Form Factors in 2D Integrable Models

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based on work with Tristan McLoughlin

"Integrable Approaches to 3pt functions in AdS/CFT"
Budapest, 15 – 19 June 2015

What's a Form Factor?

A ***Form Factor*** is a matrix element of an operator in the basis of scattering states

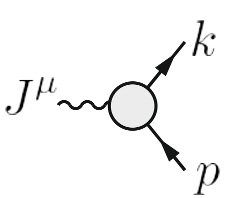
$$\langle k_1\, k_2\, \ldots | \mathcal{O}(x) | p_1\, p_2\, \ldots \rangle$$

and, thus, is a hybrid of

- a correlation function (off-shell) $\quad \langle 0 | \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \ldots | 0 \rangle$
- an S-matrix element (on-shell) $\quad \langle k_1\, k_2\, \ldots \,|\, p_1\, p_2\, \ldots \rangle$

E.g. QED vertex correction

$$\langle k | J^\mu(0) | p \rangle = -ie\bar{u}(k) \left[\gamma^\mu F_1(k-p) + \frac{i\sigma^{\mu\nu}(k-p)_\nu}{2m} F_2(k-p) \right] u(p)$$



$-ie\bar{\psi}\gamma^\mu\psi$

Dirac Form Factor $\approx$ Fourier transform of charge distribution „Form of the particle"

Pauli Form Factor

Why Form Factors?

… at a workshop for three-point correlation functions?

Indeed, correlation functions can in principle be built from form factors:

$$\langle 0|\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)|0\rangle_{\text{gauge theory}}$$

$$\sum_n \int \prod_i dp_i |p_1 \ldots p_n\rangle\langle p_1 \ldots p_n|$$

Although this procedure is much too complicated in general, people do work on ***Target Space Form Factors***

at weak coupling

[W. van Neerven, "Infrared Behavior of On-shell Form-factors in a N=4 Super-Yang-Mills Field Theory," 1986]

[Boels, Bork, Brandhuber, Engelund, Gehrmann, Gurdogan, Henn, Huber, Kazakov, Kniehl, Loebbert, Moch, Naculich, Nandan, Penante, Roiban, Sieg, Spence, Tarasov, Travaglini, Vartanov, Wilhelm, Yang, … 2011+]

at strong coupling

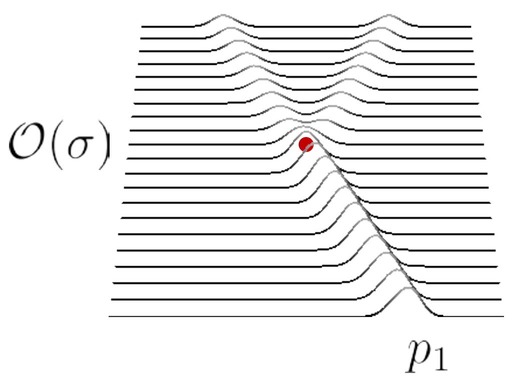
[Maldacena, Zhiboedov 2010] [Gao, Yang 2013]

Why Form Factors?

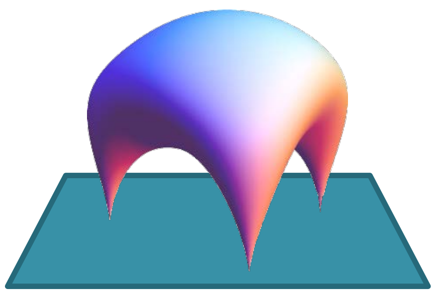
... at a workshop for three-point correlation functions?

In this talk, we are interests in ***World Sheet Form Factors:***

[TK, McLoughlin 2012, 2013]



which may eventually be used to build correlation functions of vertex operators:



$$\langle 0|\mathcal{V}_1(\sigma_1)\mathcal{V}_2(\sigma_2)\mathcal{V}_3(\sigma_3)|0\rangle_{\text{string theory}}$$

Operator product expansion coefficients of Heavy-Heavy-Light correlation functions are equal to diagonal world sheet form factors.

[Bajnok, Janik, Wereszczynski 2014]

[Hollo, Jiang, Petrovskii 2015]

Bootstrap Program

- Circumvent perturbation theory
- Write general properties of **S-Matrix** and **Form Factors** using...
  - Symmetry (e.g. Lorentz symmetry, Flavor symmetry, integrability)
  - Unitarity (total probability = 1)
  - Inspiration from Feynman integrals $\longrightarrow$ Analyticity, Crossing
- Find functions which possess these properties
- Write correlation functions as sums of products of form factors

