"Classical and Quantum symmetries in mathematics and physics"

July 25-29, 2016, Jena

The bootstrap program for the gauge/gravity duality

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Holographic QFT Group, Wigner Research Centre for Physics, Budapest

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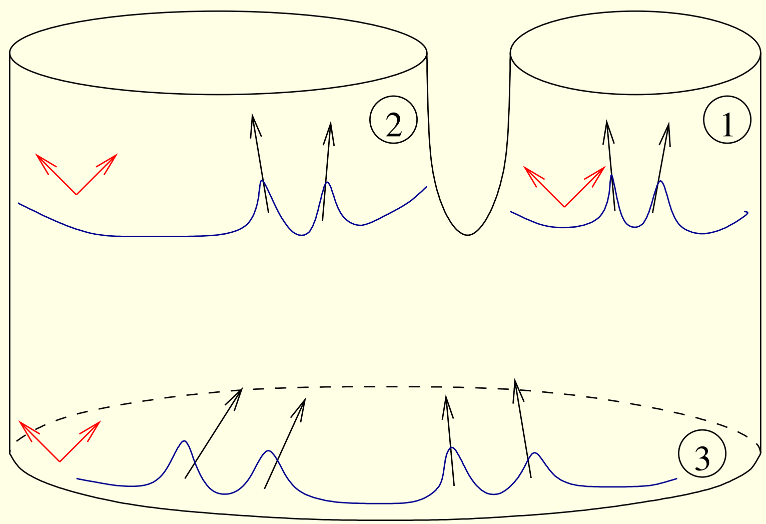
IIB strings on $AdS_5 \times S^5$	Integrability $\longleftrightarrow$	$N = 4$ SYM
SFT vertex	Form factors	3pt functions



Same light-cone gauge fixing: integrable 2D QFT with particle like excitation. Amplitude $\equiv$ string vertex $\longleftrightarrow$ 3pt functions and $1/N$ corrections in dual gauge theory.

1501.04533,1512.01471: work done in collaboration with Romuald Janik

Motivation from physics

Large hadron collider

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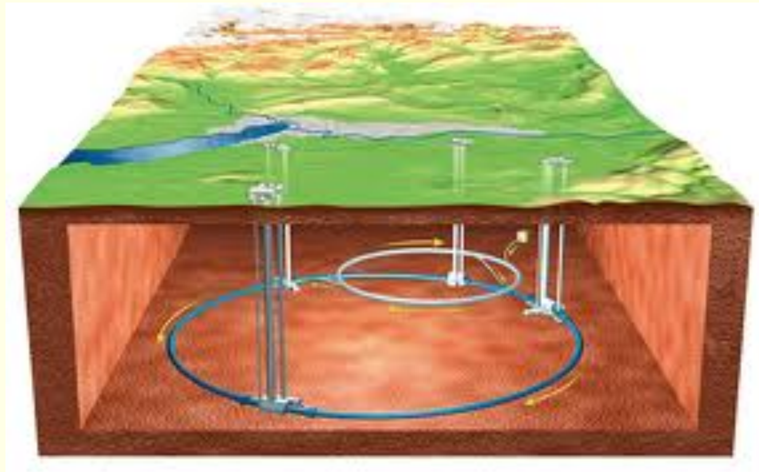

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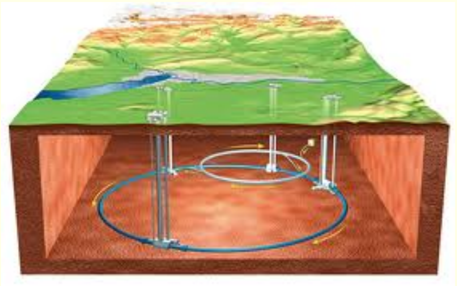

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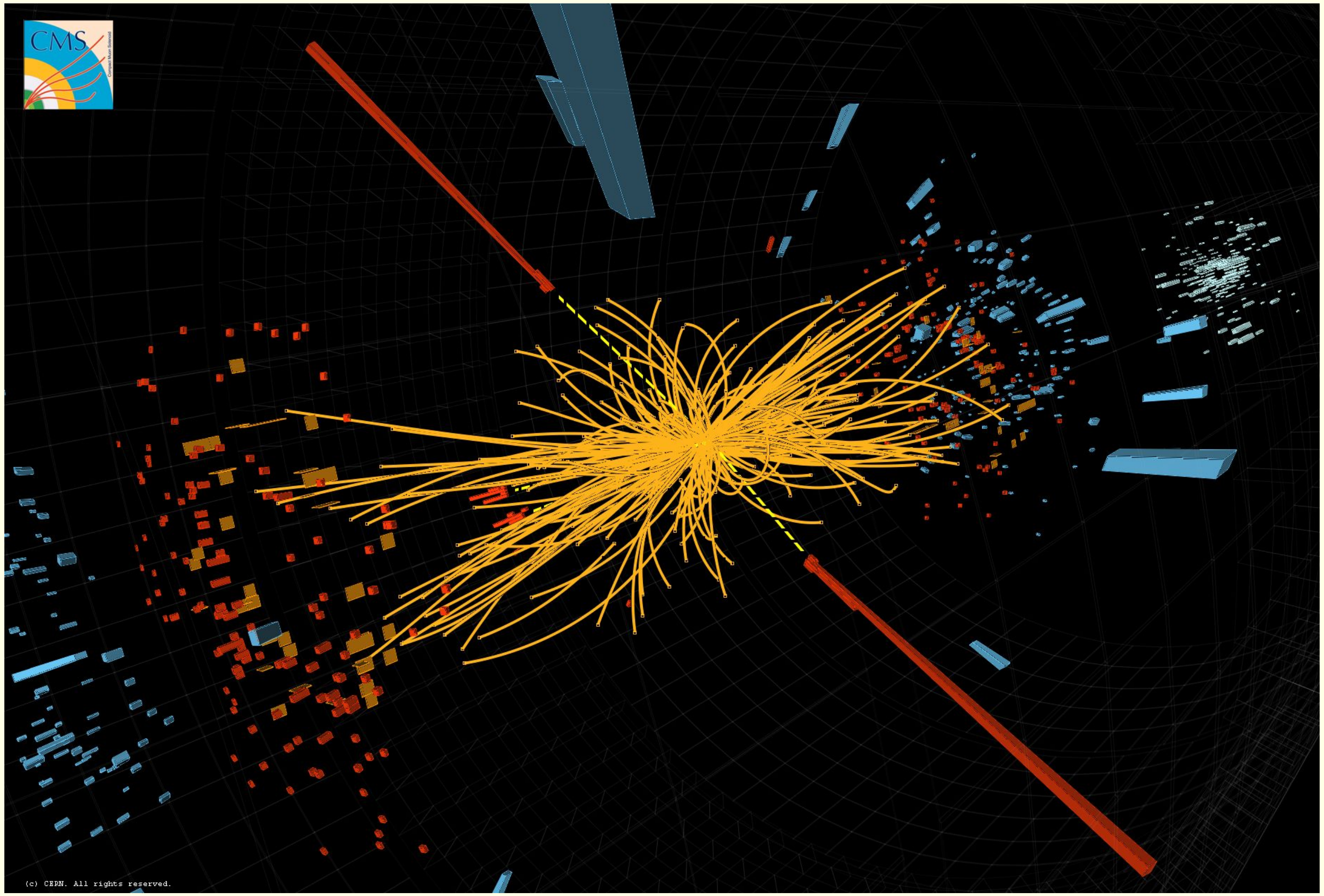

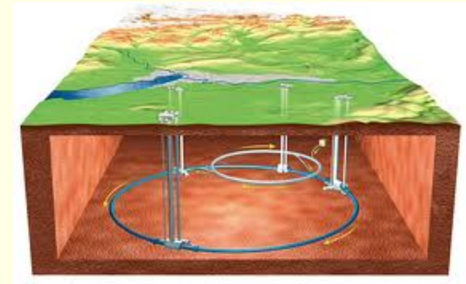

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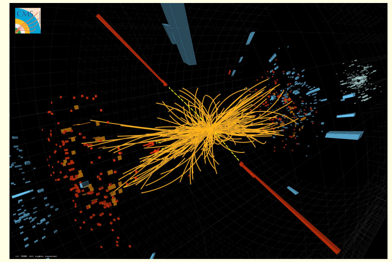

