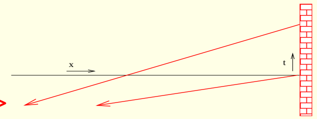

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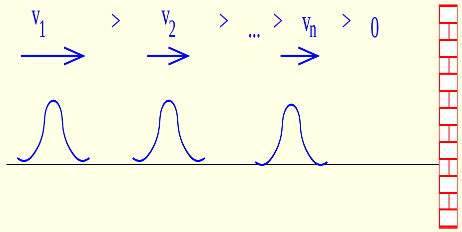

Boundary reduction formula $>$

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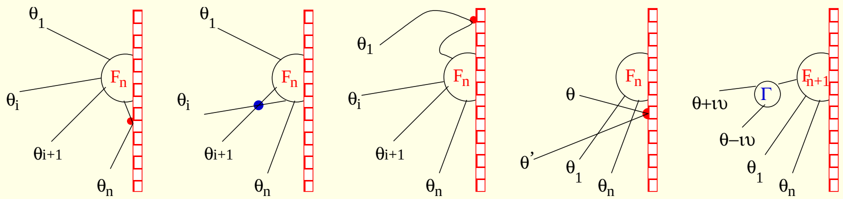

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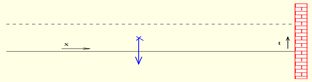

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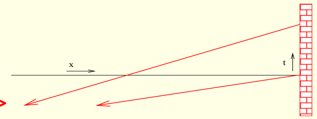

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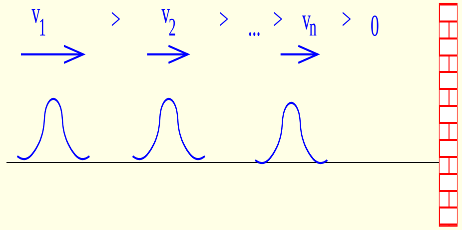
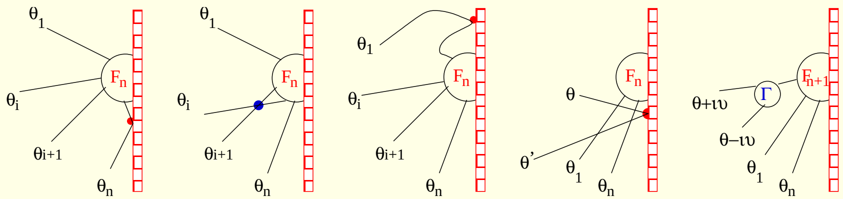


Boundary reduction formula $>$

$\downarrow$

Consistency equations !

$\longrightarrow$ Axioms $>$



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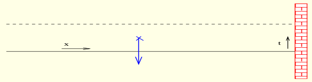

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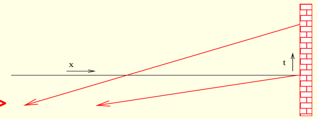

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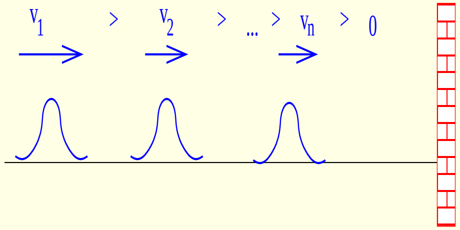

Boundary reduction formula $>$

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Consistency equations !

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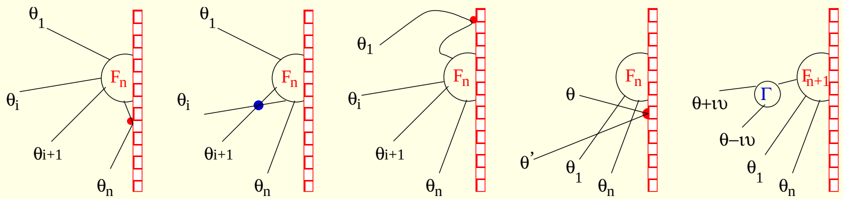
Minimal solution $>$



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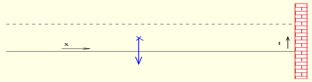

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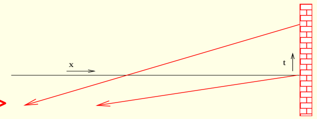

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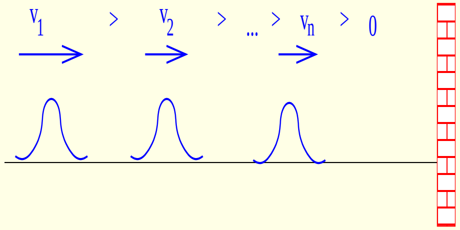


Boundary reduction formula $>$

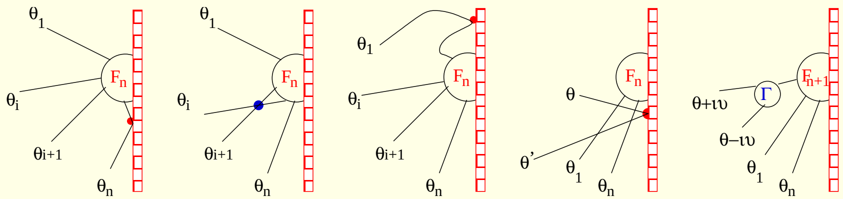
$\downarrow$

Consistency equations !

$\longrightarrow$ Axioms $>$

Minimal solution $>$

Perturbed boundary Lee-Yang



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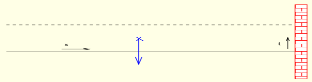

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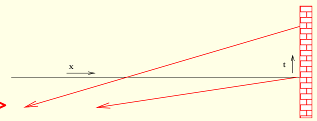

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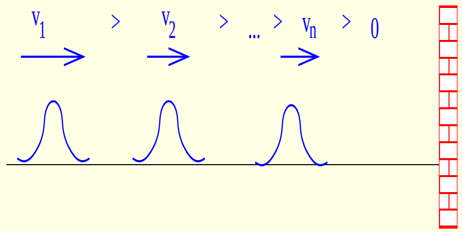

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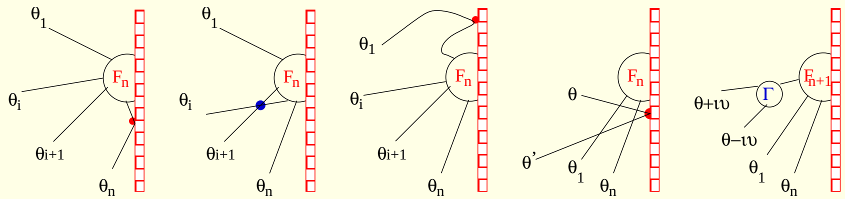

$\longrightarrow$ Axioms $>$

Minimal solution $>$

Perturbed boundary Lee-Yang

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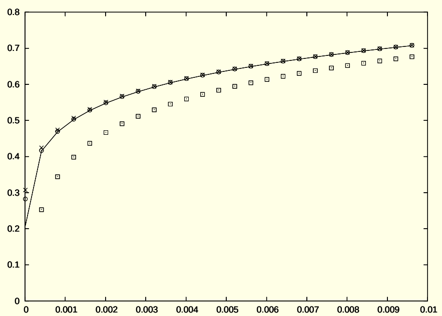
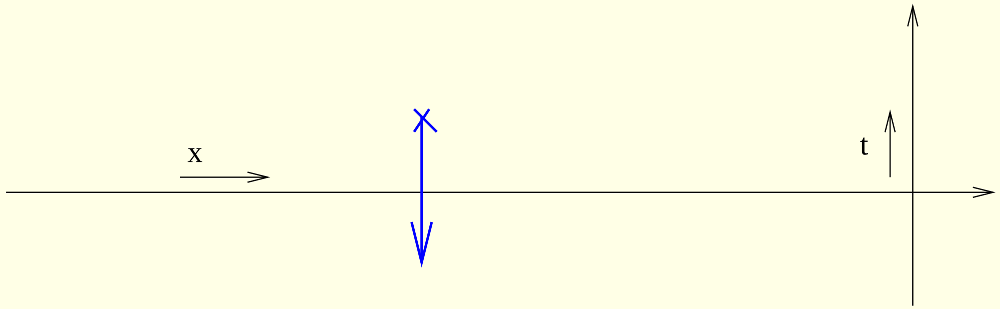


Integrable massive BQFT as perturbed BCFT

Integrable massive BQFT as perturbed BCFT

Bulk conformal field theory



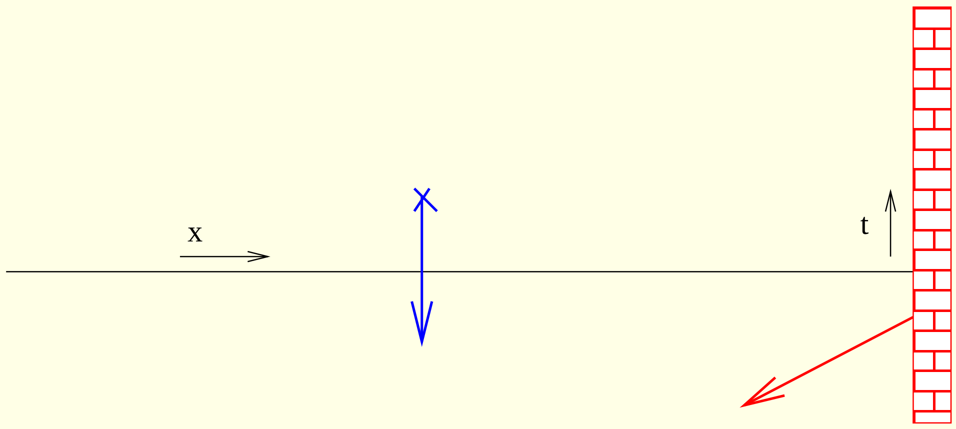
Operator - state correspondence

$$\{\Phi(x,t)\} \leftrightarrow \mathcal{H}$$

Bulk operators $\Phi(x,t)$ Operator algebra

Integrable massive BQFT as perturbed BCFT

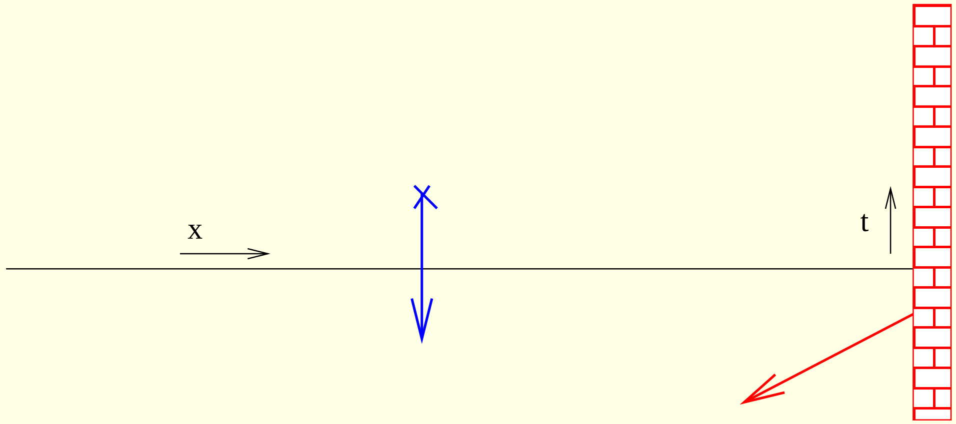
Boundary conformal field theory



Bulk operators $\Phi(x,t)$

Integrable massive BQFT as perturbed BCFT

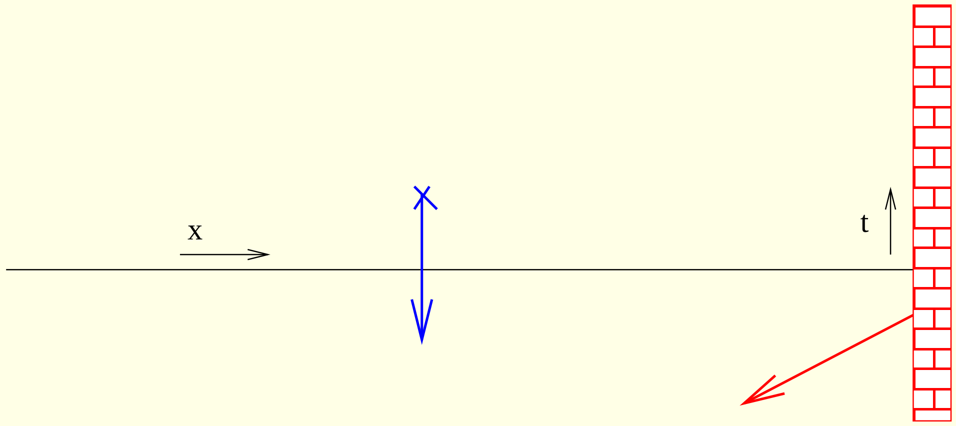
Boundary conformal field theory



Bulk operators $\Phi(x,t)$ $\varphi_B(t)$ Boundary operators Bulk - Boundary OPE

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



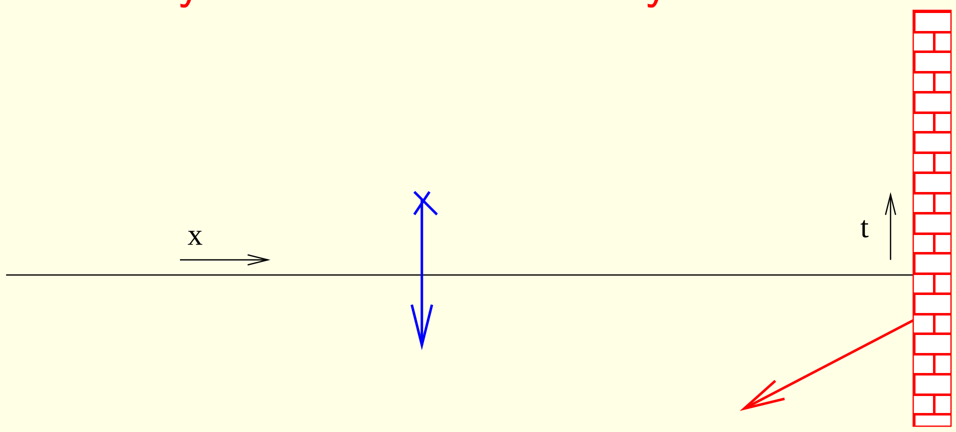
Operator - state correspondence

$$\{\varphi_B(t)\} \leftrightarrow \mathcal{H}_B$$

Bulk operators $\Phi(x,t)$ $\varphi_B(t)$ Boundary operators Bulk - Boundary OPE

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



Operator - state correspondence

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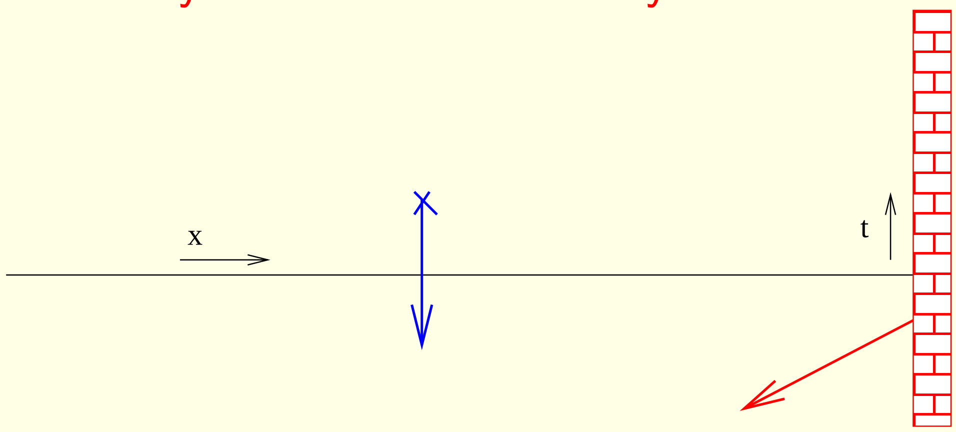
Bulk operators $\Phi(x,t)$ $\varphi_B(t)$ Boundary operators   Bulk - Boundary OPE

Integrable perturbations

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



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Bulk operators $\Phi(x,t)$

$\varphi_B(t)$ Boundary operators

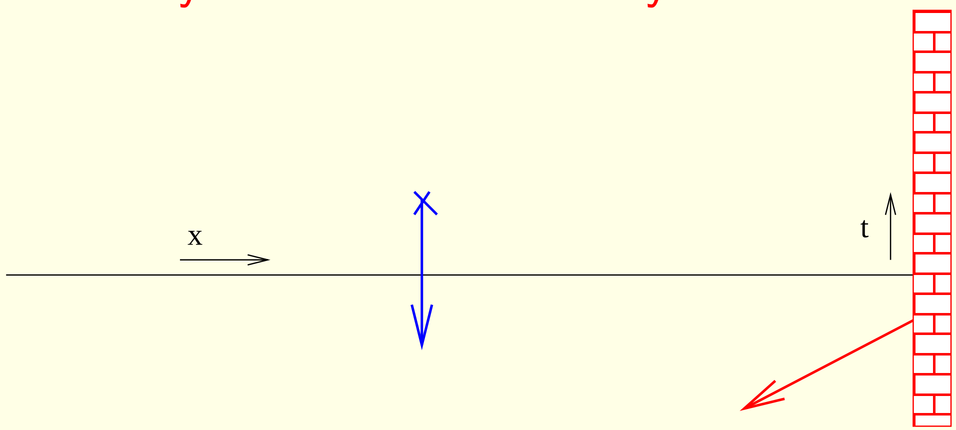
Bulk - Boundary OPE

Integrable perturbations

$$S = S_{BCFT}$$

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



Operator - state correspondence

$$\{\varphi_B(t)\} \leftrightarrow \mathcal{H}_B$$

Bulk operators $\Phi(x,t)$ $\varphi_B(t)$ Boundary operators Bulk - Boundary OPE

Integrable perturbations

$$S = S_{BCFT} - \lambda_{bulk} \int dt \int_{-\infty} dx\, \Phi(x,t)$$

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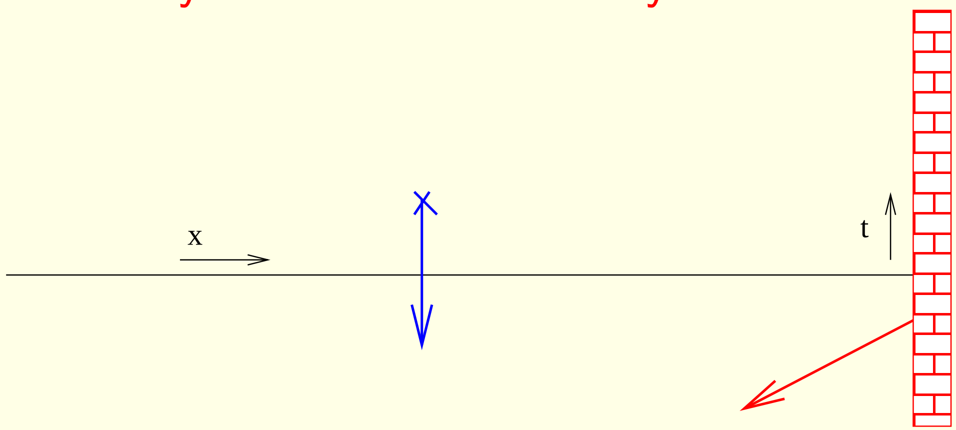

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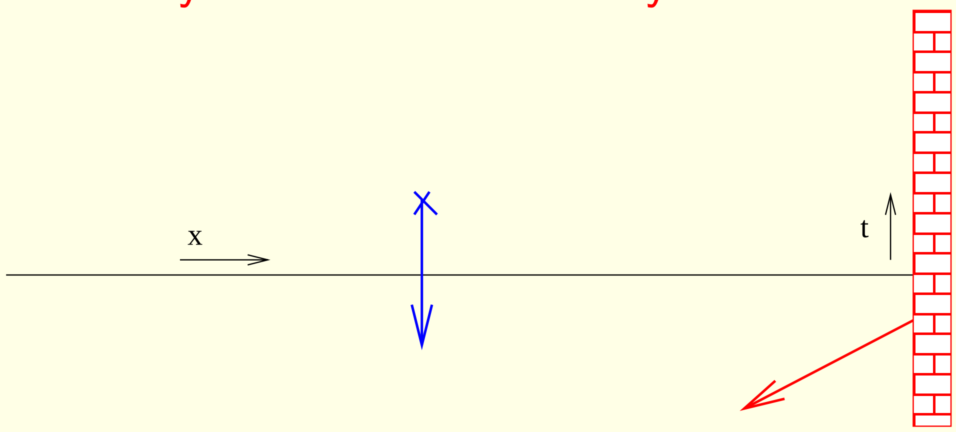

