Plane Waves as Tractor Beams


Árpád Lukács
Collaborators: Péter Forgács, Tomasz Romańczukiewicz

Wigner RCP RMKI,
Budapest, Hungary

SZTE Elméleti Fizikai Tanszék Szemináriumá
Outline

1. Introduction
   - What is negative radiation pressure?
   - NRP on kinks
   - Other approaches

2. NRP in two channel scattering
   - 1d
   - Examples
   - Force in 2d scattering

3. Applications
   - Neutron scattering on XY model vortices

4. Conclusions
Scattering of light: the pressure acting on a platelet

\[ P = (1 + R^2 - T^2)W, \]

\[ W = \frac{\epsilon_0 E_0^2}{2} \]

\( R: \) reflection, \( T: \) transmission

Can the pressure be negative?
What objects can be pulled towards the radiation source?

Unitarity (conservation of energy): \( R^2 + T^2 = 1, \ P = 2WR^2 > 0. \)

How can \( P \) be negative?
Kinks in the $\phi^4$ theory

- linearized perturbations: scattering reflectionless
- nonlinearities generate higher harmonics

More momentum in the forward direction $\rightarrow$ NRP

(Romańczukiewicz 2004, Forgács, Lukács and Romańczukiewicz 2007.)

It should be possible with 2 channels, different momenta
Other approaches

Gain media (Mizraki and Fainman, 2010)
Other approaches

Gain media (Mizraki and Fainman, 2010)

Structured beams (Sukhov and Dogariu, 2011)
Other approaches

Gain media (Mizraki and Fainman, 2010)

Structured beams (Sukhov and Dogariu, 2011)

Negative $\varepsilon, \mu$ (Veselago 1967)
Two channel scattering in 1d

Plane waves as tractor beams in unitary scattering

- Two kinds of waves (channels) \( i = 1, 2 \)
- momentum \( P_i \)
- channel 1 incoming wave

\[
p = |A_1|^2 \left[ P_1 (1 + |R_{11}|^2 - |T_{11}|^2) + P_2 (|R_{21}|^2 - |T_{21}|^2) \right],
\]

where \( R_{ij} \) and \( T_{ij} \): reflection and transmission coefficient, channel \( i \rightarrow j \).

- energy flux \( S_i \); energy conservation

\[
\sum_i S_i (|R_{ij}|^2 + |T_{ij}|^2) = S_j.
\]

- one channel: no NRP, \( p = 2|A|^2 P_1 |R|^2 > 0 \).
- NRP expected: \( P_2 > P_1 \) and large \( T_{21} \).
Birefringent media

Anisotropic dielectric tensor

\[ \varepsilon = \text{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z) \]

Two types of wave: polarizations
Wave propagation in the \( z \) direction

- Two modes: the \( x \) and \( y \) polarizations, assume \( n_x < n_y \)
- Momentum flux of plane waves \( P_i = \varepsilon_i / 2 \)
- Energy fluxes \( S_i = \sqrt{\varepsilon_i / \mu} / 2 \)

Pressure

\[ \frac{p_z}{|E_{x0}|^2} = P_x (1 + |R_{xx}|^2 - |T_{xx}|^2) + P_y (|R_{yx}|^2 - |T_{yx}|^2), \]

where \( T_{ij} \) transmission, \( R_{ij} \) reflection, channel \( j \rightarrow i \)

Energy conservation: \[ \sum_i S_i (|R_{ij}|^2 + |T_{ij}|^2) = S_j. \]
Scatterer

Plane of thickness $L$, same material, rotated by $\vartheta$

Neglect multiple surface interactions

\[ p_z = \epsilon_0 |E_{x0}|^2 \left( P_0 + P_1 \cos \frac{n_y - n_x}{c} \omega L \right), \]

- small angle

\[ P_0 = \theta^2 (n_x - n_y)(4n_x^2 + 3n_x n_y + n_y^2)/4n_y \]

\[ P_1 = -\theta^2 (n_x - n_y)(n_x + n_y)^2 / 2n_y \]

**Tractor beam:** $n_x < n_y$

- $n_{x,y} = n \pm \delta n$: $P_0 = -P_1 = -2n\delta n\theta^2$
- At 45°: $P_0 = -P_1 = -n\delta n/2$
Macroscopic example

Dielectric

\[ n_x = 3, \quad n_y = 6 \]

