Statement of research achievements and interests

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1. Dynamics of localised objects in field theory and interaction with radiation
2. Vortices, cosmic strings, and flux tubes
3. Almost time-periodic solutions
4. Skyrmions
5. Quantum mechanics of nanodevices
6. Further topics: effective field theory, perturbations of black holes and cosmological space-times

The main topic of my research is the existence, stability, and interactions of extended field theoretical objects. These objects belong to sectors of the spectrum of nonlinear field theories not available to perturbation theory. Thus they opened up a new paradigm, and bear applications in many areas of physics, from nuclear physics to cosmology, gravitation theory, high energy physics, superconductivity and superfluidity.

In my work, I apply analytical, perturbative and numerical methods, in many cases a combination of these to study the phenomena at hand, with a large set of mathematical and numerical tools.

Dynamics of localized objects in field theory and interaction with radiation Localized solutions reveal a new sector of a non-linear field theory that is not available to perturbation theory. Approximate solutions can be constructed by including the motion of the localized objects (introducing collective coordinates) and adding perturbations (waves), i.e., using *soliton perturbation theory* [1]. In the simplest case, the motion of the extended object satisfies Newton's equation, and applications include external force problems and radiation pressure for the scattering of waves.

In the 1+1 dimensional ϕ^4 model, the scattering of the waves on the extended object, the kink, is reflectionless, and therefore, the component of the radiation pressure quadratic in the amplitude vanishes. We have demonstrated that in higher orders, the radiation pressure is negative, i.e., the kink can be pulled toward a radiation source [2]. Inspired by this result, we have generalised the setting to multi-channel scatterings [3], e.g., scatterings on cosmic strings where a heavy particle undergoes cosmic string catalysed decay to a lighter one [4] or reflection of electromagnetic waves from an object rotating polarisations in a birefringent medium. Also, a similar effect is known for buckled graphene sheets [5].

Open problems related to this line of research that I am interested in include studying if the phenomenon of negative radiation pressure persists for other scattering situations, especially on localised field theoretical objects, such as the global and the local vortex and monopoles.

An additional problem is posed by the dynamics of the global vortex. As the total energy of the solution diverges due to the $1/r$ tail of its energy density, the methods of Ref. [1] cannot be applied. It is likely that different treatments of the problem of the motion of the localised objects are necessary, depending on whether the translational zero mode of the object is normalised (finite mass) or non-normalised and part of the continuous spectrum of the linearised field equations. A viable route to the solution of the problem seems to be the construction of 1+1 dimensional models with heavy kinks. The adequacy of approximation schemes shall be verified by comparison with numerical solutions.

Vortices, cosmic strings, and flux tubes are localised field theoretical objects in 2 resp. 3 spatial dimensions. A cross-section of a vortex string or flux tube is a vortex [6, 7]. The simplest vortex solutions include a complex scalar field with a nonlinear self-interaction (global vortex), and possibly a gauge field (local or Abrikosov-Nielsen-Olesen vortex). These simple solutions are topological and stable.

Additional field components may change the physics of vortices drastically. E.g., in the electroweak model, and its $\theta_{\mathrm{W}} \to 0$ limit, the semilocal model, the stability of the vortices becomes dependent on competing phenomena, and therefore on the parameters of the theory. For physical parameters, electroweak Z-strings (local vortex strings embedded in the theory with flux in the Z gauge field direction) are unstable [8]. We have shown, that the solutions which have nontrivial values of both Higgs components and a relative phase between them linearly dependent on the coordinate (twisted semilocal strings) [9] are also unstable [10].

We have constructed twisted vortex strings in non-Abelian supersymmetric theories [11], which, on the other hand, are expected to be stable. They also have unexpected mathematical properties, e.g., depending on their parameters, they can be such that they are not rotationally symmetric, although all their 2d sections orthogonal to the string axis are.

We have also shown, that in a limiting case where the size of the twisted semilocal strings diverges, stable solutions exist in a two-component extended Abelian Higgs model if the SU(2) symmetry of the two scalar field components is removed [12]. Such vortices may have applications in certain unusual superconductors, e.g., when in the proposed liquid metallic state of hydrogen both electrons and protons become paired [13] or in dense stellar matter [14], and show unusual phenomena such as type-1.5 superconductivity [15].