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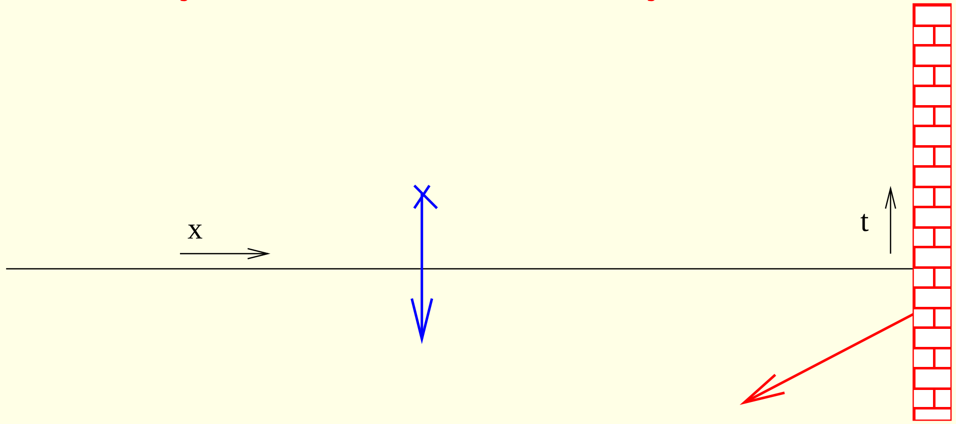

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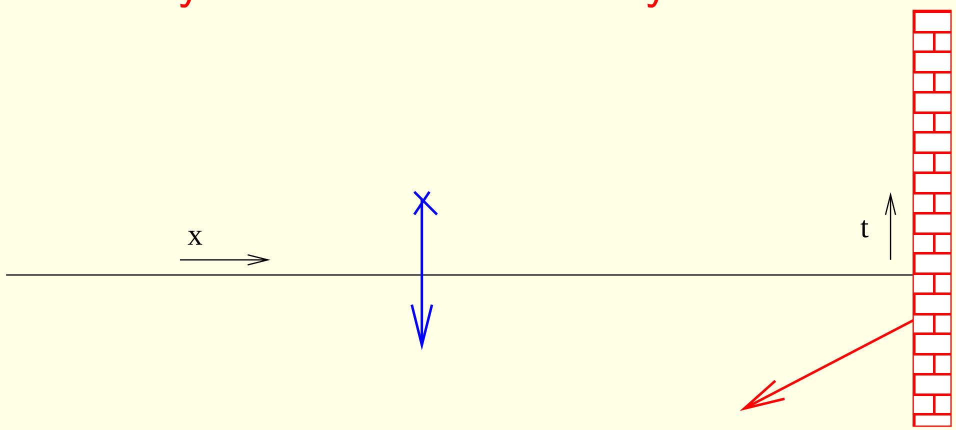
Hilbert space does not change

Assumptions: spectrum, operator algebra smoothly changes

Integrable massive BQFT as perturbed BCFT

Boundary conformal field theory



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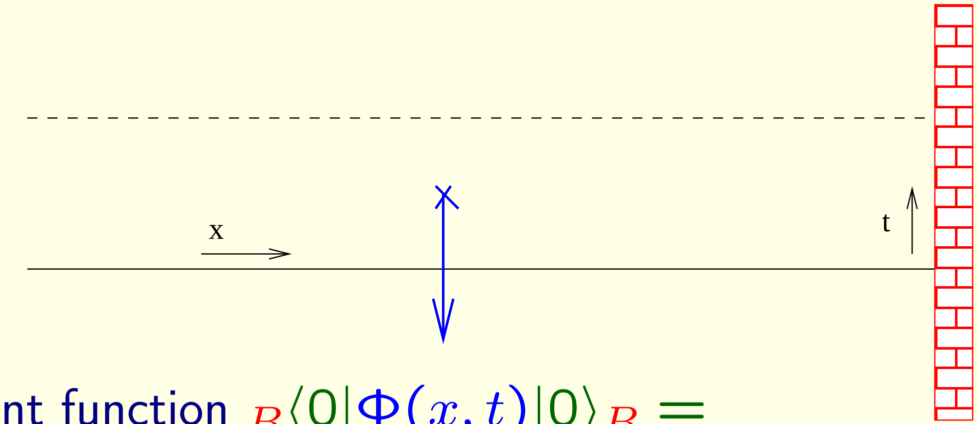
Assumptions: Hilbert space does not change

spectrum, operator algebra smoothly changes $\longrightarrow$

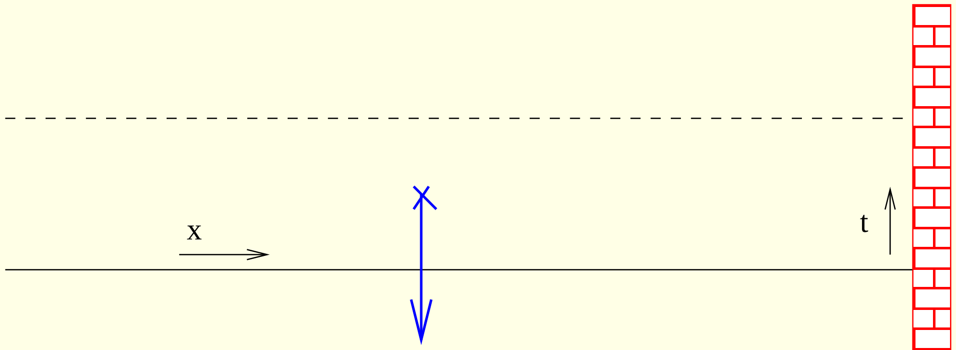
Operator content
Bulk operators $\Phi(x,t)$
Boundary operators $\varphi_B(t)$

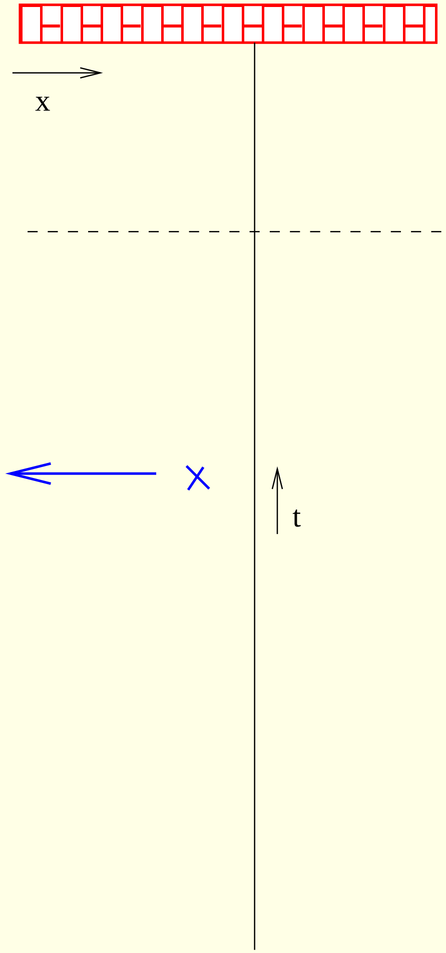
Correlation functions I: bulk operators



One point function ${}_B\langle 0|\Phi(x,t)|0\rangle_B =$

Correlation functions I: bulk operators

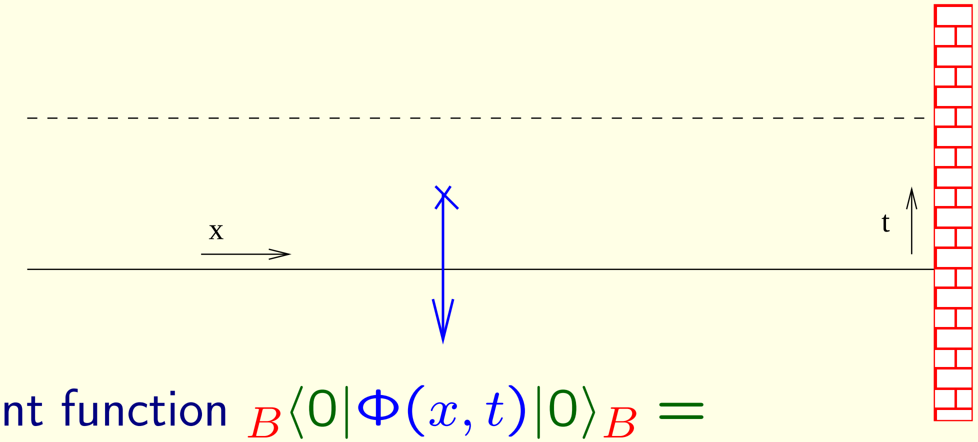




One point function ${}_B\langle 0|\Phi(x,t)|0\rangle_B =$

$$\langle 0|\Phi(x,t)|B\rangle = \sum_{n=0}\langle 0|\Phi(x,t)|n\rangle\langle n|B\rangle$$

Correlation functions I: bulk operators



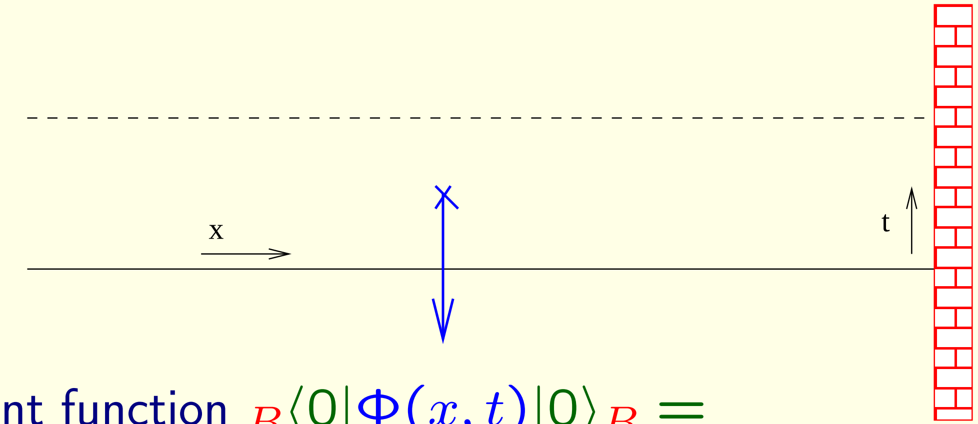
One point function ${}_B\langle 0|\Phi(x,t)|0\rangle_B =$

$$\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{d\theta_1\ldots d\theta_n}{n!(2\pi)^n}\langle 0|\Phi(x,t)|\theta_1,\ldots,\theta_n\rangle\langle\theta_1,\ldots,\theta_n|B\rangle e^{-mx\sum_{i=1}^{n}\cosh\theta_i}$$

$\updownarrow$ Bulk form factor

$\updownarrow$ boundary state

Correlation functions I: bulk operators



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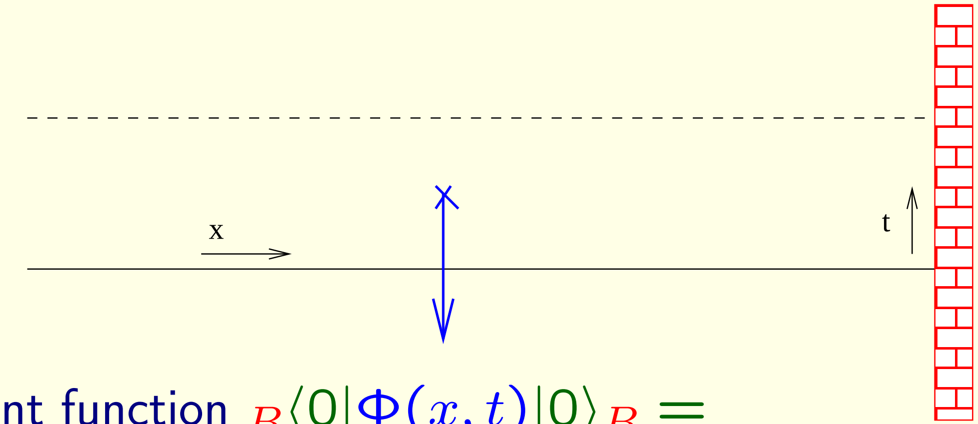
Expansion for large x using bulk form factors and the boundary state

Correlation functions I: bulk operators



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$\updownarrow$ Bulk form factor — $\updownarrow$ boundary state

Expansion for large x using bulk form factors and the boundary state

Boundary operators $x=0$ $\longrightarrow$ new (boundary) technic is needed

Correlation functions II: boundary operators

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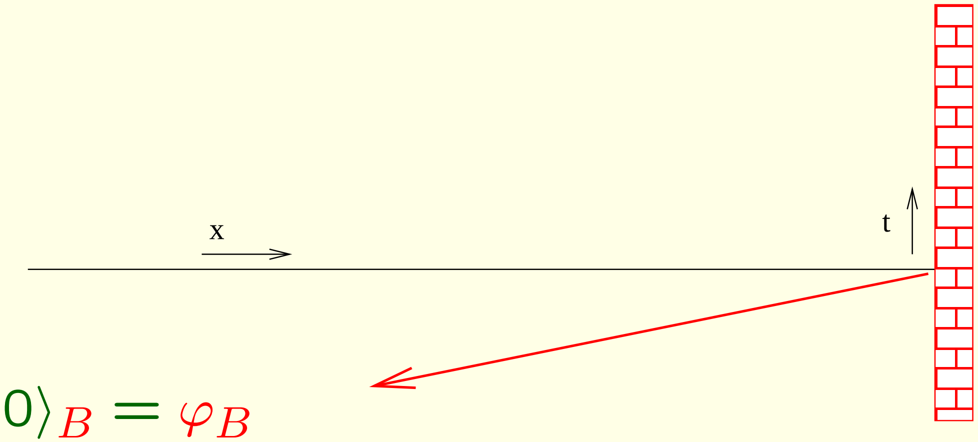
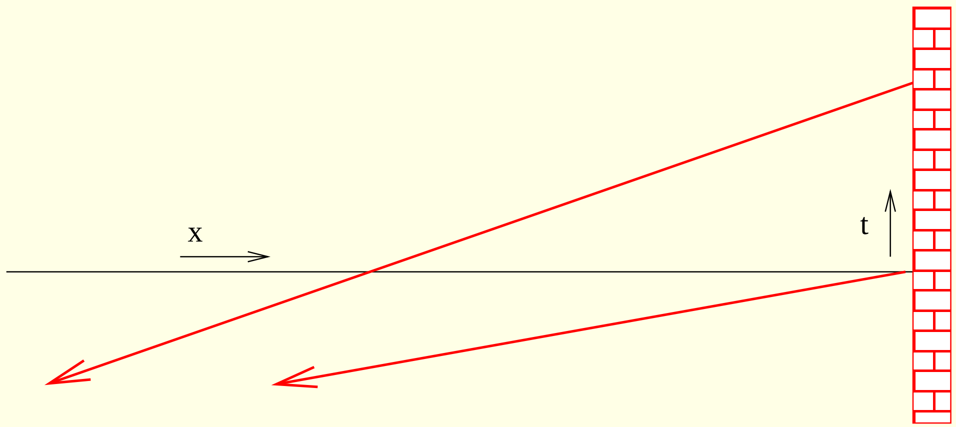


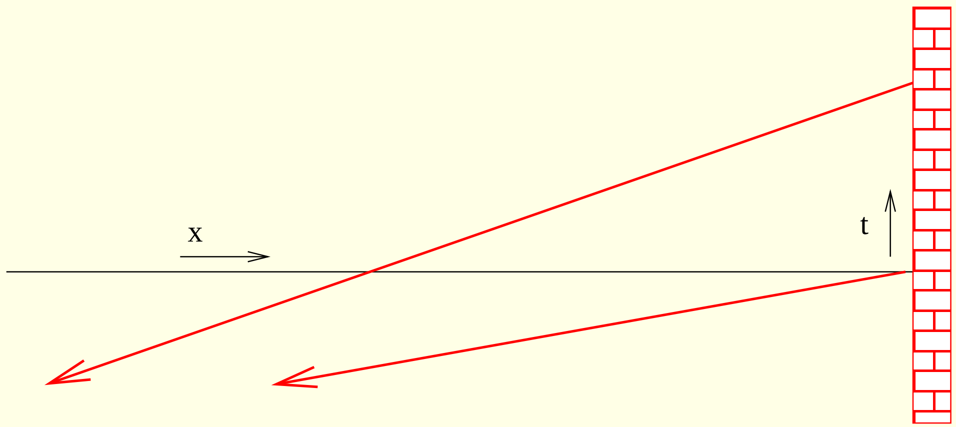
Boundary one point function ${}_B\langle 0|\varphi_B(t)|0\rangle_B = \varphi_B$

Correlation functions II: boundary operators



Boundary two point function $\qquad {}_B\langle 0|\varphi_B(t)\varphi_B(0)|0\rangle_B =$

Correlation functions II: boundary operators



Boundary two point function $\quad {}_B\langle 0|\varphi_B(t)\varphi_B(0)|0\rangle_B =$

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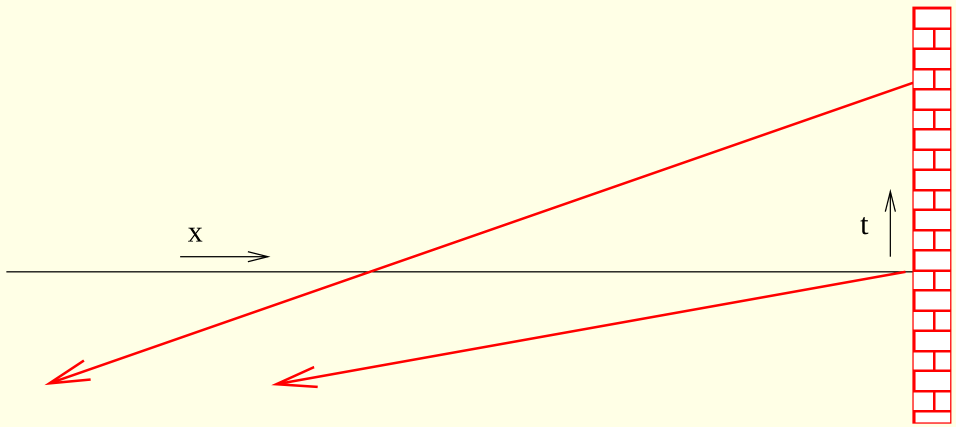
Correlation functions II: boundary operators



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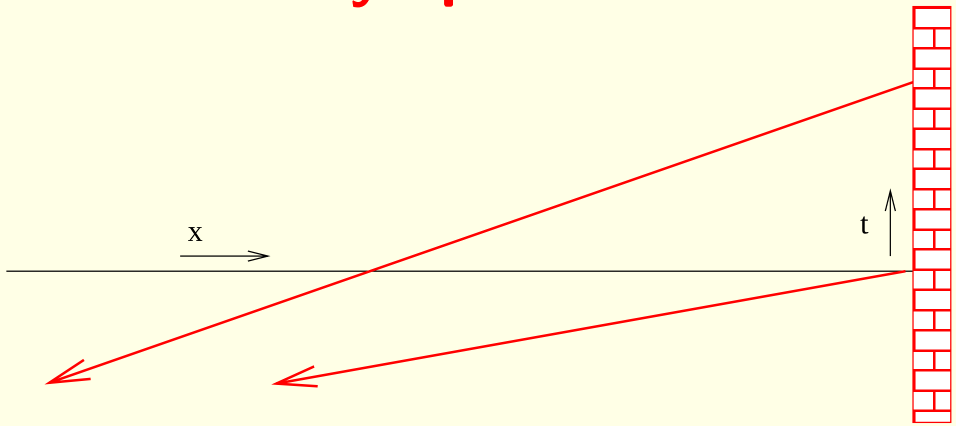
time dependence:

$$\sum_{n=0}^{\infty}\int_0^{\infty}\frac{d\theta_1\ldots d\theta_n}{n!(2\pi)^n}|F_n^{\varphi_B}(\theta_1,\ldots,\theta_n)|^2 e^{-mt\sum_{i=1}^{n}\cosh\theta_i}$$