Incoming wave: 1 kW/cm\(^2\) at \( \omega = 1\)GHz

Pressure

\[ P_0 = -1.08, \quad P_1 = 1.99, \quad \text{avg. pressure} \quad P_z = -0.072 \text{ Pa} \]

Radiation pressure at total reflection: 0.6 Pa

Accuracy: exact value \(-0.053\) Pa

This is a macroscopic effect.
An optical example

Dielectric

Liquid crystal 5CB, at 25 °C, $\lambda = 5893$ Å: $n_x = 1.53$, $n_y = 1.72$

Scatterer: rotated birefringent platelet
E.g., $L = 0.1$ mm, $\theta = \pi/4$

Pressure

- x polarization $p_z = -5.52 \times 10^{-13}$ Pa (m/V)$^2|E_0|^2$
- y polarization $p_z = 7.48 \times 10^{-13}$ Pa (m/V)$^2|E_0|^2$

Accuracy: $-4.95 \times 10^{-13}$ Pa (m/V)$^2|E_0|^2$ and $8.14 \times 10^{-13}$ Pa (m/V)$^2|E_0|^2$

A few percents of radiation pressure on a totally reflecting mirror!
Force in 2d scattering

Scattering in 2d, rotational invariance
- Partial wave expansion (Fourier trf. in $\vartheta$)
- S-matrix elements $S_\ell$: $n \times n$ matrix ($n$: channels)
- $S_\ell$ unitary (conservation of energy)

Momentum balance: force master formula

$$F = F_x + iF_y = -4 \sum_\ell \left\{ A^\dagger S^\dagger_{\ell+1} KS_\ell A - A^\dagger KA \right\},$$

$A = (A_1, \ldots, A_n)^T$ amplitude, $K = \text{diag}(k_1, \ldots, k_n)$ wave numbers
Force in 2d scattering

Scattering in 2d, rotational invariance
- Partial wave expansion (Fourier trf. in $\vartheta$)
- S-matrix elements $S_\ell$: $n \times n$ matrix ($n$: channels)
- $S_\ell$ unitary (conservation of energy)

Momentum balance: force master formula

$$F = F_x + iF_y = -4 \sum_\ell \left\{ A^\dagger S^\dagger_{\ell+1} K S_\ell A - A^\dagger K A \right\},$$

$A = (A_1, \ldots, A_n)^T$ amplitude, $K = \text{diag}(k_1, \ldots, k_n)$ wave numbers

A consequence of unitarity:

$$\text{Re} |A_a|^2 k_a (1 - S^*_{aa,\ell+1} S_{aa\ell}) > 0$$

one channel radiation pressure positive

NRP: $k_b > k_a$ necessary
Scattering on vortices

- Two types of wave (particle species/spin/etc.)
- Neglect vortex core
  
  \( r \to \infty \) asymptotic form of \( A \)
  
  \( \to \) a two channel Aharonov-Bohm scattering problem

\[
\left( \nabla + iA \frac{\sigma_2}{2} \right)^2 \rho - K^2 \rho = 0, \quad \sigma_2 = \begin{pmatrix} i & -i \end{pmatrix},
\]

where

- \( A = e^\vartheta / r \)
- \( \rho = (u, d) \), \( u \): heavy and \( d \) light mode
- fermionic bdry cond. \( \rho(r, \vartheta + 2\pi) = -\rho(r, \vartheta) \)
Scattering on vortices

- Two types of wave (particle species/spin/etc.)
- Neglect vortex core
  \[ r \rightarrow \infty \text{ asymptotic form of } A \]
  \[ \rightarrow \text{ a two channel Aharonov-Bohm scattering problem} \]

\[
\left( \nabla + iA\frac{\sigma_2}{2} \right)^2 \rho - K^2 \rho = 0, \quad \sigma_2 = \begin{pmatrix} i & -i \end{pmatrix},
\]

where
- \( A = e\vartheta / r \)
- \( \rho = (u, d) \), \( u \): heavy and \( d \) light mode
- fermionic bdry cond. \( \rho(r, \vartheta + 2\pi) = -\rho(r, \vartheta) \)

Solution: partial waves, radial eq. numerically
- NRP for large range of parameters
- Cross section \( \gg \) geometric, \( 1/k \)
- \( 1/sin\vartheta/2 \) in scattering amplitude
- Large cross section from one channel to another
Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

\[ B + \text{string} \rightarrow \ell + \text{string} \]
Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

\[ B + \text{string} \rightarrow \ell + \text{string} \]