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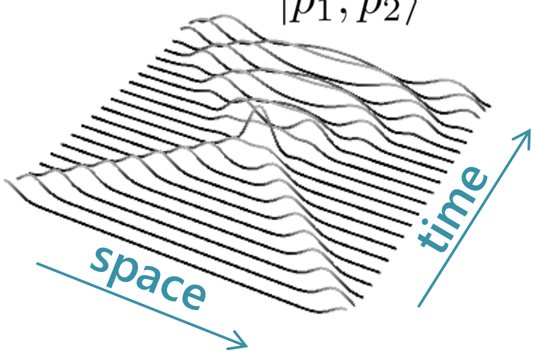
S-Matrix in 2D models

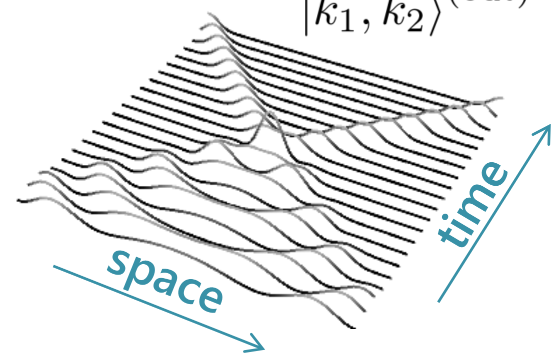
In the Hilbert-space of states, we can choose a basis in terms of

In-states $|p_1, p_2\rangle^{(\mathrm{in})}$



or

Out-states $|k_1, k_2\rangle^{(\mathrm{out})}$



The S-matrix mediates between the two bases

$$|p_1, p_2, \ldots\rangle^{(\mathrm{in})} = |k_1, k_2, \ldots\rangle^{(\mathrm{out})} \mathcal{S}(p_1, p_2, \ldots, k_1, k_2, \ldots)$$

At asymptotic times, the in- and out-states are Fock states

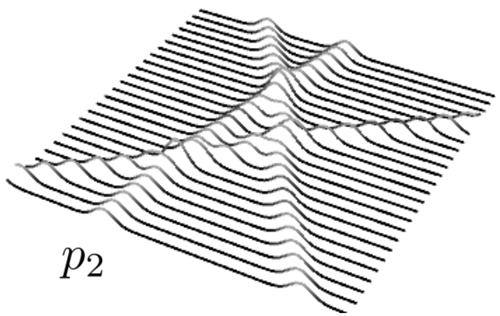
$$a^{\dagger(\mathrm{in})}(p_1) a^{\dagger(\mathrm{in})}(p_2)|0\rangle^{(\mathrm{in})} \longleftrightarrow a^{\dagger(\mathrm{out})}(k_1) a^{\dagger(\mathrm{out})}(k_2)|0\rangle^{(\mathrm{out})}$$

in general, very non-trivial relation

S-Matrix in 2D *integrable* models

In integrable models, the in- and out-states *are* related in a simple way:

Let $p_1 > p_2 > \ldots > p_n$, then one can write [Zamolodchikov 1977]



$$|p_1, p_2, \ldots, p_n\rangle^{(\mathrm{out})} = A^\dagger(p_n) \ldots A^\dagger(p_1)|\Omega\rangle$$

$$|p_1, p_2, \ldots, p_n\rangle^{(\mathrm{in})} = A^\dagger(p_1) \ldots A^\dagger(p_n)|\Omega\rangle$$

with $$A^\dagger(p_1)A^\dagger(p_2) = A^\dagger(p_2)A^\dagger(p_1)\,\mathcal{S}(p_1, p_2)$$ "Zamolodchikov-algebra"

This is cooked up such that $|p_1, p_2\rangle^{(\mathrm{in})} = |p_1, p_2\rangle^{(\mathrm{out})}\mathcal{S}(p_1, p_2)$

S-Matrix in 2D *integrable* models

Even if the interacting oscillators, $A^\dagger$, cannot be constructed explicitly, the Zamolodchikov algebra can be used to "derive" relations for $\mathcal{S}(p_1, p_2)$
(Chicken-or-egg-dilemma)

- Unitarity

$$A^\dagger(p_1)A^\dagger(p_2) = \underbrace{A^\dagger(p_2)A^\dagger(p_1)}\,\mathcal{S}(p_1,p_2)$$

$$= A^\dagger(p_1)A^\dagger(p_2)\,\mathcal{S}(p_2,p_1)$$

$$\Rightarrow \quad \boxed{\mathcal{S}(p_2,p_1)\mathcal{S}(p_1,p_2) = \mathbb{1}}$$

- Yang-Baxter equation

$$A^\dagger(p_1)A^\dagger(p_2)A^\dagger(p_3) \;\xrightarrow[123 \to 132 \to 312 \to 321]{123 \to 213 \to 231 \to 321}\; A^\dagger(p_3)A^\dagger(p_2)A^\dagger(p_1)$$

$$\Rightarrow \quad \boxed{\mathcal{S}(p_1,p_2)\mathcal{S}(p_1,p_3)\mathcal{S}(p_2,p_3) = \mathcal{S}(p_2,p_3)\mathcal{S}(p_1,p_3)\mathcal{S}(p_1,p_2)}$$

S-Matrix in 2D *integrable* models

In the previous discussion, flavor indices were ignored.

