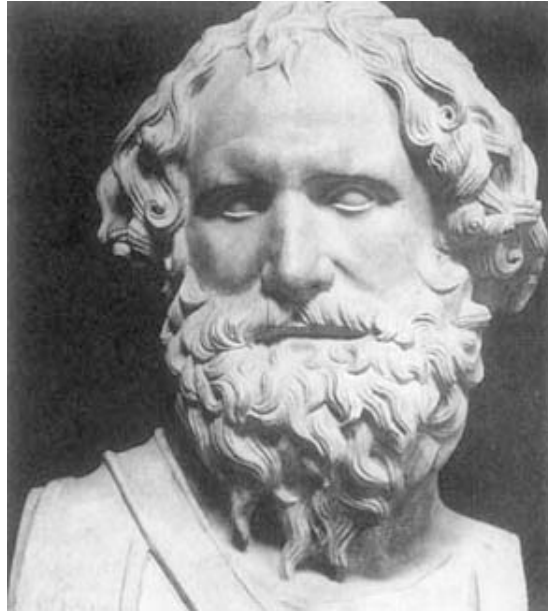
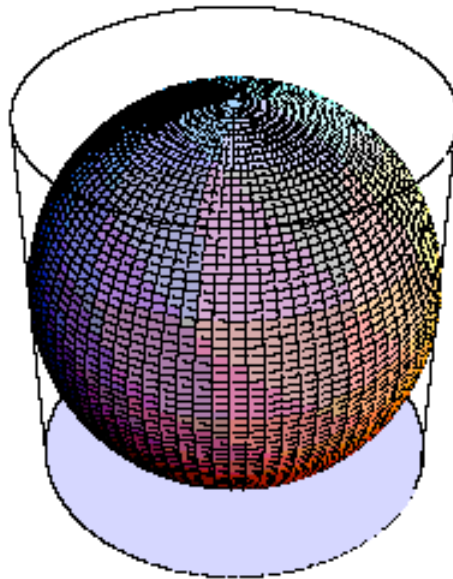


ERATOSTHENES AND ARCHIMEDES



THE SPHERE AND THE CYLINDER



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TOURS University (France)

Inst. Mod. Phys. Lanzhou, CAS (China)



Ancient Greece (−700 – +350) spread around Mediterranean sea

Alexandria (Egypte): cradle of European Science

Archimedes in Syracuse Sicily ("Big Greece")



Alexander the Great

356 - 323 BC

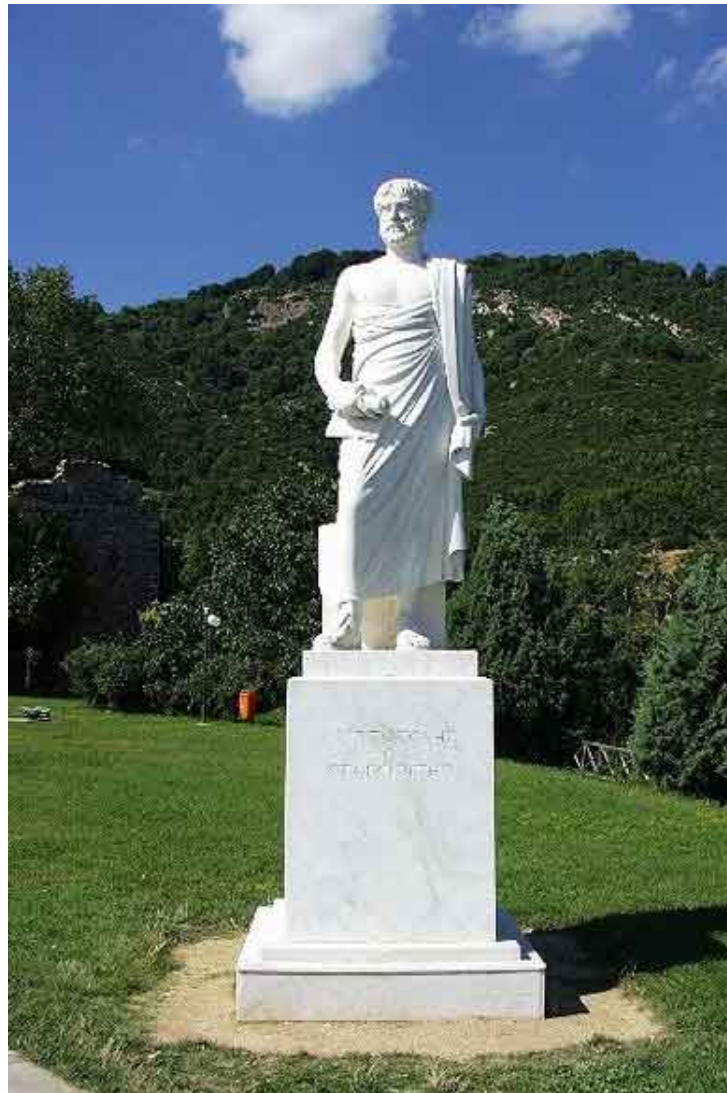


Ta sage instruction sert de riche couronne
 A Trajan, esleué par dessus tous humains.
 Si les grands te porroient au cœur & dans leurs mains,
 Vertu viuroit au lieu de Venus & Bellone

(≈ 100 AD)

Plutarchos

greco-roman historian : Alexander educated by



Aristoteles

the most

illustrious and knowledgable of all philosophers

After the death of his father, Alexander undertook to conquer the entire world.

In 333 BC defeats the Persian army





mosaic Pompei – Napoli



Conquests of Alexander

Occupies also Egypte.

Realizes synthesis of Greek and Egyptian civilisations.





Nile mosaic 2nd A.D (Palestrina, Italy)



331 BC founds of Alexandria



Plutarchos recounts the legend of founding Alexandria.

“He went to Pharos, . . . a narrow strip of earth and traced the plan of a city in harmony with the form of the terri-



tory.” Aristoteles leaves his private collection of books to Alexander, who founds “Museion” (Library).

Demetrios (another pupil of Aristoteles) becomes first Librarian. 500 000 parchments !

[Parchment is a thin material made from calf-skin, sheepskin or goatskin. Its most common use was as a material for writing on, for documents, notes, or the pages of a book, codex or manuscript. Substitute for papyrus, which was temporarily not being exported from Alexandria]



Bibliotheca Alexandrina

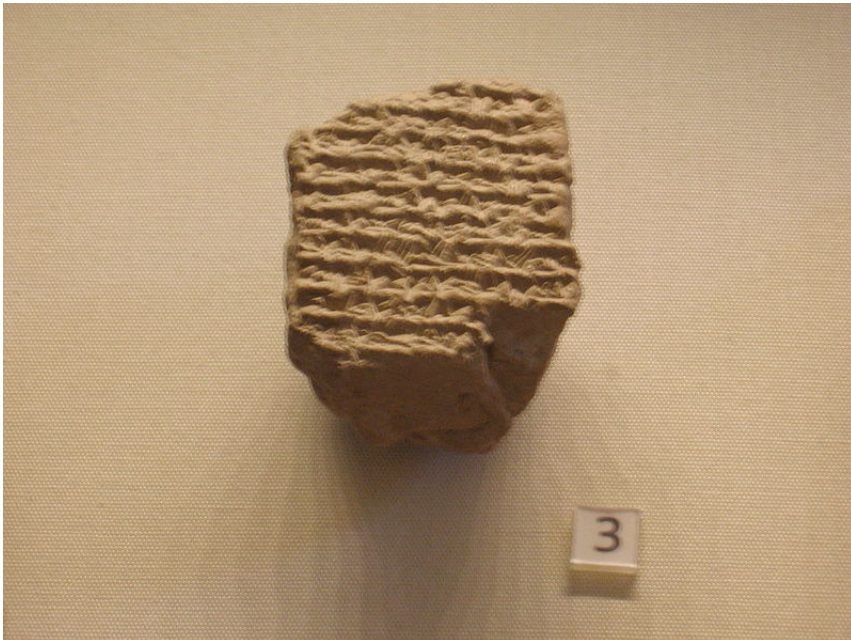
TI·CLAVD I·OTI·CLAVD
 I·F· QVIR
 BALBILLO
 PROCAS IAE·ET·A·EDIVM·DIVI·AVG·ET
 ET·LVCORVM·SACRÓ
 5 RVMQVE·OMNIVM·QVAE·SVNT·ALEXAN
 DREA·ET·INTÓTÁ·AEGYPTO·ET·SVPR·A·MV
 SEVM·ET·ALEXANDRINA·BYBLIOTHECE
 ET·ARCHI·EREI·ET·AD·HERMEN·ALEXAN
 10 DREON·PER·ANNOS· ET·AD·LEGATI
 ONES·ET·RESPONSA· CAESARIS·AVG·
 DIVI·CLAVD·ET·TRIB·MILIT·LEG·XX·ET·PRAE
 FABR·DIVI·CLAVD·ET·DD·INTRI·VAMP·HO·ADIVÓ
 CLAVDIO·
 15 PVR·

(1st BC)

Alexander -323 dies in Babylon (Mesopotamia)



aged 33 years



clay tablet found in Babylon telling death of Alexander.

