Plane Waves as Tractor Beams

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   - NRP on kinks
   - Other approaches
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3. Applications
   - Neutron scattering on XY model vortices
4. Conclusions

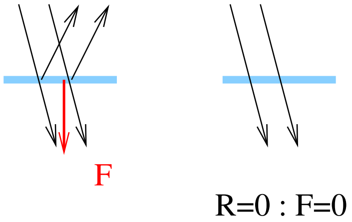
Radiation pressure

Scattering of light: the pressure acting on a platelet

$$P = (1 + R^2 - T^2)W\,,$$

$$W = \epsilon_0 E_0^2/2$$

R: reflection, T: transmission



Can the pressure be negative?
What objects can be pulled towards the radiation source?
Unitarity (conservation of energy): $R^2 + T^2 = 1$, $P = 2WR^2 > 0$.
How can P be negative?

NRP on kinks

Kinks in the ϕ^4 theory

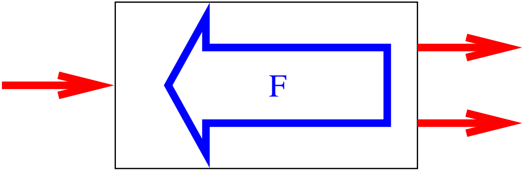
- linearized perturbations: scattering reflectionless
- nonlinearities generate higher harmonics

(Video: Tomasz Romańczukiewicz)

- More momentum in the forward direction $\rightarrow$ NRP
  (Romańczukiewicz 2004, Forgács, Lukács and Romańczukiewicz 2007.)
- It should be possible with 2 channels, different momenta

Other approaches

Gain media (Mizraki and Fainman, 2010)



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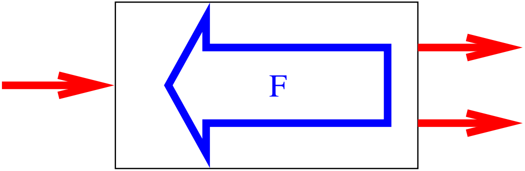
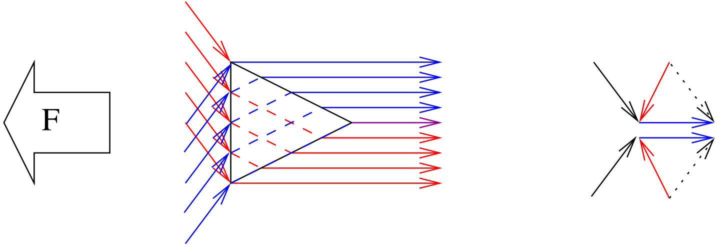


Structured beams (Sukhov and Dogariu, 2011)



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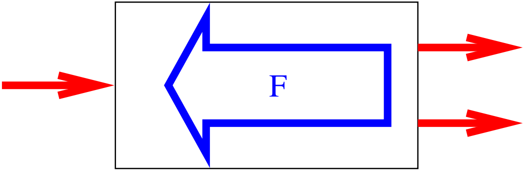

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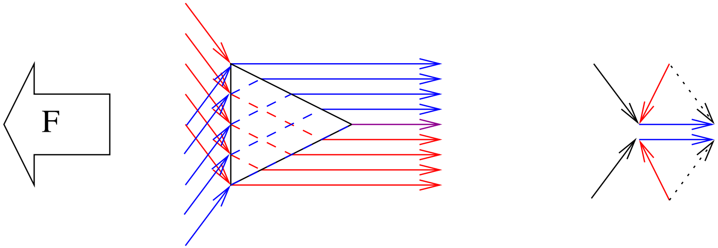

Negative ε, μ (Veselago 1967)

Two channel scattering in 1d

Plane waves as tractor beams in unitary scattering

- Two kinds of waves (channels) $i = 1, 2$
- momentum P_i
- channel 1 incoming wave

$$p = |A_1|^2 \left[P_1 (1 + |R_{11}|^2 - |T_{11}|^2) + P_2 (|R_{21}|^2 - |T_{21}|^2) \right] ,$$

  where R_{ij} and T_{ij} : reflection and transmission coefficient, channel $i \to j$.
- energy flux S_i; energy conservation

$$\sum_i S_i (|R_{ij}|^2 + |T_{ij}|^2) = S_j .$$

- one channel: no NRP, $p = 2|A|^2 P_1 |R|^2 > 0$.
- NRP expected: $P_2 > P_1$ and large T_{21}.

