Finite volume matrix elements in integrable QFT: Some exact results

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Relevant articles

- **B. Pozsgay: Form factor approach to diagonal finite volume matrix elements in Integrable QFT**
  `arXiv:1305.3373`

Earlier works:

- B. Pozsgay, G. Takacs: Characterization of resonances using finite size effects
  `arXiv:hep-th/0604022`
- B. Pozsgay, G. Takacs: Form factors in finite volume I: form factor bootstrap and truncated conformal space
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The setting

Object: Finite volume matrix elements of local operators

- 1+1 D Integrable QFT
- Massive theories, diagonal scattering
- IR approach (using infinite volume quantitities)

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Infinite volume theory

- One particle with mass m.
- Exact scattering phase shift: $S(\theta) = e^{i\delta(\theta)}$
- Example: sinh-Gordon theory

$$S(\theta) = e^{i\delta(\theta)} = \frac{\sinh\theta - i\sin b\pi}{\sinh\theta + i\sin b\pi} \qquad b > 0$$

(no bound states)

- We assume that the form factors are known

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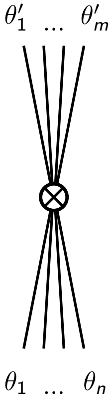
- Form factors:

$$F^{\mathcal{O}}_{mn}(\theta_1^{'},\ldots,\theta_m^{'}|\theta_1,\ldots,\theta_n) = \langle\theta_1^{'},\ldots,\theta_m^{'}|\mathcal{O}(0,0)|\theta_1,\ldots,\theta_n\rangle$$

- Crossing:

$$F^{\mathcal{O}}_{mn}(\theta_1^{'},\ldots,\theta_m^{'}|\theta_1,\ldots,\theta_n) = F^{\mathcal{O}}_{m-1,n+1}(\theta_2',\ldots,\theta_m^{'}|\theta_1'+i\pi,\theta_1,\ldots,\theta_n)$$

+disconnected terms...



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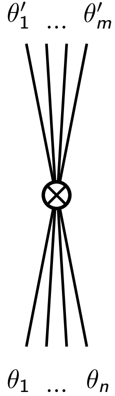


- Watson's equations: (exchange and periodicity)

$$F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_k, \theta_{k+1}, \ldots, \theta_n) = S(\theta_k - \theta_{k+1}) F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{k+1}, \theta_k, \ldots, \theta_n)$$

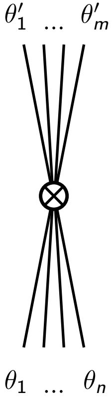
$$F_n^{\mathcal{O}}(\theta_1 + 2i\pi, \theta_2, \ldots, \theta_n) = F_n^{\mathcal{O}}(\theta_2, \ldots, \theta_n, \theta_1)$$

- Kinematical pole:

$$\mathrm{Res}_{\theta_1 = \theta_1'} F_{n+1}^{\mathcal{O}}(\theta_1' + i\pi, \theta_1, \theta_2, \ldots, \theta_n) = i \left(1 - \prod_{k=2}^{n} S(\theta_1' - \theta_k) \right) F_n^{\mathcal{O}}(\theta_2, \ldots, \theta_n).$$

- Bound state structure...

- Analitic properties...



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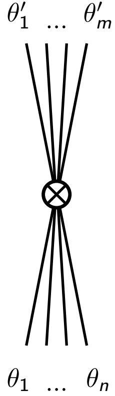

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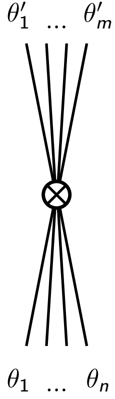

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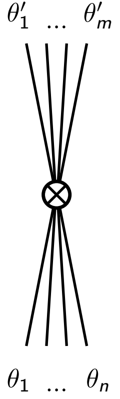

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$$ {}_L\langle m|\mathcal{O}|n\rangle_L = ? $$

1. Step: Bethe Ansatz in large volume
All orders in $1/L$

2. Step: Lüschers corrections
$\mathcal{O}(e^{-mL})$

3. Step: Exact results?

Possible for the diagonal elements ${}_L\langle n|\mathcal{O}|n\rangle_L$

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$$Q_j(\theta_1,\dots,\theta_n) = mL\sinh\theta_j + \sum_{k\neq j}\delta(\theta_j-\theta_k) = 2\pi I_j \qquad j=1\dots n$$