His empire is divided among his generals. Egypt

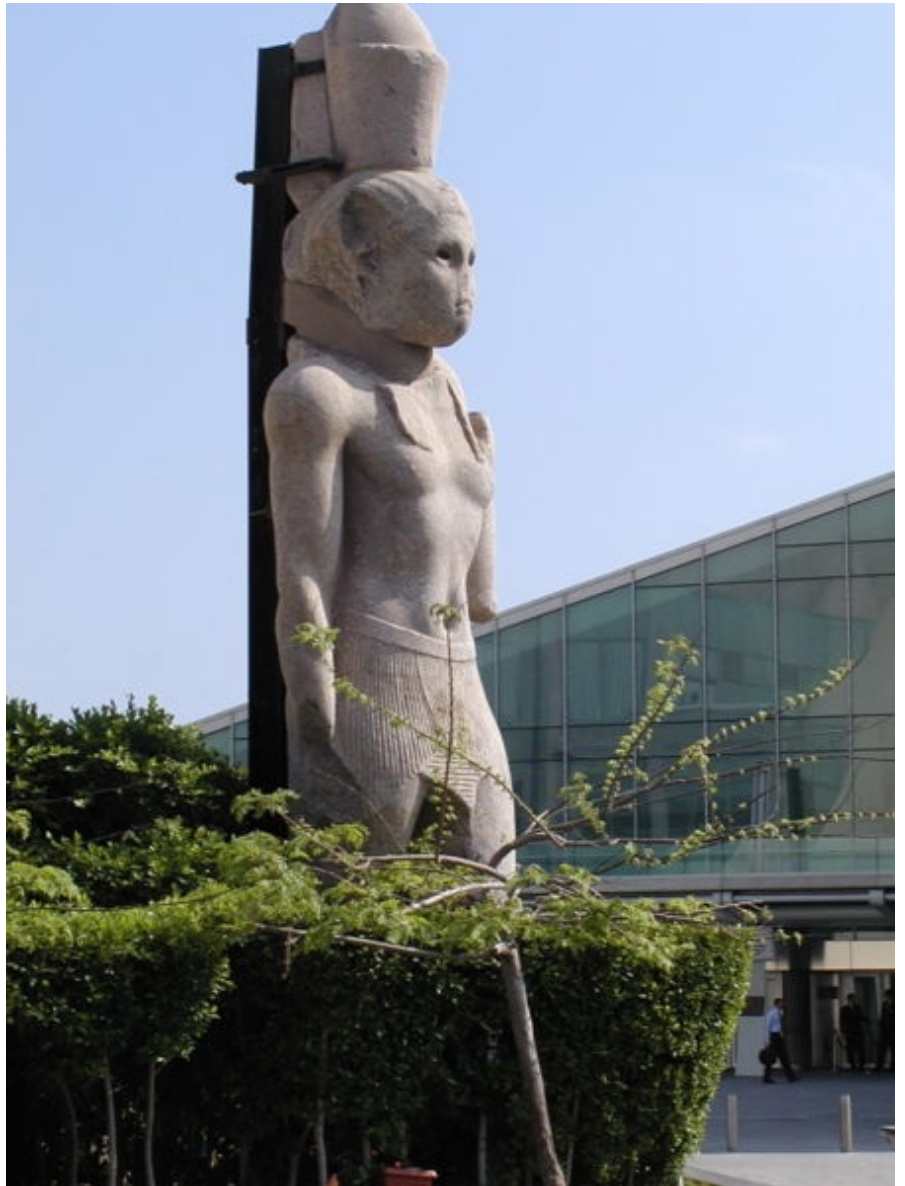


inherited by **Ptolemaios** who
founds greco-egyptian dynasty of king-pharaohs.





Ptolemaios II – Arsinoe II (Museo Gregoriano, Rome)



Stele of **Ptolemaios II** found in the sea, put in front of the new [Library of Alexandria](#).

lighthouse (Pharos)





maquette (reconstruction)



Qaitbay



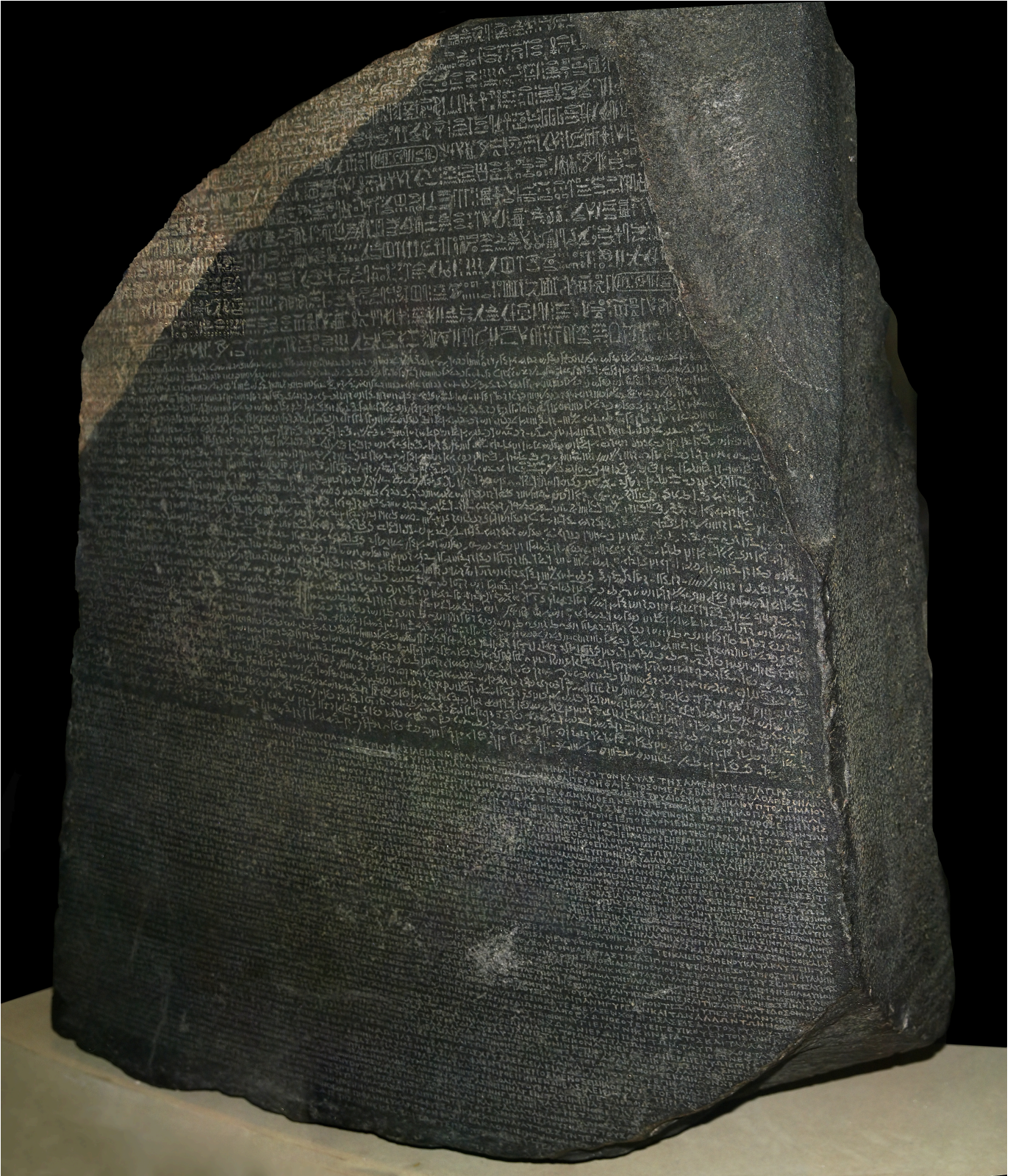
Ptolemaios III

founds

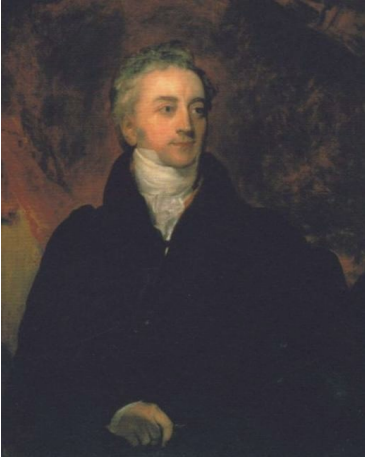


Serapeum [“annex University” to Library]

Ptolemaios V



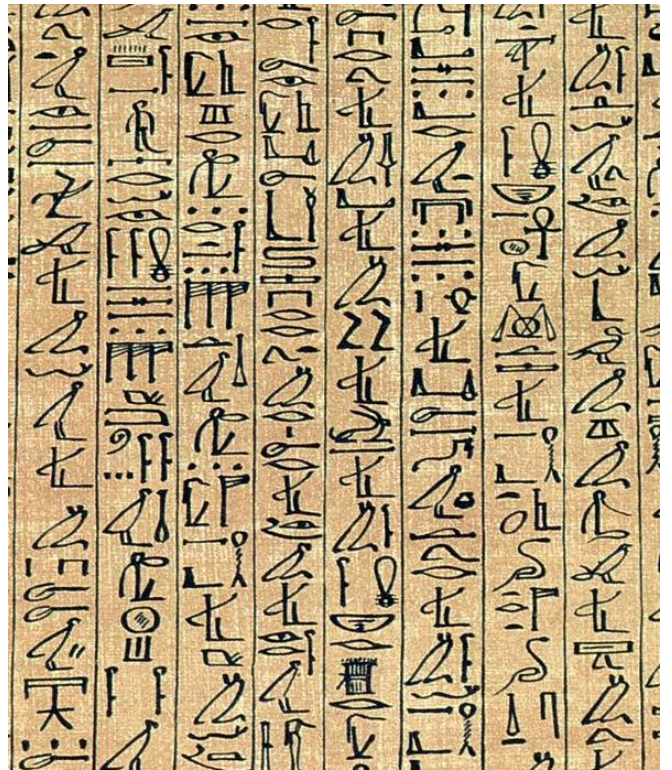
Rosetta stone [written in 3 languages – key to reading Egyptian hieroglyphs]



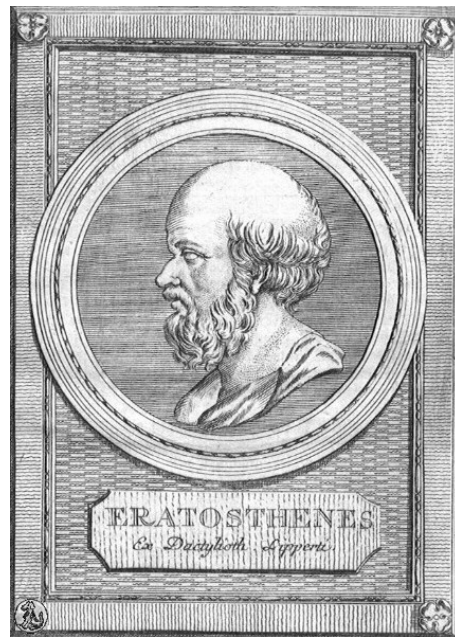
Thomas Young (double-slit experiment \sim wave optics !)