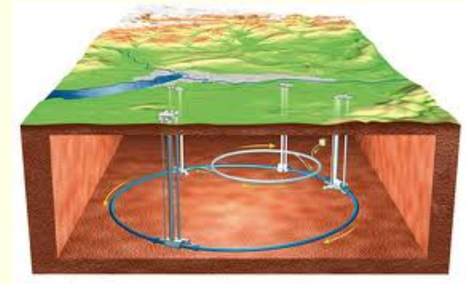

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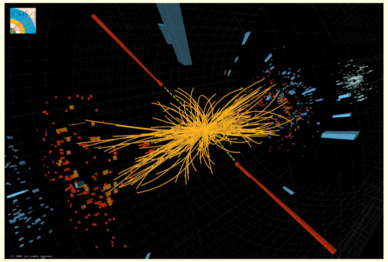

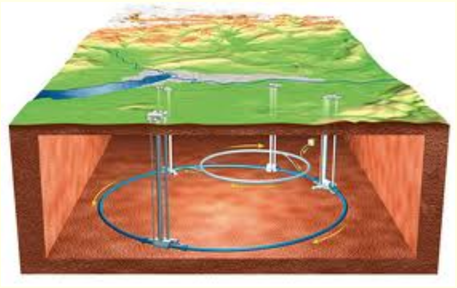

Large hadron collider

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Many "elementary particles" $\longrightarrow$ classification

Motivation from physics

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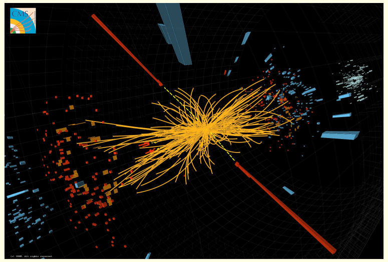

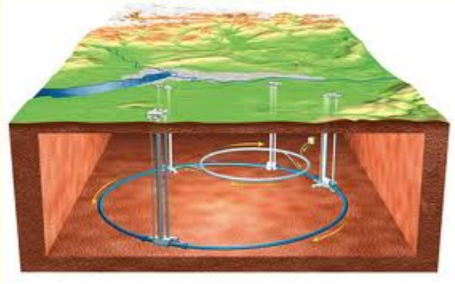

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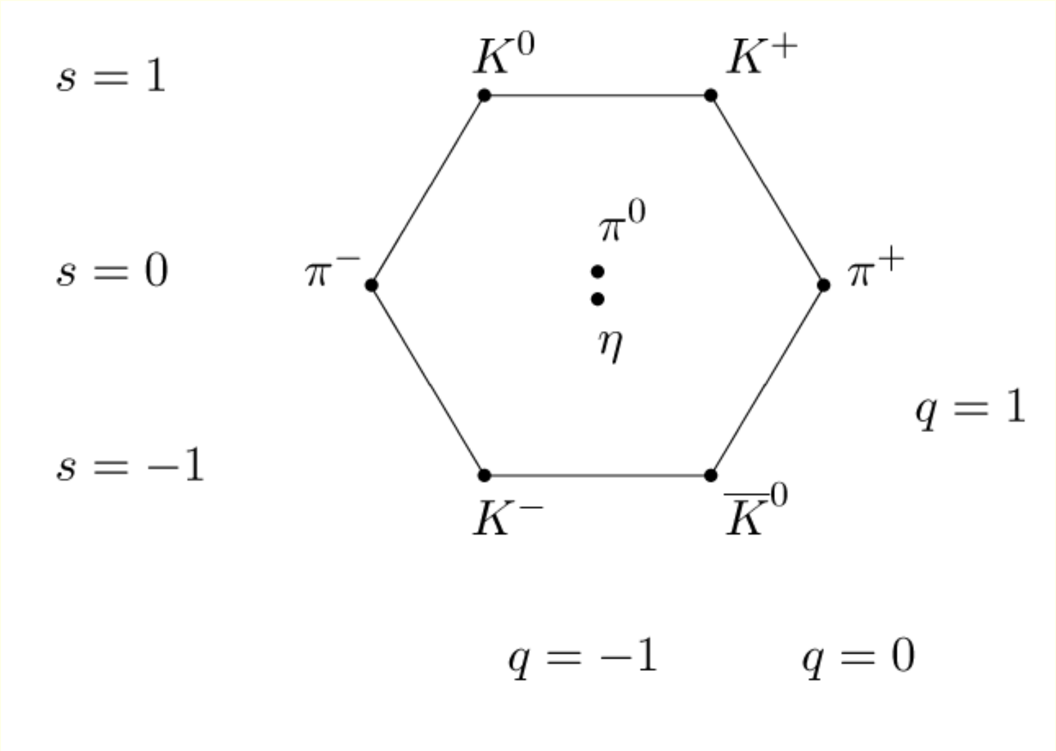
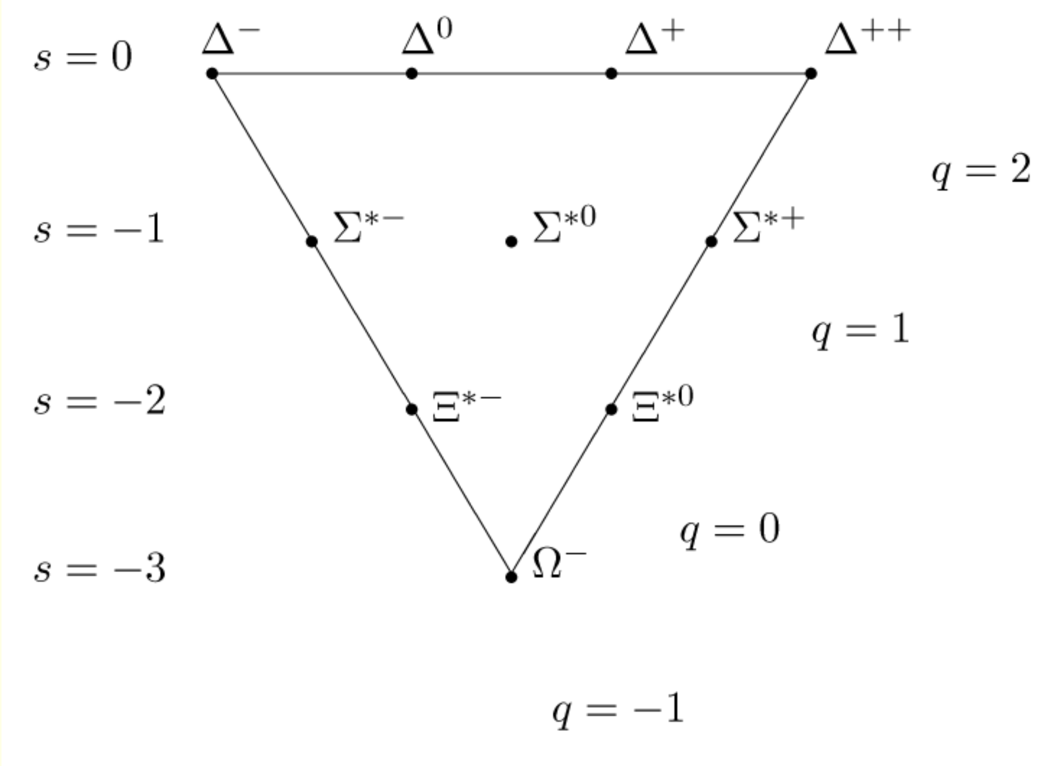
Many "elementary particles" → classification





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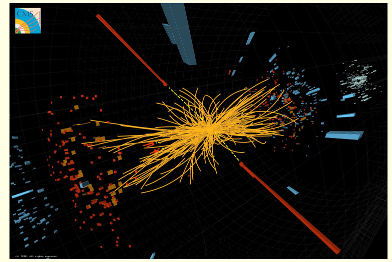
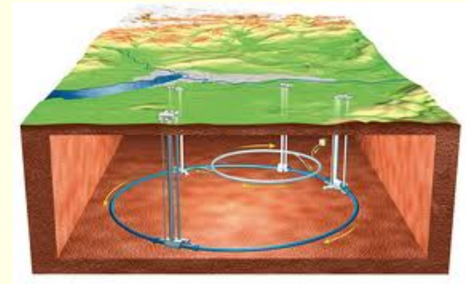
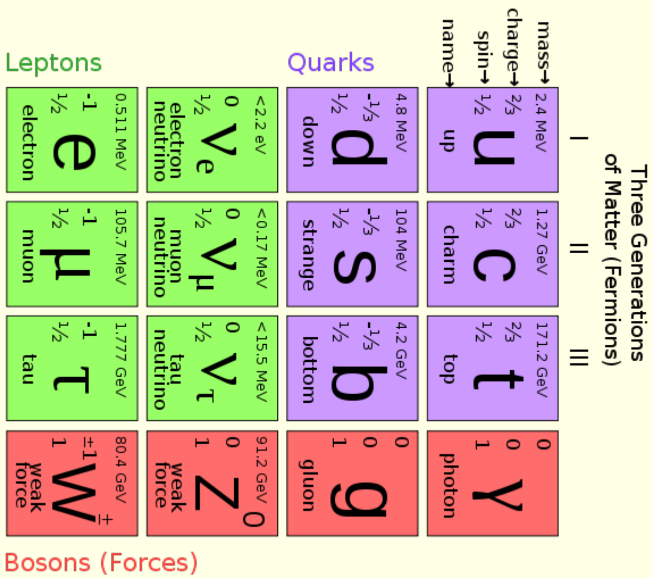