The stable solutions in the non-SU(2) symmetric extended Abelian Higgs models motivated extension to larger models. Certain models of dark matter contain additional scalar fields [16], and string solutions with their flux in the dark sector (and their string tension on a scale determined by its parameters) have been considered [17]. However, as the instability of the semilocal string can be understood as the system lowering its potential energy by filling up the string core (a false vacuum tube) with the Higgs component zero far from the string, and thereby unwinding, semilocal and electroweak strings (with the flux in the electroweak sector, and string tension determined by the known electroweak scale) may be stabilized by filling up the string core with the dark scalar. This we demonstrated for the semilocal model extended with a dark sector [18].

We have considered string solutions in the electroweak model extended with a dark Higgs field and a dark sector U(1) gauge field (the unified model of dark matter, Ref. [16]) [19]. It is found, that the stability properties of the strings are significantly enhanced in the semilocal limit (vanishing $SU(2)$ gauge coupling), but upon approaching the semilocal limit, the instabilities reappear. This is explained by the fact that it is the $SU(2)$ gauge field (W) which plays the major role in the instability.

Open problems in this line of research which I would like to study in the future is whether stabilisation persists fot physical values of the couplings in other models of the dark sector, and couplings of the electroweak sector to the dark one, e.g., in models where the dark sector is coupled to the W field in the visible sector. If it does, that opens up the road to the study of the interaction of such strings, and their formation and dynamics.

Almost time-periodic solutions In 1+1 spacetime dimensions, the only scalar field theory with a smooth potential admitting time-periodic localised solutions is the sine-Gordon model [20]. However, numerically investigations have shown, that in many non-linear field theories, from the simple ϕ^4 model to the Standard Model, long-lived, almost periodic solutions exist. These objects have a radiation tail, which is, however, very weak. Time-periodic solutions can be constructed, if one allows for an incoming wave feeding back the energy to the centre (i.e., without localisation); these are termed quasi-breathers [21]. The corresponding object with a purely outgoing wave-tail, termed oscillon, or pulson, is unstable, but often long-lived. In Ref. [22], we have constructed an universal expansion for the quasi-breather, and demonstrated that the so constructed quasi-breathers provide good initial date for oscillons.

The lifetime of oscillons is very hard to obtain numerically. However, a well-tested framework based on asymptotic expansions, and a beyond-all-orders correction, including an outgoing radiation tail [23]. This method has been generalized to arbitrary self-interaction potentials of the scalar field theory [24].

A related concept is that of quasi-integrability [25]. In quasi-integrable theories, similarly to integrable ones, an infinite system of quantities exist, which are, however, only almost conserved, e.g., for solutions consisting of solitons that are well separated (and the violation of the conservation is larger during collisions). An important open question is the relation of quasi-integrability, and the existence of long-lived almost periodic solutions.

Skyrmions In a modification of the Skyrme model, where the Bogomolnyi bound is saturated by the solutions (termed BPS Skyrme model)[26], with T. Ioannidou, we have studied the radial oscillations of the solutions with soliton perturbation theory, and compared the results to numerical simulations [27]. In a further paper, I have shown how to use the symmetries of the model, which include all volume preserving diffeomorphisms, to construct solutions with fractional topological charge [28].

In this model, a remaining interesting project would be to consider if slow time dependent solutions satisfy an ideal fluid equation of motion, as suggested by the fact, that volume preserving diffeomorphisms are also the symmetry of incompressible ideal fluids.

Quantum mechanics of nanodevices With Y.V. Nazarov, I have worked on the quantum mechanics of nanodevices. Josephson junctions with four or mode terminals are known to exhibit Weyl points (level crossings) in their Andreev bound state spectrum [29]. With J. Erdmanis and Y.V. Nazarov, we have shown that if the device is in an external circuit, the degeneracy in the tip of the Weyl cone spreads out to a finite region of codimension one in the parameter space. The possibility to manipulate the device in this degenerate region has possible applications in adiabatic quantum computing [30].

Further topics With Z. Perjés, I have worked on perturbations of black holes, and the possibility to quantize these via finding a Lagrangian formulation of Teukolsky's equation; and later, on cosmological perturbation theory. With G. Wolf, I worked on low-energy effective meson models.

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