François Champollion

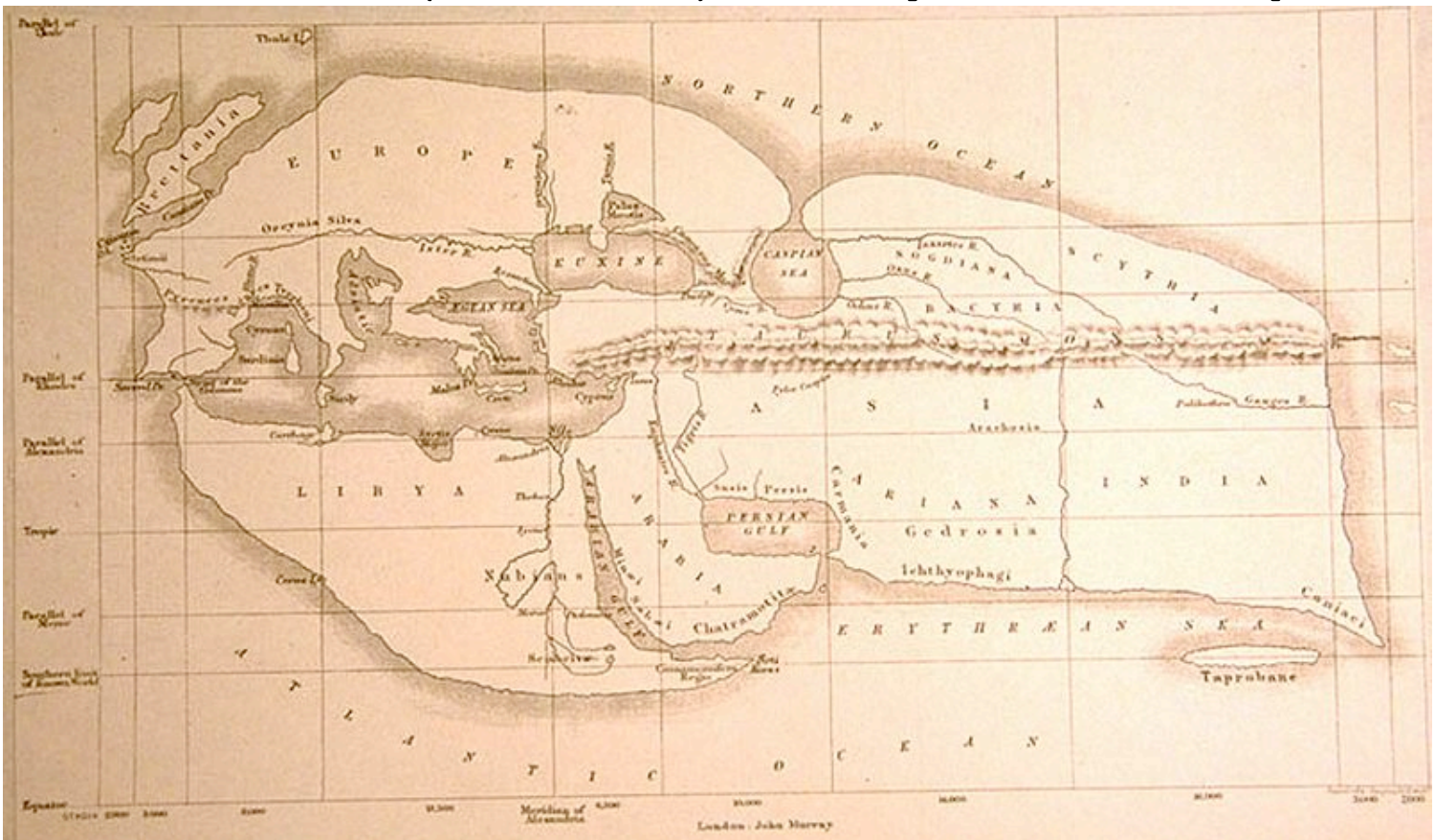


SCIENCE IN ALEXANDRIA

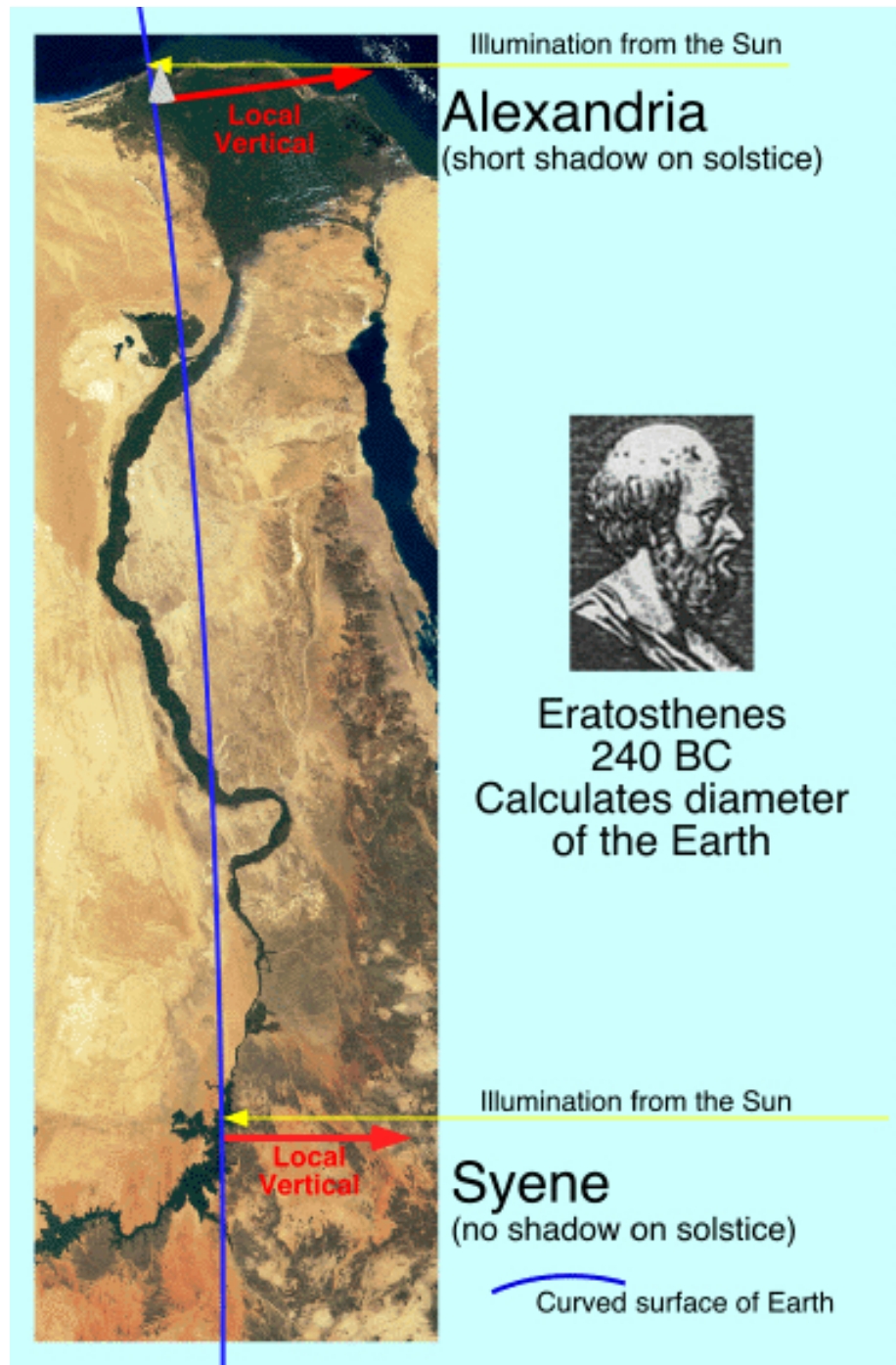


Eratosthenes

3rd Librarian of Alexandria, appointed by Ptolemy III in 236 BC. Draws map of (Hellenistic) World [reconstructed]



- measures **diameter of Earth** :
 - on the summer solstice at local noon in **Syene**, (Aswan – on Tropic of Cancer), sun would appear at zenith, directly overhead.