In a theory with different particle species, we would have to write

$$|p_1, p_2, \ldots\rangle^{(\text{in})}_{ij\ldots} = |k_1, k_2, \ldots\rangle^{(\text{out})}_{mn\ldots} \mathcal{S}^{mn\ldots}_{ij\ldots}(p_1, p_2, \ldots, k_1, k_2, \ldots)$$

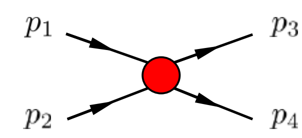
and

$$A^\dagger_i(p_1) A^\dagger_j(p_2) = A^\dagger_m(p_2) A^\dagger_n(p_1)\, \mathcal{S}^{mn}_{ij}(p_1, p_2)$$

or more specifically, e.g., for O(N)-type indices

$$\begin{aligned} A^\dagger_i(p_1) A^\dagger_j(p_2) = {} & A^\dagger_j(p_2) A^\dagger_i(p_1)\, \mathcal{S}_T(p_1, p_2) && \text{(transmission)} \\ & + A^\dagger_i(p_2) A^\dagger_j(p_1)\, \mathcal{S}_R(p_1, p_2) && \text{(reflection)} \\ & + \delta_{ij} A^\dagger_k(p_2) A^\dagger_k(p_1)\, \mathcal{S}_A(p_1, p_2) && \text{(annihilation)} \end{aligned}$$

The Analytic S-Matrix



In a relativistic theory, the two-particle S-matrix is a function of the Mandelstam variables

$$s = (\mathbf{p}_1 + \mathbf{p}_2)^2 \geq 4m^2 \qquad t = (\mathbf{p}_1 - \mathbf{p}_3)^2 \leq 0 \qquad u = (\mathbf{p}_1 - \mathbf{p}_4)^2 \leq 0$$

In 2D, only one of $\{s, t, u\}$ is independent. Let that be s .

1. Consider the process

$$A + B \longrightarrow A + B$$

Say, you find $\mathcal{S}_{AB}(s_{AB} \geq 4m^2)$

2. Now, consider the process

$$A + \bar{B} \longrightarrow A + \bar{B}$$

Say, now you find $\mathcal{S}_{A\bar{B}}(s_{A\bar{B}} \geq 4m^2)$

In both cases, you will have drawn the same Feynman diagrams and solved the same integrals --- although in different kinematical regions.

As a consequence, we have the crossing relation

$$\mathcal{S}_{A\bar{B}}(s_{A\bar{B}} \geq 4m^2) = \mathcal{S}_{AB}(t_{A\bar{B}} \leq 0)$$

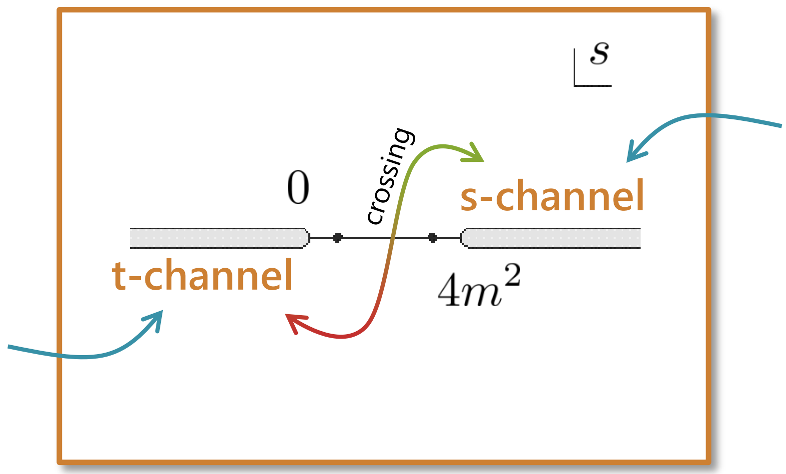
The Analytic S-Matrix

There is ONE S-matrix for ALL channels

$$\mathcal{S}_{AB}(s)$$

$A + \bar{B} \longrightarrow A + \bar{B}$

$\bar{A} + B \longrightarrow \bar{A} + B$



$A + B \longrightarrow A + B$

$\bar{A} + \bar{B} \longrightarrow \bar{A} + \bar{B}$

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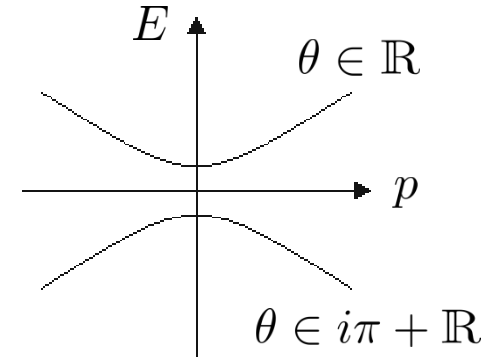
The Rapidity Plane