A simplified description:
- Neglect spin degrees of freedom
- 2 channel Aharonov–Bohm scattering
  1 heavy baryon, 1 light lepton, mass ratio 1.5 : 2.
Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

\[ B + \text{string} \rightarrow \ell + \text{string} \]

A simplified description:
- Neglect spin degrees of freedom
- 2 channel Aharonov–Bohm scattering
  - 1 heavy baryon, 1 light lepton, mass ratio 1.5 : 2.
- Large cross section:
  - cosmic string catalyzed baryon number violation
- Force: string friction (moving in a plasma)
  - Decoupled approximation

\[ F_i = -4n_i \nu (1 - \exp(2\pi i \nu_i \Phi)) \]

\( n_i \) density, \( \nu_i \) coupling to the X-boson
- only valid for light modes, heavy modes give negative contribution
- at \( \nu = 0.65 \)

\[ F_x^u = -6.09|A|^2, \quad F_x^d = 7.44|A|^2. \]
- scattering energy \( m_i/\sqrt{1 - \nu^2} \)
**Force: x**

### x component

![Graph showing the relationship between $F_x / (\text{Amplitude})^2$ and $\omega$.](image)

- **$F_{x,u}$**: Represented by a dot symbol (+)
- **$F_{x,d}$**: Represented by an 'x' symbol
- **$F_{x,dc}$**: Represented by a dashed line

- The graph plots $F_x / (\text{Amplitude})^2$ against $\omega$ for different values of $\omega$.

- The x-axis represents $\omega$ ranging from 2 to 2.9.
- The y-axis represents $F_x / (\text{Amplitude})^2$ ranging from -10 to 15.
Force: y

y component

\[ F_y/(\text{Amplitude})^2 \]

\[ \omega \]

\[ F_{y,u} \]

\[ F_{y,d} \]
Neutron scattering: XY model vortices I

XY model
rotators (spins) in a plane, with nearest neighbor interaction

Magnetic vortex: singularity of magnetization $\mathbf{M}$

$gM$ energy difference between parallel and antiparallel spin neutrons

Diagonalize Hamiltonian locally
- 2 modes, $\hbar^2/2m(k_d^2 - k_u^2) = gM$
- small momentum transfer

Measurable manifestation of the same phenomenon as NRP
- Large cross sections, $1/k$ (A-B)
- Large spin-flip cross section
  $E = 4.1 \times 10^{-5}$ eV (45 Å neutrons), $\sigma_{du} = 1.19 \times 10^{-4}$ m
- Can be calculated perturbatively
Conclusions

- Many approaches to tractor beams (e.g., structured beams)
- Multi-channel scattering, $k_i \neq k_j$ fairly general
- One-dimensional examples:
  - polarizations of EM waves
  - higher harmonics (kink)
- Two dimensions
  - Cosmic strings: baryon decay
  - Magnetic XY-vortex
Conclusions

- Many approaches to tractor beams (e.g., structured beams)
- Multi-channel scattering, $k_i \neq k_j$ fairly general
- One-dimensional examples:
  - polarizations of EM waves
  - higher harmonics (kink)
- Two dimensions
  - Cosmic strings: baryon decay
  - Magnetic XY-vortex

THANK YOU FOR YOUR ATTENTION!
Cosmic strings

- Classical field theoretical solution
- Thin, elongated object
- String core: a zero of a Higgs field
- Energy density localized in the core

Important parameter: string tension

\[ \mu = \frac{E}{L} \]

- Electroweak string: \( G_\mu \approx 10^{-32} \) (\( \mu \): 10 mg/Solar diam)
- GUT string: \( G_\mu \approx 10^{-6} \) (\( \mu \): Solar mass/Solar diam)

Cosmic strings are high energy localized objects that provide a link between astrophysics and particle physics.
Physics of cosmic strings

- Formation: during phase transitions
- Evolution of a string network
  - Friction dominated era: scattering of particles
  - Scaling $\nu \sim 0.65$
  - collisions, interlinking
  - radiation (e.g. at cusps formed in collisions)
  - string tension contracts loops
  - expansion: the network becomes more diluted
- Signatures of cosmic strings
  - Scattering of material off strings: structure formation: galaxies, voids, filaments (fractal dimension)
  - Contribution to CMB anisotropy: best fit with GUT strings
    \[ G_\mu = (2.04 \pm 0.13) \times 10^{-6}, \text{ Contribution to multipole } \ell = 10: f_{10} = 0.11 \pm 0.05 \] (Hindmarsh et al., 2007, 2008)
  - Gravitational lensing
  - Gravitational radiation
Artificial gauge potential