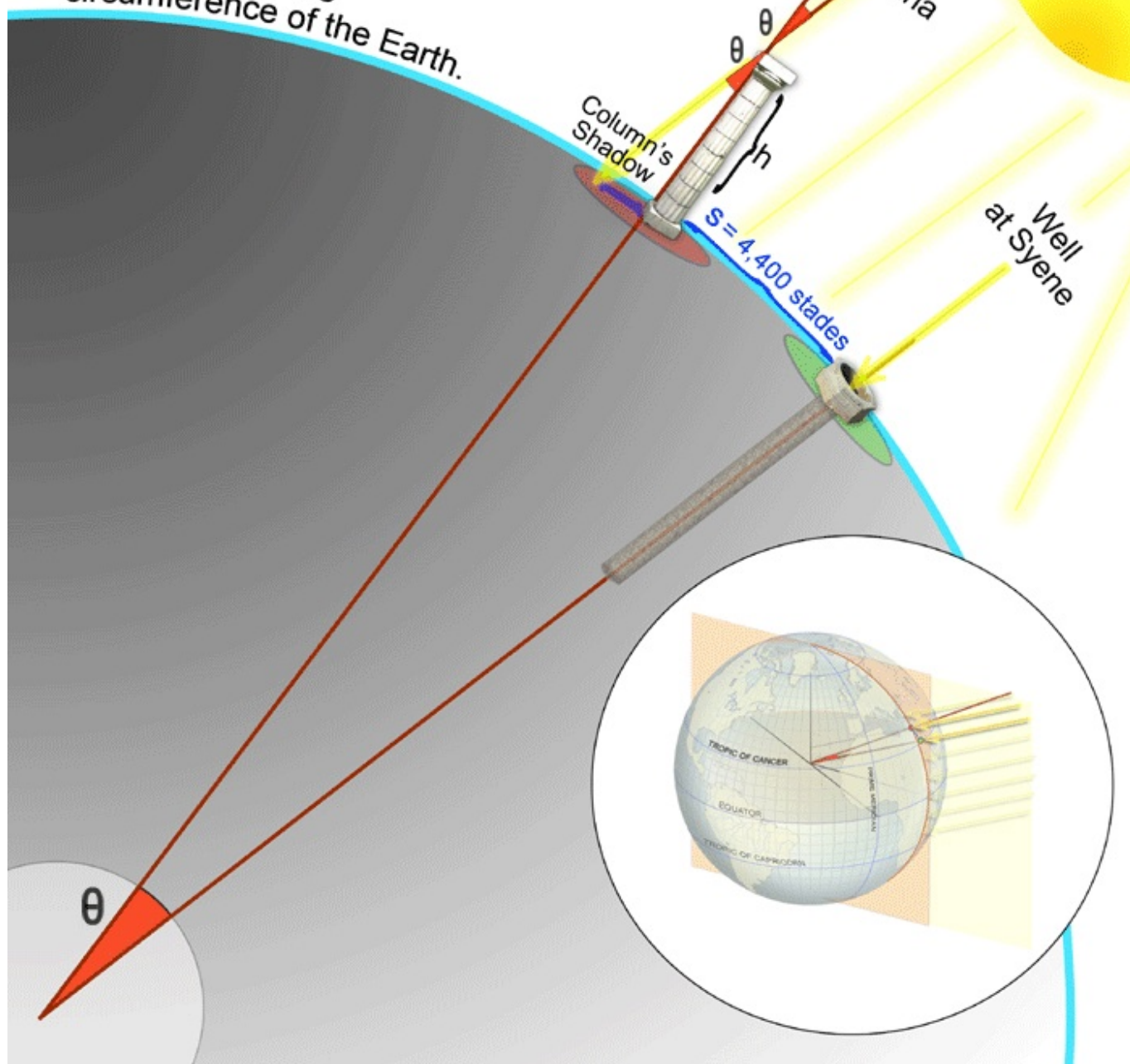


With h and s known,
you can solve for θ .

With θ known,
you can use the equation:

$$(360^\circ/\theta) \times (s)$$

... to measure the
circumference of the Earth.



- in his hometown of Alexandria, the angle of elevation of the sun would be $1/50$ of a full circle south of the zenith.

$$\theta = \frac{360^\circ}{50}$$

Assuming that Alexandria was due north of Syene
 \Rightarrow meridian arc distance from Alexandria to Syene must be $1/50$ of the total circumference of the earth,

$$\text{circumference} = 50 \times \underbrace{(\text{Alexandria-Syene})}_{\approx 800 \text{ km}}$$

Eratosthenes \Rightarrow

$$\text{circumference} \approx 39,690 \text{ km.}$$

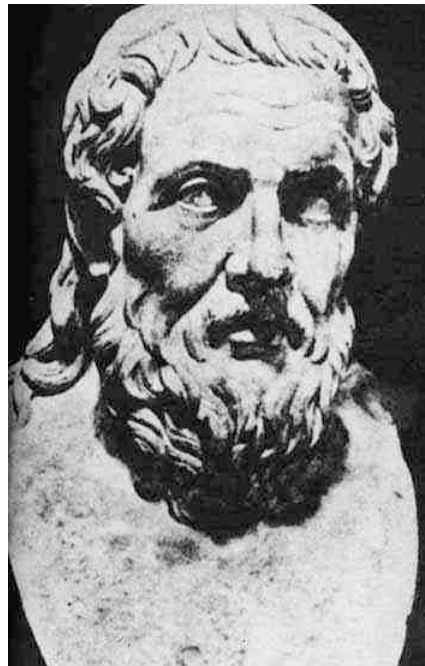
$$\text{Error} < 1 \% !$$

Leading scientists in Alexandria :

Geometry

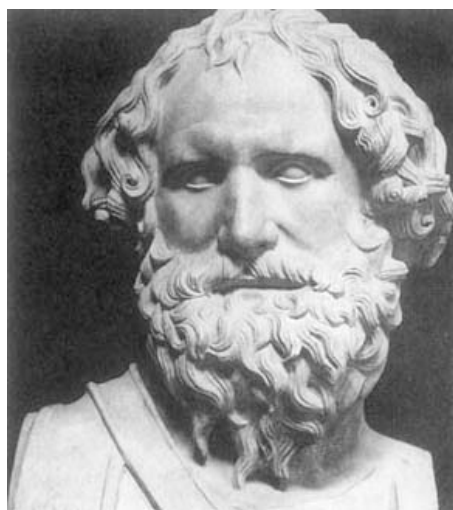


Euclides



Appollonios

Mechanics



Archimedes



Heron



Archimedes

287-212 BC



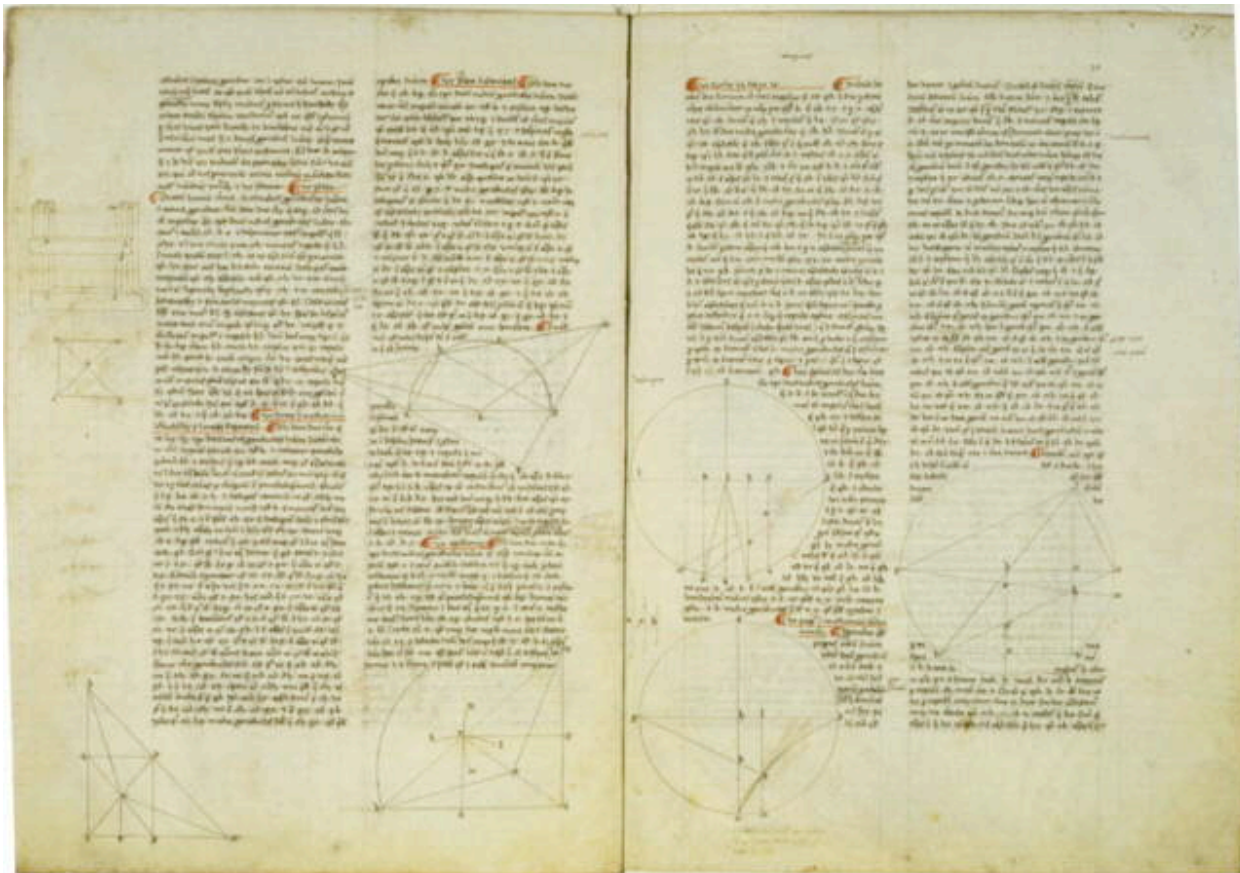
Siracusa



Archimedes ... in writing to **King Hiero**, whose friend and near relation he was ... (Plutarchos)