- Off-diagonal matrix elements:

$$\langle\{I'_1,\dots,I'_m\}|\mathcal{O}(0,0)|\{I_1,\dots,I_n\}\rangle_L = \frac{F^{\mathcal{O}}_{m+n}(\tilde\theta'_m,\dots,\tilde\theta'_1|\tilde\theta_1,\dots,\tilde\theta_n)}{\sqrt{\rho_n(\tilde\theta_1,\dots,\tilde\theta_n)\rho_m(\tilde\theta'_1,\dots,\tilde\theta'_m)}} + O(\mathrm{e}^{-\mu L}),$$

where ρ_N is the density of states:

$$\rho_n(\theta_1,\dots,\theta_n) = \det\mathcal{J}^{(n)}, \qquad \mathcal{J}^{(n)}_{jk} = \frac{\partial Q_j}{\partial\theta_k}, \qquad \rho_n \sim L^n$$

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arXiv:0706.1445

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- Diagonal limit ill-defined:

  $$F_{2n}^{\mathcal{O}}(\theta_n+\varepsilon_n,\dots,\theta_1+\varepsilon_1|\theta_1,\dots,\theta_n) = ?$$

  Terms with combinations of $\varepsilon_j/\varepsilon_k$

- Connected diagonal form factor:

  $$F_{2n,c}^{\mathcal{O}}(\tilde{\theta}_1,\dots,\tilde{\theta}_n)$$

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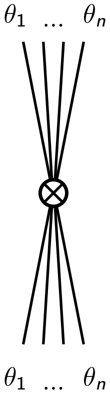

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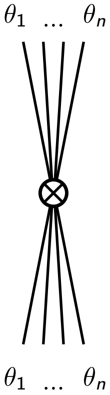

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$$\langle\theta_1,\ldots,\theta_K|\mathcal{O}|\theta_1,\ldots,\theta_K\rangle_L = \sum_{\{\theta_+\}\cup\{\theta_-\}} F^{\mathcal{O}}_{2n,c}(\{\theta_-\})\,\frac{\tilde\rho_{K-n}(\{\theta_+\}|\{\theta_-\})}{\rho_K(\theta_1,\ldots,\theta_K)} + \mathcal{O}(e^{-\mu L})$$

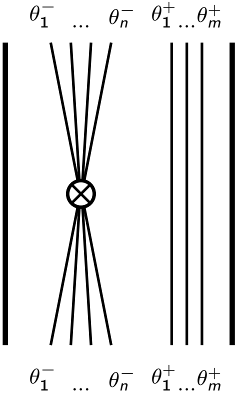
Sum over partitions:

$$\{\theta\}_K = \{\theta_+\}_m \cup \{\theta_-\}_n$$

with the sub-determinants

$$\tilde\rho_{K-n}(\{\theta_+\}|\{\theta_-\}) = \det \mathcal{J}_+$$

Important: $\tilde\rho$ depends on both sets!



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2. Step: Lüschers corrections

Main idea: Small perturbation towards non-integrability

$$H' = H + \lambda \int dx\ \mathcal{O}(x) \qquad \rightarrow \qquad \langle n|\mathcal{O}|n\rangle = \frac{1}{L}\frac{dE_n}{d\lambda}$$

Lüscher's correction for E_n can be computed for one-particle states.
Even the $\mathcal{O}(\lambda)$ perturbation:

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Result for a standing one-particle state:

$$\begin{aligned}\langle\theta=0|\mathcal{O}|\theta=0\rangle_L = \frac{F_2^{\mathcal{O}}}{mL} + \langle\mathcal{O}\rangle_L + \\ + \frac{1}{mL}\int\frac{d\theta}{2\pi}F_{4,c}^{\mathcal{O}}(\theta,-i\pi/2)S(\theta+i\pi/2)e^{-mL\cosh\theta} \\ - \frac{1}{mL}F_2^{\mathcal{O}}\int\frac{d\theta}{2\pi}i\varphi(\theta+i\pi/2)S(\theta+i\pi/2)\sinh\theta e^{-mL\cosh\theta} \\ + F_2^{\mathcal{O}}\int\frac{d\theta}{2\pi}(S(\theta+i\pi/2)-1)e^{-mL\cosh\theta}\end{aligned}$$