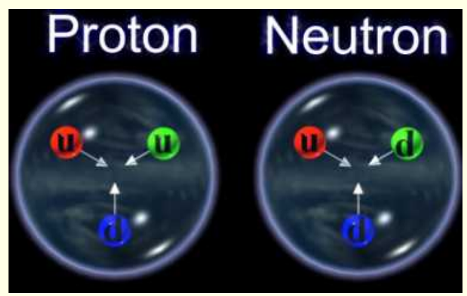


Many "elementary particles" $\longrightarrow$ classification

Fundamental representation $\longrightarrow$ quarks $\longrightarrow$ Standard Model: Calculate scatterings



Interaction	gauge group
electromagnetic	$U(1)$
weak	$SU(2)$
strong	$SU(3)$



Symmetries = space-time and inner

Space-time symmetries

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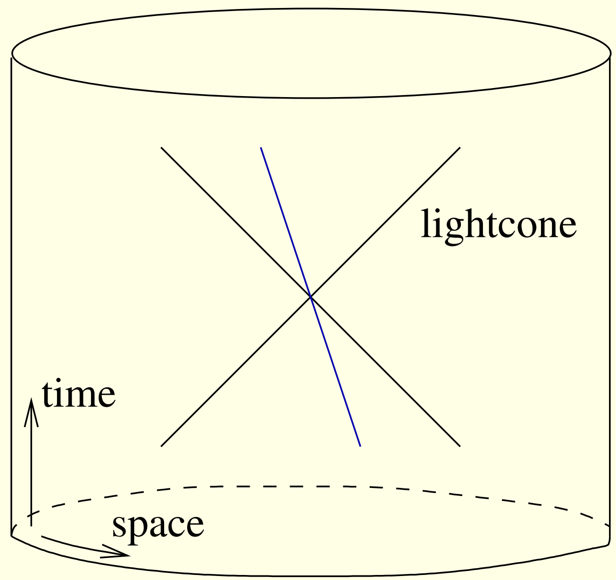

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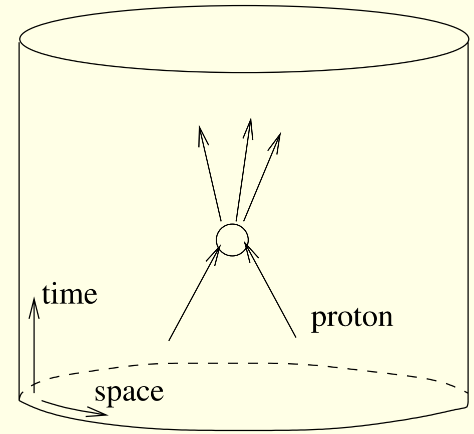
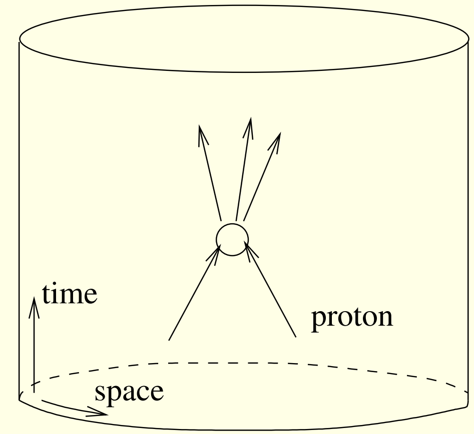


Space-time symmetries

Translations: space and time: $\mathbb{R}^{1,d}$ mostly $d = 1$

LHC

Symmetries = space-time and inner



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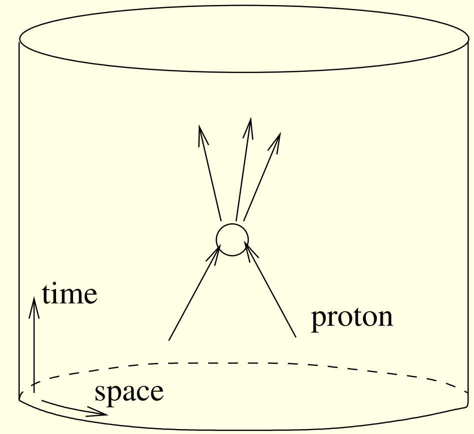
Space-time symmetries

Translations: space and time: $\mathbb{R}^{1,d}$ mostly $d = 1$

conserved charges: E energy and P momentum

generate time/space shifts

Symmetries = space-time and inner



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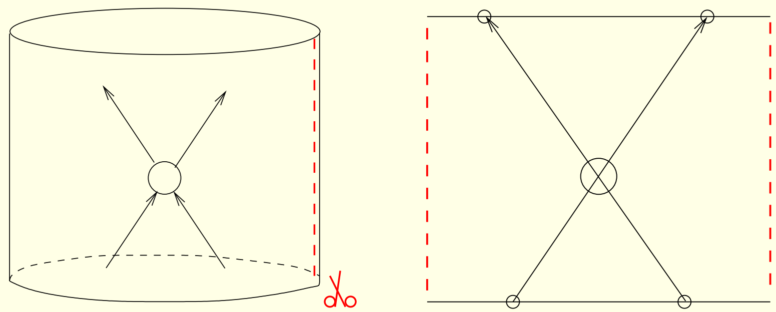

Lorentz emerges

$SO(1, d)$

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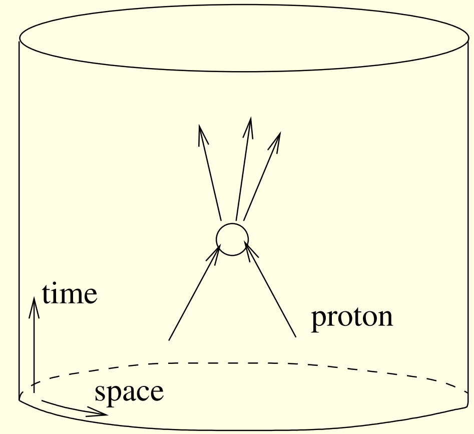

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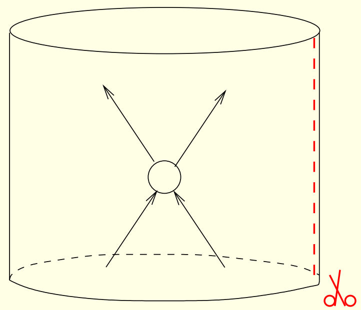

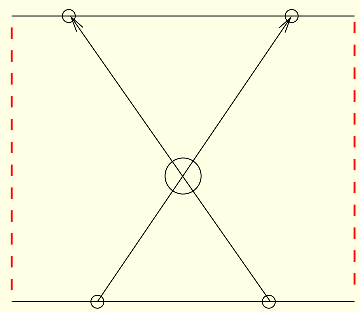

1 particle state
$p = m \sinh \theta$
$E = m \cosh \theta$
$v = \tanh \theta$

Lorentz emerges

$SO(1, d)$

Full symmetry: $SO(1, d) \ltimes \mathbb{R}^{1,d}$

trajectory: $x(t) = v\,(t - t_0)$

dispersion relation $E^2 - P^2 = m^2$

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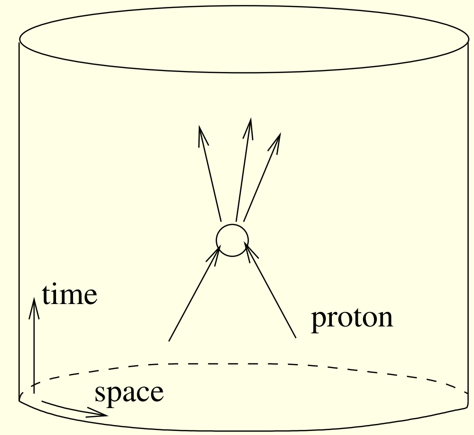

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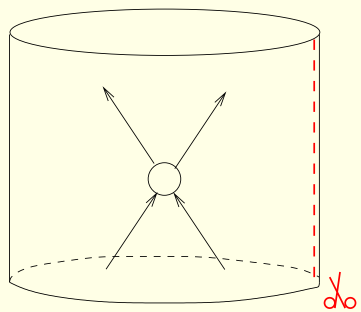