Correlation functions II: boundary operators



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boundary form factor

$$\boxed{F_n^{\varphi_B}(\theta_1,\ldots,\theta_n) = {}_B\langle 0|\varphi_B(t)|\theta_1,\ldots,\theta_n\rangle_B^{in}}$$

Boundary correlation functions, large t expansion$\longrightarrow$ boundary form factors |>

Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

$$v_i = \sinh \theta_i$$

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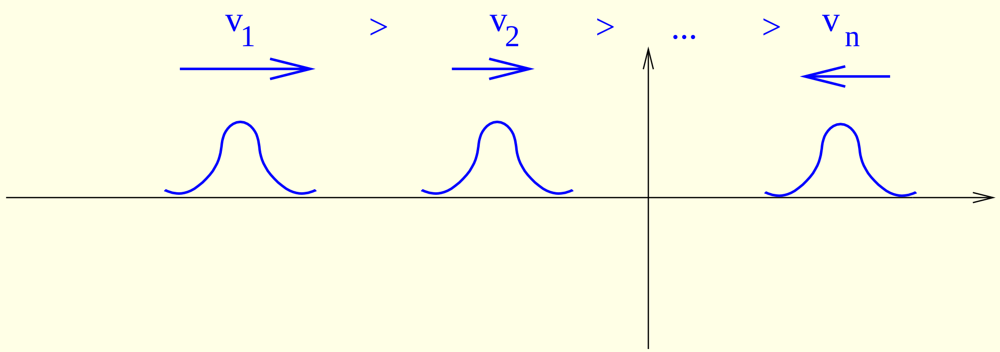
Bulk multiparticle state: with n particles



Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

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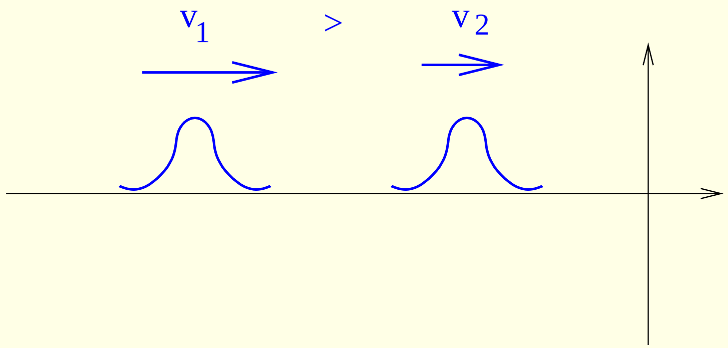
Bulk two particle state:



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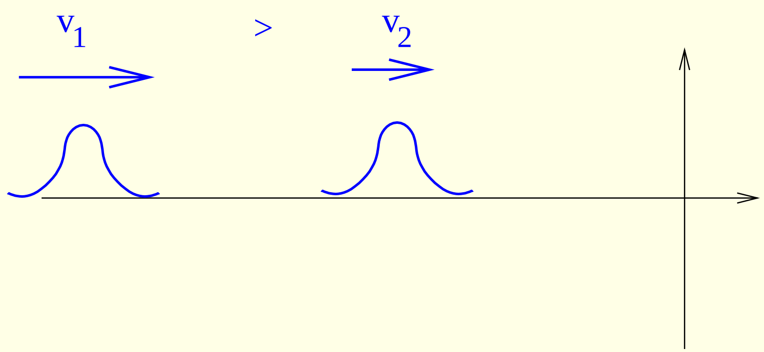
Bulk two particle in state: $t \rightarrow -\infty$



Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

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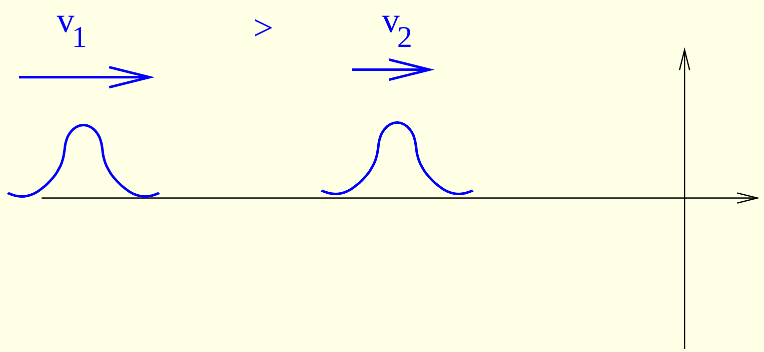
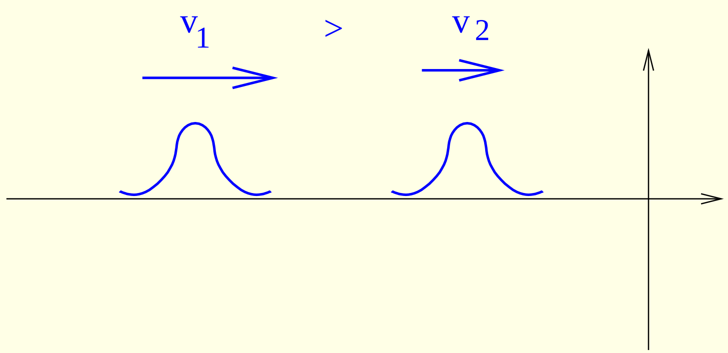
Bulk two particle in state: $t \to -\infty$



times develop



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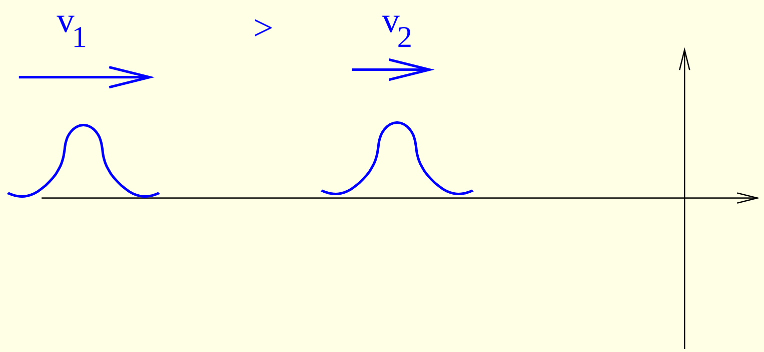
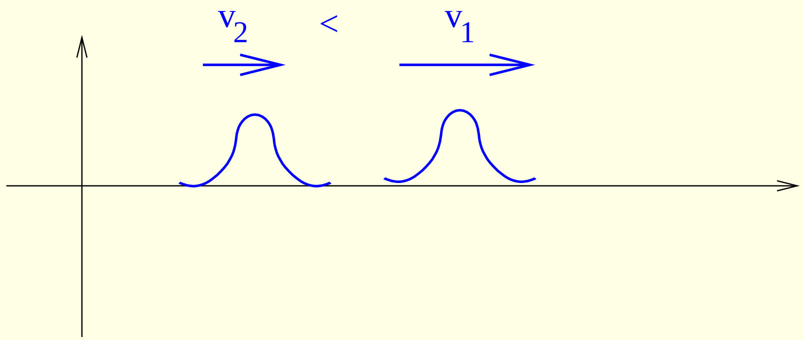


times develop further



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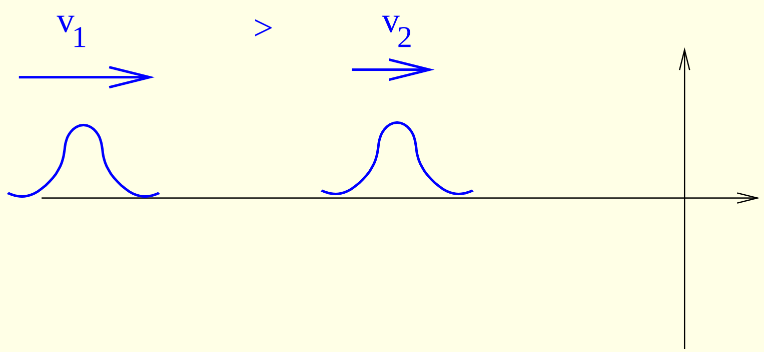
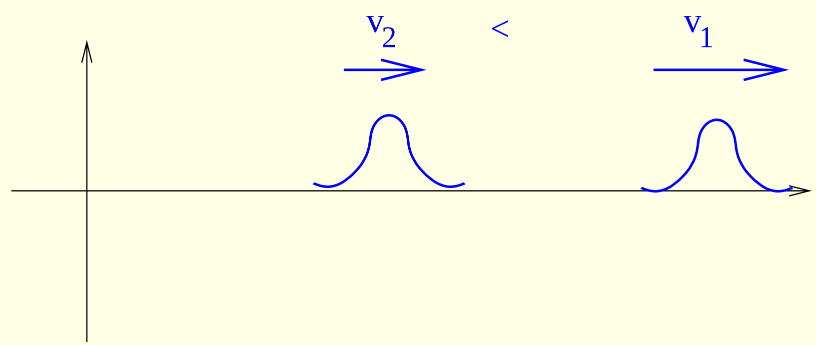


Bulk two particle out state: $t \to \infty$



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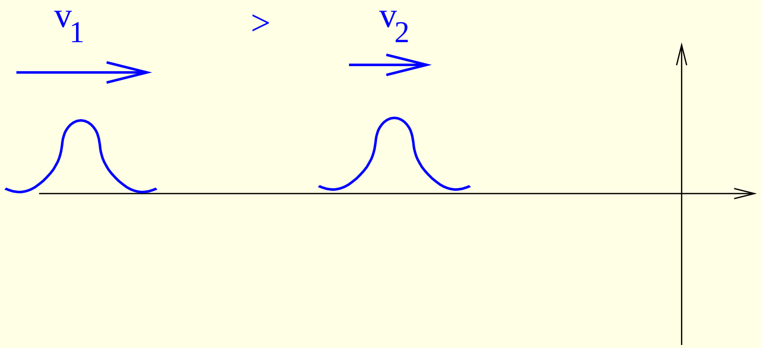
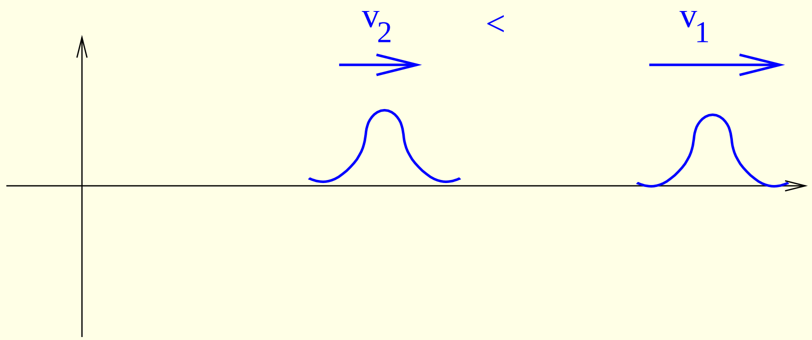


Free, noninteracting in particles

Bulk two particle out state: $t \to \infty$



Free, noninteracting out particles

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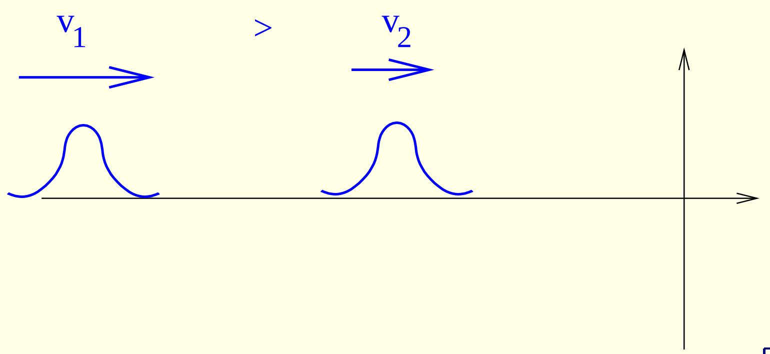

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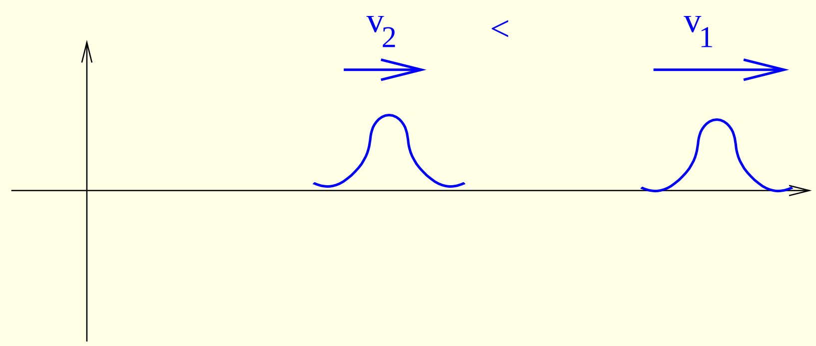

Free, noninteracting in particles $\longleftarrow$ **S-matrix** $\longrightarrow$ Free, noninteracting out particles

Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

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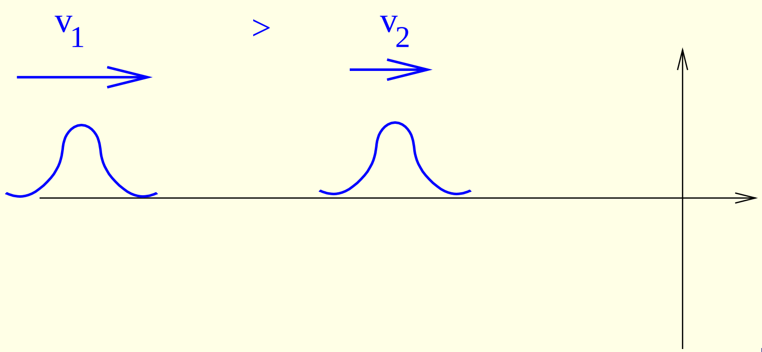
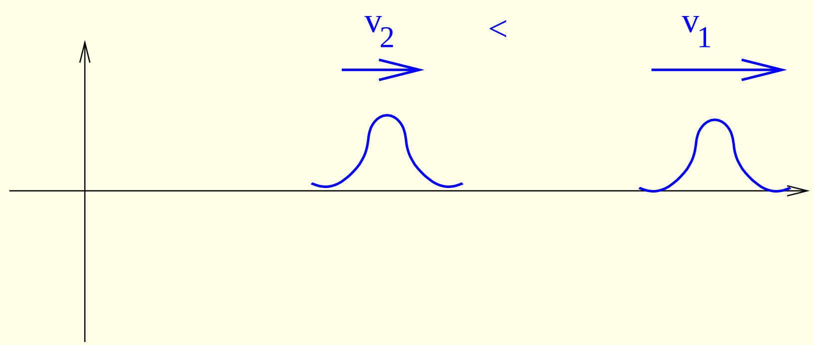
Bulk two particle in state: $t \to -\infty$



Bulk two particle out state: $t \to \infty$

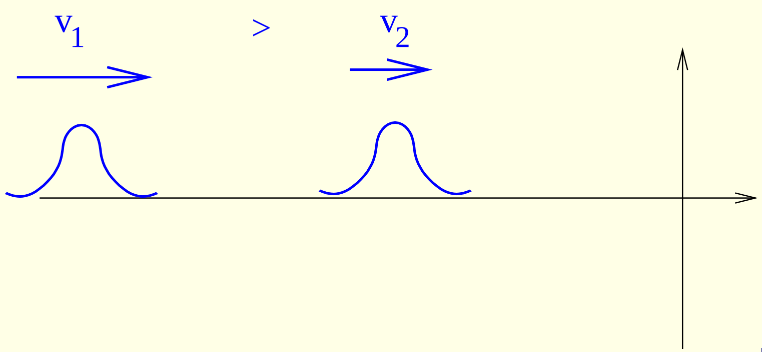


Free, noninteracting in particles $\longleftarrow$ **S-matrix** $\longrightarrow$ Free, noninteracting out particles

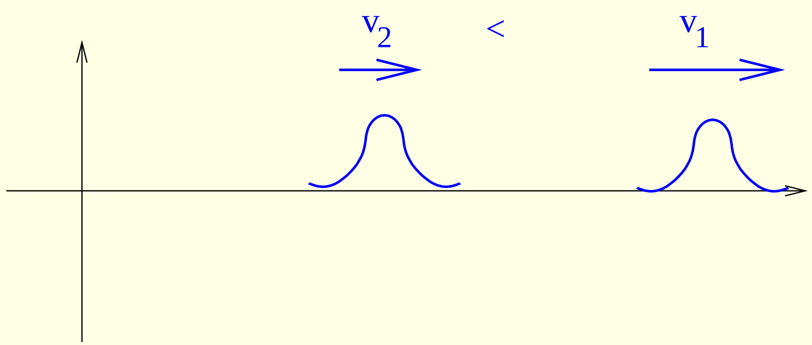
$$|\theta_1, \theta_2\rangle^{in} \quad = \quad S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$ $\boxed{v_i = \sinh\theta_i}$

Bulk two particle in state: $t \to -\infty$



Bulk two particle out state: $t \to \infty$



Free, noninteracting in particles $\leftarrow$ **S-matrix** $\rightarrow$ Free, noninteracting out particles

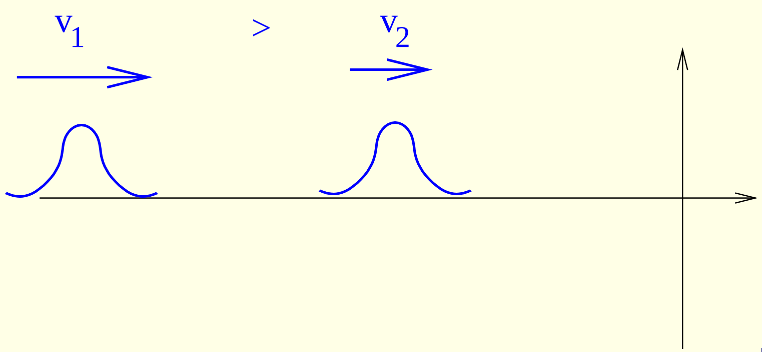
$$|\theta_1, \theta_2\rangle^{in} = S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

$$\boxed{\theta_1 > \theta_2}$$

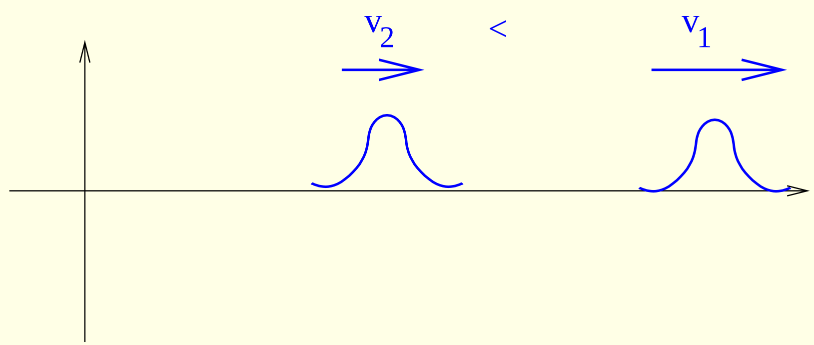
Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

$v_i = \sinh\theta_i$

Bulk two particle in state: $t \to -\infty$



Bulk two particle out state: $t \to \infty$



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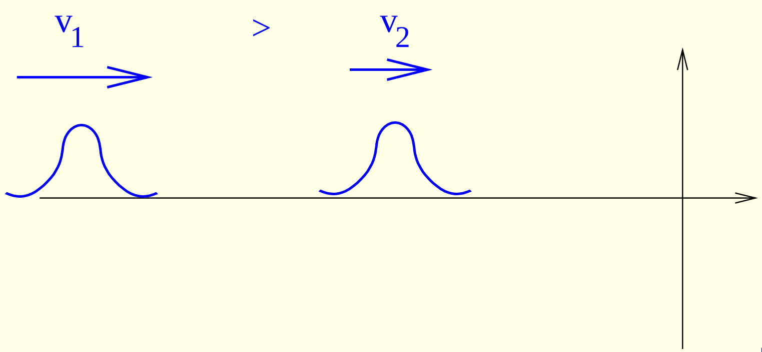
Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$