The relativistic dispersion relation $E^2 = m^2 + p^2$

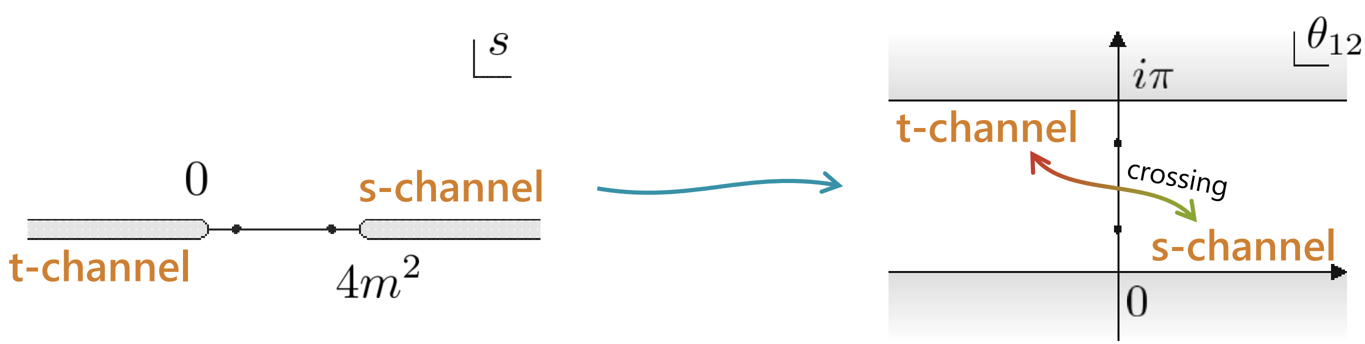
can be uniformized by introducing the rapidity θ as

$$E = m\cosh\theta \qquad p = m\sinh\theta$$



For two particles, we obtain

$$s = (\mathbf{p}_1 + \mathbf{p}_2)^2 = 2m^2(1 + \cosh\theta_{12}) \qquad \text{where} \qquad \theta_{12} = \theta_1 - \theta_2$$



Now, the crossing relation reads $\mathcal{S}_{12}(\theta) = \mathcal{S}_{2\bar{1}}(i\pi - \theta) = \mathcal{S}_{\bar{2}1}(i\pi - \theta) = \mathcal{S}_{\bar{1}\bar{2}}(\theta)$

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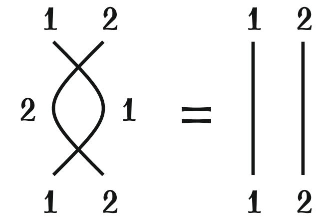
S-Matrix Properties

- Unitarity

$$\mathcal{S}_{12}(\theta)\mathcal{S}_{21}(-\theta) = \mathbb{1}$$

- Hermitian analyticity

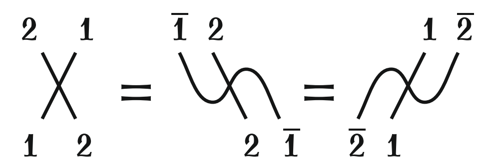
$$\mathcal{S}^\dagger_{12}(\theta) = \mathcal{S}_{21}(-\theta)$$



$$\mathcal{S}_{12}\mathcal{S}^\dagger_{12} = \mathcal{S}^\dagger_{12}\mathcal{S}_{12} = \mathbb{1}$$

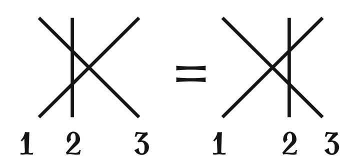
- Crossing symmetry

$$\mathcal{S}_{12}(\theta) = \mathcal{S}_{2\bar{1}}(i\pi - \theta) = \mathcal{S}_{\bar{2}1}(i\pi - \theta)$$



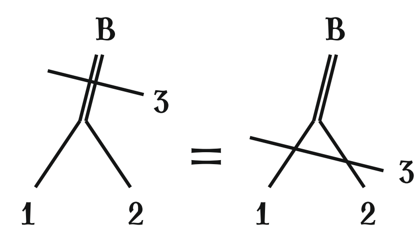
- Yang-Baxter equation

$$\mathcal{S}_{12}\mathcal{S}_{13}\mathcal{S}_{23} = \mathcal{S}_{23}\mathcal{S}_{13}\mathcal{S}_{12}$$



- Bound states

$$\mathcal{S}_{B3}\varphi^B_{12} = \varphi^B_{12}\mathcal{S}_{13}\mathcal{S}_{23}$$



Exact Relativistic S-Matrices

By solving the previous list of properties under the assumption of certain symmetries, various exact S-matrices were found.