- Diagonalize locally the Hamiltonian

\[ U^\dagger (V - \omega) U = K \]

\[ K = \text{diag}(k_1, \ldots, k_n) \]

- kinetic term:

\[ U^\dagger \partial_i U = \nabla + U^\dagger (\partial_i U) = \nabla - iA_i \]

- \( A \): artificial gauge potential
- \( U = U(\vartheta) \): Aharonov-Bohm form

\[ A = \frac{A_\vartheta}{r} e^\vartheta, \]

where \( A_\vartheta = U^\dagger \partial U / \partial \vartheta \)
Scalar perturbations of the global vortex

One massive, one Goldstone (massless) mode
Perturbations of the superfluid vortex

Asymptotics: $H_{\nu}(\omega r/2), K_{i\eta}(2r)$, one channel
The Aharonov–Bohm effect:
Motion of a charged particle in a region with $B = 0$

Double slit experiment: Scattering:

- Both experiments show flux dependence
- Holonomy is also physical not just field strength $\mathbb{P}e^{i \oint A \, dr}$
- Reaction force (deflected beam): Force acting on the scatterer
Aharonov–Bohm scattering II

Schrödinger-equation

\[-i\dot{\psi} = (\nabla - iA)^2\psi,\]

with electromagnetic vector potential

\[A(r, \vartheta, z) = \frac{A_0}{r} e^{i\vartheta}.\]

$2\pi A_0$ flux; outside $B = 0$.

Fixed energy: $\psi(r, t) = e^{-i\omega t}\psi(r)$.

Scattering asymptotics ()

\[\psi \sim e^{ikx} + \frac{f(\vartheta)}{\sqrt{r}} e^{ikr}\]

Cross sections:

\[d\sigma/d\vartheta = |f(\vartheta)|^2\]

Scattering amplitude:

\[f(\vartheta) \sim \frac{\sin \pi A_0}{2\pi} \frac{1}{\sin(\vartheta/2)}.\]
Aharonov–Bohm scattering III

Partial waves: $s_\ell \propto J_{|\ell - A_0|}$ (note index shift wrt plane wave)

Effect of scattering in the outgoing wave: phase shift $\delta_\ell$,

$S_\ell = \exp(2i\delta_\ell)$

$(J_\nu(z) \sim \cos(z - \nu \pi/2 - \pi/4)/\sqrt{2\pi z})$

\[
\ell \geq 0 : \delta_\ell = \frac{A_0 \pi}{2}, \quad \ell \leq -1 : \delta_\ell = \pi \ell - \frac{A_0 \pi}{2}.
\]

thus

\[
F_x = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \{\cos[2(\delta_\ell - \delta_{\ell-1})] - 1\}
\]

\[
= -|\phi_0|^2 4k (\cos(2\pi A_0) - 1) \approx 16|\phi_0|^2 \pi^2 A_0^2 k,
\]

\[
F_y = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \sin[2(\delta_\ell - \delta_{\ell-1})] = |\phi_0|^2 4k \sin(2\pi A_0) \approx 8|\phi_0|^2 \pi A_0 k.
\]

Analog problem: force acting on a superfluid vortex (GPe)
Iordanskii force controversy


\[ f(-\vartheta) = f(\vartheta) \Rightarrow F_y = 0 \]


scattering asymptotics does not hold in forward direction

\[
\psi(r, t) = e^{-i(\omega t - kx)}\phi(r)
\]

and for \( y \ll \sqrt{x} \)

\[
\phi(x > 0, y) \sim \cos(A_0 \pi) - \frac{2i^{1/2}}{\sqrt{\pi}} \sin(A_0 \pi) \sqrt{\frac{k}{2}} \frac{y}{\sqrt{x}}
\]

transversal force \( F_y \) from this region (although not \( F_x \))


A. Dogariu, S. Sukhov, and J.J. Sáenz *Optically induced ‘negative forces’* *Nat. Pho.* **7**, 24 (2012);


V.G. Veselago, *The electrodynamics of substances with simultaneously negative values of $\epsilon$ and $\mu$* Sov. Phys. Usp. **10** 509 (1968).