3. Step: Exact results

- Strategy: VEV's first, excited states afterwards
- Vacuum energy from Thermodynamic Bethe Ansatz (TBA)

$$E_0(L) = -m \int \frac{d\theta}{2\pi} \cosh(\theta) \log(1 + e^{-\varepsilon_0(\theta)})$$

where

$$\varepsilon_0(\theta) = mL\cosh(\theta) - \int_{-\infty}^{\infty} \frac{d\theta'}{2\pi} \varphi(\theta - \theta') \log(1 + e^{-\varepsilon_0(\theta')})$$

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- LeClair-Mussardo series for the VEVs:

$$\langle \mathcal{O} \rangle_L = \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d\theta_1}{2\pi} \dots \frac{d\theta_n}{2\pi} \left(\prod_j \frac{1}{1+e^{\varepsilon_0(\theta_j)}} \right) F^{\mathcal{O}}_{2n,c}(\theta_1, \dots, \theta_n)$$

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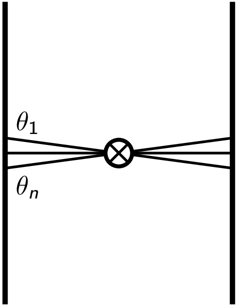

A. LeClair, G. Mussardo: Finite Temperature Correlation Functions in Integrable QFT
`arXiv:hep-th/9902075`

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  B. Pozsgay: Mean values of local operators in highly excited Bethe states

  arXiv:1009.4662

- No summation!
  Exception: Non-relativistic 1D Bose gas, see

  M. Kormos, G. Mussardo, A. Trombettoni: Expectation Values in the Lieb-Liniger Bose Gas

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- Excited state TBA in sinh-Gordon for $\left|\bar{\theta}_1,\ldots,\bar{\theta}_K\right\rangle_L$:

$$\varepsilon(\theta) = mL\cosh\theta + \sum_{j=1}^{K}\log S(\theta-\bar{\theta}_i-i\pi/2) - \int\frac{d\theta'}{2\pi}\varphi(\theta-\theta')\log(1+e^{-\varepsilon(\theta')})$$

Condition for the Bethe roots:

$$1+e^{\varepsilon(\bar{\theta}_j+i\pi/2)}=0, \qquad j=1\ldots K$$

Exact finite size energies:

$$E=\sum_{j=1}^{K} m\cosh\bar{\theta}_j - \int\frac{d\theta}{2\pi}m\cosh(\theta)\log(1+e^{-\varepsilon(\theta)})$$

- Derived in:

J. Teschner: On the spectrum of the Sinh-Gordon model in finite volume
`arXiv:hep-th/0702214`

3. Step: Exact results

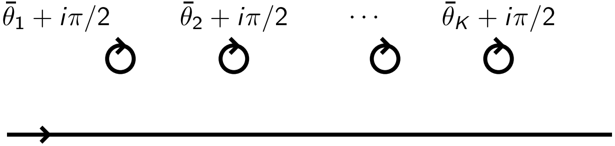
- Excited state TBA in sinh-Gordon:

$$\varepsilon(\theta) = mL\cosh(\theta) - \int_C \frac{d\theta'}{2\pi}\varphi(\theta-\theta')\log(1+e^{-\varepsilon(\theta')})$$

and

$$E(L) = -m\int_C \frac{d\theta}{2\pi}\cosh(\theta)\log(1+e^{-\varepsilon(\theta)})$$

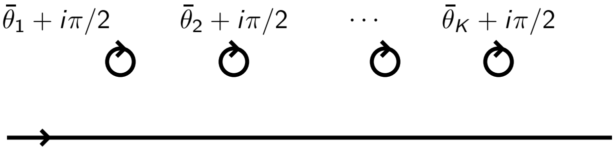
New contour C:



3. Step: Exact results

- Big conjecture: LeClair-Mussardo series for excited states

$$\langle \bar\theta_1, \bar\theta_2, \dots, \bar\theta_K | \mathcal{O} | \bar\theta_1, \bar\theta_2, \dots, \bar\theta_K \rangle_L = \sum_n \frac{1}{n!} \int_{\mathcal{C}} \frac{d\theta_1}{2\pi} \cdots \frac{d\theta_n}{2\pi} \left(\prod_j \frac{1}{1+e^{\varepsilon(\theta_j)}} \right) F^{\mathcal{O}}_{2n,c}(\theta_1, \dots, \theta_n)$$



- Strategy: Evaluate the residues and do a partial resummation...