$v_i = \sinh\theta_i$

Bulk two particle in state: $t \to -\infty$



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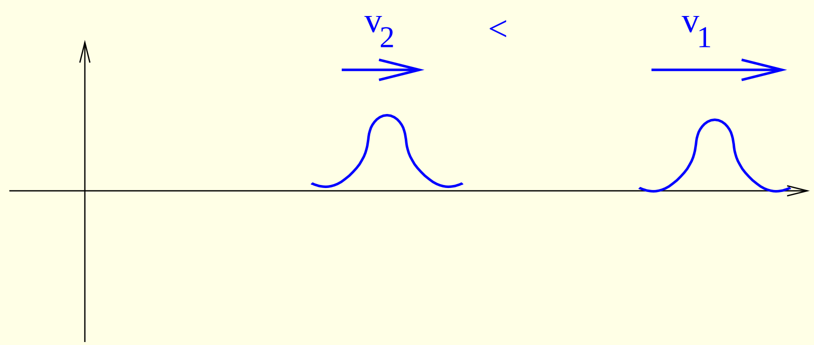

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$\theta_1 > \theta_2$

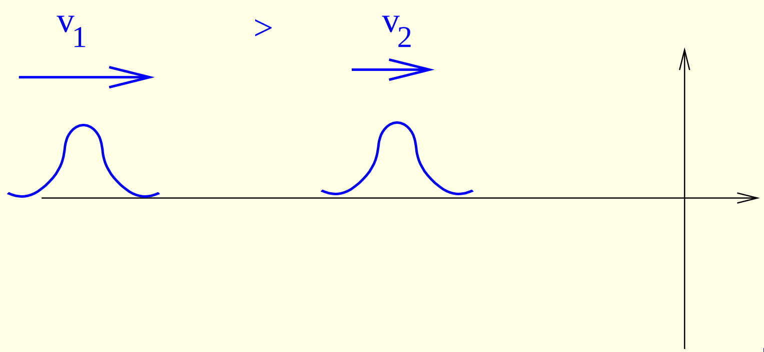
$$|\theta_1, \theta_2\rangle = S(\theta_1 - \theta_2)|\theta_2, \theta_1\rangle$$

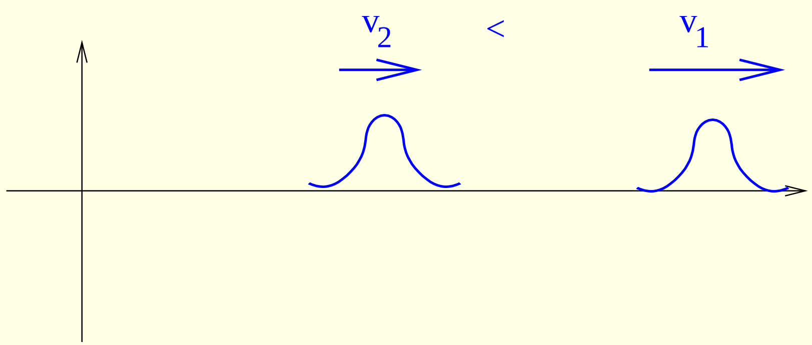
Unitarity $\quad S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2)$

Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$ $\boxed{v_i = \sinh\theta_i}$

Bulk two particle in state: $t \rightarrow -\infty$

Bulk two particle out state: $t \rightarrow \infty$





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$$\boxed{\theta_1 > \theta_2}$$

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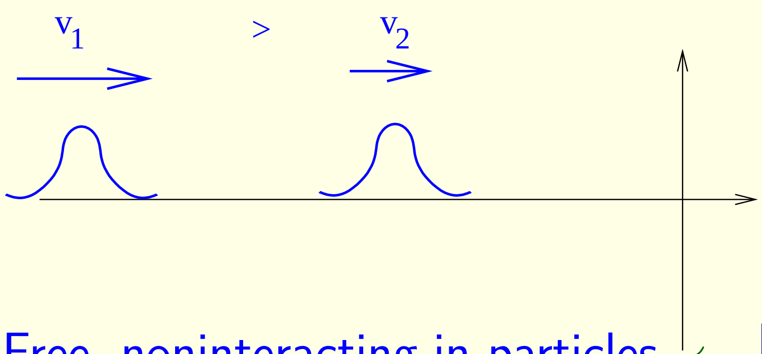
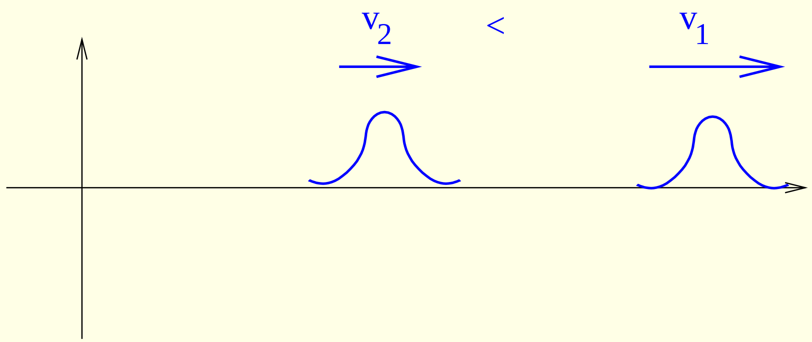
Unitarity $\quad S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$

Asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle^{in}$ $\boxed{v_i = \sinh \theta_i}$

Bulk two particle in state: $t \to -\infty$



Bulk two particle out state: $t \to \infty$



Free, noninteracting in particles $\longleftarrow$ **S-matrix** $\longrightarrow$ Free, noninteracting out particles

$$|\theta_1, \theta_2\rangle^{in} = S(|\theta_1 - \theta_2|)|\theta_1, \theta_2\rangle^{out}$$

$$\boxed{\theta_1 > \theta_2}$$

$$|\theta_1, \theta_2\rangle = S(\theta_1 - \theta_2)|\theta_2, \theta_1\rangle$$

Unitarity $\quad S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$

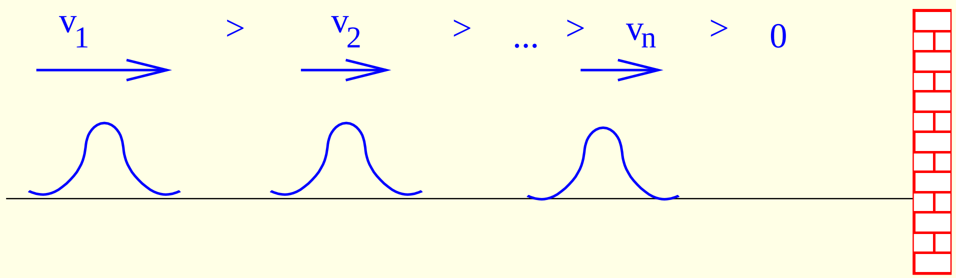
Integrability $\longrightarrow$ factorizability:

$$|\theta_1, \theta_2, \ldots, \theta_n\rangle = \prod_{i<j} S(\theta_i - \theta_j)|\theta_n, \theta_{n-1}, \ldots, \theta_1\rangle$$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

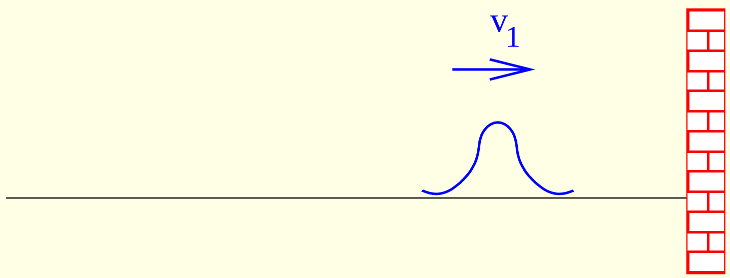
Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary multiparticle state: with n particles



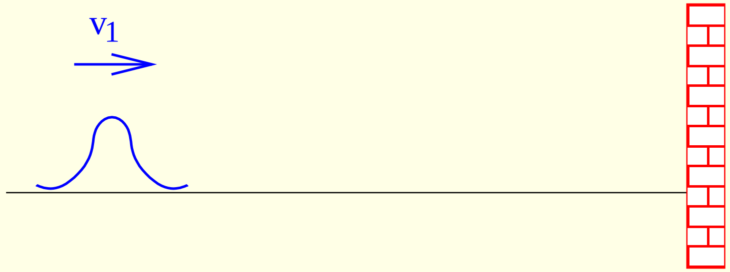
Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle state:



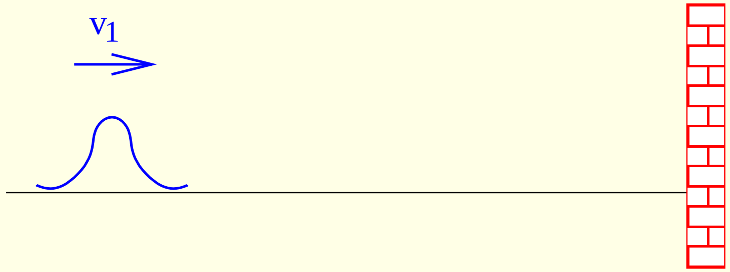
Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \longrightarrow -\infty$

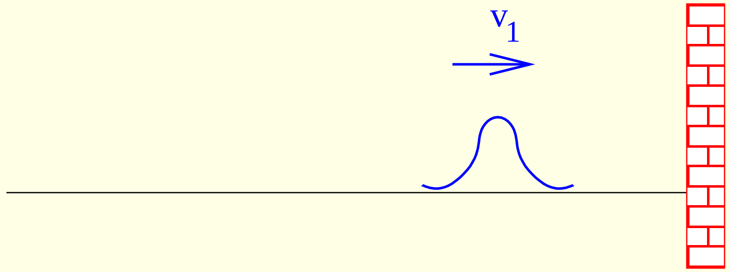


Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \longrightarrow -\infty$

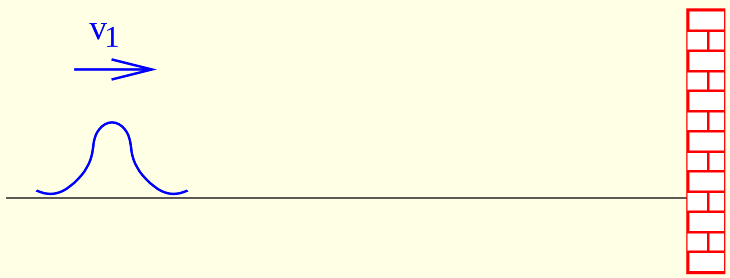


times develop

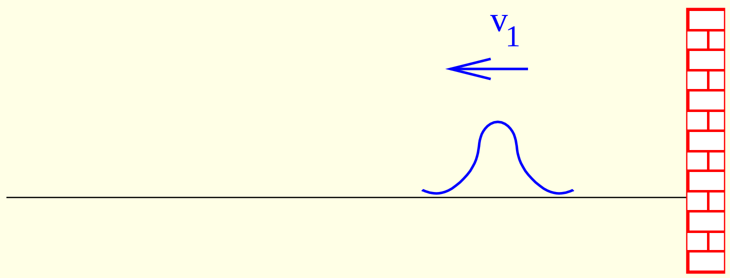


Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \longrightarrow -\infty$

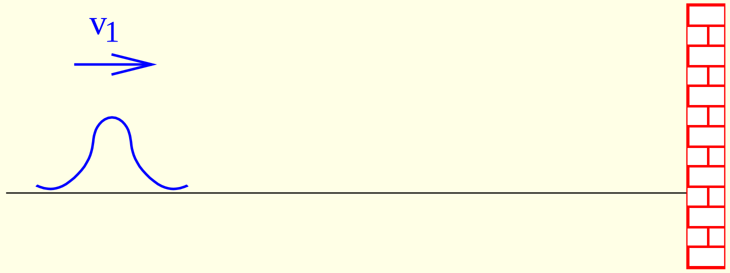


times develop further



Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

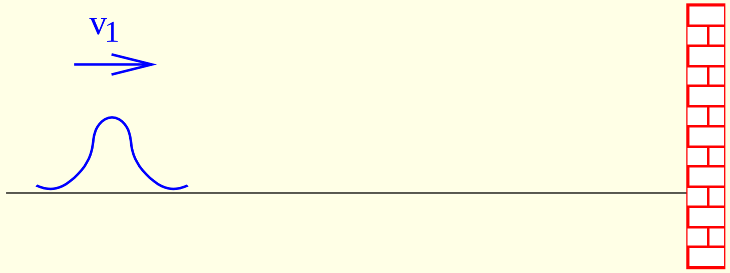
Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Free in particle

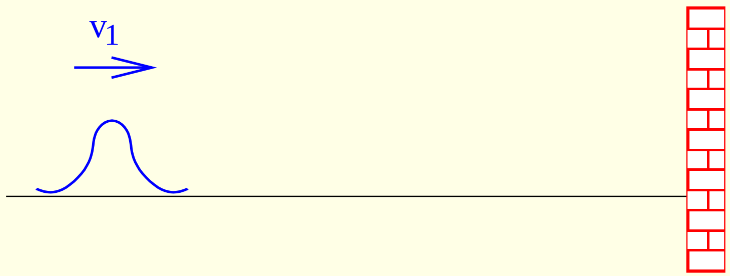
Boundary one pt out state: $t \to \infty$

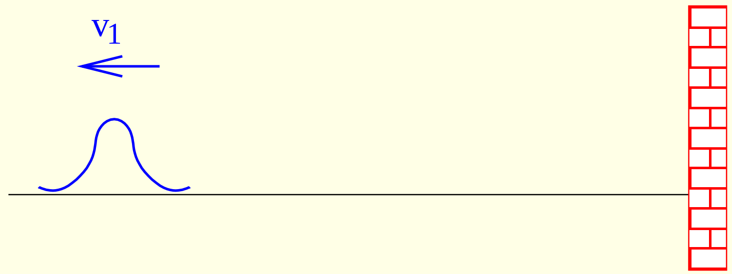
Free out particle

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \rightarrow -\infty$

Boundary one pt out state: $t \rightarrow \infty$

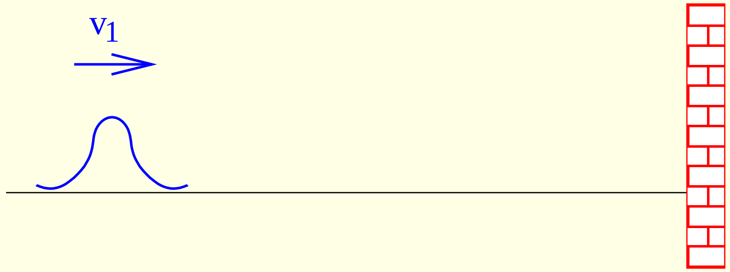




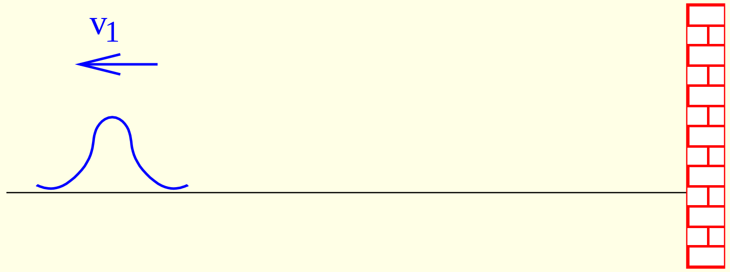
Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$

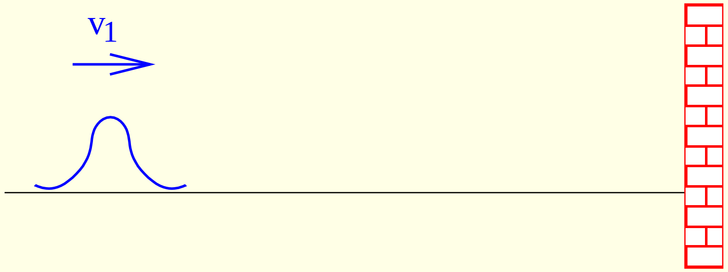


Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

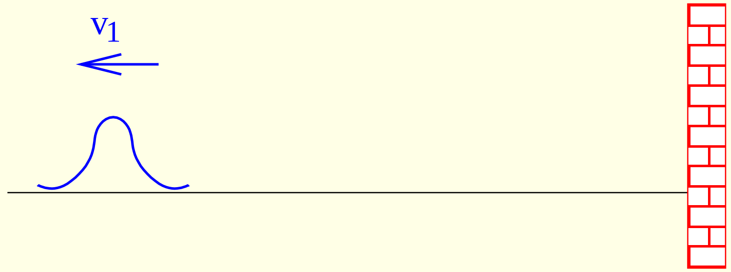
$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$



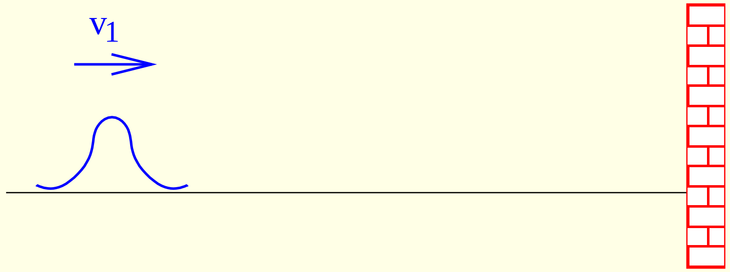
Free in particle ← **R-matrix** → Free out particle

$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

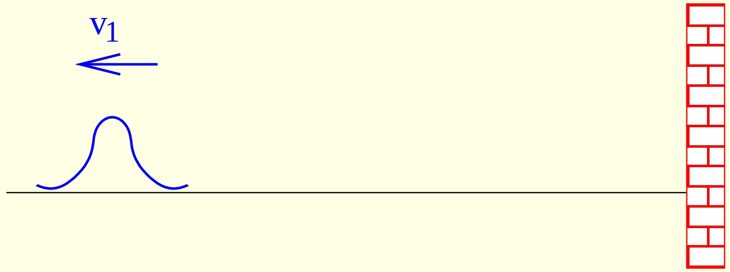
$\theta_1 > 0$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$



Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

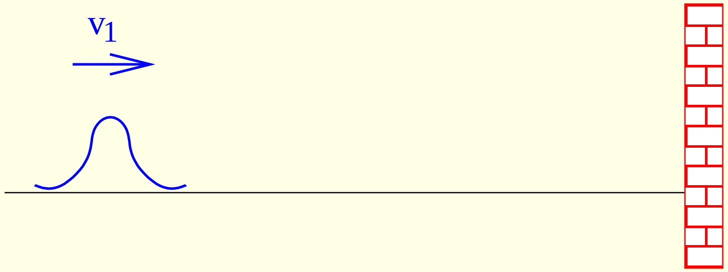
$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

$$\boxed{\theta_1 > 0}$$

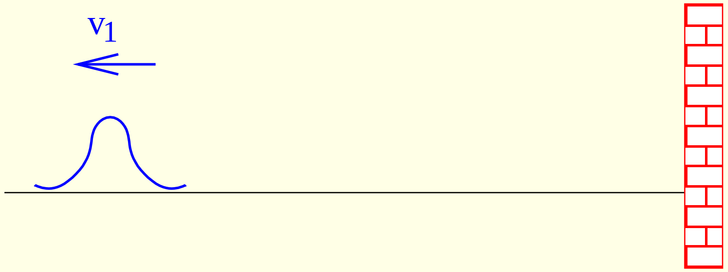
$$|\theta_1\rangle_B = R(\theta_1)|-\theta_1\rangle_B$$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$



Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

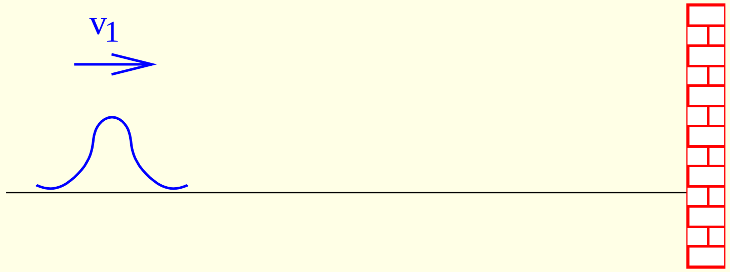
$$\boxed{\theta_1 > 0}$$

$$|\theta_1\rangle_B = R(\theta_1)|-\theta_1\rangle_B$$

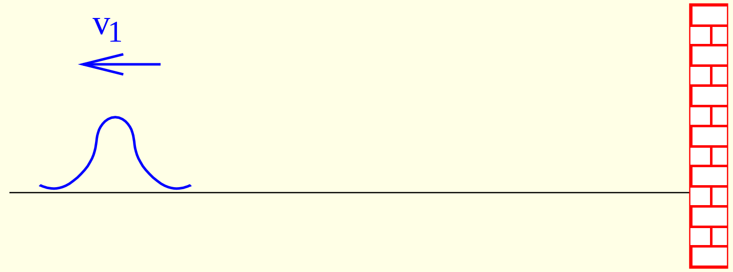
Unitarity: $R^*(\theta) = R^{-1}(\theta)$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$



Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

$$\theta_1 > 0$$

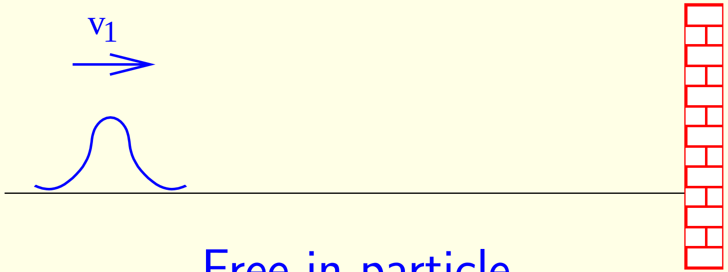
$$|\theta_1\rangle_B = R(\theta_1)|-\theta_1\rangle_B$$

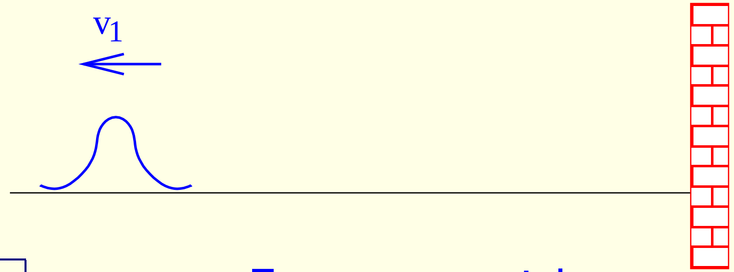
Unitarity: $R^*(\theta) = R^{-1}(\theta) = R(-\theta)$

Boundary asymptotic states: $|\theta_1, \theta_2, \ldots, \theta_n\rangle_B^{in}$

Boundary one particle in state: $t \to -\infty$

Boundary one pt out state: $t \to \infty$





Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

$$|\theta_1\rangle_B^{in} = R(|\theta_1|)|\theta_1\rangle_B^{out}$$

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$$|\theta_1\rangle_B = R(\theta_1)|-\theta_1\rangle_B$$

Unitarity: $R^*(\theta) = R^{-1}(\theta) = R(-\theta)$

Integrability $\longrightarrow$ factorizability:

$$|\theta_1, \theta_2, \ldots, \theta_n\rangle_B = \prod_{i<j} S(\theta_i - \theta_j) \prod_i R(\theta_i)|-\theta_1, -\theta_2, \ldots, -\theta_n\rangle_B$$

!>

Analytic structure: reduction formula

Unitarity $\quad S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of
S matrix

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of
S matrix

known: analytic structure of
correlators

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of S matrix	$\leftrightarrow$	reduction formula	$\leftrightarrow$	known: analytic structure of correlators

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of	↔	reduction formula	↔	known: analytic structure of
S matrix				correlators

$$< \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in > = \delta(\theta_1 - \theta) \langle \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in \setminus \theta >$$

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

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$$< \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(x,t) | in > = \delta(\theta_1 - \theta) \langle \theta_2, \dots, \theta_n | \mathcal{O}(x,t) | in \setminus \theta >$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{\infty} dx' e^{ip(\theta_1)x'} \{\partial_{t'}^2 - \partial_{x'}^2 + m^2\}$$

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of S matrix	$\leftrightarrow$	reduction formula	$\leftrightarrow$	known: analytic structure of correlators

$$< \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in > = \delta(\theta_1 - \theta) \langle \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in \setminus \theta >$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{\infty} dx' e^{ip(\theta_1)x'} \{\partial^2_{t'} - \partial^2_{x'} + m^2\}$$

$$< \theta_2, \ldots, \theta_n | T(\Phi(x', t') \mathcal{O}(x,t)) | in >$$

Analytic structure: reduction formula

Unitarity $S^*(\theta_1 - \theta_2) = S^{-1}(\theta_1 - \theta_2) = S(\theta_2 - \theta_1)$ not restrictive enough

need: analytic structure of S matrix	$\leftrightarrow$	reduction formula	$\leftrightarrow$	known: analytic structure of correlators

$$< \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in > = \delta(\theta_1 - \theta) \langle \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in \setminus \theta >$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{\infty} dx' e^{ip(\theta_1)x'} \{\partial_{t'}^2 - \partial_{x'}^2 + m^2\}$$

$$< \theta_2, \ldots, \theta_n | T(\Phi(x', t') \mathcal{O}(x,t)) | in >$$

Crossing $S(\theta_1 - \theta_2) = S(i\pi - \theta_1 + \theta_2)$

Analytic structure: reduction formula

Unitarity $S^*(\theta_1-\theta_2) = S^{-1}(\theta_1-\theta_2) = S(\theta_2-\theta_1)$ not restrictive enough

need: analytic structure of	↔	reduction formula	↔	known: analytic structure of
S matrix				correlators

$$< \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in > = \delta(\theta_1 - \theta) \langle \theta_2, \ldots, \theta_n | \mathcal{O}(x,t) | in \setminus \theta >$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{\infty} dx' e^{ip(\theta_1)x'} \{\partial_{t'}^2 - \partial_{x'}^2 + m^2\}$$

$$< \theta_2, \ldots, \theta_n | T(\Phi(x', t')\mathcal{O}(x,t)) | in >$$

Crossing $S(\theta_1-\theta_2) = S(i\pi - \theta_1 + \theta_2)$

perturbed Lee Yang model $S_{LY}(\theta) = \frac{\sinh\theta + i \sin\frac{\pi}{3}}{\sinh\theta - i\sin\frac{\pi}{3}}$

Analytic structure: reduction formula

Unitarity $S^*(\theta_1-\theta_2)=S^{-1}(\theta_1-\theta_2)=S(\theta_2-\theta_1)$ not restrictive enough

need: analytic structure of S matrix $\leftrightarrow$ reduction formula $\leftrightarrow$ known: analytic structure of correlators

$$<\theta_1,\theta_2,\dots,\theta_n|\mathcal{O}(x,t)|in> = \delta(\theta_1-\theta)\langle\theta_2,\dots,\theta_n|\mathcal{O}(x,t)|in\setminus\theta>$$

$$iZ^{-1/2}\int_{-\infty}^{\infty}dt'\,e^{-i\omega(\theta_1)t'}\int_{-\infty}^{\infty}dx'\,e^{ip(\theta_1)x'}\{\partial_{t'}^2-\partial_{x'}^2+m^2\}$$

$$<\theta_2,\dots,\theta_n|T(\Phi(x',t')\mathcal{O}(x,t))|in>$$

Crossing $S(\theta_1-\theta_2)=S(i\pi-\theta_1+\theta_2)$

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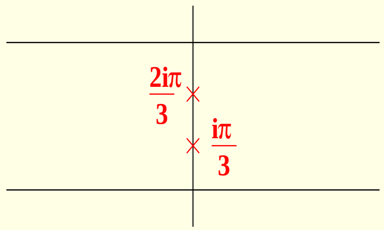

Analytic structure: reduction formula

Unitarity $S^*(\theta_1-\theta_2)=S^{-1}(\theta_1-\theta_2)=S(\theta_2-\theta_1)$ not restrictive enough

need: analytic structure of S matrix $\leftrightarrow$ reduction formula $\leftrightarrow$ known: analytic structure of correlators

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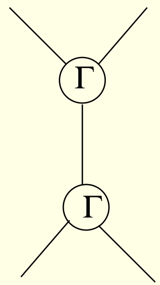
Crossing $S(\theta_1-\theta_2)=S(i\pi-\theta_1+\theta_2)$



perturbed Lee Yang model $S_{LY}(\theta)=\frac{\sinh\theta+i\sin\frac{\pi}{3}}{\sinh\theta-i\sin\frac{\pi}{3}}$

Analytic structure: boundary reduction formula

Unitarity $\quad R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of
R matrix

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of
R matrix

known: analytic structure of
boundary correlators

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of R matrix	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	known: analytic structure of boundary correlators

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of		boundary reduction formula		known: analytic structure of
R matrix	$\leftrightarrow$		$\leftrightarrow$	boundary correlators

$$ {}_B < \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta)\; {}_B < \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B $$

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	known: analytic structure of
R matrix				boundary correlators

$${}_B < \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta) \, {}_B < \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B$$

$$i2Z^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{ \partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'} \}$$

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	known: analytic structure of
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$$ {}_B < \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta) \, {}_B < \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B $$

$$ i2Z^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{ \partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'} \} $$

$$ {}_B < \theta_2, \dots, \theta_n | T(\Phi(x', t') \mathcal{O}(t)) | in >_B $$

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of R matrix	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	known: analytic structure of boundary correlators

$$
{}_B < \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta)\, {}_B < \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B
$$

$$
i2Z^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{\partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'}\}
$$

$$
{}_B < \theta_2, \ldots, \theta_n | T(\Phi(x', t')\mathcal{O}(t)) | in >_B
$$

Boundary crossing $R(\theta_1) = S(i\pi - \theta_1) R(i\pi - \theta_1)$

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of R matrix	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	known: analytic structure of boundary correlators

$$_B < \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta)\, _B < \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B$$

$$i2Z^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{\partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'}\}$$

$$_B < \theta_2, \dots, \theta_n | T(\Phi(x', t')\mathcal{O}(t)) | in >_B$$

Boundary crossing $R(\theta_1) = S(i\pi - \theta_1) R(i\pi - \theta_1)$

perturbed boundary Lee Yang

$$S(\theta) = -\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = -\left[\frac{1}{3}\right] \quad , \quad [x] = (x)(1-x) \quad ; \quad (x) = \frac{\sinh\left(\frac{\theta}{2} + \frac{i\pi x}{2}\right)}{\sinh\left(\frac{\theta}{2} - \frac{i\pi x}{2}\right)}$$

Analytic structure: boundary reduction formula

Unitarity $\quad R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of				known: analytic structure of
R matrix	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	boundary correlators

$$
{}_B\langle \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \rangle_B = \delta(\theta_1 - \theta)\, {}_B\langle \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \setminus \theta \rangle_B
$$

$$
i2Z^{-1/2} \int_{-\infty}^{\infty} dt'\, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{\partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'}\}
$$

$$
{}_B\langle \theta_2, \dots, \theta_n | T(\Phi(x', t')\mathcal{O}(t)) | in \rangle_B
$$

Boundary crossing $\quad R(\theta_1) = S(i\pi - \theta_1) R(i\pi - \theta_1)$

perturbed boundary Lee Yang

$$
S(\theta) = -\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = -\left[\frac{1}{3}\right] \quad , \quad [x] = (x)(1-x) \quad ; \quad (x) = \frac{\sinh\left(\frac{\theta}{2} + \frac{i\pi x}{2}\right)}{\sinh\left(\frac{\theta}{2} - \frac{i\pi x}{2}\right)}
$$

$$
R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\left(-\frac{2}{3}\right)
$$

Analytic structure: boundary reduction formula

Unitarity $R^*(\theta_1) = R^{-1}(\theta_1) = R(-\theta_1)$ not restrictive enough

need: analytic structure of R matrix $\leftrightarrow$ boundary reduction formula $\leftrightarrow$ known: analytic structure of boundary correlators

$$
{}_B < \theta_1, \theta_2, \dots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta)\; {}_B < \theta_2, \dots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B
$$

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$$

$$
R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\left(-\frac{2}{3}\right)\left[\frac{b+1}{6}\right]\left[\frac{b-1}{6}\right]
$$

Analytical properties of the bulk form factors

Bulk form factor

$$\boxed{\theta_1 > \theta_2 > \ldots > \theta_n}$$

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Crossing from reduction formula

$$\begin{aligned} F_{1n}^{\mathcal{O}}(\theta|\theta_1, \ldots, \theta_n) &= F_{n+1}^{\mathcal{O}}(\theta_1, \ldots, \theta_n, \theta - i\pi) \\ &\quad + \delta(\theta - \theta_n) F_{n-1}^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}) \end{aligned}$$

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!>

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$$F_1^{\mathcal{O}}(\theta_1) = _B\langle 0|\mathcal{O}(0)|\theta_1\rangle_B = R(\theta_1)\, _B\langle 0|\mathcal{O}(0)|-\theta_1\rangle_B$$

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Reflection

$$F_{m1}^{\mathcal{O}}(\theta_1', \ldots, \theta_m'|\theta_1) = R(\theta_1) F_{m1}^{\mathcal{O}}(\theta_1', \ldots, \theta_m'| - \theta_1)$$

$$F_{1n}^{\mathcal{O}}(\theta_1'|\theta_1, \ldots, \theta_n) = R(-\theta_1') F_{1n}^{\mathcal{O}}(-\theta_1'|\theta_1, \ldots, \theta_n)$$

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$$F_{1n}^{\mathcal{O}}(\theta'|\theta_1,\ldots,\theta_n) = F_{n+1}^{\mathcal{O}}(\theta'+i\pi,\theta_1,\ldots,\theta_n) + \delta(\theta-\theta_1)F_{n-1}^{\mathcal{O}}(\theta_2,\ldots,\theta_n)$$

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!>

Comparison to other approaches

Consistency eqs.:
derived

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Similar equations in spin models:

XXZ,XYZ	M. Jimbo, R. Kedem, H. Konno T. Miwa, R. Weston	Nucl.Phys. B448 (1995) 429-456
Higher rank XXZ	Y.-H. Quano	Int.J.Mod.Phys. A15 (2000) 3699-3716
Belavin's Z_n-symmetric	Y.-H. Quano	J.Phys. A33 (2000) 8275
$A^{(1)}_{n-1}$ face model	Y.-H. Quano	J.Phys. A34 (2001) 8445-8464

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Equations are DERIVED $\longrightarrow$ valid for any QFT for form factors of local operators!

Bulk form factor axioms

Permutation

$$F_n^{\mathcal{O}}(\theta_1, \ldots \theta_i, \theta_{i+1}, \ldots, \theta_n) =$$
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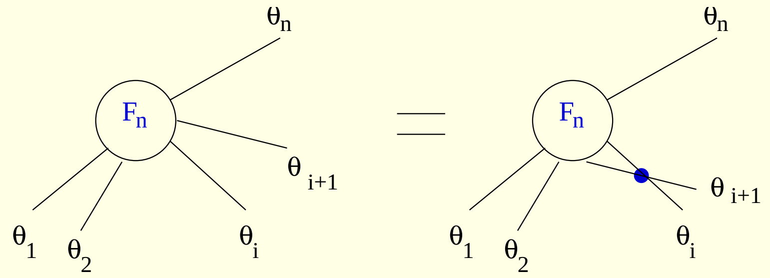

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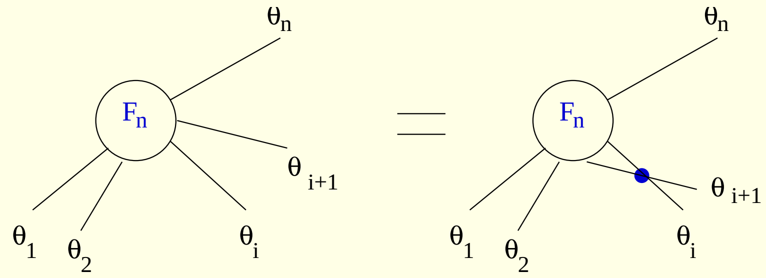

Periodicity

$$F_n^{\mathcal{O}}(\theta_1,\ldots,\theta_{n-1},\theta_n) = F_n^{\mathcal{O}}(\theta_n+2\pi i,\theta_1,\ldots,\theta_{n-1})$$

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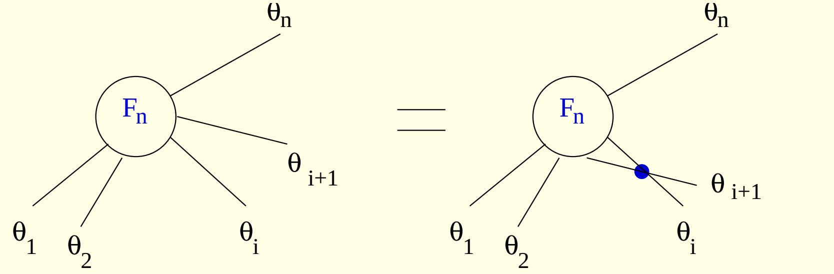

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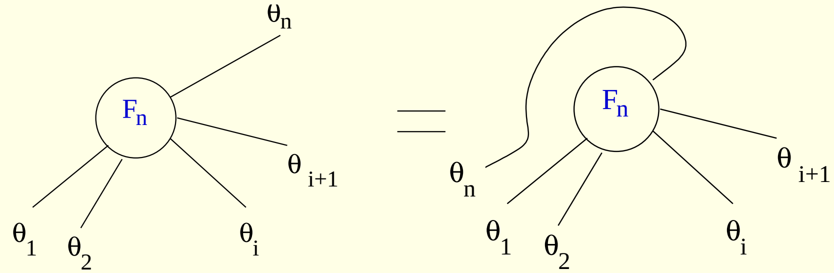
$$F_n^{\mathcal{O}}(\theta_n + 2\pi i, \theta_1, \ldots, \theta_{n-1})$$



Bulk form factor axioms

Permutation

$$F_n^{\mathcal{O}}(\theta_1,\dots\theta_i,\theta_{i+1},\dots,\theta_n) =$$
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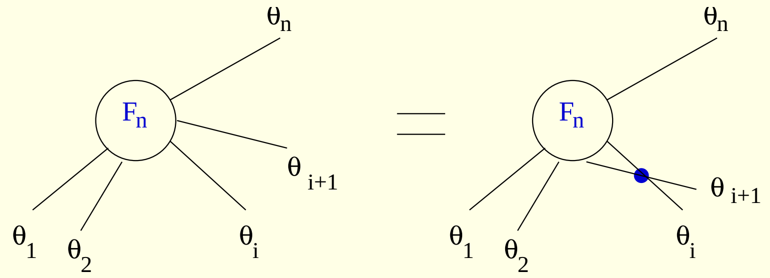

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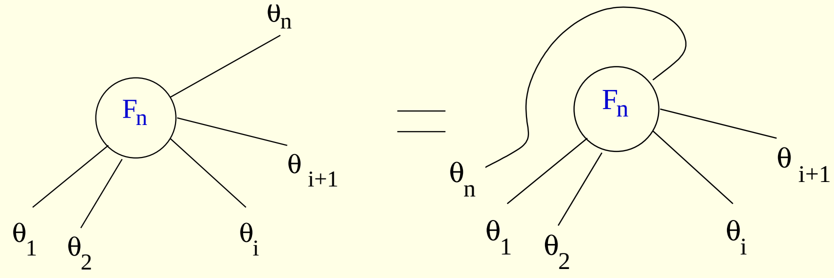

Kinematical singularities

$$i\,\mathrm{res}_{\theta=\theta'}F_{n+2}^{\mathcal{O}}(\theta+i\pi,\theta',\theta_1,\dots,\theta_n) =$$
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Bulk form factor axioms

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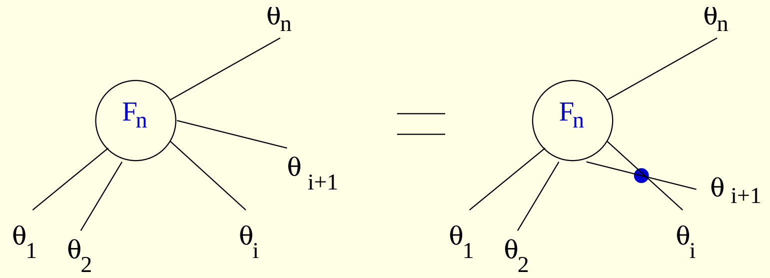