- O(2)=U(1) [Zamolodchikov 1977] [Karowski, Thun, Truong, Weisz 1977]
  → Sine-Gordon / Massive Thirring
- U(n>1) [Berg, Karowski, Kurak, Weisz 1978]
- O(n>2) [Zamolodchikov, Zamolodchikov 1978]
  → O(n) chiral field (minimal sol.) / Gross-Neveu (non-min. sol.)

Use perturbation theory to confirm that the constructed S-matrix is really the S-matrix of the model to which is has been associated.

World-Sheet and Spin-Chain S-Matrix

The world-sheet theory and the spin-chain model are ***non***-relativistic.

The dispersion relation is $E^2 = 1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}$

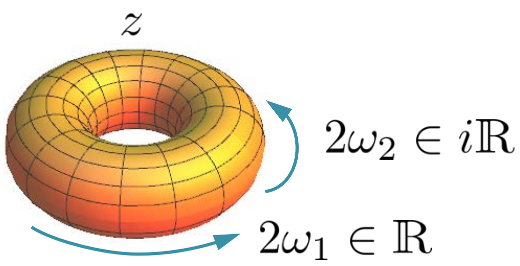
The S-matrix is a function of *two* variables $\mathcal{S}(p_1, p_2)$, not just one $\mathcal{S}(\theta_1 - \theta_2)$

Q: Is there a non-relativistic crossing relation? How does crossing act?

A: Introduce Rapidity *Torus*

$$E = \mathrm{dn}(z, -\tfrac{\lambda}{\pi^2})$$

$$\sin \frac{p}{2} = \mathrm{sn}(z, -\tfrac{\lambda}{\pi^2})$$



[Janik, 2006]
[Arutyunov, Frolov, Zamaklar, 2006]

Then, crossing acts as $z \mapsto z \pm \omega_2$ $\quad (E \mapsto -E\ ,\ p \mapsto -p)$

World-Sheet and Spin-Chain S-Matrix

[Arutyunov, Frolov, Zamaklar, 2006]

- Unitarity

$$\mathcal{S}_{12}(z_1, z_2)\mathcal{S}_{21}(z_2, z_1) = \mathbb{1}$$

Generalized unitarity

$$\mathcal{S}_{12}(z_1^*, z_2^*)\,[\mathcal{S}_{12}(z_1, z_2)]^\dagger = \mathbb{1}$$

- Hermitian analyticity

$$[\mathcal{S}_{12}(z_1, z_2)]^\dagger = \mathcal{S}_{21}(z_2^*, z_1^*)$$

- Crossing symmetry

$$\mathcal{S}_{\bar{1}2}(z_1 + \omega_2, z_2) = \mathcal{S}_{1\bar{2}}(z_1, z_2 - \omega_2) = [\mathcal{S}_{12}(z_1, z_2)]^{-1}$$

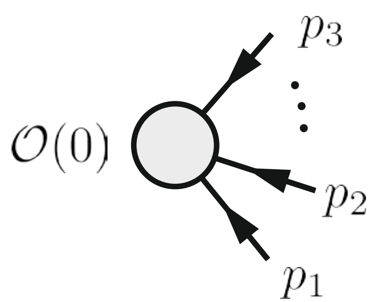
[Janik, 2006]

- Yang-Baxter

$$\mathcal{S}_{12}\mathcal{S}_{13}\mathcal{S}_{23} = \mathcal{S}_{23}\mathcal{S}_{13}\mathcal{S}_{12}$$

Form Factors in Relativistic Theory

For momenta $p_1 > p_2 > \ldots > p_n$ define the Form Factor as



$$F(\theta_{ij}) = \langle \Omega | \mathcal{O}(0) | p_1, \ldots, p_n \rangle^{(\mathrm{in})}$$

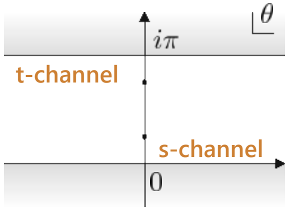
and consider it as a function of all pairs $\theta_{ij} = \theta_i - \theta_j > 0$ with $i < j$

where $p = m \sinh \theta$

Then define the Form Factor $F(\theta_{ij})$ by analytic extension for complex θ_{ij}

Form Factors in Relativistic Theory

Analytic properties:



- CPT invariance $(+i\varepsilon \leftrightarrow -i\varepsilon)$

$$\langle\Omega|\mathcal{O}|p_1,\ldots,p_n\rangle^{(\mathrm{out})} = F(-\theta_{ij}^*) = F(-\theta_{ij})$$

- Crossing symmetry

$$^{(\mathrm{out})}\langle \underset{\substack{\Psi \\ i,j,k}}{p_1,\ldots,p_m}|\mathcal{O}|\underset{\substack{\Psi \\ r,s,t}}{p_{m+1},\ldots,p_n}\rangle^{(\mathrm{in})} = F(\theta_{ij}, i\pi-\theta_{kr}, \theta_{st})$$