Siracusa





Manuscripts of Archimedes (Vatican)

- Mechanics :

Archimedes : *“if there were another earth, he could remove this”*



Archimedes' Law (levers) EQUILIBRIUM:

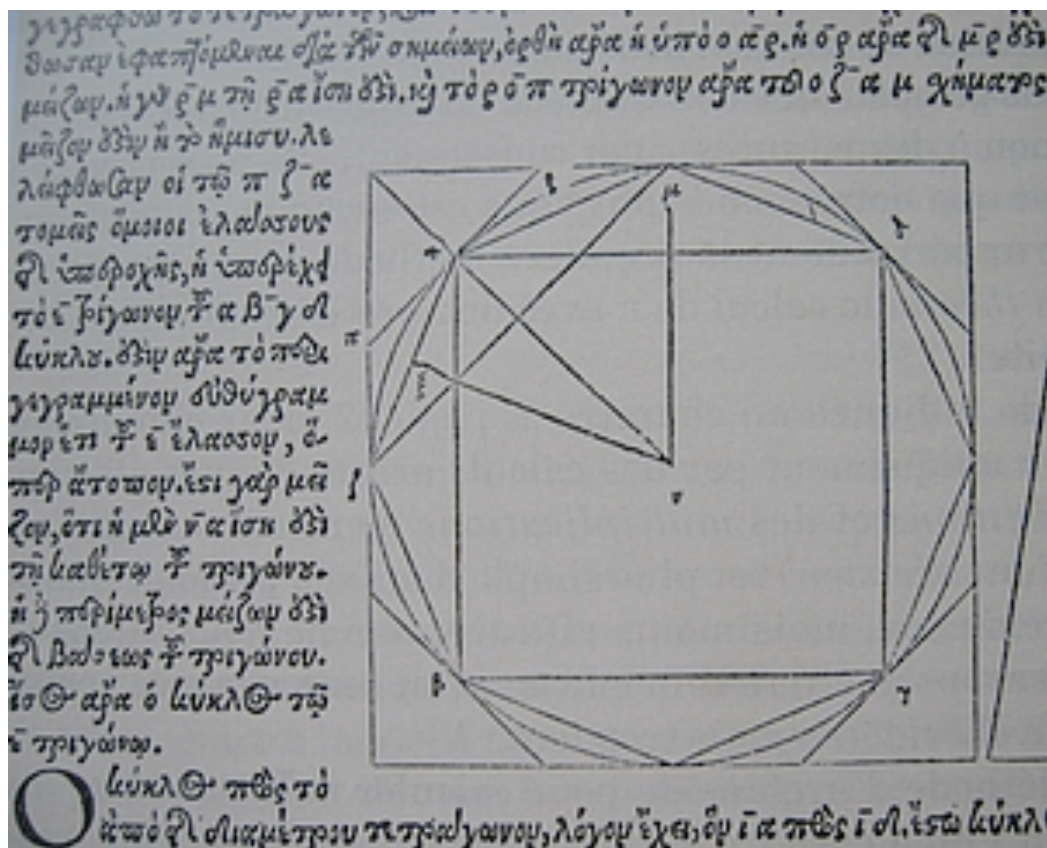
$$(\text{distance}) \times (\text{weight}) = \text{same on both sides}$$

pulley : Demonstration by experiment :

... he fixed upon a ship ... which could not be drawn out of the dock without great labour and many men; ... sitting himself ... holding the head of the pulley in his hand and drawing the cords by degrees, he drew the ship as smoothly and evenly as if she had been in the sea.



These machines [Archimedes] had designed not as matters of any importance, but as mere amusements in geometry.



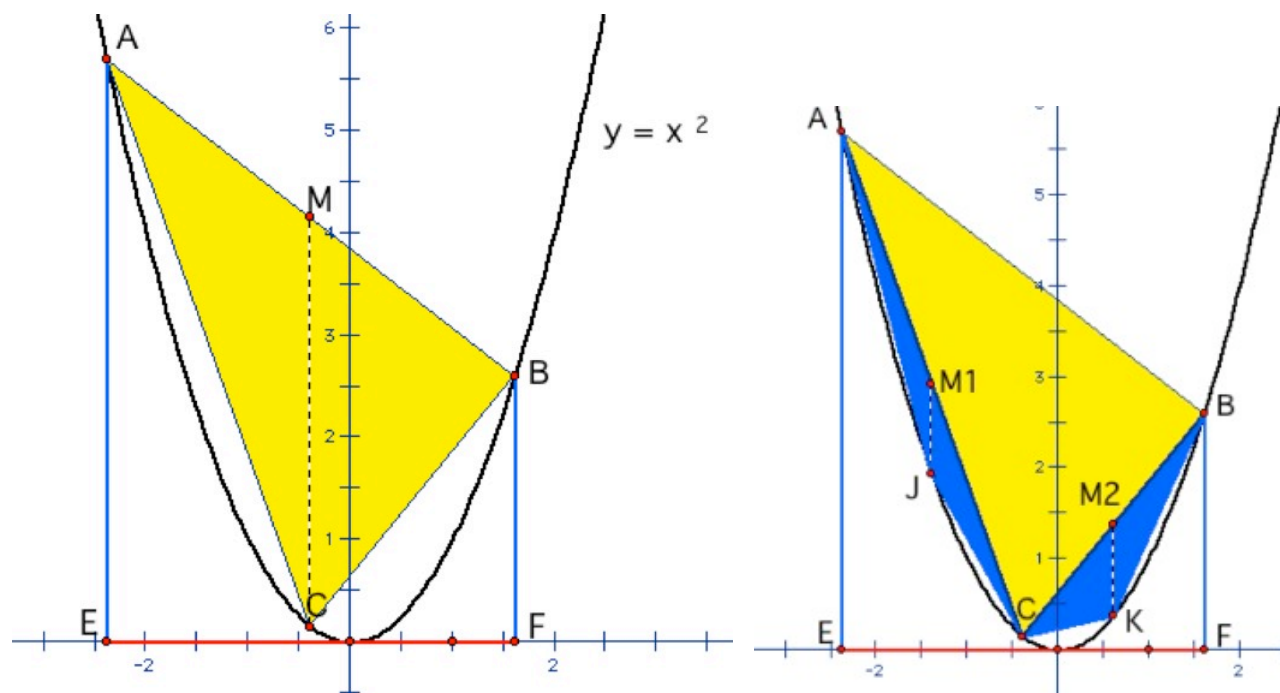
(\Rightarrow infinitesimal calculus !)

- “Measurement of the Circle” area and circumference of circle.

$$3 \frac{10}{71} < \pi < 3 \frac{1}{7}$$

obtained by circumscribing and inscribing a circle with regular polygons having 96 sides.

- Area under parabola using method of “exhaustion”



$$T = \Delta + \frac{1}{4}\Delta + \frac{1}{4} \cdot \frac{1}{4}\Delta + \dots = \Delta \frac{1}{1 - \frac{1}{4}}$$

$$\text{Area} = \frac{4}{3} \times \text{Area of triangle}$$

[sum of infinite geometric series !]

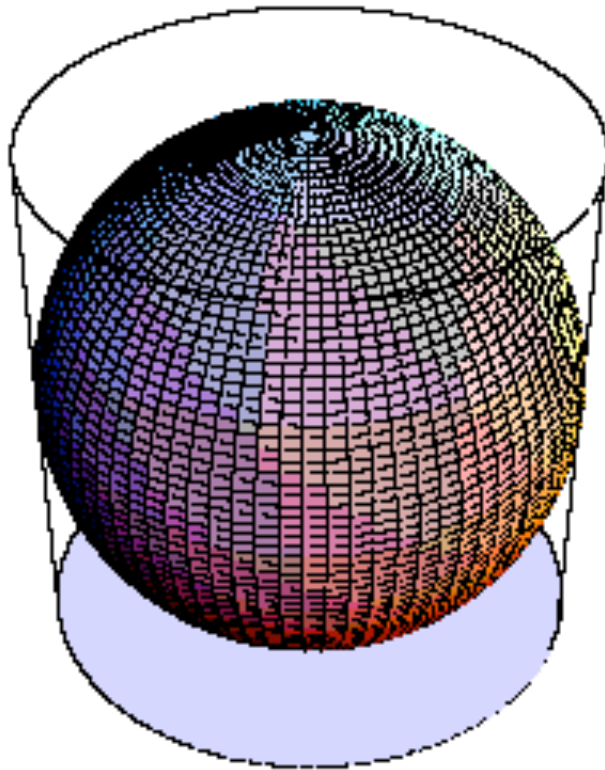
N.B. Can **not** calculate area under **hyperbola** or **ellipses** [integration requires **transcendental** functions].

“On the sphere and the cylinder”

calculates (by “exhaustion”) area and volume of the sphere :

$$\text{area} = S = 4\pi r^2 = 4 \times \underbrace{(\pi r^2)}_{\text{area of circle}}$$

$$\text{volume} = V = \frac{4\pi r^3}{3} = \frac{2}{3} \times \underbrace{((\pi r^2) \times 2r)}_{\text{vol of cylinder}}$$





It is not possible to find in all geometry more difficult and intricate questions, or more simple and lucid explanations . . .