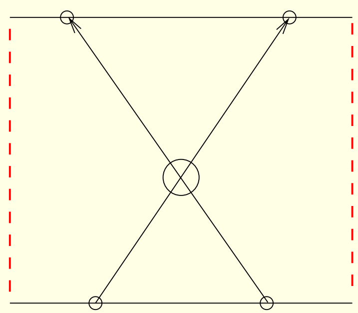

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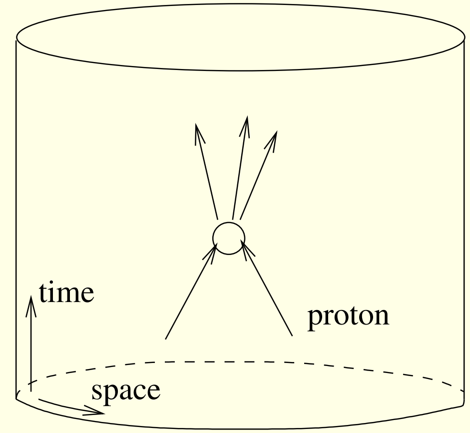
Inner symmetries typically Lie group

Nontrivial combination with boosts

$[B, Q_s] = sQ_s$

Symmetries = space-time and inner



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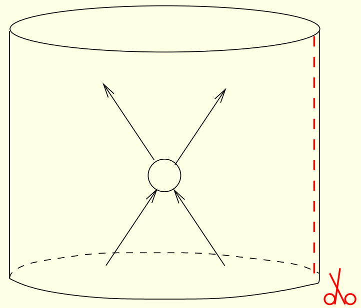

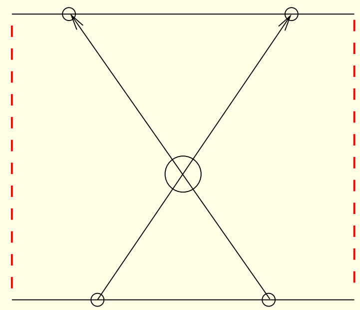

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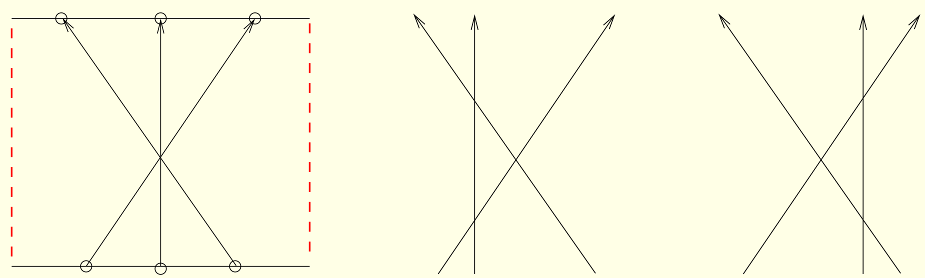

Nontrivial combination with boosts

$[B, Q_s] = sQ_s$

$s = \frac{1}{2}$ SUSY, $s = 1$, energy and momentum, $s > 1$ momentum dependent time-shift:

$d = 1$ factorization and YBE, $d > 1$ trivial scattering [Coleman-Mandula]

Perturbative S-matrix

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The simplest interacting QFT: $\mathcal{L} = \frac{1}{2}(\partial_t \varphi)^2 - \frac{1}{2}(\partial_x \varphi)^2 - V(\varphi) \quad V(\varphi) = \frac{m^2}{b^2}(\cosh b\varphi - 1)$

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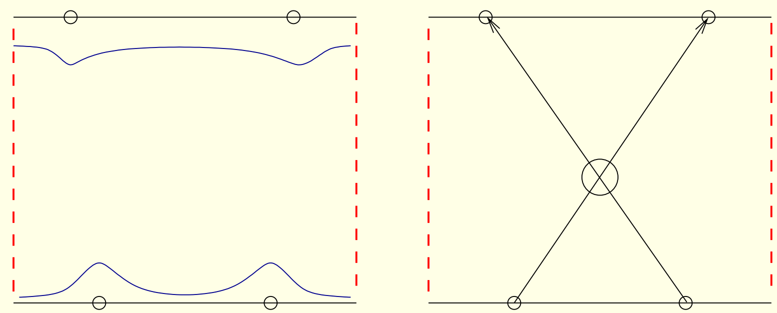
S-matrix connects initial and final states

asymptotic states are multiparticle states

$\rightarrow$ LSZ reduction formula



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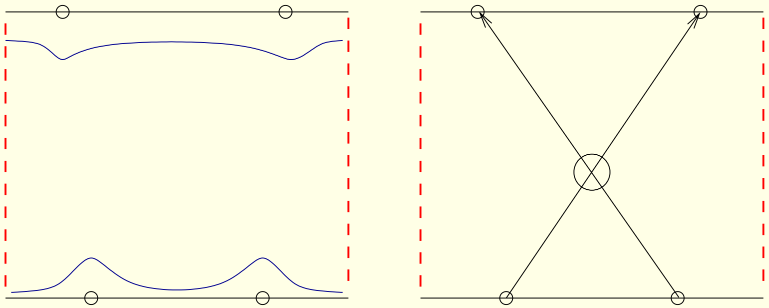

$$\langle p_1', p_2'|\mathcal{O}|p_1, p_2\rangle = \bar{\mathcal{D}}_1'\bar{\mathcal{D}}_2'\mathcal{D}_1\mathcal{D}_2\langle 0|T(\mathcal{O}\varphi(1)\varphi(2)\varphi(3)\varphi(4))|0\rangle$$

where $\mathcal{D}_j = i\int d^2x_j e^{ip_jx - i\omega_j t}\left\{\partial_t^2 - \partial_x^2 + m^2\right\}$ amputes a leg and puts it onshell

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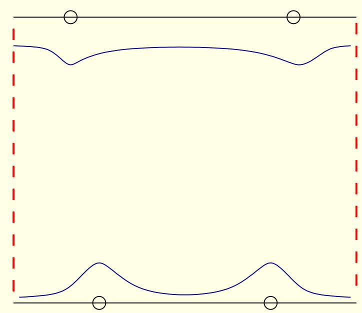

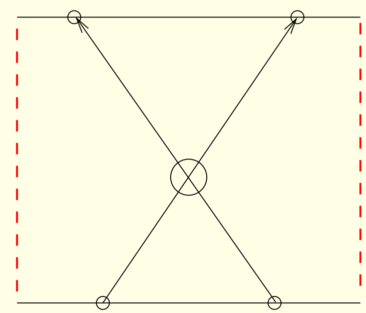

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Consequence: perturbative definition, convergent expansion, calculational tool :

$$S(\theta) = 1 - \frac{1}{4}ib^2\text{csch}\theta - \frac{b^4(\text{csch}\theta(\pi\text{csch}\theta - i))}{32\pi} + \frac{ib^6\text{csch}\theta(\pi\text{csch}\theta - i)^2}{256\pi^2} + O\left(b^8\right)$$

Lorentz transformation $\theta \rightarrow \theta + \Lambda$: invariant combination: $\theta = \theta_1 - \theta_2$

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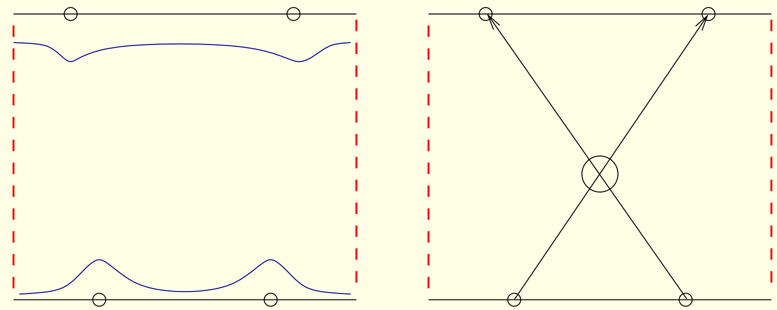

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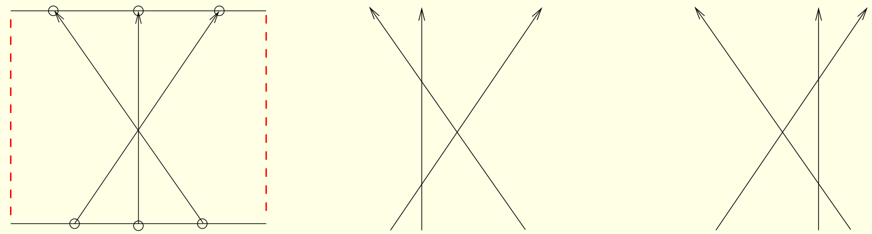
control over analytical properties: unitarity, crossing $S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$