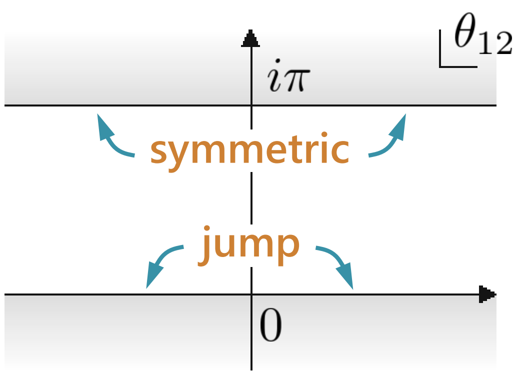
Watson's Equations

named after
[Watson 1954]

Let's focus on the two-particle case and compute...

- $$F(\theta_{12}) = \langle 0|\mathcal{O}|p_1,p_2\rangle^{(\text{in})} \quad = \sum_n \langle 0|\mathcal{O}|n\rangle^{(\text{out})\,(\text{out})}\langle n|p_1,p_2\rangle^{(\text{in})}$$
$$= F(-\theta_{12})\mathcal{S}(\theta_{12})$$

- $$F(i\pi-\theta_{12}) = {}^{(\text{out})}\langle p_1|\mathcal{O}|p_2\rangle^{(\text{in})} \quad = \sum_{n,m} {}^{(\text{o})}\langle p_1|n\rangle^{(\text{i})\,(\text{i})}\langle n|\mathcal{O}|m\rangle^{(\text{o})\,(\text{o})}\langle m|p_2\rangle^{(\text{i})}$$
$$= F(i\pi+\theta_{12})$$



Solving Watson's Equations

We are looking for a solution of $F(\theta) = F(-\theta)\mathcal{S}(\theta) = F(\theta + 2\pi i)\mathcal{S}(\theta)$

assuming that $F(\theta)$
- is meromorphic in the physical strip
- has poles only along the imaginary axis
- falls off sufficiently rapidly for $|\mathrm{Re}\,\theta\,|$

Factorize $F(\theta) = NK(\theta)F^{\min}(\theta)$ with $F^{\min}(\theta)$ analytic in physical strip.

[Karowski, Weisz 1978]

❶ By Cauchy's theorem for a contour around the strip $0 \leq \mathrm{Im}\,\theta \leq 2\pi$

$$\ln F^{\min}(\theta) = \int_C \frac{dz}{4\pi i} \coth\frac{z-\theta}{2} \ln F^{\min}(z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi i} \coth\frac{z-\theta}{2} \ln \underbrace{\frac{F^{\min}(z)}{F^{\min}(z+2\pi i)}}_{\mathcal{S}(z)}$$

⇒ Integral formula for the Form Factor in terms of the S-matrix !

❷ see Tristan's talk...

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Auxiliary Form Factor Function

[Babujian, Fring, Karowski, Zapletal 1998]

Definition: The auxiliary form factor function is defined as

$$f(\theta_1, \ldots, \theta_n) = F(\theta_{ij}) = \langle \Omega | \mathcal{O}(0) | p_1, \ldots, p_n \rangle^{(\text{in})} \quad \underline{\text{for } \theta_1 > \ldots > \theta_n}$$

and by analytic continuation for all other orderings.

Theorem: From LSZ reduction formalism and maximal analyticity, it follows:

- Permutation $f_{\ldots ij \ldots}(\ldots, \theta_i, \theta_j, \ldots) = f_{\ldots ji \ldots}(\ldots, \theta_j, \theta_i, \ldots) \mathcal{S}_{ij}(\theta_i - \theta_j)$
- Periodicity $f_{12 \ldots n}(\theta_1, \theta_2, \ldots, \theta_n) = f_{2 \ldots n1}(\theta_2, \ldots, \theta_n, \theta_1 - 2\pi i)$
- One-particle pole

$$\operatorname*{Res}_{\theta_{12} = i\pi} f_{12 \ldots n}(\theta_1, \theta_2, \ldots, \theta_n) = 2iC_{12} f_{3 \ldots n}(\theta_3, \ldots, \theta_n) \Big(\mathbf{1} - \mathcal{S}_{2n} \cdots \mathcal{S}_{23} \Big)$$

- Bound states

$$\operatorname*{Res}_{\theta_{12} = \theta_B} f_{12 \ldots n}(\theta_1, \theta_2, \ldots, \theta_n) = f_{B \ldots n}(\theta_B, \theta_3, \ldots, \theta_n) \sqrt{2i \operatorname*{Res}_{\theta_{12} = \theta_B} \mathcal{S}_{12}}$$