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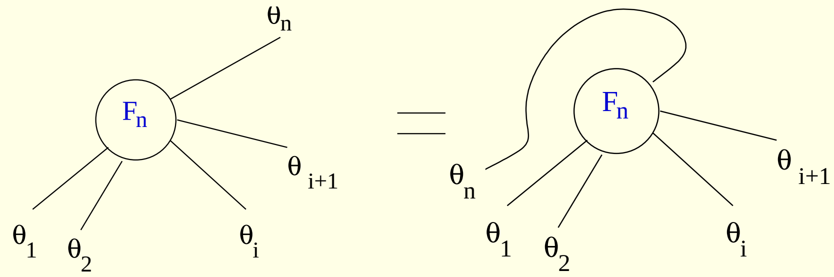

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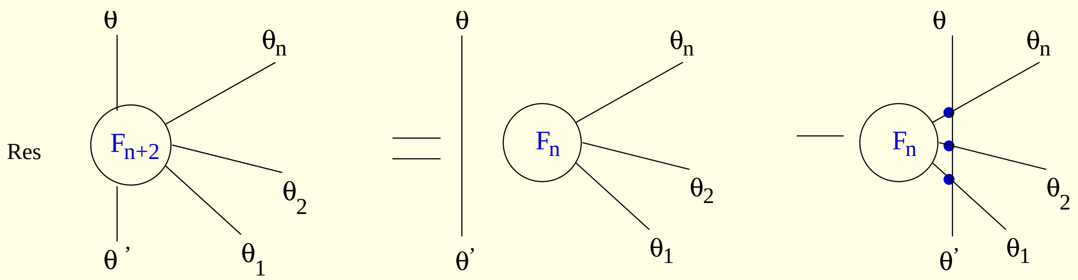

Bulk form factor axioms

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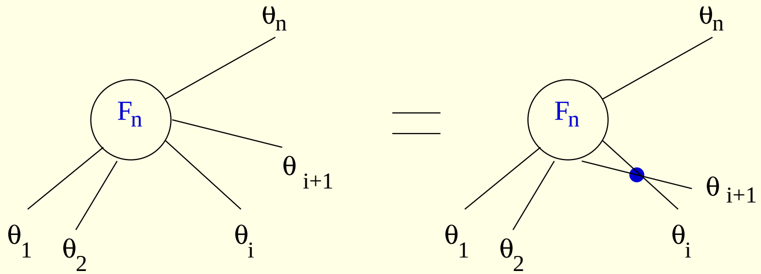

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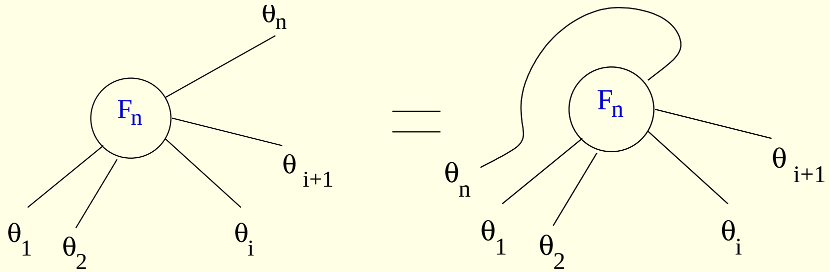

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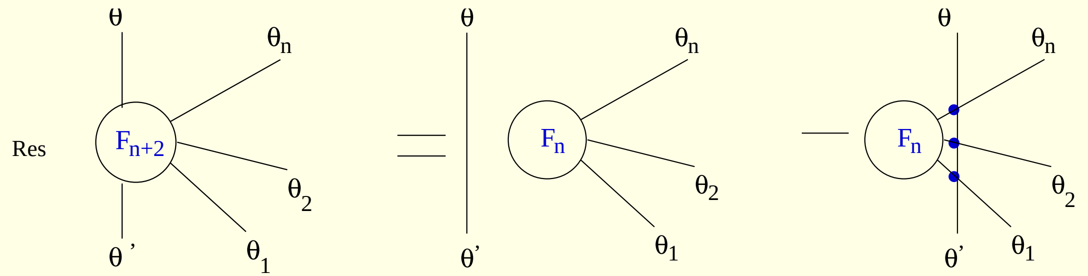

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Bulk form factor axioms

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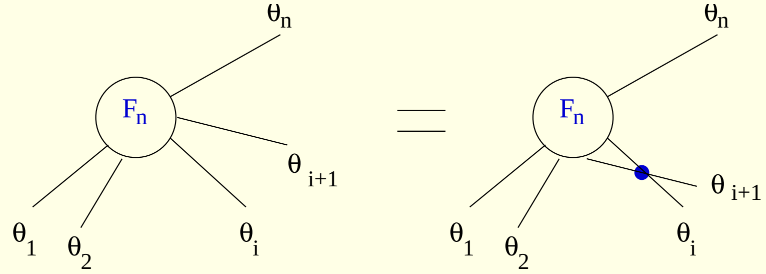

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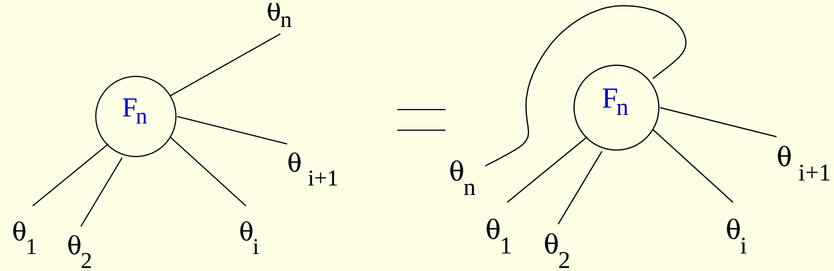

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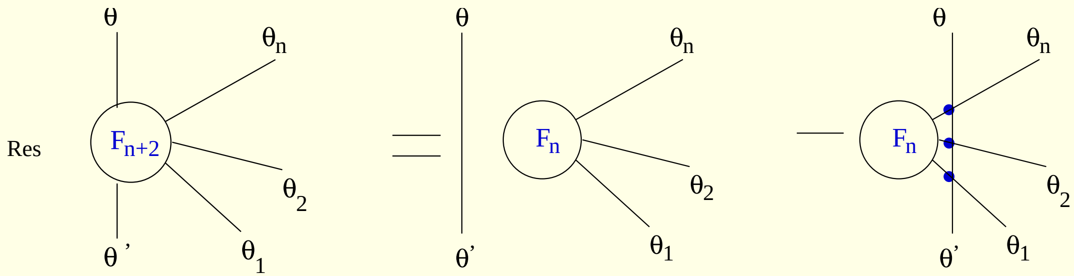

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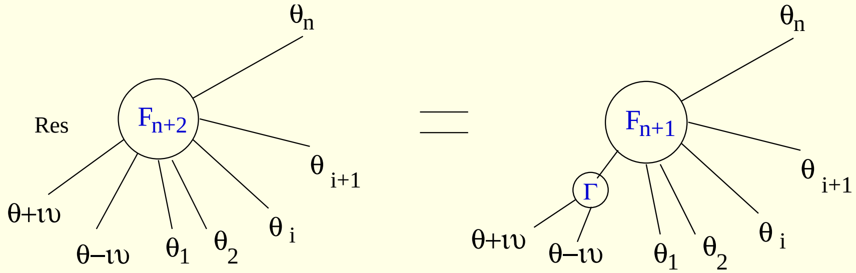

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Boundary form factor axioms I

Reflection

$$F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}, \theta_n) =$$
$$R(\theta_n) F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}, -\theta_n)$$

Boundary form factor axioms I

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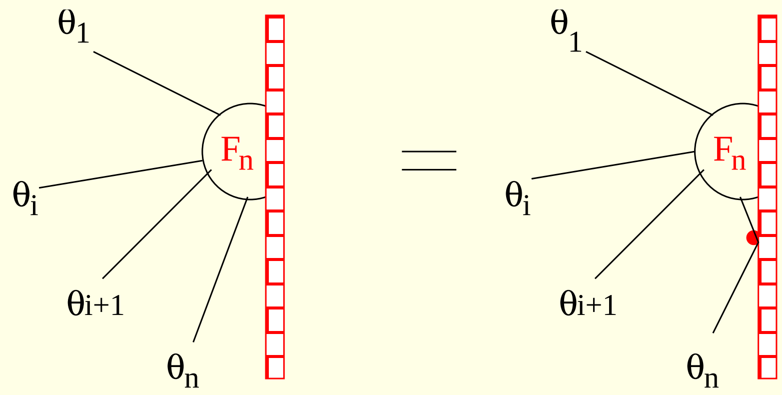

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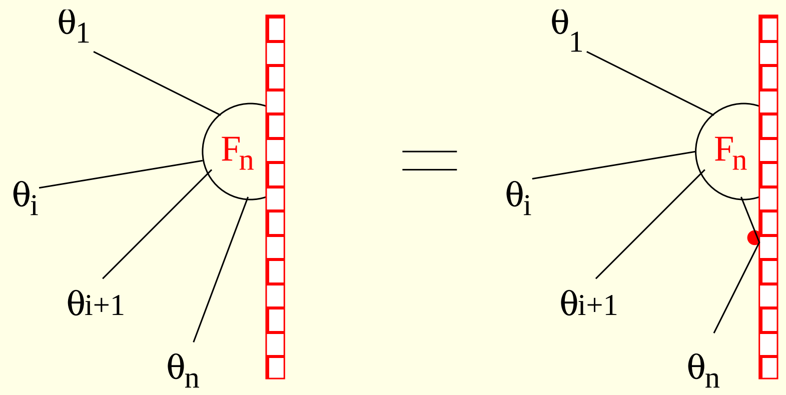

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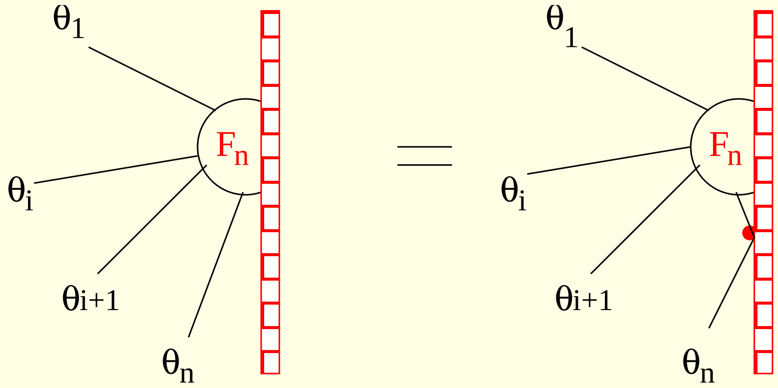

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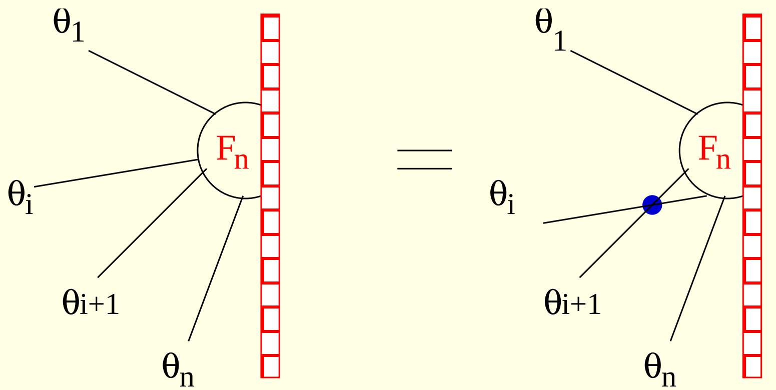

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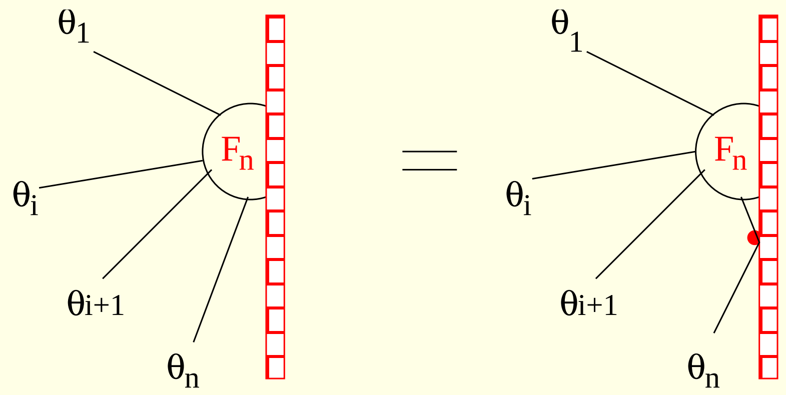

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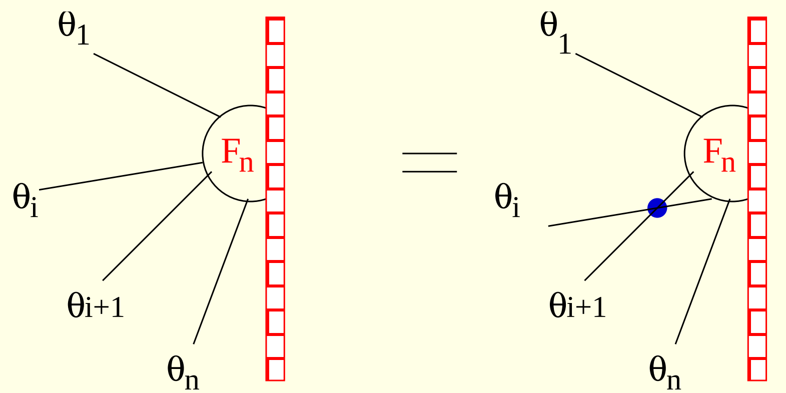

Boundary periodicity

$$F_n^{\mathcal{O}}(\theta_1,\theta_2,\dots,\theta_n) = R(i\pi-\theta_1)F_n^{\mathcal{O}}(2i\pi-\theta_1,\theta_2,\dots,\theta_n)$$

Boundary form factor axioms I

Reflection

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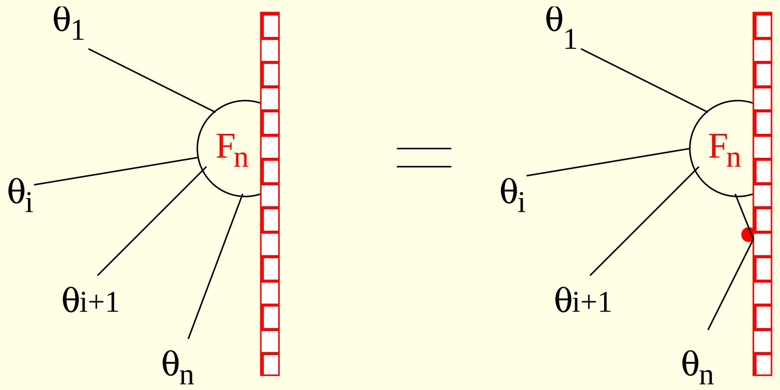

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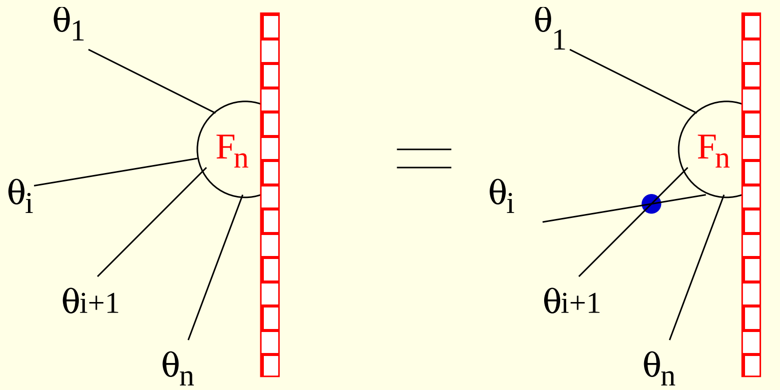

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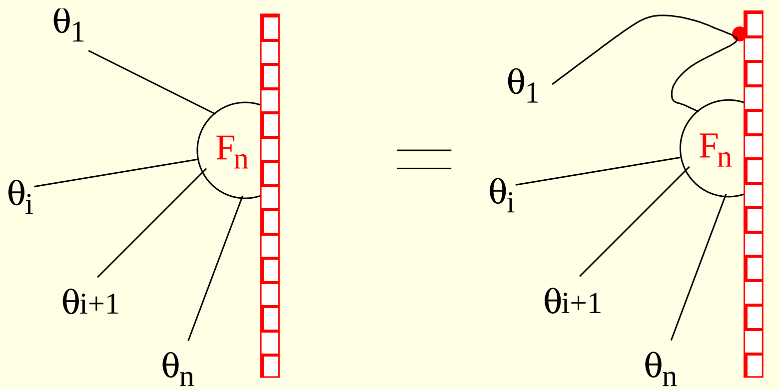

Boundary form factor axioms II: Singularities

Kinematical singularities
$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta+i\pi,\theta',\theta_1,\ldots,\theta_n) = \left(1-\prod_{i=1}^{n} S(\theta-\theta_i)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\ldots,\theta_n)$
$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(-\theta+i\pi,\theta',\theta_1,\ldots,\theta_n) = \left(R(\theta)-\prod_{i=1}^{n} S(\theta-\theta_i)R(\theta)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\ldots,\theta_n)$

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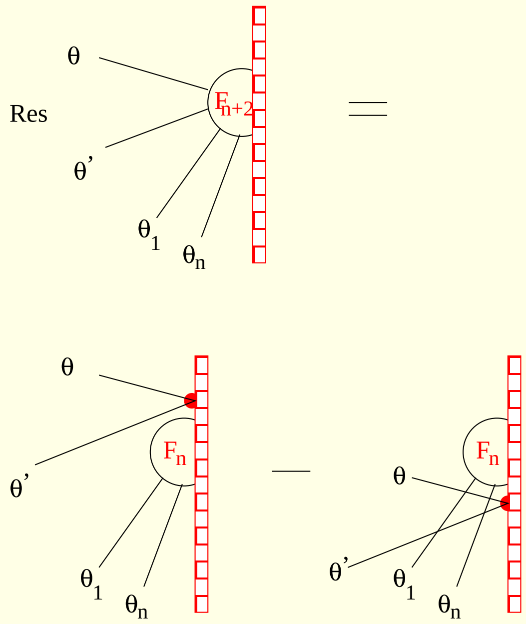

Boundary form factor axioms II: Singularities

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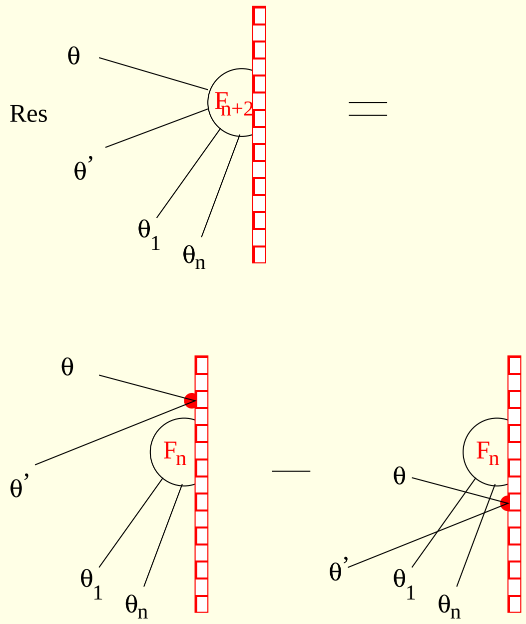

Bulk dynamical singularities

$$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta + iu, \theta' - iu, \theta_1, \dots, \theta_n) =$$
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Boundary form factor axioms II: Singularities

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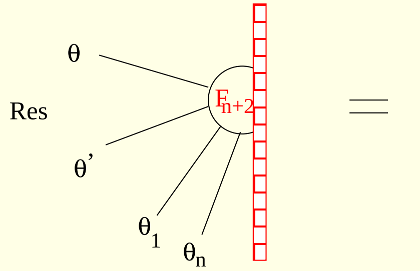