S-matrix bootstrap

S-matrix bootstrap

S-matrix bootstrap: find the two particle S-matrix which satisfies [Zamolodchikov[2] '79]

1. Yang-Baxter equation:



$S_{ij}(\theta_i - \theta_j) : V_i \otimes V_j \to V_j \otimes V_i$

$$S_{12}S_{13}S_{23} = S_{23}S_{13}S_{12}$$

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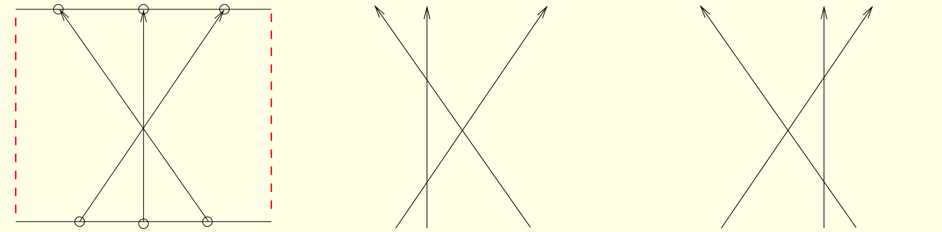

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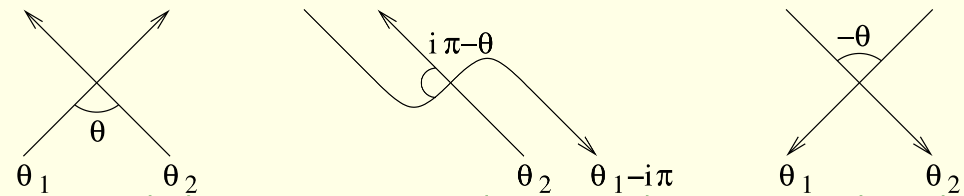
2. Unitarity and crossing symmetry



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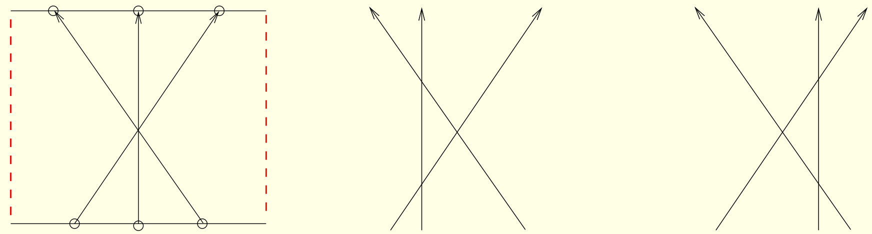

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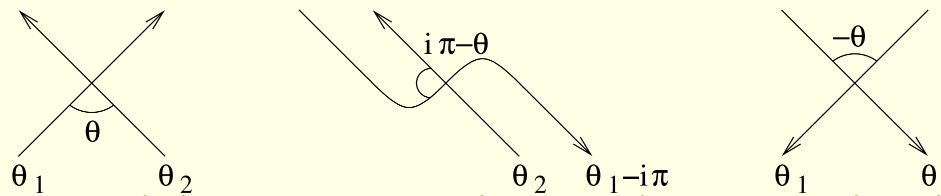

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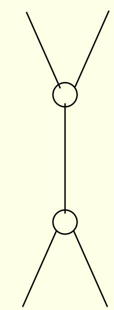
3. Maximal analyticity: $S_{12}(\theta)$ is meromorphic for $\Im m(\theta) \in [0, \pi]$, with possible poles at $\Re e(\theta) = 0$. For each pole $\exists$ a Coleman-Thun diagram, in which particles propage on-shell interacting at 3pt or 4pt vertices, preserving all conserved charges.



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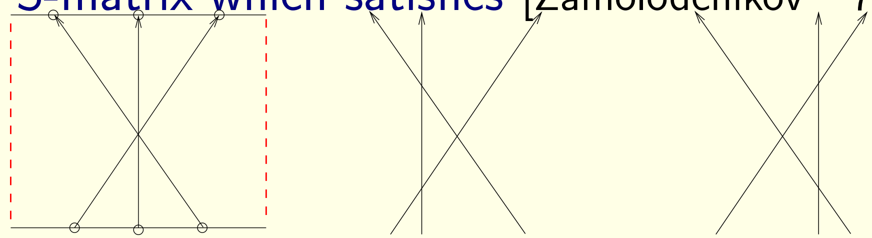

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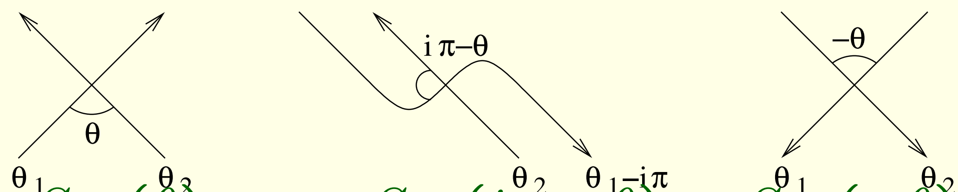

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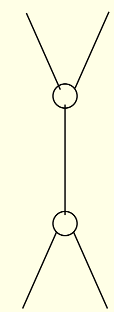
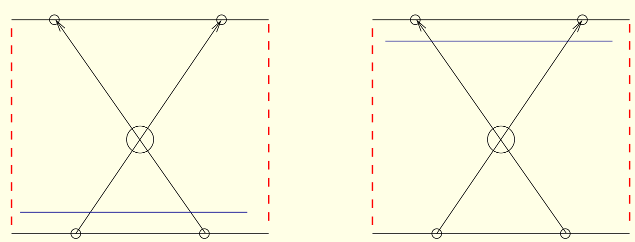


4. Inner symmetry:
for any conserved charge, Q



$$S_{12}\Delta_{12}(Q) = \Delta_{21}(Q)S_{12}$$ qtriang. (w) Hopf algebra

S-matrix solutions

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No inner symmetry, scalar particle $S(\theta) = S(-\theta)^{-1} = S(i\pi - \theta)$

no pole $S(\theta) = \frac{\sinh\theta - i\sin a}{\sinh\theta + i\sin a}$ sinh-Gordon: $a = \frac{\pi b^2}{8\pi + b^2}$ $V(\varphi) = \frac{m^2}{b^2}(\cosh b\varphi - 1)$

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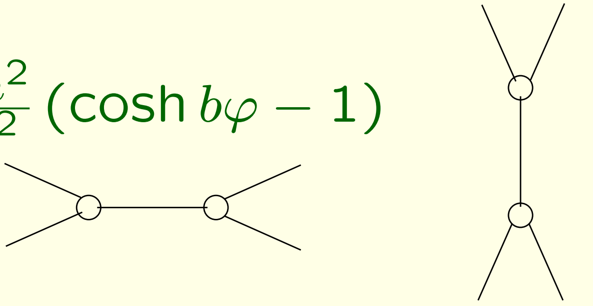

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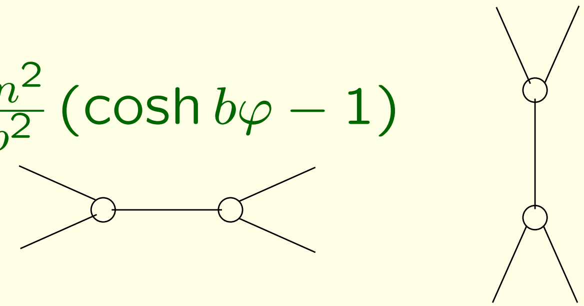

Inner symmetry$= U_q(\widehat{sl}_2)$

2d evaluation representation:

soliton doublet $(s, \bar{s})$

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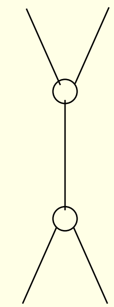

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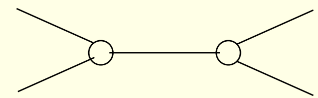

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---

Inner symmetry $= U_q(\widehat{sl}_2)$

2d evaluation representation:

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Nondiagonal scattering: $S(\theta) = S_0(\theta) R(\theta)$

$$R(\theta) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\lambda\pi}{\sin\lambda(\pi + i\theta)} & \frac{\sin i\lambda\theta}{\sin\lambda(\pi + i\theta)} & 0 \\ 0 & \frac{\sin i\lambda\theta}{\sin\lambda(\pi + i\theta)} & \frac{-\sin\lambda\pi}{\sin\lambda(\pi + i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

R-matrix of $U_{q=e^{i\pi\lambda}}(\widehat{sl}_2)$ XXZ

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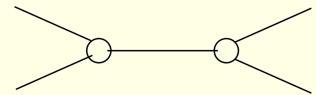