Birefringent media

Anisotropic dielectric tensor

$$\varepsilon = \mathrm{diag}(\varepsilon_x, \varepsilon_y, \varepsilon_z)$$

Two types of wave: polarizations
Wave propagation in the z direction

- Two modes: the x and y polarizations, assume $n_x < n_y$
- Momentum flux of plane waves $P_i = \varepsilon_i/2$
- Energy fluxes $S_i = \sqrt{\varepsilon_i/\mu}/2$

Pressure

$$\frac{p_z}{|E_{x0}|^2} = P_x(1 + |R_{xx}|^2 - |T_{xx}|^2) + P_y(|R_{yx}|^2 - |T_{yx}|^2)\,,$$

where T_{ij} transmission, R_{ij} reflection, channel $j \to i$

Energy conservation: $\sum_i S_i(|R_{ij}|^2 + |T_{ij}|^2) = S_j$.

Force formula

Scatterer

Plane of thickness L, same material, rotated by ϑ

Neglect multiple surface interactions

$$p_z = \epsilon_0 |E_{x0}|^2 \left(P_0 + P_1 \cos \frac{n_y - n_x}{c} \omega L \right) ,$$

- small angle

$$P_0 = \theta^2 (n_x - n_y)(4n_x^2 + 3n_x n_y + n_y^2)/4n_y$$

$$P_1 = -\theta^2 (n_x - n_y)(n_x + n_y)^2/2/n_y$$

Tractor beam: $n_x < n_y$

- $n_{x,y} = n \pm \delta n$: $P_0 = -P_1 = -2n\delta n \theta^2$
- At 45°: $P_0 = -P_1 = -n\delta n/2$

Macroscopic example

Dielectric

$$n_x = 3, \quad n_y = 6$$

Incoming wave: 1 kW/cm^2 at $\omega = 1$GHz

Pressure

$P_0 = -1.08$, $P_1 = 1.99$, avg. pressure $P_z = -0.072$ Pa

Radiation pressure at total reflection: 0.6 Pa
Accuracy: exact value -0.053 Pa

This is a macroscopic effect.

An optical example

Dielectric

Liquid crystal 5CB, at 25 °C, $\lambda = 5893$ Å : $n_x = 1.53$, $n_y = 1.72$

Scatterer: rotated birefringent platelet
E.g., $L = 0.1$ mm, $\theta = \pi/4$

Pressure

- x polarization $p_z = -5.52 \times 10^{-13}$Pa (m/V)$^2|E_0|^2$
- y polarization $p_z = 7.48 \times 10^{-13}$Pa (m/V)$^2|E_0|^2$

Accuracy: -4.95×10^{-13}Pa (m/V)$^2|E_0|^2$ and
8.14×10^{-13}Pa (m/V)$^2|E_0|^2$
A few percents of radiation pressure on a totally reflecting mirror!

Force in 2d scattering

Scattering in 2d, rotational invariance

- Partial wave expansion (Fourier trf. in ϑ)
- S-matrix elements S_ℓ: $n \times n$ matrix (n: channels)
- S_ℓ unitary (conservation of energy)

Momentum balance: force master formula

$$F = F_x + iF_y = -4\sum_\ell \left\{ A^\dagger S^\dagger_{\ell+1} K S_\ell A - A^\dagger K A \right\} ,$$

$A = (A_1, \ldots, A_n)^T$ amplitude, $K = \mathrm{diag}(k_1, \ldots, k_n)$ wave numbers

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A consequence of unitarity:

$$\mathrm{Re}\, |A_a|^2 k_a (1 - S_{aa,\ell+1}^* S_{aa\ell}) > 0$$

one channel radiation pressure positive

NRP: $k_b > k_a$ necessary

Scattering on vortices

- Two types of wave (particle species/spin/etc.)
- Neglect vortex core
  $r \to \infty$ asymptotic form of A
  $\to$ a two channel Aharonov-Bohm scattering problem

$$\left(\nabla + i\mathbf{A}\frac{\sigma_2}{2}\right)^2 \rho - K^2 \rho = 0\,, \quad \sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}\,,$$

where

- $\mathbf{A} = \mathbf{e}_\vartheta / r$
- $\rho = (u, d)$, u: heavy and d light mode
- fermionic bdry cond. $\rho(r, \vartheta + 2\pi) = -\rho(r, \vartheta)$

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Solution: partial waves, radial eq. numerically

- NRP for large rane of parameters
- Cross section $\gg$ geometric, $1/k$
- $1/sin\vartheta/2$ in scattering amplitude
- Large cross section from one channel to another

Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

$$B + \text{string} \to \ell + \text{string}$$

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A simplified description:

- Neglect spin degrees of freedom
- 2 channel Aharonov–Bohm scattering
  1 heavy baryon, 1 light lepton, mass ratio 1.5 : 2.