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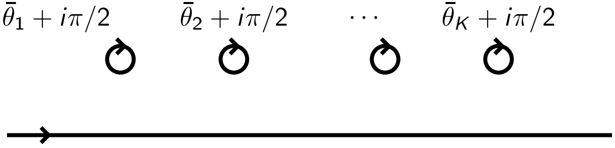
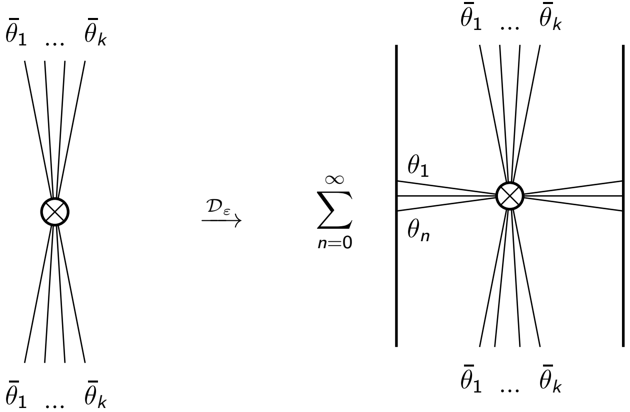


- Strategy: Evaluate the residues and do a partial resummation...

3. Step: Exact results

- For a form factor $F_{2k}^{\mathcal{O}}(\bar{\theta}_1, \ldots, \bar{\theta}_k)$ define the „dressed version" as

$$\mathcal{D}_{\varepsilon}^{\mathcal{O}}(\bar{\theta}_1, \ldots, \bar{\theta}_k) \equiv$$
$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_j \int_{-\infty}^{\infty} \frac{d\theta_j}{2\pi(1+e^{\varepsilon(\theta_j)})} \right) F_{2(n+k),c}^{\mathcal{O}}(\bar{\theta}_1 + i\pi/2, \ldots, \bar{\theta}_k + i\pi/2, \theta_1, \ldots, \theta_n)$$



3. Step: Exact results

- After partial resummation:

$$
{}_L\big\langle \bar\theta_1,\dots,\bar\theta_K \big|\mathcal{O}\big|\bar\theta_1,\dots,\bar\theta_K\big\rangle_L = \sum_{\{\bar\theta_+\}\cup\{\bar\theta_-\}} \mathcal{D}^{\mathcal{O}}_{\varepsilon}\big(\{\bar\theta_-\}\big)\ \frac{\bar\rho_{K-n}\big(\{\bar\theta_+\}|\{\bar\theta_-\}\big)}{\bar\rho_K(\bar\theta_1,\dots,\bar\theta_K)}
$$

Here

$$
\bar\rho_K(\bar\theta_1,\dots,\bar\theta_K) = \det \mathcal{K}_{jk} \qquad\qquad \mathcal{K}_{jk} = \frac{d\varepsilon(\bar\theta_j + i\pi/2)}{d\bar\theta_k}
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\bar\rho_{K-n}(\{\bar\theta_+\}|\{\bar\theta_-\}) = \det \mathcal{K}_+,
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- Can be regarded as the dressed version of the asymptotic result

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Checks and To Do list

Checks:

- Agrees with the Lüscher's correction for the one-particle states
- Proven for $\mathcal{O} = \Theta$
- Numerical checks in

  B. Pozsgay, I.M. Szecsenyi, G. Takacs: Exact finite volume expectation values of local operators in excited states

  `arXiv:1412.8436`

To Do:

- Resummation?
- UV limit
- Other derivation?

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Thank you for your attention!