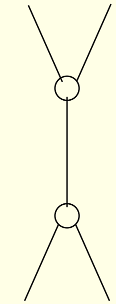

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Unitarity: $S_0(\theta)S_0(-\theta) = 1$

$$S_0(\theta) = \prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\lambda + \frac{\lambda i\theta}{\pi})\Gamma(2l\lambda + 1 + \frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda + \frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda + 1 + \frac{\lambda i\theta}{\pi})}/(\theta \to -\theta)\right]$$

Crossing symmetry: $S_0(i\pi - \theta) = S_0(\theta)\frac{-\sin\lambda\pi}{\sin\lambda(\pi + i\theta)}$

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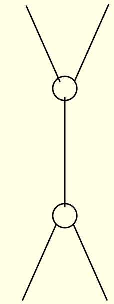

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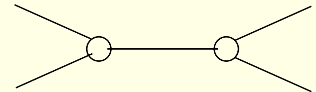

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No poles for $a = \lambda^{-1} > 1$ for $a < 1$ for all poles $\exists$ a CT diagram [Zamolodchikov[2]]

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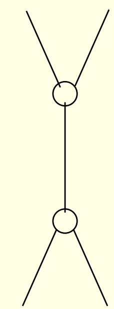

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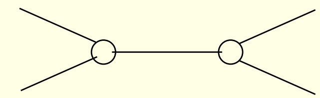

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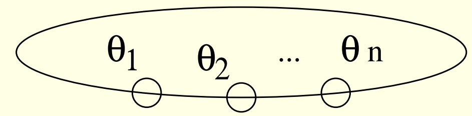
AdS_5/CFT_4 duality: Inner symmetry: $su(2|2)^{\otimes 2}$, particles: short representations

Finite size spectrum

Finite size spectrum

No inner symmetry, scalar particle



Finite volume spectrum [Bethe-Yang]
upto $O(e^{-mL})$, polynomial in L^{-1} :

$$e^{i\Phi_j} = -e^{ip_jL} S(\theta_j - \theta_1) \dots S(\theta_j - \theta_n) = 1$$

$$E_n(L) = \sum_i m \cosh \theta_i$$

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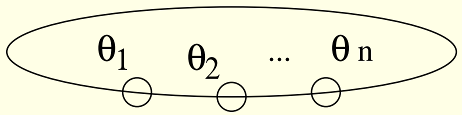

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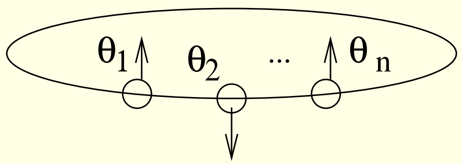

diagonalize: $BY_j(\theta_j|\{\theta_i\}) = S_{j1}(\theta_j - \theta_1) \dots S_{jn}(\theta_j - \theta_n)$ for all j

Finite size spectrum

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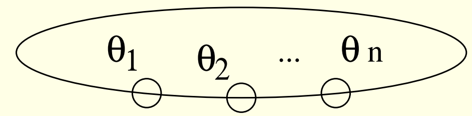

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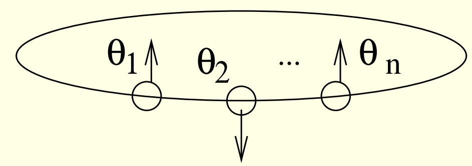

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commutes $[T(\theta), T(\theta')] = 0$ and $T(\theta_j|\{\theta_i\}) = BY_j(\theta_j|\{\theta_i\})$ sine-Gordon $\leftrightarrow$ XXZ

eigenvalue: $\lambda(\theta|\{\theta_i\})$ Bethe-Yang: $-e^{ip_jL}\lambda(\theta_j|\{\theta_i\}) = -1$

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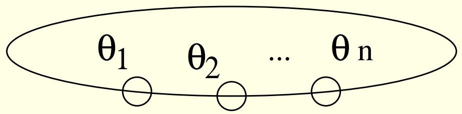

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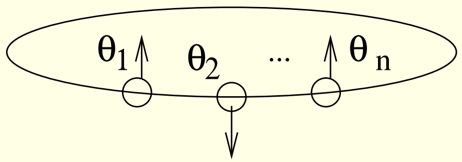

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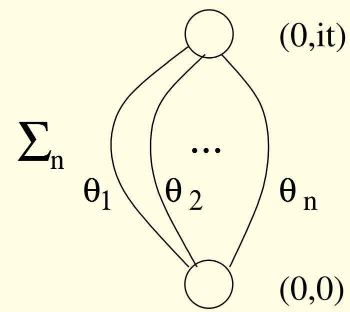
Bethe-Yang equations = Beisert-Staudacher equations

Form factor bootstrap

Form factor bootstrap



Correlation functions: [Smirnov, Karowszki] $\langle 0|\mathcal{O}(it)\mathcal{O}(0)|0\rangle =$

$\sum_n \frac{1}{n!}\int \frac{d\theta_1}{2\pi}\ldots\int \frac{d\theta_n}{2\pi}|\langle 0|\mathcal{O}(0)|\theta_1,\ldots,\theta_n\rangle|^2 e^{-m(\sum_i \cosh\theta_i)t}$

Form factor bootstrap

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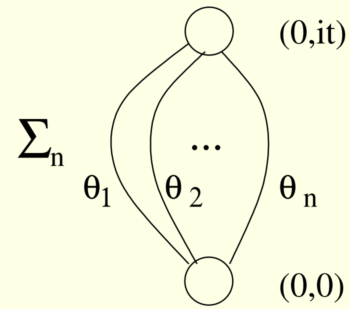
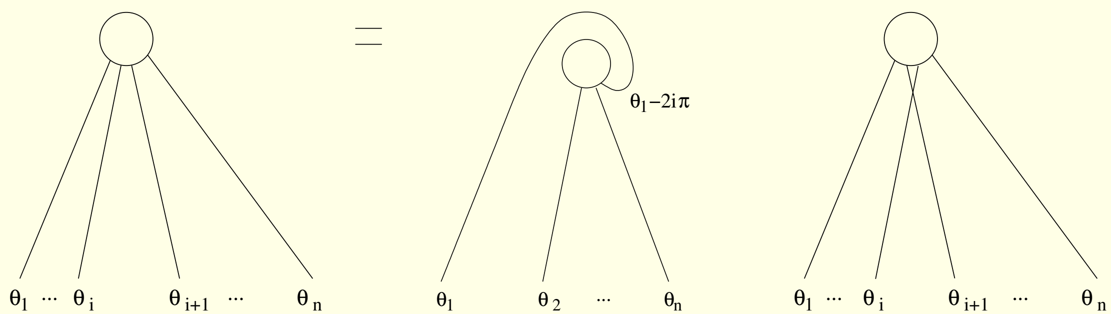


Form factor bootstrap:



$$\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle = \langle 0|\mathcal{O}|\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1})\langle 0|\mathcal{O}|\dots,\theta_{i+1},\theta_i,\dots\rangle$$

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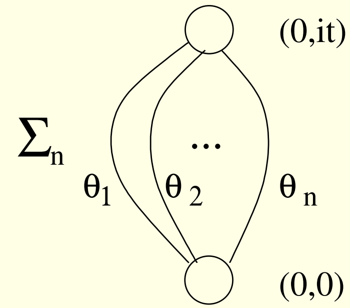

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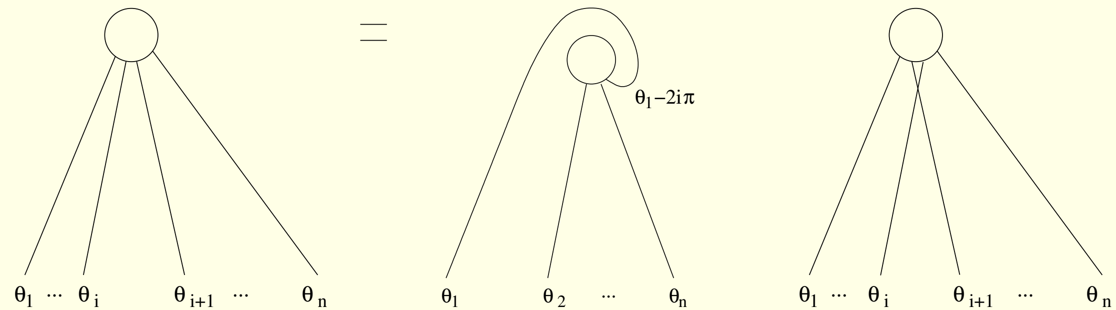