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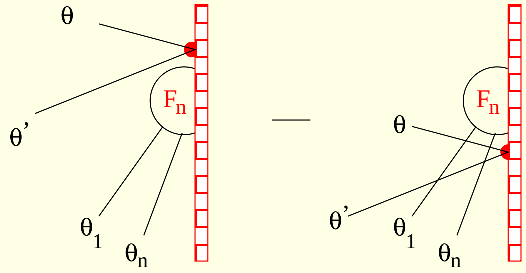

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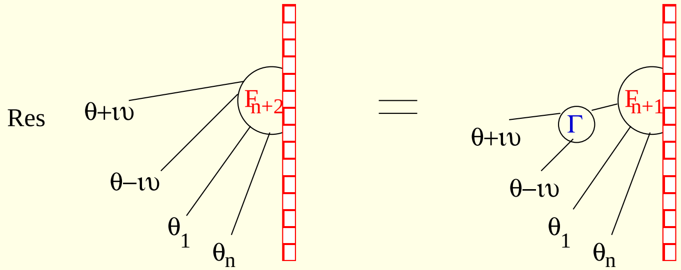

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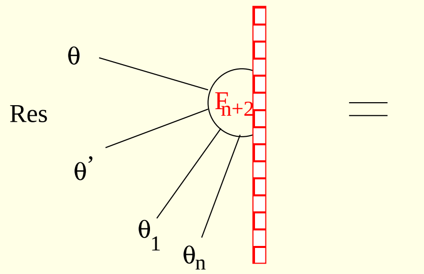

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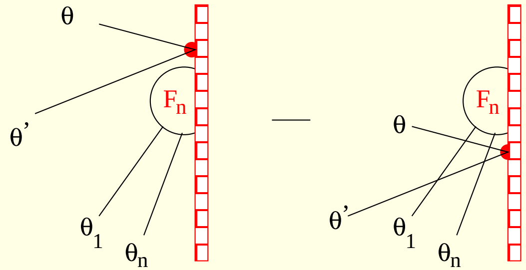

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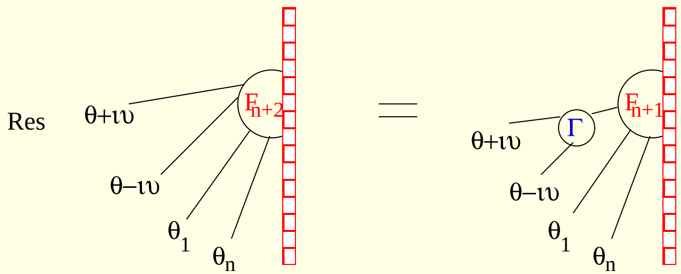

Boundary dynamical singularities

$$i\mathrm{res}_{\theta=iu} F^{\mathcal{O}}_{n+1}(\theta_1,\dots,\theta_n,\theta) =$$
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Boundary form factor axioms II: Singularities

Kinematical singularities

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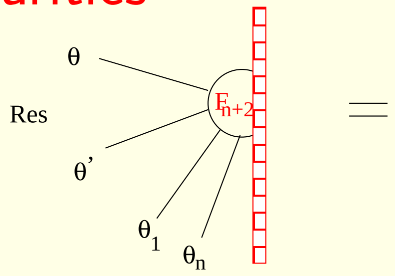

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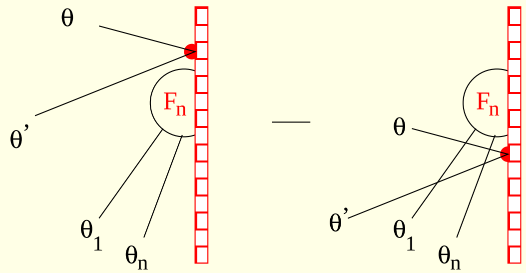

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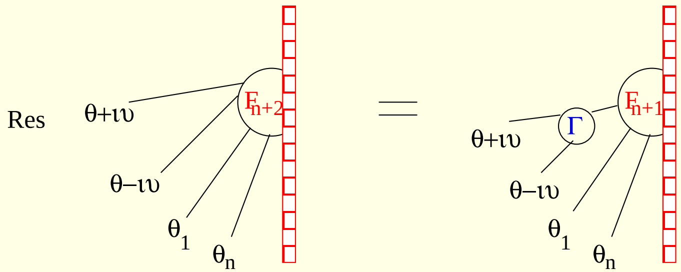

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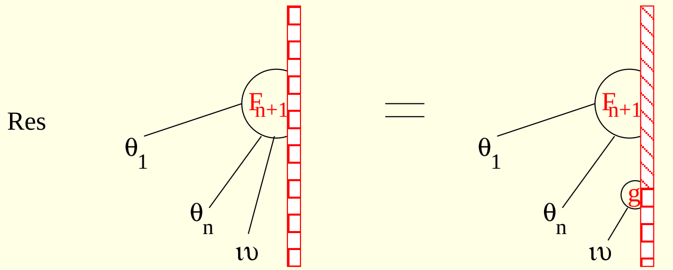

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Solving the bulk form factor equations

Bulk theory with $S(\theta)$:

Solving the bulk form factor equations

Bulk theory with $S(\theta)$: FF axioms + minimality

Solving the bulk form factor equations

Bulk theory with $S(\theta)$: FF axioms + minimality $\longrightarrow$ Solution of the FF equations

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Step 1. Solve first the two particle case $F_2(\theta_1 - \theta_2) = f(\theta)$

Solving the bulk form factor equations

Bulk theory with $S(\theta)$: FF axioms + minimality $\longrightarrow$ Solution of the FF equations

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$$f(\theta) = S(\theta)f(-\theta) \quad ; \quad f(i\pi + \theta) = f(i\pi - \theta)$$

minimality: only dynamical poles, one zero at $\theta = 0$, minimal growth at $\theta \rightarrow \infty$

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$$F_n(\theta_1, \dots, \theta_n) = \prod_{i<j} f(\theta_i - \theta_j) \quad ; \quad x = e^{\theta}$$

Solving the bulk form factor equations

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$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(x_1, \dots, x_n) \prod_{i<j} f(\theta_i - \theta_j) \quad ; \quad x = e^{\theta}$$

Solving the bulk form factor equations

Bulk theory with $S(\theta)$: FF axioms + minimality $\rightarrow$ Solution of the FF equations

Step 1. Solve first the two particle case $F_2(\theta_1-\theta_2)=f(\theta)$

$$f(\theta)=S(\theta)f(-\theta) \quad ; \quad f(i\pi+\theta)=f(i\pi-\theta)$$

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$$F_n(\theta_1,\dots,\theta_n)=H_nQ_n(x_1,\dots,x_n)\prod_{i<j}f(\theta_i-\theta_j)\frac{1}{x_i+x_j} \quad ; \quad x=e^{\theta}$$

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Step 4. Solve recursion, classify the solutions: operator content

Perturbed Lee-Yang model

Bulk theory with $S(\theta) = \frac{\sinh\theta + i\sin\frac{\pi}{3}}{\sinh\theta - i\sin\frac{\pi}{3}}$

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minimality: dynamical pole at $\theta = \frac{2i\pi}{3}$, zero at $\theta = 0$, growth: $f(\theta) \to 1$ for $\theta \to \infty$

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$$P_n(x|x_1, \ldots, x_n) = \frac{(-1)^n x^2}{2(x_+ - x_-)} \left(\prod_{i=1}^n (x_i + x_+)(x_i - x_-) - \prod_{i=1}^n (x_i - x_+)(x_i + x_-) \right) \quad x_\pm = e^{\mp i\frac{\pi}{3}} x$$

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dynamical $Q_{n+1}(x_+, x_-, x_1, \ldots, x_n) = x \prod_{i=1}^n (x + x_i) Q_n(x, x_1, \ldots, x_n)$

Perturbed bulk Lee-Yang: minimal form factor solution

Step 4. Solve recursion and classify them: 2pt function and operator content

$$Q_1 = 1; \quad Q_2 = \sigma_1; \quad Q_3 = \sigma_1 \sigma_2 \ldots \quad , \qquad \prod_{i=1}^{n}(x + x_i) = \sum_{k=0}^{n} x^k \sigma_{n-k}$$

The solution with minimal degree is unique.

Perturbed bulk Lee-Yang: minimal form factor solution

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The solution with minimal degree is unique. Which operator is it?

Perturbed bulk Lee-Yang: minimal form factor solution

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The solution with minimal degree is unique. Which operator is it? Compute 2pt:

$$\langle 0|\mathcal{O}(x,0)\mathcal{O}(0,0)|0\rangle = \sum_{n=0}^{\infty}(-1)^n \int_0^{\infty} \frac{d\theta_1 \dots d\theta_n}{n!(2\pi)^n} |F_n^{\mathcal{O}}(\theta_1, \dots, \theta_n)|^2 e^{-mx \sum_{i=1}^{n} \cosh\theta_i}$$

Large distance expansion

Perturbed bulk Lee-Yang: minimal form factor solution

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Large distance expansion $\leftrightarrow$ Compare to short distance (UV) expansion

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Large distance expansion $\leftrightarrow$ Compare to short distance (UV) expansion

$$\mathcal{A} = \mathcal{A}_{(2,5)} + g \int d^2 x \Phi_{(\frac{1}{5},\frac{1}{5})}$$

$\Phi_{(\frac{1}{5},\frac{1}{5})} = \Phi$ field with smallest scaling dimension

$$\langle 0|\Phi(x,0)\Phi(0,0)|0\rangle = x^{\frac{4}{5}} + x^{\frac{2}{5}} C_{\Phi\Phi}^{\Phi} \langle \Phi \rangle + \dots$$

Perturbed bulk Lee-Yang two point function

Conformal limit compared to form factor expansion

$\langle 0|\Phi(x,0)\Phi(0,0)|0\rangle$

Perturbed bulk Lee-Yang two point function

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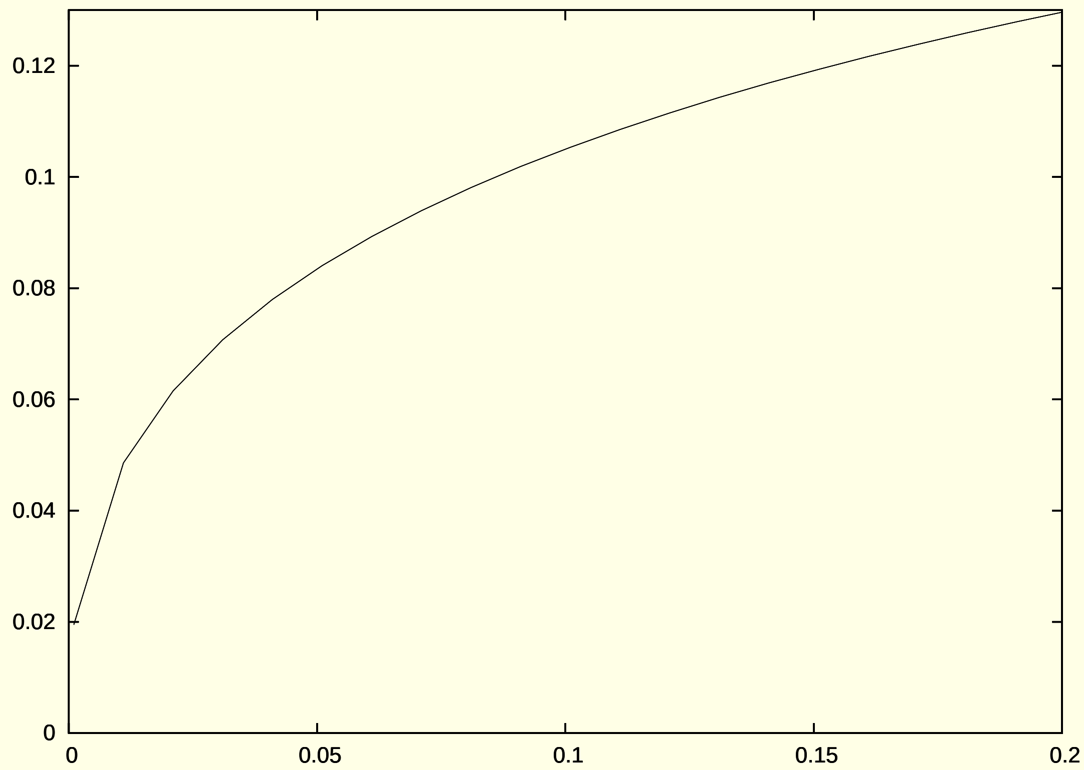

Conformal limit: $x^{\frac{4}{5}} + x^{\frac{2}{5}} C_{\Phi\Phi}^{\Phi}\langle\Phi\rangle$

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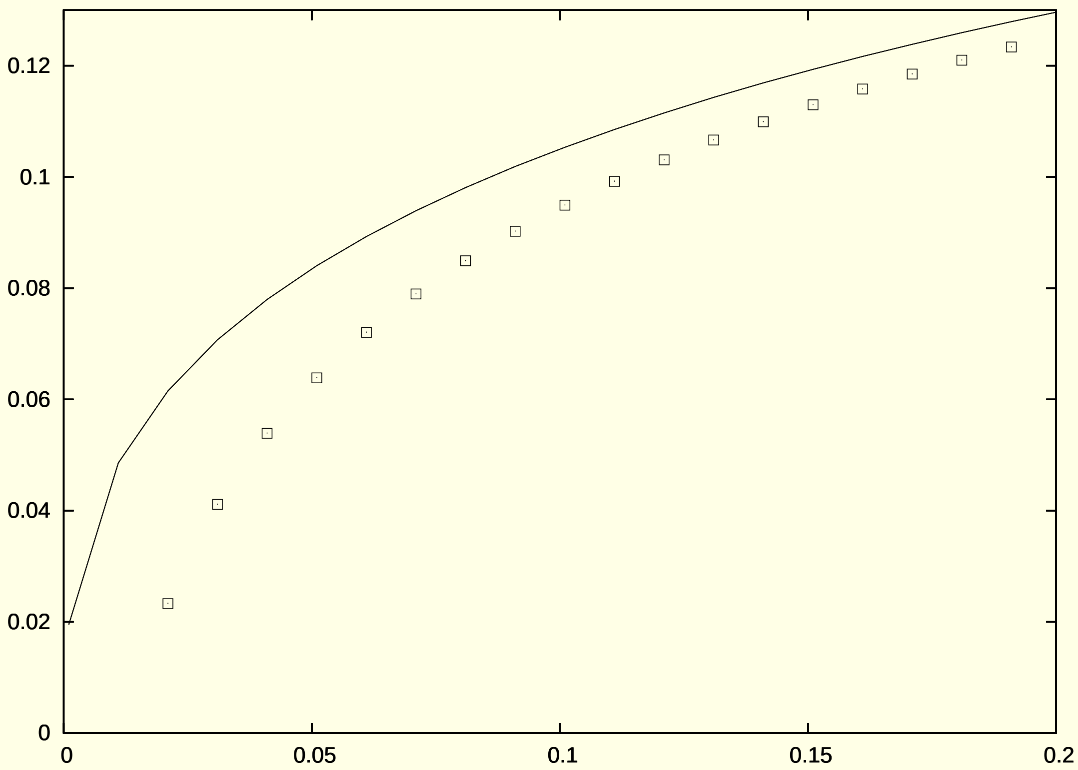

Conformal limit: $x^{\frac{4}{5}} + x^{\frac{2}{5}} C_{\Phi\Phi}^{\Phi} \langle \Phi \rangle$

FF expansion 0+1pt: $|F_0^{\Phi}|^2 +$

$$-\int_{-\infty}^{\infty} \frac{d\theta}{2\pi} |F_1^{\Phi}|^2 e^{-mx\cosh\theta}$$

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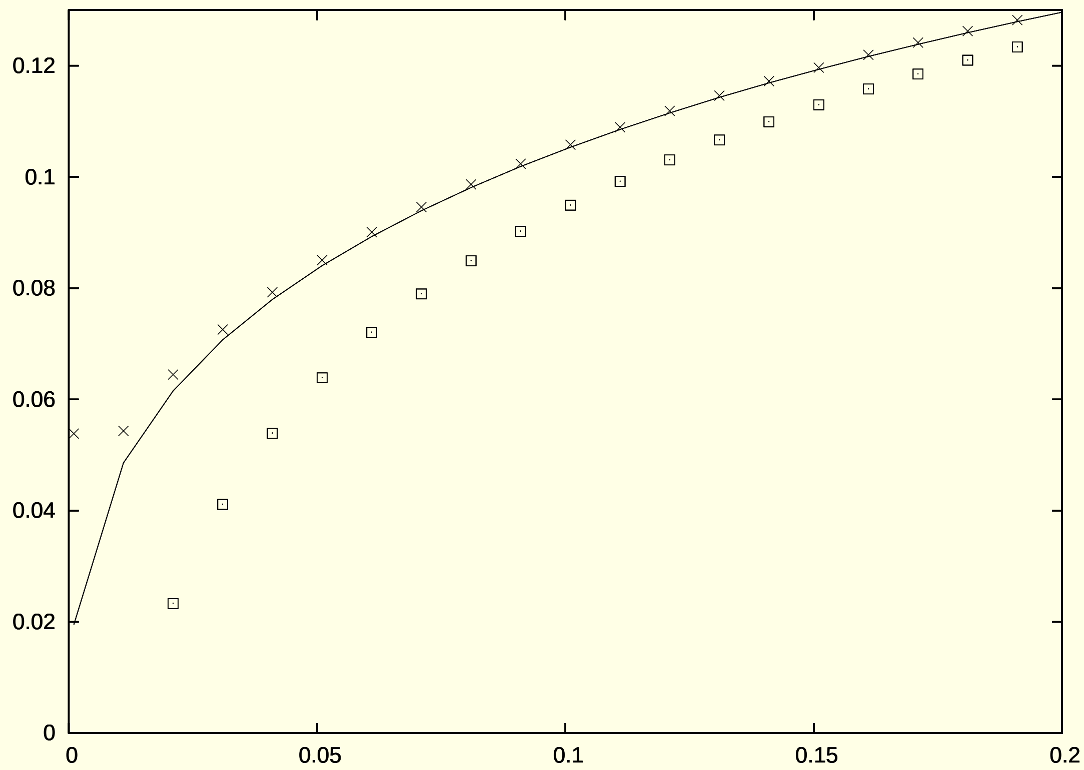

Conformal limit: $x^{\frac{4}{5}} + x^{\frac{2}{5}} C^{\Phi}_{\Phi\Phi} \langle\Phi\rangle$

FF expansion 0+1+2pt: $|F_0^{\Phi}|^2 +$

$$-\int_{-\infty}^{\infty} \frac{d\theta}{2\pi} |F_1^{\Phi}|^2 e^{-mx\cosh\theta}$$

$$+\int_{-\infty}^{\infty} \frac{d\theta_1 d\theta_2}{2(2\pi)^2} |F_2^{\Phi}(\theta_1,\theta_2)|^2 \cdot e^{-mx(\cosh\theta_1+\cosh\theta_2)}$$

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$$\langle 0|\Phi(x,0)\Phi(0,0)|0\rangle$$

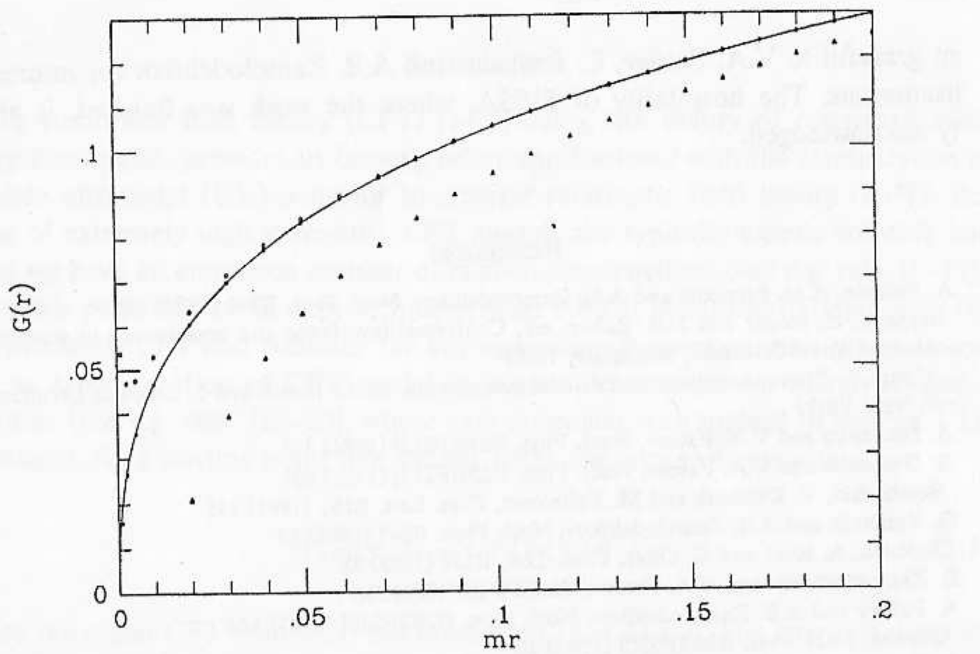


Fig. 3. Convergence of the large-distance expansion for small mr. Empty triangles: zero- and one-particle contributions. Empty circles: the same plus two-particle term. Full circles: up to three-particle state contributions. Full curve: the short-distance data.