No amount of investigation of yours would succeed in attaining the proof, and yet, once seen, you immediately believe you would have discovered it; by so smooth and so rapid a path he leads you to the conclusion required. (Plutarchos)

“The method” – Archimedes’ palimpsest

Archimedes sent a letter (now lost) to the [Alexandria Librarian Erathosthenes](#), in which he explains how he had found some of his results.

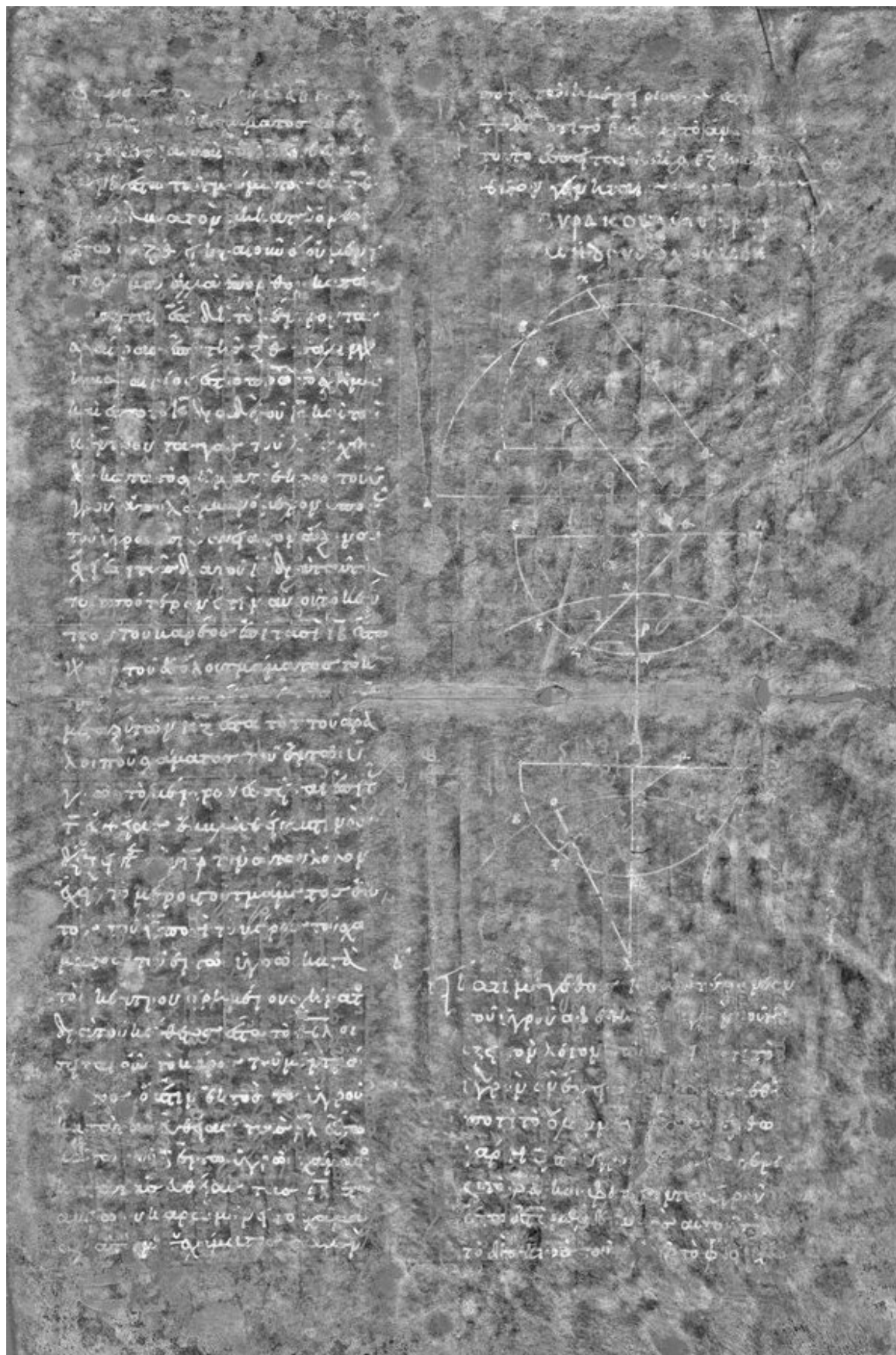
In **1906** in Istanbul (Constantinople - now Turkey) **9th** century parchment partially erased and overwritten in **12th** with religious text.





not completely erased !

Turning by 90 degrees . . .



mathematical figures !

→ can read original text !

Contains : books of Archimedes only copy !!!

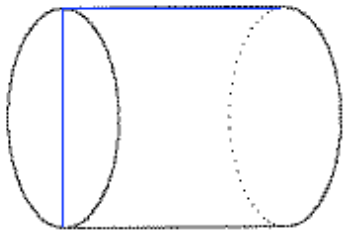
— “*Archimedes greets Eratosthenes . . .*”

“The Method of Mechanical Theorems”

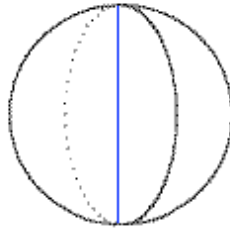
. . . certain things first became clear to me by a mechanical method, although they had to be proved by geometry afterwards.

It is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge.

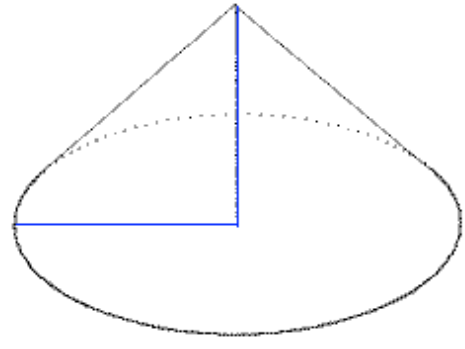
- Volume of a sphere. Archimedes considers