Worldsheet Form Factors

- Permutation

$$f_{\dots ij \dots}(\dots, z_i, z_j, \dots) = f_{\dots ji \dots}(\dots, z_j, z_i, \dots)\mathcal{S}_{ij}(z_i, z_j)$$

- Periodicity

$$f_{12\dots n}(z_1, z_2, \dots, z_n) = f_{2\dots n1}(z_2, \dots, z_n, z_1 - 2\omega_2)$$

- One-particle pole

$$\operatorname*{Res}_{\mathbf{p}_{12}=0} f_{1\dots n}(z_1, z_2, \dots, z_n) = 2iC_{12} f_{3\dots n}(z_3, \dots, z_n)\Big(\mathbf{1} - \mathcal{S}_{2n} \cdots \mathcal{S}_{23}\Big)$$

- Bound states

$$\operatorname*{Res}_{z_{12}=z_B} f_{12\dots n}(z_1, z_2, \dots, z_n) = f_{B\dots n}(z_B, z_3, \dots, z_n)\sqrt{2i \operatorname*{Res}_{z_{12}=z_B} \mathcal{S}_{12}}$$

Perturbative Checks

- Strong coupling: tree-level in *near-plane-wave model,* [Berenstein, Maldacena, Nastase 2002]
  and two-loop in *near-flat-space model* [Maldacena, Swanson 2006]
  - Vacuum $|0\rangle$
  - Particle excitation $|p\rangle = \bar{Y}(p)|0\rangle$
  - Anti-particle excitation $|\bar{p}\rangle = Y(p)|0\rangle$

  We chose to restrict ourselves to external states with these types of excitations

- Weak coupling: leading-λ in SU(2)-sector = *Heisenberg XXX* [Minahan, Zarembo 2002]
  - Vacuum $|0\rangle = |\uparrow\uparrow\uparrow\uparrow\cdots\uparrow\rangle$
  - Particle excitation $|\psi(p)\rangle = \sum_x e^{ipx} S_{-,x}|0\rangle = \sum_x e^{ipx}|\uparrow\cdots\uparrow\underset{x}{\downarrow}\uparrow\cdots\uparrow\rangle$

- Relation via Landau-Lifshitz model [Kruczenski 2003]

$$S_{\mp} \hat{=} \frac{1 - Y\bar{Y}/2}{(1 + Y\bar{Y}/2)^2} \times \begin{cases} \bar{Y} \\ Y \end{cases}$$

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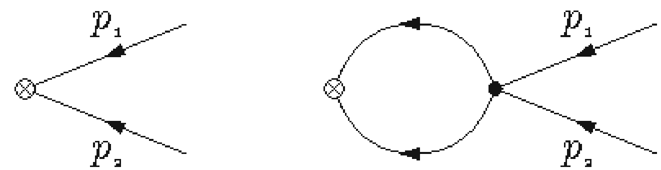
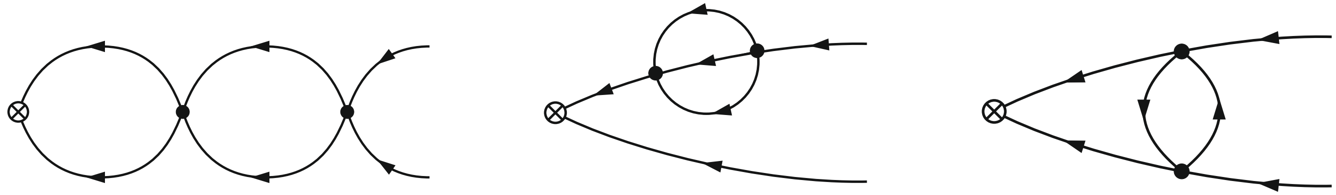
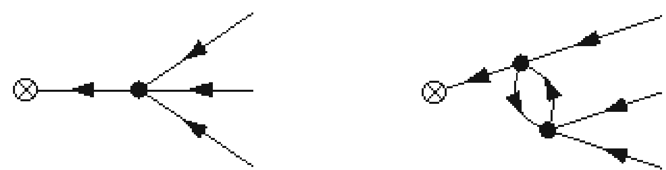
Perturbative Computations