Conformal limit: $x^{\frac{4}{5}} + x^{\frac{2}{5}} C^{\Phi}_{\Phi\Phi}\langle\Phi\rangle$

FF expansion 0+1+2+3pt: $|F^{\Phi}_0|^2 +$

$$-\int_{-\infty}^{\infty}\frac{d\theta}{2\pi}|F^{\Phi}_1|^2 e^{-mx\cosh\theta}$$

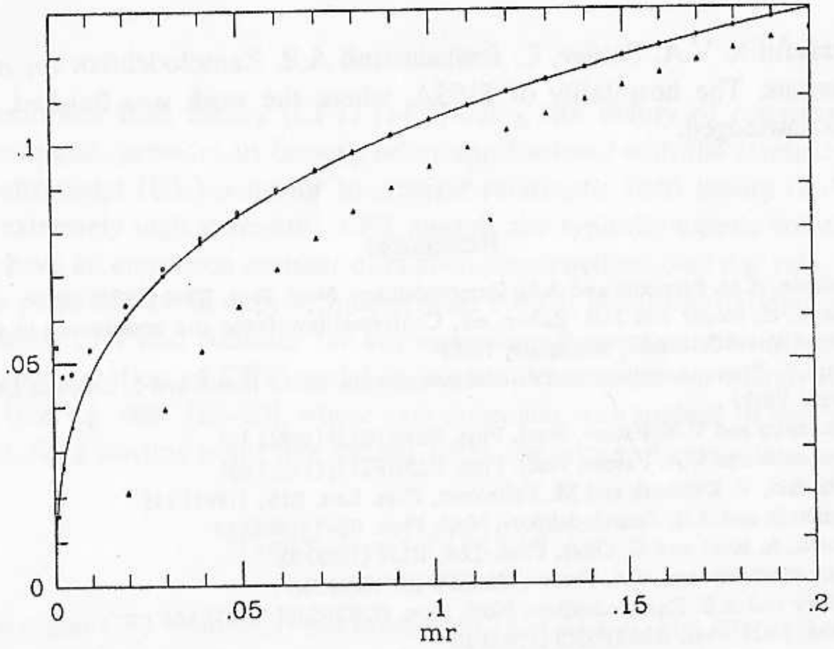
$$+\int_{-\infty}^{\infty}\frac{d\theta_1 d\theta_2}{2(2\pi)^2}|F^{\Phi}_2(\theta_1,\theta_2)|^2 \cdot e^{-mx(\cosh\theta_1+\cosh\theta_2)}$$

$$+\int_{-\infty}^{\infty}\frac{d\theta_1 d\theta_2 d\theta_3}{6(2\pi)^3}|F^{\Phi}_3(\theta_1,\theta_2,\theta_3)|^2\ldots$$

Perturbed bulk Lee-Yang two point function

Conformal limit compared to form factor expansion

$\langle 0|\Phi(x,0)\Phi(0,0)|0\rangle$



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FF expansion 0+1+2+3pt: $|F_0^{\Phi}|^2 +$

$- \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} |F_1^{\Phi}|^2 e^{-mx\cosh\theta}$

$+ \int_{-\infty}^{\infty} \frac{d\theta_1 d\theta_2}{2(2\pi)^2} |F_2^{\Phi}(\theta_1,\theta_2)|^2$
$\cdot e^{-mx(\cosh\theta_1 + \cosh\theta_2)}$

$+ \int_{-\infty}^{\infty} \frac{d\theta_1 d\theta_2 d\theta_3}{6(2\pi)^3} |F_3^{\Phi}(\theta_1,\theta_2,\theta_3)|^2 \dots$

Al. B. Zamolodchikov NPB348 (1991) 619.

Solving the boundary form factor equations

Boundary theory with $S(\theta)$,$R(\theta)$:

Solving the boundary form factor equations

Boundary theory with $S(\theta)$,$R(\theta)$: BFF axioms + minimality

Solving the boundary form factor equations

Boundary theory with $S(\theta)$,$R(\theta)$: BFF axioms + minimality $\longrightarrow$ Solution of the BFF eqs

Solving the boundary form factor equations

Boundary theory with $S(\theta)$,$R(\theta)$: BFF axioms + minimality $\longrightarrow$ Solution of the BFF eqs

Step 1. Solve first the one particle case $F_1(\theta) = r(\theta)$

Solving the boundary form factor equations

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$$r(\theta) = R(\theta) r(-\theta) \quad ; \quad r(i\pi - \theta) = R(\theta) r(i\pi + \theta)$$

minimality: only dynamical poles, one zero at $\theta = 0$, minimal growth at $\theta \to \infty$

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$$r(\theta) = R(\theta)r(-\theta) \quad ; \quad r(i\pi - \theta) = R(\theta)r(i\pi + \theta)$$

minimality: only dynamical poles, one zero at $\theta = 0$, minimal growth at $\theta \to \infty$

Step 2. Take the Ansatz to satisfy the reflection, permutation and the bperiodicity axioms

$$F_n(\theta_1, \dots, \theta_n) = H_n Q_n(y_1, \dots, y_n) \prod_i r(\theta_i) \prod_{i<j} f(\theta_i + \theta_j) f(\theta_i - \theta_j)$$

Solving the boundary form factor equations

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Step 4. Solve recursion, classify the solutions: operator content

Perturbed boundary Lee-Yang model

Boundary theory with $S(\theta) = -\left[\frac{1}{3}\right]$, $R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)\left(-\frac{2}{3}\right)\left[\frac{b+1}{6}\right]\left[\frac{b-1}{6}\right]$

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Perturbed boundary Lee-Yang: minimal form factor solution

Step 4. Solve recursion and classify them: 2pt function and operator content

$$Q_1 = \sigma_1; \quad Q_2 = \sigma_2 + 3 - \beta^2_{-3}(b);$$

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The solution with minimal degree is unique.

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The solution with minimal degree is unique. Which operator is it? Compute 2pt:

$$\langle 0|\mathcal{O}(t)\mathcal{O}(0)|0\rangle = \sum_{n=0}^{\infty}(-)^n \int_0^{\infty} \frac{d\theta_1 \ldots d\theta_n}{n!(2\pi)^n} |F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_n)|^2 e^{-mt\sum_{i=1}^{n}\cosh\theta_i}$$

Large distance expansion.

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Large distance expansion. $\leftrightarrow$ Compare to short distance (UV) expansion

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Large distance expansion. $\leftrightarrow$ Compare to short distance (UV) expansion

$$\mathcal{A} = \mathcal{A}_{BCFT(2,5)} + g\int_{-\infty}^{\infty} dt \int_{-\infty}^{0} dx \, \Phi_{(\frac{1}{5},\frac{1}{5})}(x,t) + h\int_{-\infty}^{\infty} dt \, \varphi_{\frac{1}{5}}(t)$$

$\varphi_{\frac{1}{5}} = \varphi$ boundary field with smallest scaling dimension

$$\langle 0|\varphi(t)\varphi(0)|0\rangle = -t^{\frac{2}{5}} + t^{\frac{1}{5}} C_{\varphi\varphi}^{\varphi}\langle\varphi\rangle + \dots$$

Perturbed boundar Lee-Yang two point function

Conformal limit compared to form factor expansion

$\langle 0|\varphi(t)\varphi(0)|0\rangle$

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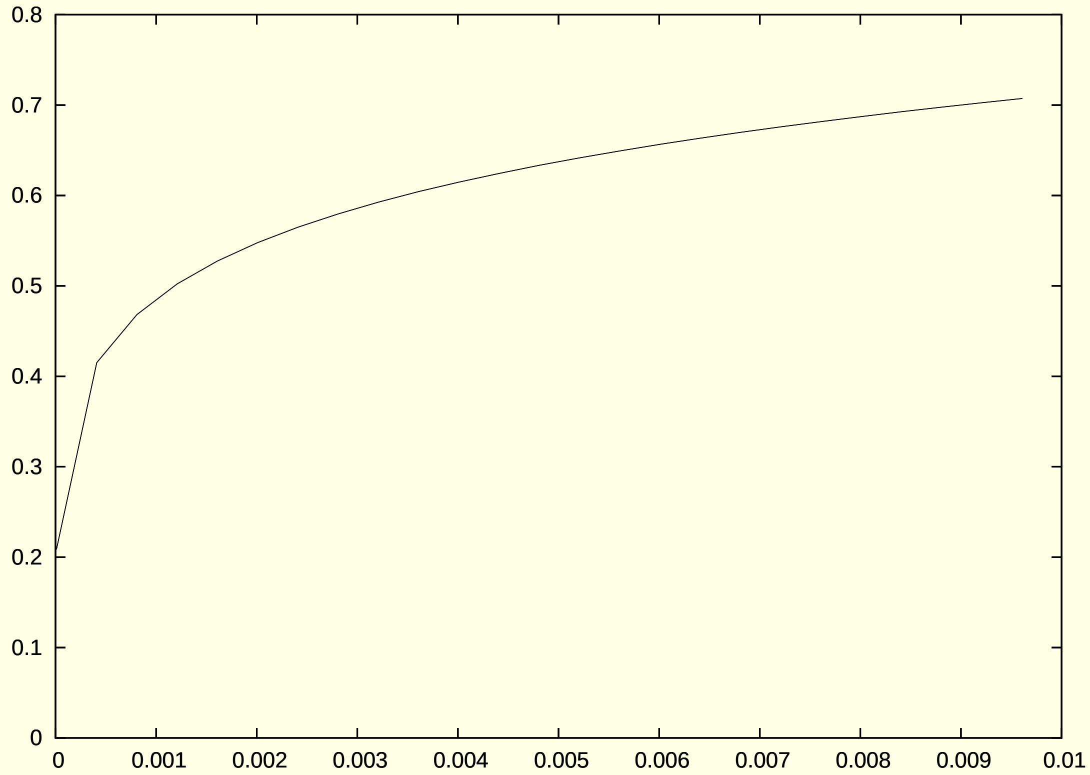

Conformal limit: $-t^{\frac{2}{5}}+t^{\frac{1}{5}}C_{\varphi\varphi}^{\varphi}\langle\varphi\rangle$

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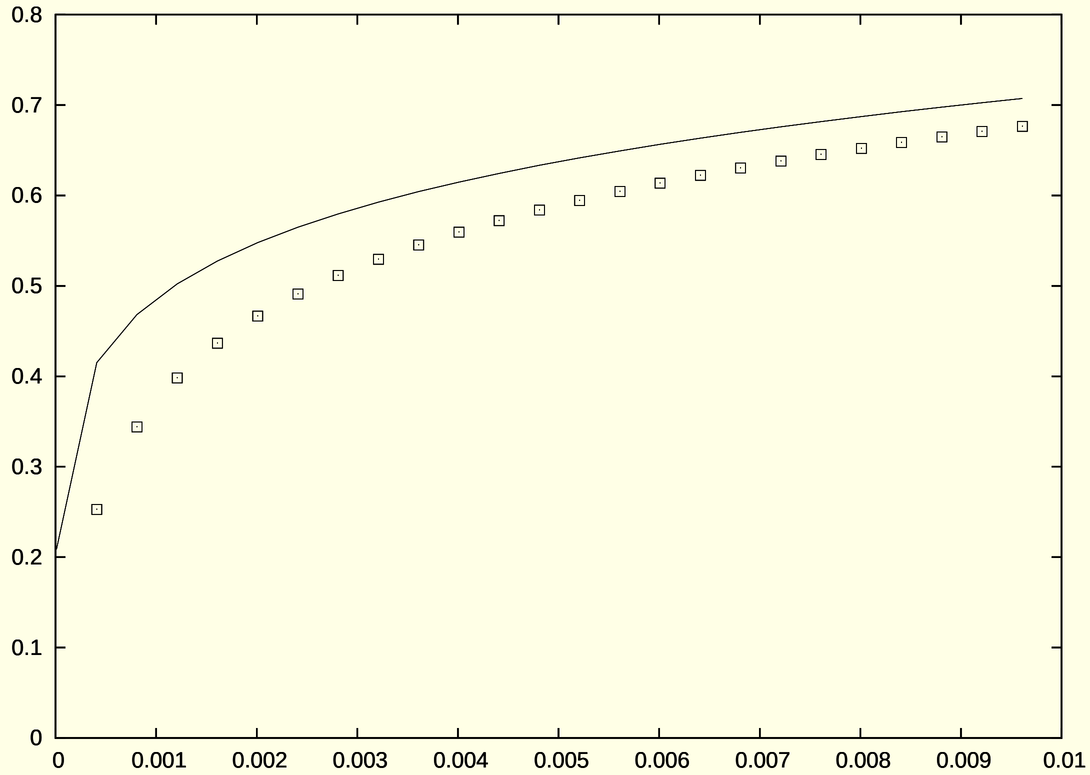

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BFF expansion 0+1pt: $|F_0^{\varphi}|^2 +$

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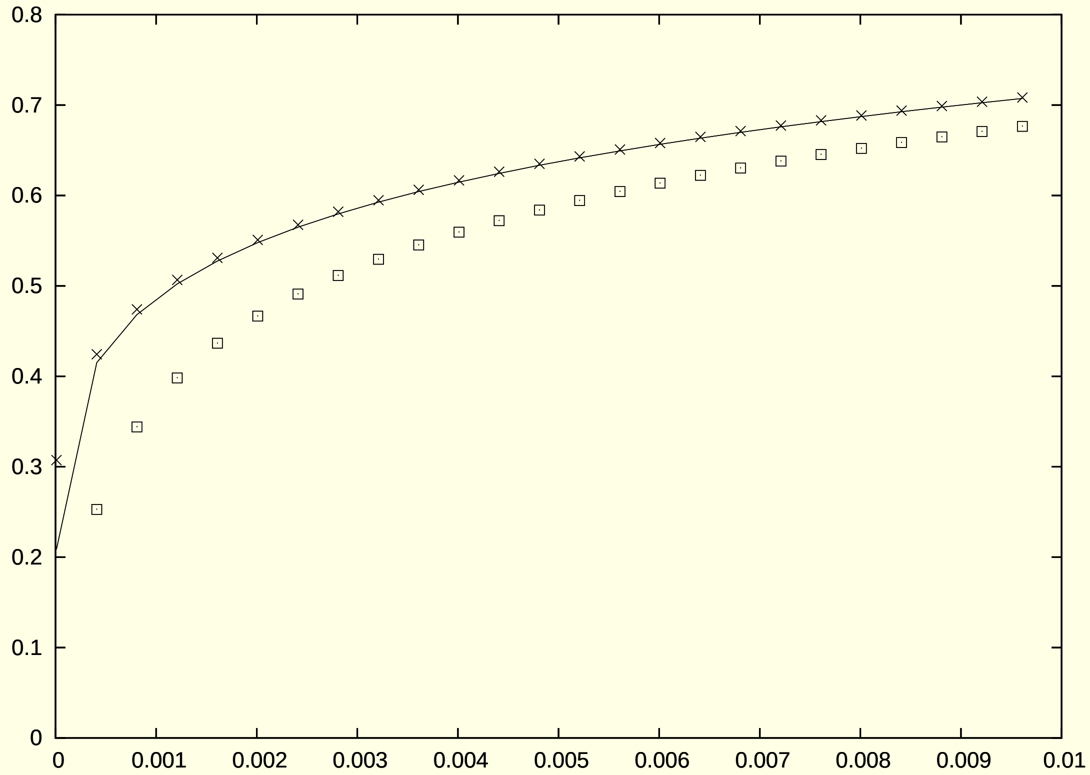

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BFF expansion 0+1+2pt: $|F_0^{\varphi}|^2+$

$$-\int_0^{\infty}\frac{d\theta}{2\pi}|F_1^{\varphi}|^2 e^{-mt\cosh\theta}$$

$$+\int_0^{\infty}\frac{d\theta_1 d\theta_2}{2(2\pi)^2}|F_2^{\varphi}(\theta_1,\theta_2)|^2 \cdot e^{-mt(\cosh\theta_1+\cosh\theta_2)}$$

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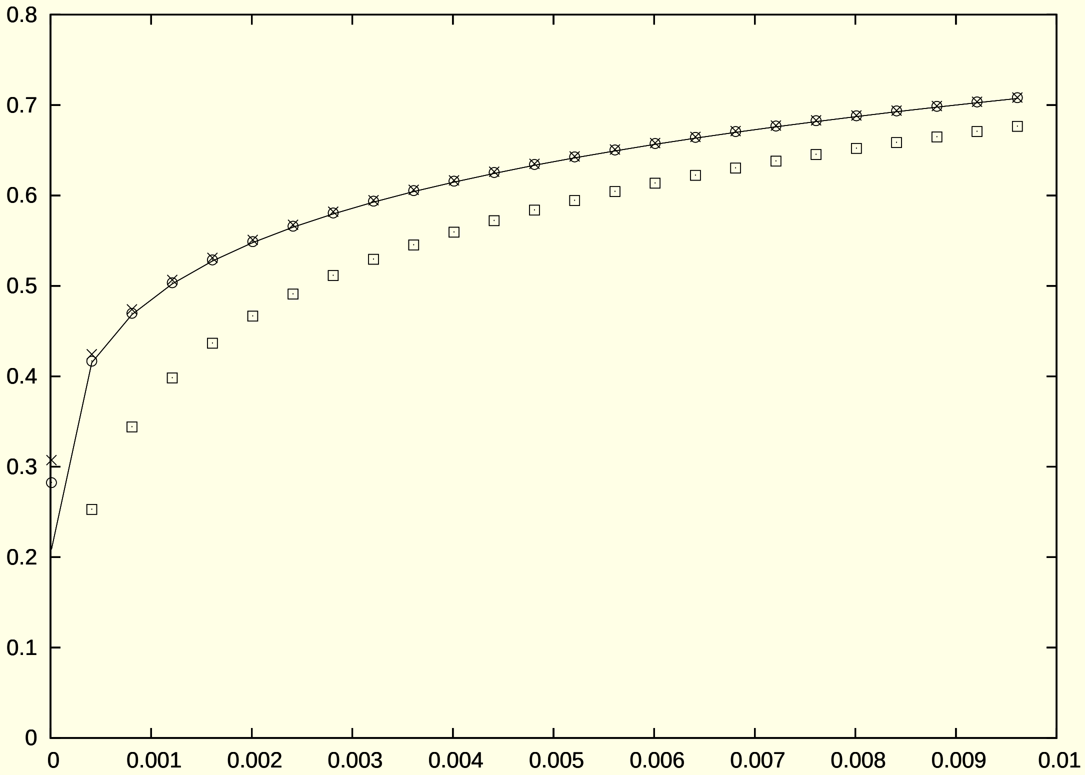

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BFF expansion 0+1+2+3pt: $|F_0^{\varphi}|^2 +$

$$- \int_0^\infty \frac{d\theta}{2\pi}|F_1^{\varphi}|^2 e^{-mt\cosh\theta}$$

$$+ \int_0^\infty \frac{d\theta_1 d\theta_2}{2(2\pi)^2}|F_2^{\varphi}(\theta_1,\theta_2)|^2 \cdot e^{-mt(\cosh\theta_1+\cosh\theta_2)}$$

$$+ \int_0^\infty \frac{d\theta_1 d\theta_2 d\theta_3}{6(2\pi)^3}|F_3^{\varphi}(\theta_1,\theta_2,\theta_3)|^2 \dots$$

>