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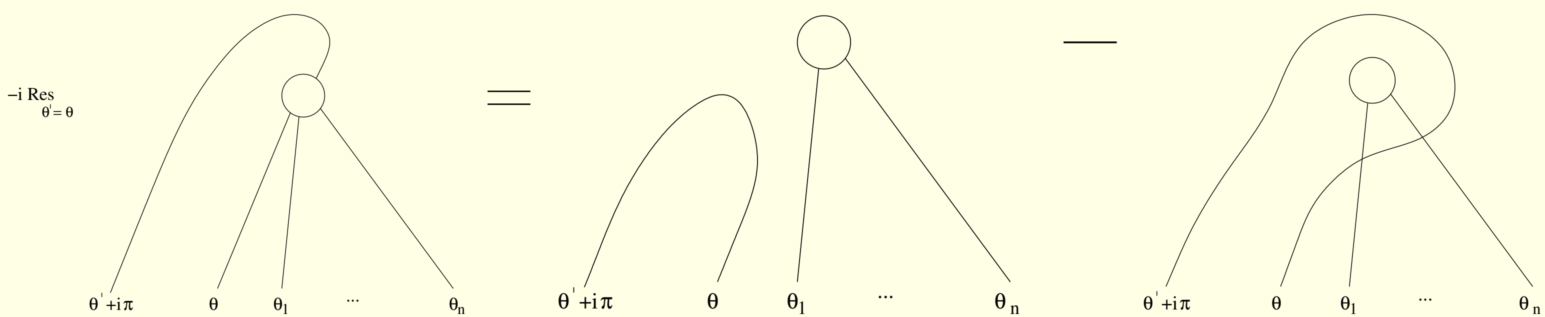
Singularity stucture



$$-i\mathrm{Res}_{\theta'=\theta}\langle 0|\mathcal{O}|\theta' + i\pi,\theta,\theta_1\dots,\theta_n\rangle = (1-\prod_i S(\theta-\theta_i))\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle$$

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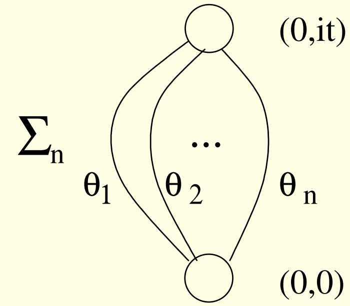

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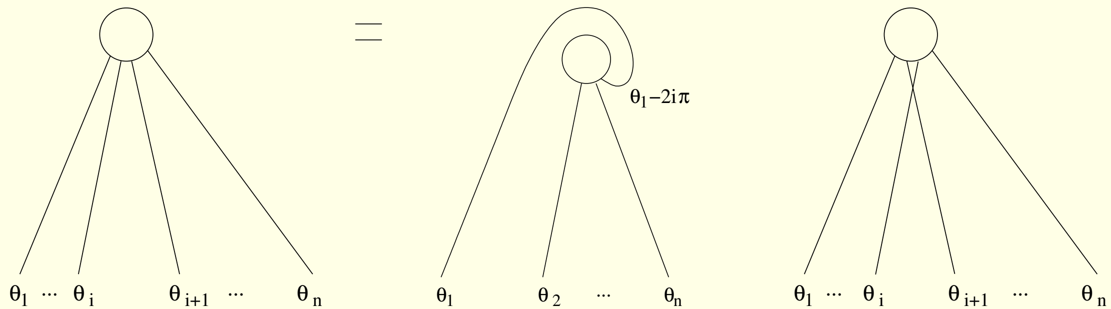

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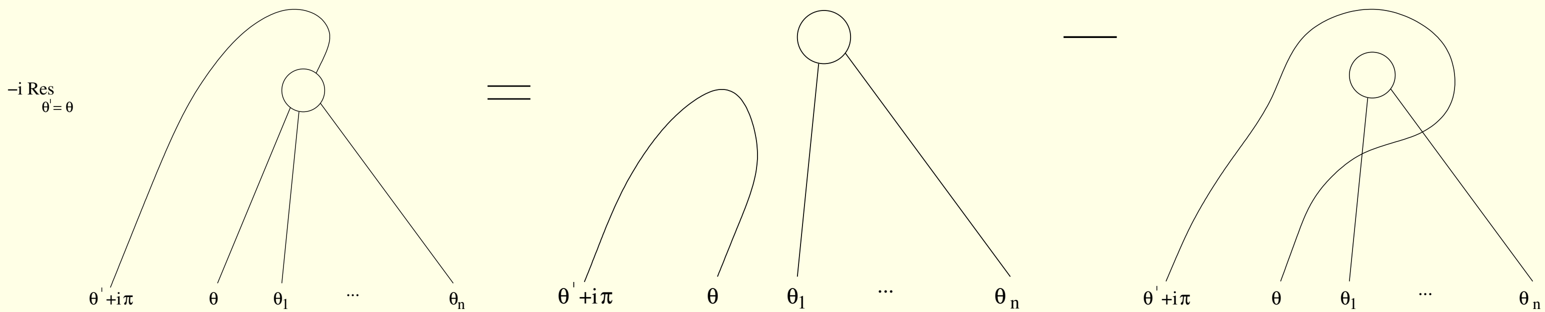

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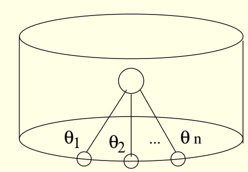
Solution for sinh-Gordon: $\langle 0|\mathcal{O}|\theta_1,\theta_2\rangle = e^{(D+D^{-1})^{-1}\log S}$; $Df(\theta)=f(\theta+i\pi)$

Finite volume form factors: polynomial in L^{-1}: $\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle_L = \frac{\langle 0|\mathcal{O}|\theta_1,\dots,\theta_n\rangle}{\sqrt{\det[\frac{\partial\Phi_i}{\partial\theta_j}]}}$



Decompactification limit of the string vertex

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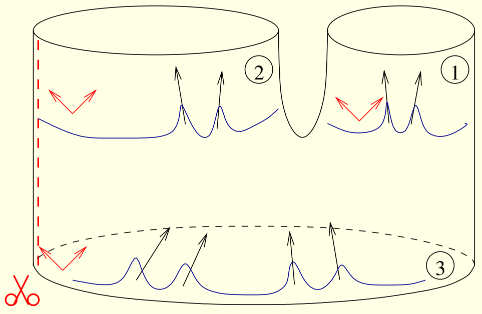
Decompactify string 2 & 3:



Decompactification limit of the string vertex

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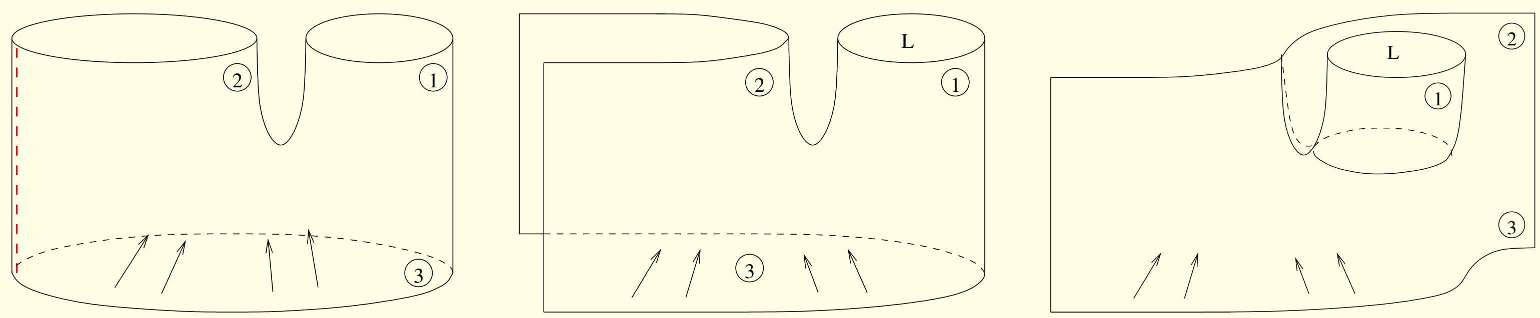