- $f(p_1, p_2) = \langle \Omega | Y^2 | p_1, p_2 \rangle$

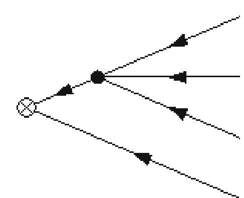


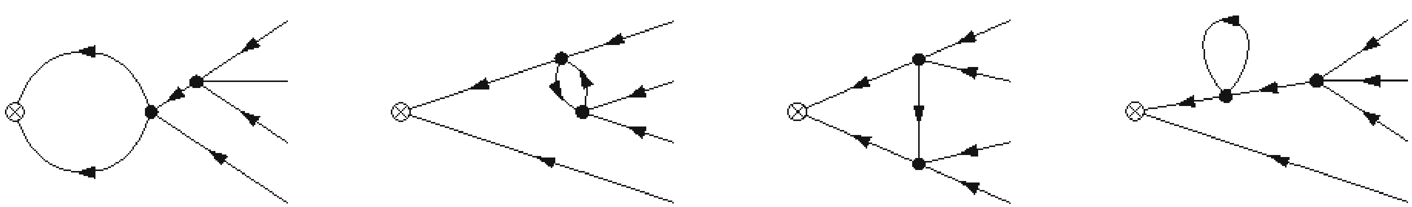


- $f(\bar{p}_1, p_2, p_3) = \langle \Omega | Y | \bar{p}_1, p_2, p_3 \rangle$

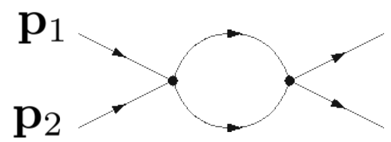


- $f(\bar{p}_1, p_2, p_3, p_4) = \langle \Omega | Y^2 | \bar{p}_1, p_2, p_3, p_4 \rangle$





Bubble Integral



The bubble integral with two in-flowing momenta evaluates to

$$B(p_1,p_2) = \frac{i}{2\pi}\frac{p_1 p_2}{p_1^2 - p_2^2}\ln\left|\frac{p_2}{p_1}\right| - \frac{p_1 p_2}{4(p_1+p_2)|p_1-p_2|}\left(\frac{p_1}{|p_1|}+\frac{p_2}{|p_2|}\right)$$

The form factor is defined with a certain ordering, say $p_1 > p_2 > 0$, s.t.

$$B_>(p_1,p_2) = \frac{i}{2\pi}\frac{p_1 p_2}{p_1^2 - p_2^2}\left[\ln\frac{p_2}{p_1} + i\pi\right]$$

and by analytic continuation we use the same expression for *any* p_1 and p_2.

Permutation and periodicity are then a consequence of

$$B_>(p_2,p_1) - B_>(p_1,p_2) = B_>(p_1 e^{2\pi i},p_2) - B_>(p_1,p_2) = \frac{p_1 p_2}{p_1^2 - p_2^2}$$

Spin-Chain Form Factors

Form factors $\langle\psi(p_1',\ldots)|\mathcal{O}_x|\psi(p_1,\ldots)\rangle$, typically $\mathcal{O}_x = S_x^+$, $\mathcal{O}_x = S_x^+ S_{x+1}^+$

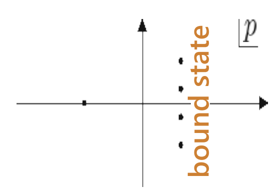
Bethe states:

$$|\psi(p_1,p_2,\ldots)\rangle = \sum_{1\le x_1<x_2<\ldots\le L} \chi(p_1,p_2,\ldots)_{x_1,x_2,\ldots} |\underset{1}{\uparrow}\uparrow\cdots\uparrow\uparrow\underset{x_1}{\downarrow}\uparrow\uparrow\cdots\uparrow\uparrow\underset{x_2}{\downarrow}\uparrow\uparrow\uparrow\cdots\uparrow\uparrow\uparrow\underset{L}{\uparrow}\rangle$$

where, e.g. for two magnons, the wave-function is

$$\chi(p_1,p_2)_{x_1,x_2} = e^{i(p_1x_1+p_2x_2)} + \mathcal{S}(p_2,p_1)\, e^{i(p_2x_1+p_1x_2)}$$

- Permutation $|\psi(p_2,p_1)\rangle = |\psi(p_1,p_2)\rangle\, \mathcal{S}(p_1,p_2)$
- Bound states $\operatorname*{Res}_{p_{1,2}=\hat{p}_{1,2}} |\psi(p_1,p_2)\rangle = |\psi_B(\hat{p}_1,\hat{p}_2)\rangle \operatorname*{Res}_{p_{1,2}=\hat{p}_{1,2}} S(p_1,p_2)$



- Periodicity
- One-particle pole

} Not observable at weak coupling, rapidity torus degenerates for λ→0

Conclusions and Outlook

- Formulated a set of Consistency Conditions for Form Factors in the worldsheet theory of light-cone gauge-fixed strings in AdS_5xS^5: two-dimensional, massive, integrable, *non*-relativistic QFT.

- Perturbative computation of various worldsheet form factors at strong and weak coupling, SU(2) sector.
  This provides a first check of the axioms.

- Showed match of thermodynamic limit of spin-chain form factors and small momentum limit of worldsheet form factors (via Landau-Lifshitz).

- *Hope* of *possibility* to find all-loop or non-perturbative results by solving form factor equations directly (analogous to S-matrix).

- *M*ake contact with holographic correlation functions by considering form factors of vertex operators.