Scattering off cosmic strings

Inside a cosmic string: GUT Higgs zero, flux of broken gauge field
Cosmic string catalyzed baryon number violation:

$$B + \text{string} \to \ell + \text{string}$$

A simplified description:

- Neglect spin degrees of freedom
- 2 channel Aharonov–Bohm scattering
  1 heavy baryon, 1 light lepton, mass ratio 1.5 : 2.
- Large cross section:
  cosmic string catalyzed baryon number violation
- Force: string friction (moving in a plasma)
  - Decoupled approximation

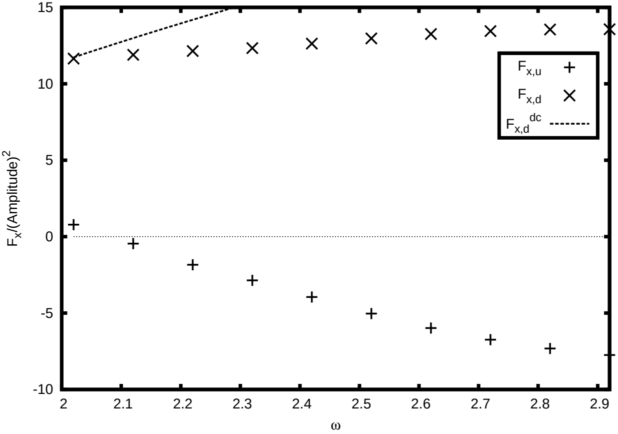
    $$F_i = -4n_i v(1 - \exp(2\pi i \nu_i \Phi))$$

    n_i density, ν_i coupling to the X-boson
  - only valid for light modes, heavy modes give negative contribution
  - at $v = 0.65$

    $$F_x^u = -6.09|A|^2\,, \quad F_x^d = 7.44|A|^2\,.$$
  - scattering energy $m_i/\sqrt{1 - v^2}$

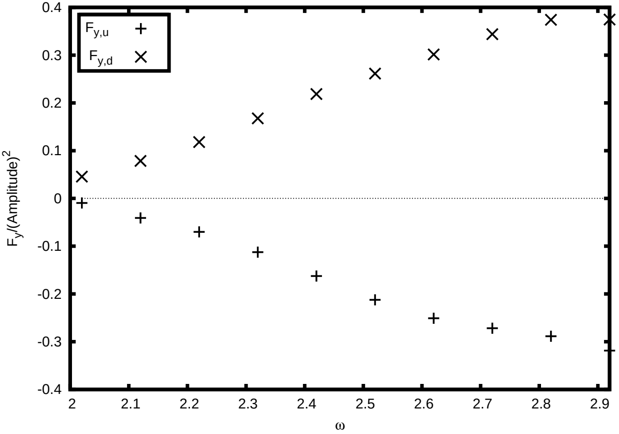
Force: x

x component



Force: y

y component



Neutron scattering: XY model vortices I

XY model

rotators (spins) in a plane, with nearest neighbor interaction

Magnetic vortex: singularity of magnetization **M**

gM energy difference between parallel and antiparallel spin neutrons

Diagonalize Hamiltonian locally

- 2 modes, $\hbar^2/2/m(k_d^2 - k_u^2) = gM$
- small momentum transfer

Measurable manifestation of the same phenomenon as NRP

- Large cross sections, $1/k$ (A-B)
- Large spin-flip cross section
  $E = 4.1 \times 10^{-5}$ eV (45 Å neutrons), $\sigma_{du} = 1.19 \times 10^{-4}$ m
- Can be calculated perturbatively

Conclusions

- Many approaches to tractor beams (e.g., structured beams)
- Multi-channel scattering, $k_i \neq k_j$ fairly general
- One-dimensional examples:
  - polarizations of EM waves
  - higher harmonics (kink)
- Two dimensions
  - Cosmic strings: baryon decay
  - Magnetic XY-vortex

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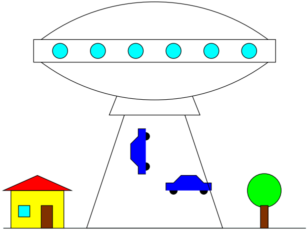


THANK YOU FOR YOUR ATTENTION!