(a) Cylinder

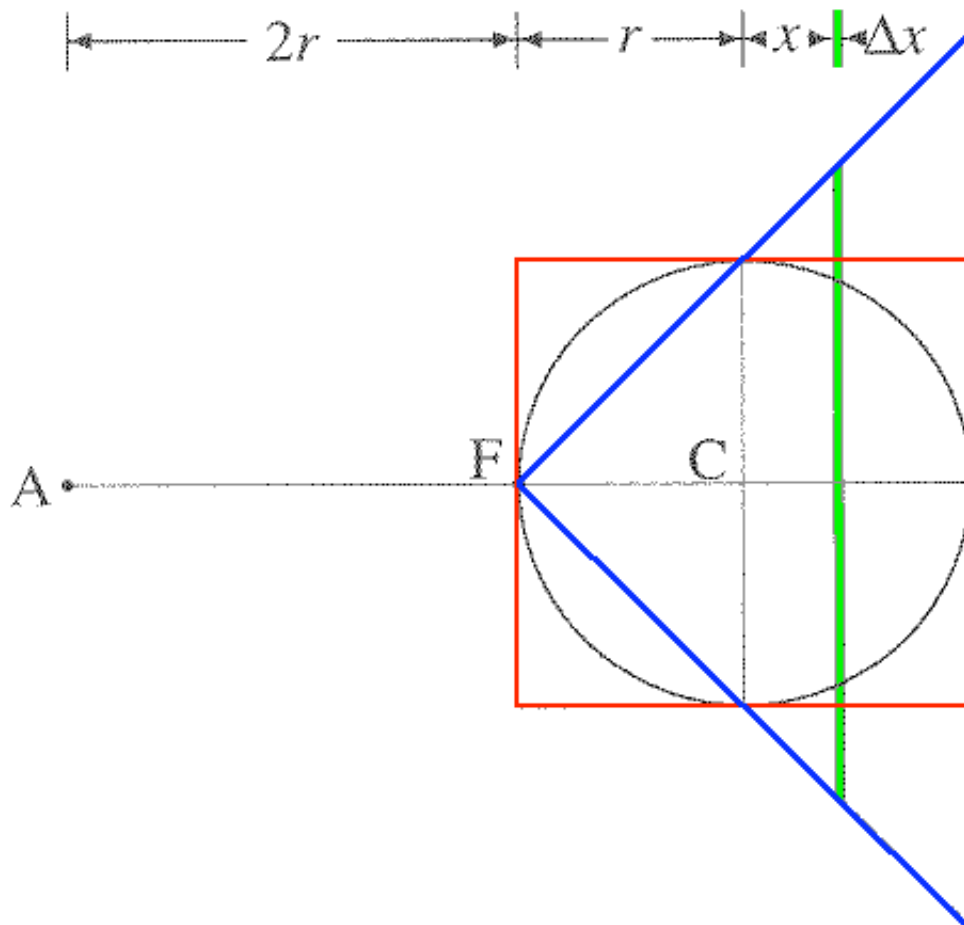


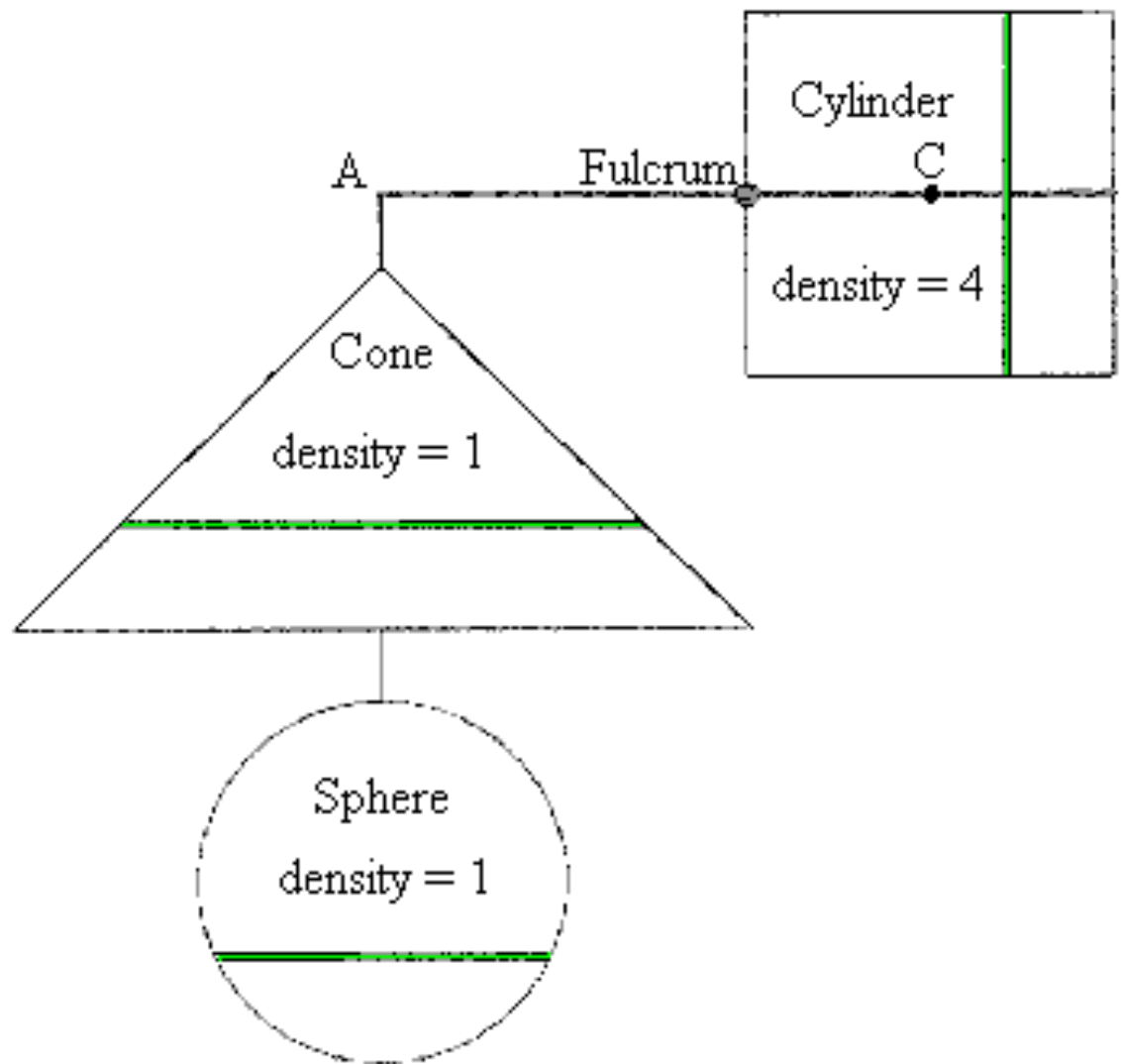
(b) Sphere



(c) Cone

- cylinder of radius r , heights $2r$, density 4
- sphere radius r , density 1
- cone of circular base radius $2r$, heights $2r$, density 1





$$AF = 2r$$

Archimedes' law: applied to sections :

$$2r \left(\underbrace{(2r - h)^2 \pi}_{\text{cone}} + \underbrace{[r^2 - (r - h)^2] \pi}_{\text{sphere}} \right) = \underbrace{(2r - h)}_{\text{distance}} (r^2 \pi) \times 4$$

h sweeps from $0 \rightarrow 2r \Rightarrow$

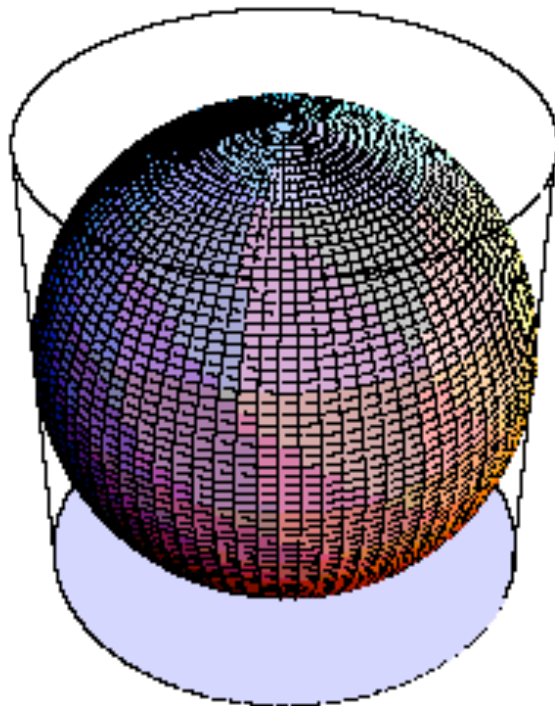
$$2r \times \left(\underbrace{\frac{1}{3} 2r (2r)^2 \pi}_{\text{cone vol}} + \boxed{V} \right) \times 1 = r \times \underbrace{2r \times r^2 \pi}_{\text{cyl vol}} \times 4 \Rightarrow$$

$$\boxed{V = \frac{4r^3 \pi}{3}}$$

Archimedes : Both the **surface** and the **volume** of **sphere** inscribed into a cylinder are

2/3

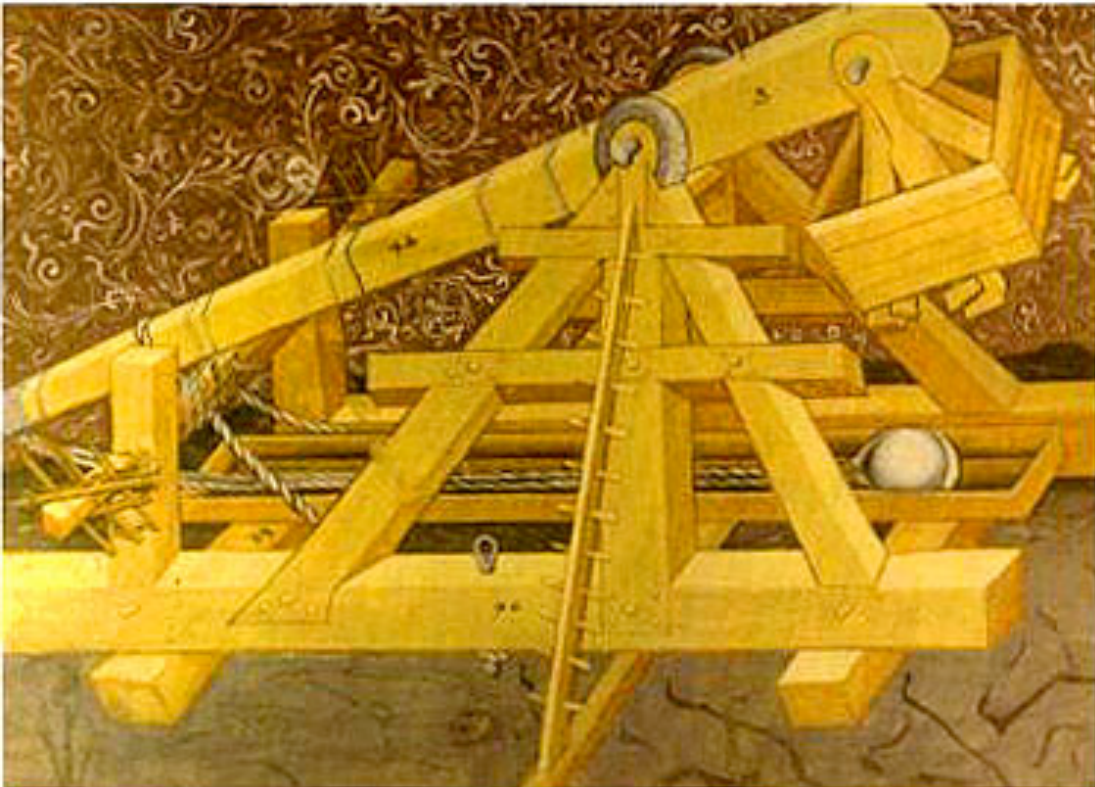
of that of the **cylinder**.



Archimedes considered as his most significant accomplishments . . . his result concerning a cylinder circumscribing a sphere, and he asked for a representation of this together with his result on the ratio of the two, to be inscribed on his tomb.