Decompactification limit of the string vertex

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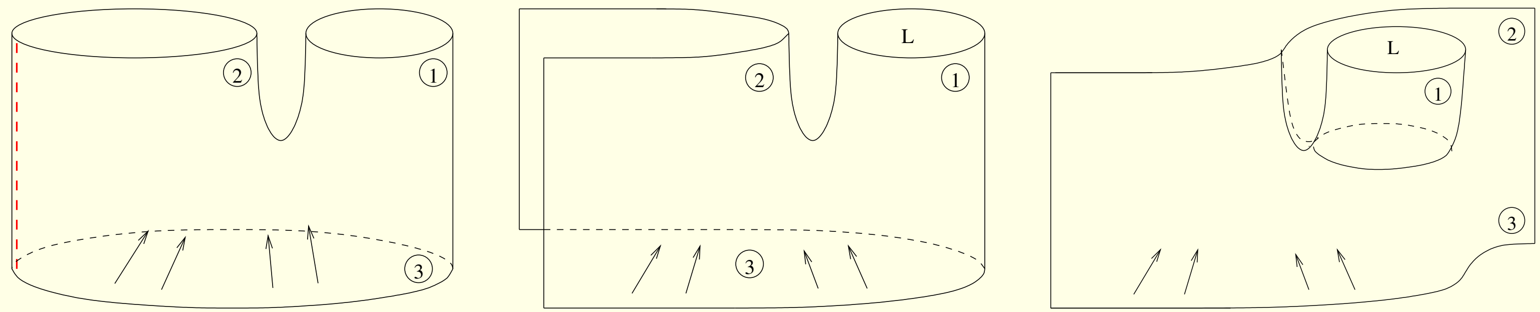
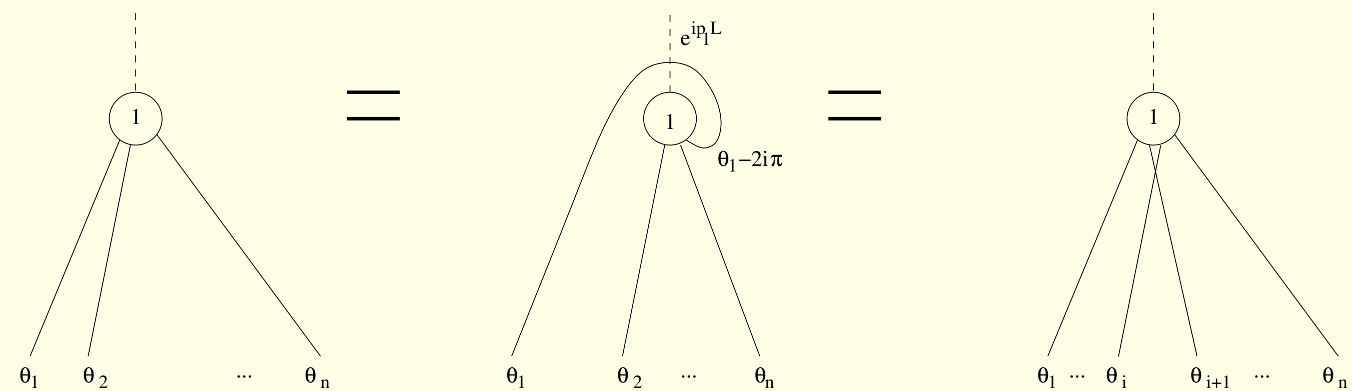


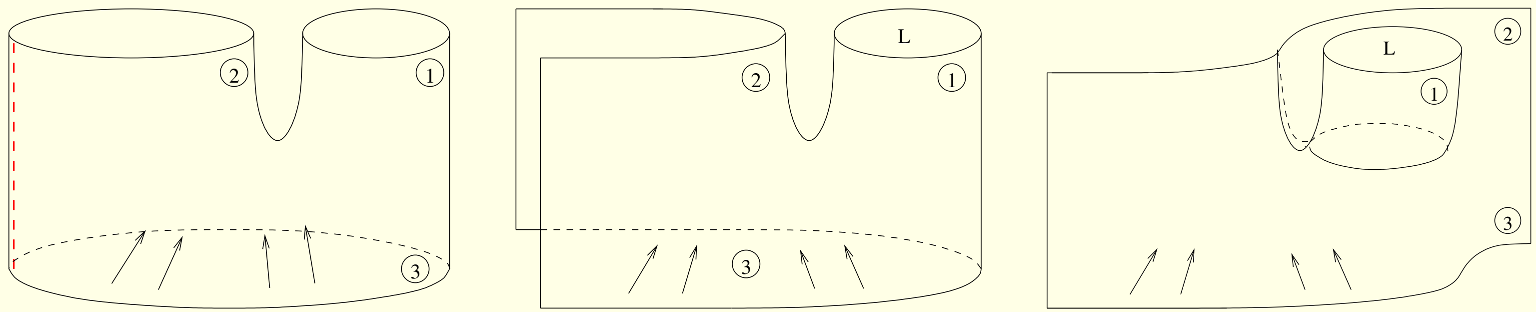
Form factor equations:



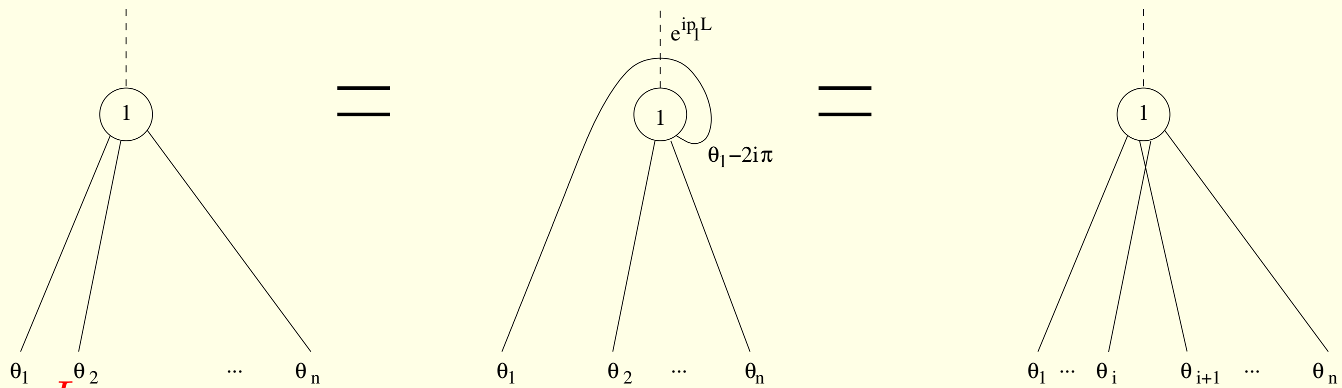
$$N_L(\theta_1,\dots,\theta_n\rangle = e^{-ip_1L} N_L(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1}) N_L(\dots,\theta_{i+1},\theta_i,\dots)$$

Decompactification limit of the string vertex

Decompactify string 2 & 3:

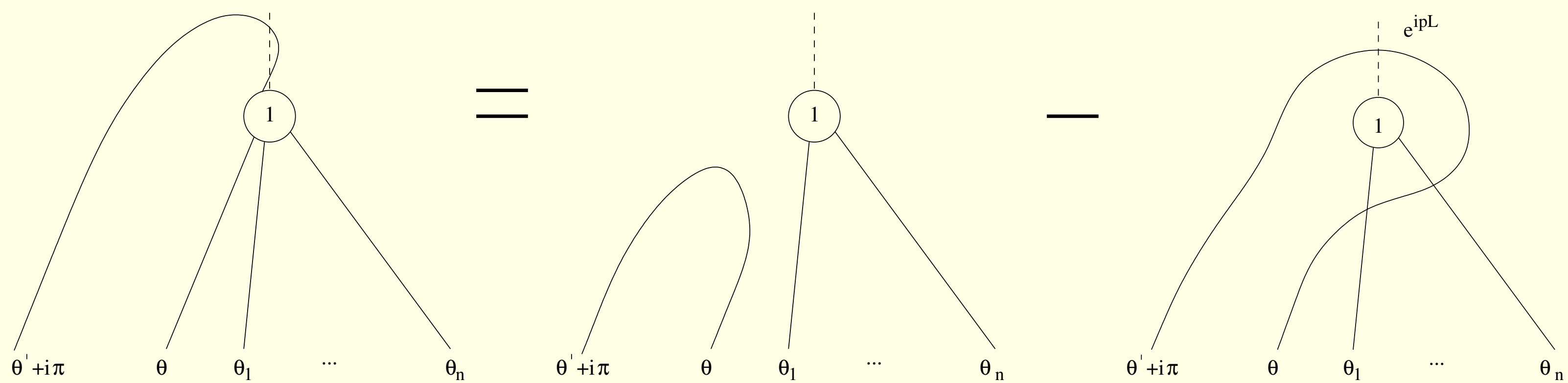


Form factor equations:



$$N_L(\theta_1,\dots,\theta_n\rangle = e^{-ip_1L} N_L(\theta_2,\dots,\theta_n,\theta_1-2i\pi\rangle = S(\theta_i-\theta_{i+1})N_L(\dots,\theta_{i+1},\theta_i,\dots)$$

Singularity stucture



$$-i\mathrm{Res}_{\theta'=\theta} N_L(\theta'+i\pi,\theta,\theta_1\dots,\theta_n) = (1-e^{ipL}\prod_i S(\theta-\theta_i))N_L(\theta_1,\dots,\theta_n)$$