Cosmic strings

- Classical field theoretical solution
- Thin, elongated object
- String core: a zero of a Higgs field
- Energy density localized in the core

Important parameter: string tension

$$\mu = E/L$$

- Electroweak string: $G\mu \approx 10^{-32}$ (μ: 10 mg/Solar diam)
- GUT string: $G\mu \approx 10^{-6}$ (μ: Solar mass/Solar diam)

Cosmic strings are high energy localized objects that provide a link between astrophysics and particle physics.

Physics of cosmic strings

- Formation: during phase transitions
- Evolution of a string network
  - Friction dominated era: scattering of particles
  - Scaling $v \sim 0.65$
  - collisions, interlinking
  - radiation (e.g. at cusps formed in collisions)
  - string tension contracts loops
  - expansion: the network becomes more diluted
- Signatures of cosmic strings
  - Scattering of material off strings: structure formation: galaxies, voids, filaments (fractal dimension)
  - Contribution to CMB anisotropy: best fit with GUT strings $G\mu = (2.04 \pm 0.13) \times 10^{-6}$, Contribution to multipole $\ell = 10$:$f_{10} = 0.11 \pm 0.05$ (Hindmarsh et al., 2007, 2008)
  - Gravitational lensing
  - Gravitational radiation

Artificial gauge potential

- Diagonalize locally the Hamiltonian

$$U^\dagger (V - \omega) U = K$$

$K = \mathrm{diag}(k_1, \ldots, k_n)$

- kinetic term:

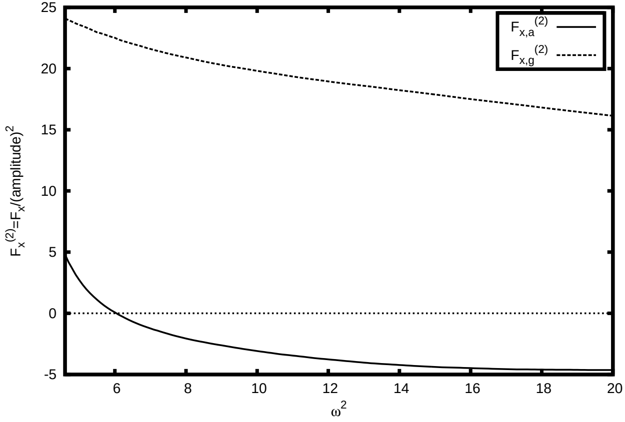
$$U^\dagger \partial_i U = \nabla + U^\dagger (\partial_i U) = \nabla - iA_i$$

- **A** : artificial gauge potential
- $U = U(\vartheta)$: Aharonov-Bohm form

$$\mathbf{A} = \frac{A_\vartheta}{r} \mathbf{e}_\vartheta \,,$$

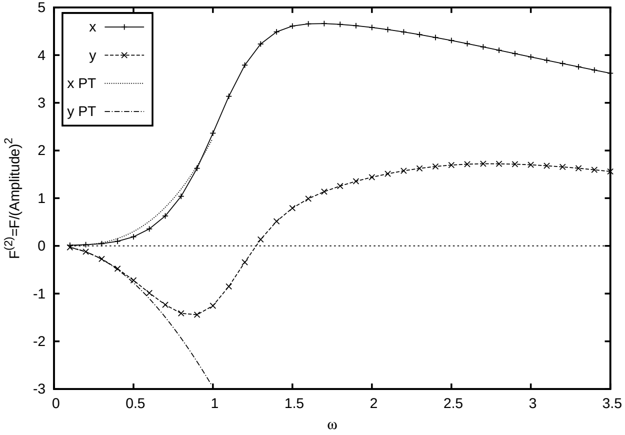
where $A_\vartheta = U^\dagger \partial U / \partial \vartheta$