Plutarchos

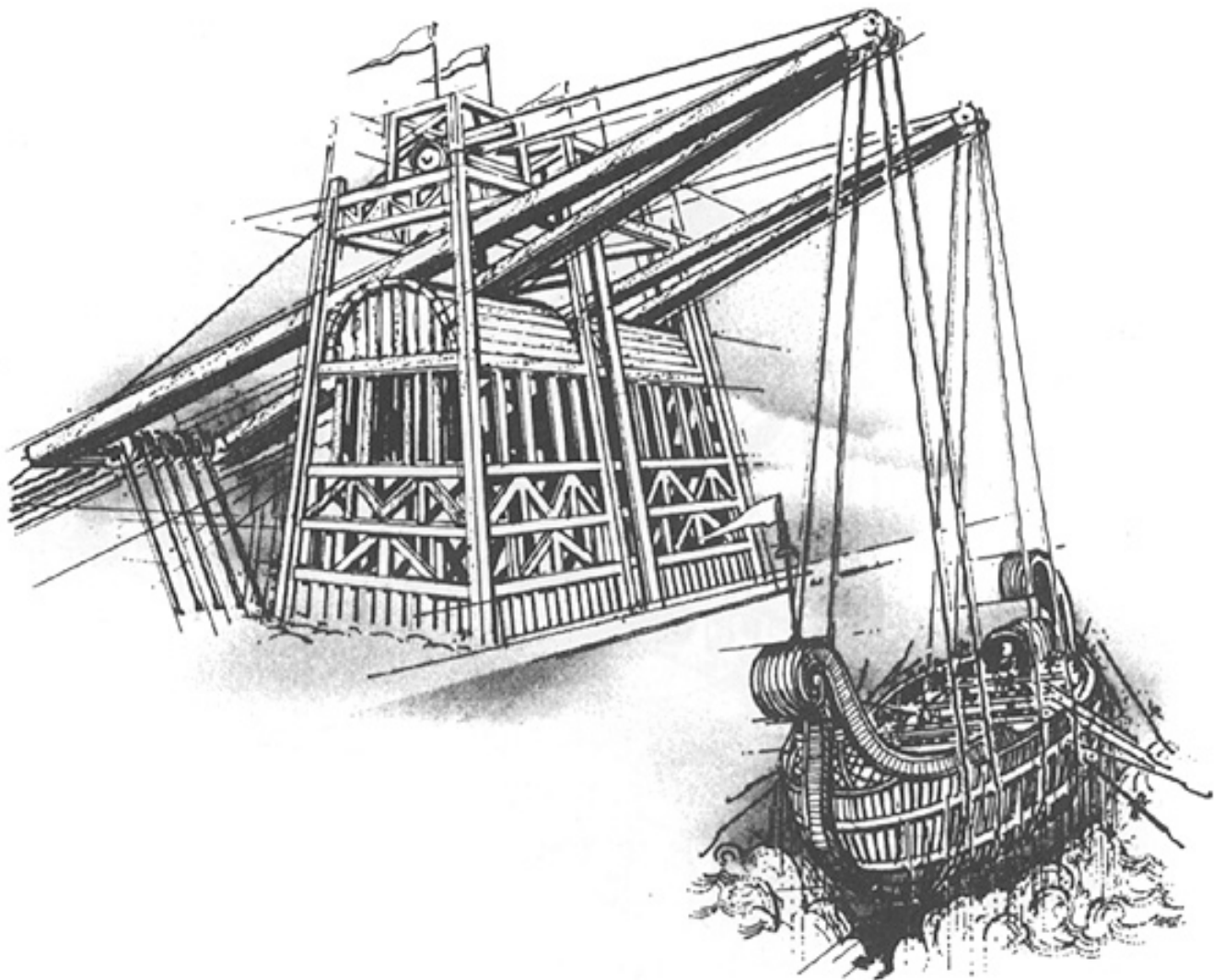
In 212 BC, during the 2nd punic war the romans put siege on Siracusa.



catapult

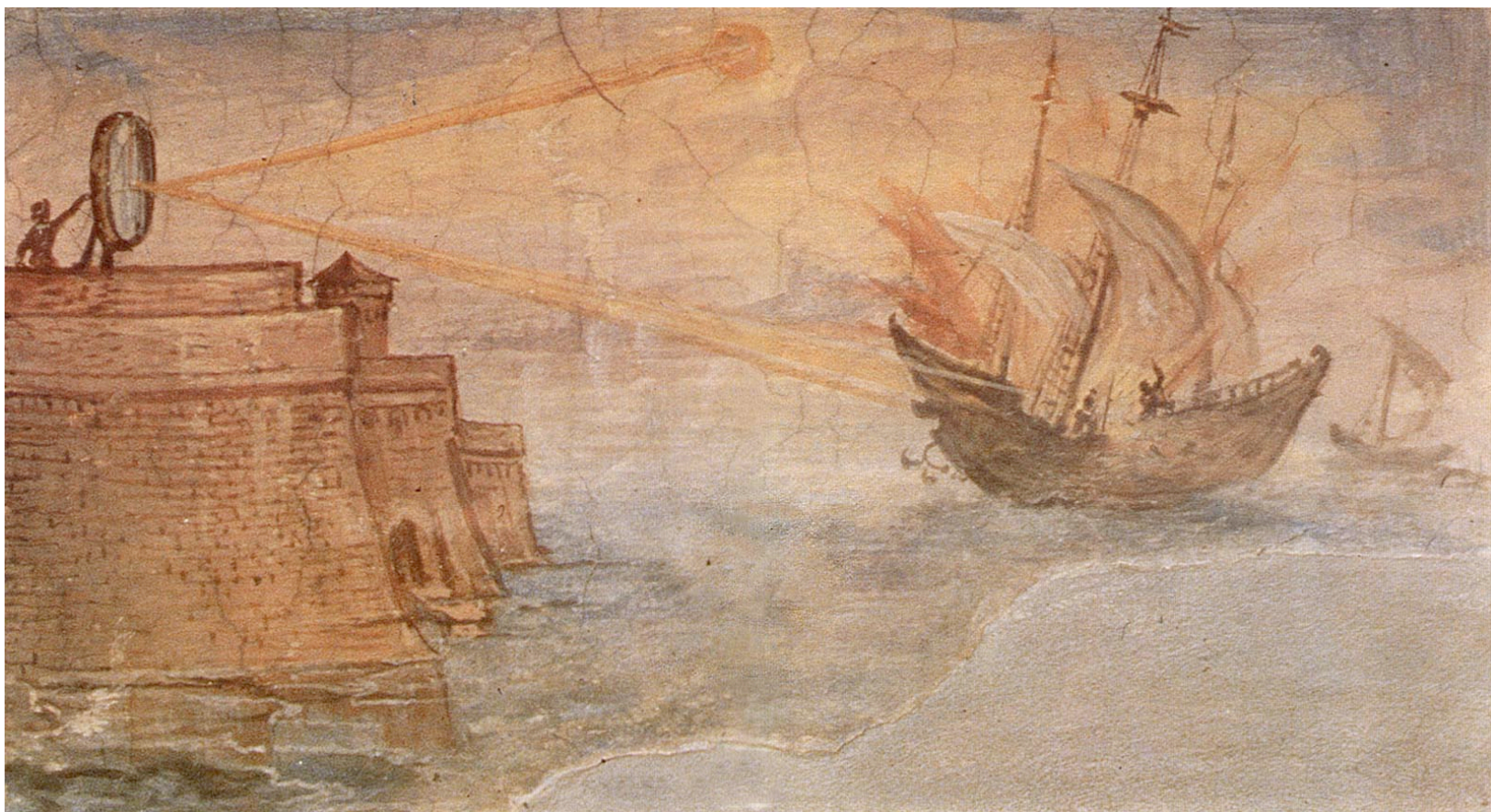
when Archimedes began to ply his engines, he at once shot against the land forces all sorts of missile weapons, and immense masses of stone that came down with incredible noise and violence; against which no man could stand . . .

... huge poles thrust out from the walls over the ships and sunk some by great weights which they let down from on high upon them; others they lifted up into the air by an iron hand





also used mirror

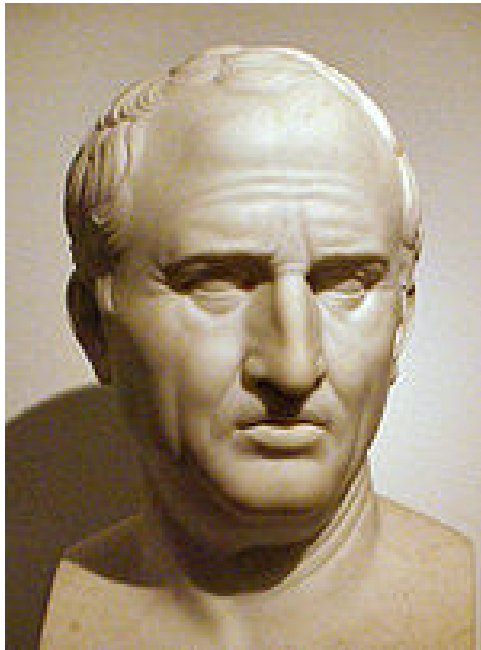


Archimedes was intent upon working out some problem by a diagram, and he never noticed the incursion of the Romans, nor that the city was taken.

In this transport of study and contemplation, a soldier, unexpectedly coming up to him, commanded him to follow to Marcellus; which he declining to do before he had worked out his problem to a demonstration, the soldier, drew his sword and ran him through.



mosaic found in Pompei



Cicero

(106 BC – 43 BC).

roman Governor of Sicily recounts :

I remembered certain lines inscribed upon his tomb, which stated that a sphere along with a cylinder had been put on top of his grave.

Accordingly, after taking a good look all around ... I noticed a small column on which there was a figure of a sphere and a cylinder ... the epigram was traceable with about half of the lines legible, as the latter portion was worn away.

Antikythera mechanism : “mechanical computer”, made before 87 BC, discovered in a shipwreck under the sea off a Greek island.



82 elements from which about 30 gears.

Reconstruction :



Allows to “calculate” the position of the stars in a given moment.

Another version of the death of Archimedes that Plutarchos had heard:

... as Archimedes was carrying to Marcellus [the Roman general] ... mathematical instruments, dials, spheres, and angles, by which the magnitude of the sun might be measured to the sight, some soldiers seeing him, and thinking that he carried gold in a vessel, slew him.

Cicero mentions TWO similar machines.

The first, constructed by **Archimedes**, was brought to Rome by Marcellus and was kept in his family. Cicero has seen and described it 150 years later, as being able to reproduce the motion of the **sun**, the **moon** and the **planets**.