Scalar perturbations of the global vortex



One massive, one Goldstone (massless) mode

Perturbations of the superfluid vortex

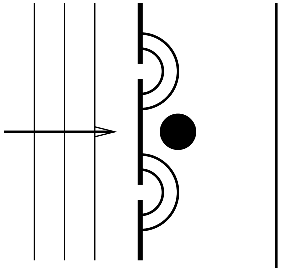


Asymptotics: $H_\nu(\omega r/2)$, $K_{i\eta}(2r)$, one channel

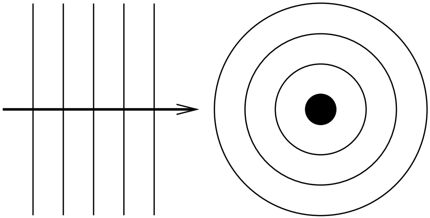
Aharonov–Bohm scattering I

The Aharonov–Bohm effect:
Motion of a charged particle in a region with $\mathbf{B} = 0$

Double slit expreriment:



Scattering:



- Both experiments show flux dependence
- Holonomy is also physical not just field strength $\mathbb{P}e^{i\int \mathbf{A}d\mathbf{r}}$
- Reaction force (deflected beam): Force acting on the scatterer

Aharonov–Bohm scattering II

Schrödinger-equation

$$-i\dot{\psi} = (\nabla - i\mathbf{A})^2\psi\,,$$

with electromagnetic vector potential

$$\mathbf{A}(r,\vartheta,z) = \frac{A_0}{r}\mathbf{e}_\vartheta\,.$$

$2\pi A_0$ flux; outside $\mathbf{B} = 0$.
Fixed energy: $\psi(\mathbf{r},t) = \mathrm{e}^{-i\omega t}\psi(\mathbf{r})$.
Scattering asymptotics (?)

$$\psi \sim \mathrm{e}^{ikx} + \frac{f(\vartheta)}{\sqrt{r}}\mathrm{e}^{ikr}$$

Cross sections: $\mathrm{d}\sigma/\mathrm{d}\vartheta = |f(\vartheta)|^2$
Scattering amplitude: $f(\vartheta) \sim \frac{\sin \pi A_0}{2\pi}\frac{1}{\sin(\vartheta/2)}$.

Aharonov–Bohm scattering III

Partial waves: $s_\ell \propto J_{|\ell - A_0|}$ (note index shift wrt plane wave)
Effect of scattering in the outgoing wave: phase shift δ_ℓ,
$S_\ell = \exp(2i\delta_\ell)$
$(J_\nu(z) \sim \cos(z - \nu\pi/2 - \pi/4)/\sqrt{2\pi z})$

$$\ell \geq 0 \;:\; \delta_\ell = \frac{A_0 \pi}{2}\,, \quad \ell \leq -1 \;:\; \delta_\ell = \pi\ell - \frac{A_0\pi}{2}\,.$$

thus

$$F_x = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \{\cos\left[2(\delta_\ell - \delta_{\ell-1})\right] - 1\}$$
$$= -|\phi_0|^2 4k\,(\cos(2\pi A_0) - 1) \approx 16|\phi_0|^2 \pi^2 A_0^2 k\,,$$
$$F_y = -|\phi_0|^2 4k \sum_{\ell=-\infty}^{\infty} \sin\left[2(\delta_\ell - \delta_{\ell-1})\right] = |\phi_0|^2 4k \sin(2\pi A_0) \approx 8|\phi_0|^2 \pi A_0 k\,.$$

Analog problem: force acting on a superfluid vortex (GPe)

Aharonov–Bohm scattering IV

Iordanskii force controversy C. Wexler, D.J. Thouless, *Phys. Rev.* **B58** R8897–R8900 (1998):

$$f(-\vartheta) = f(\vartheta) \quad \Rightarrow \quad F_y = 0$$

A.L. Shelankov *Europhys. Lett.* **43** (1998) 623
M.V. Berry *J. Phys. A: Math. Gen.* **32** (1999) 5627:
scattering asymptotics does not hold in forward direction

$$\psi(\mathbf{r}, t) = \mathrm{e}^{-i(\omega t - kx)}\phi(\mathbf{r})$$

and for $y \ll \sqrt{x}$

$$\phi(x > 0, y) \sim \cos(A_0\pi) - \frac{2i^{1/2}}{\sqrt{\pi}}\sin(A_0\pi)\sqrt{\frac{k}{2}}\frac{y}{\sqrt{x}}$$

transversal force F_y from this region (although not F_x)

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