

Holográfia a részecskefizikában

Bajnok Zoltán



Lendület Holografikus Kvantumtérelmélet Csoport

Középiskolai feladat

$$x^2 - 2x + 1 = 0$$

Valódi probléma

$$x^3 + 1.3463x^2 + 4.129846x + 0.142935 = 0$$

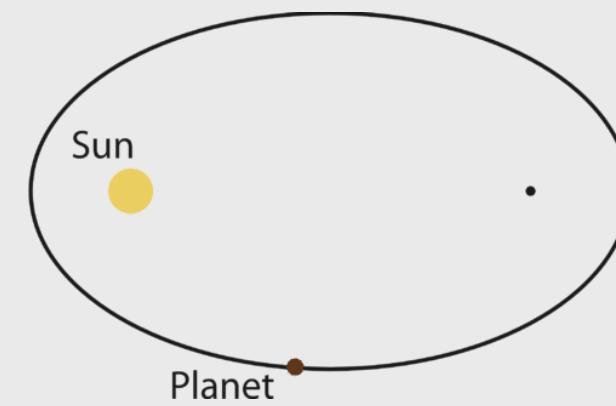
A pályakezdő elméleti fizikus kognitív disszonanciája az elméleti fizikai példatár feladatai és a valódi világban felmerülő problémák között



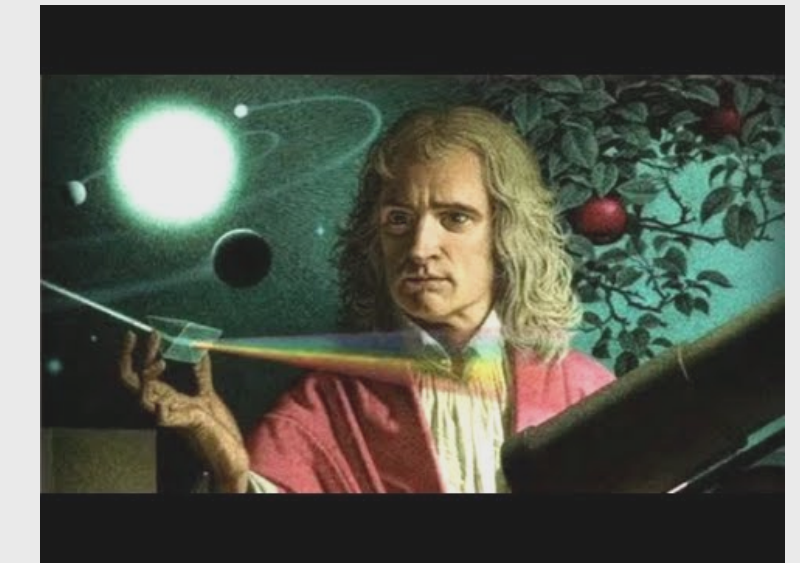
Feloldása:
Egzaktul megoldható
problémák
tanulmányozása

Egzaktul megoldható problémák ösztönző hatásai

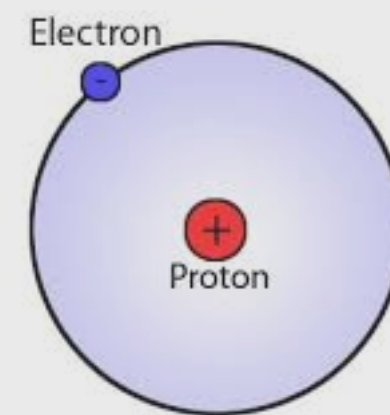
Kepler probléma



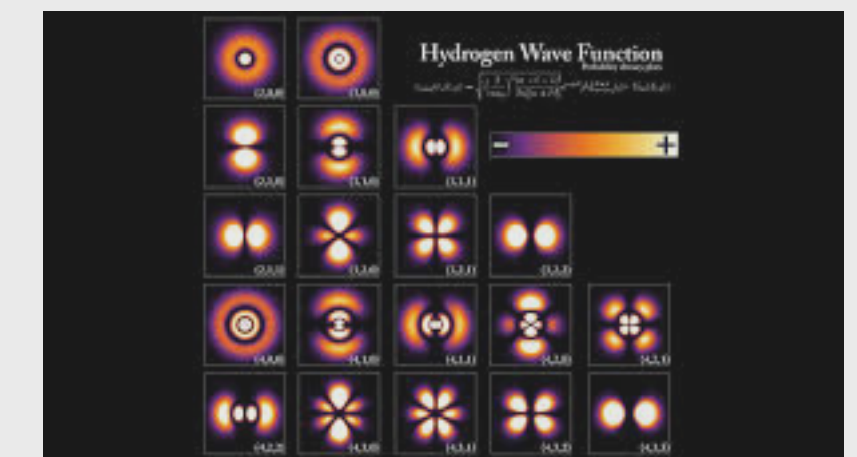
Newtoni mechanika



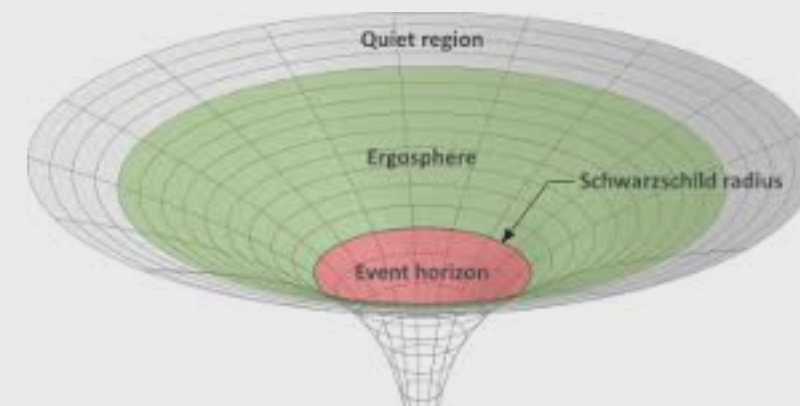
Hidrogén atom



Kvantummechanika

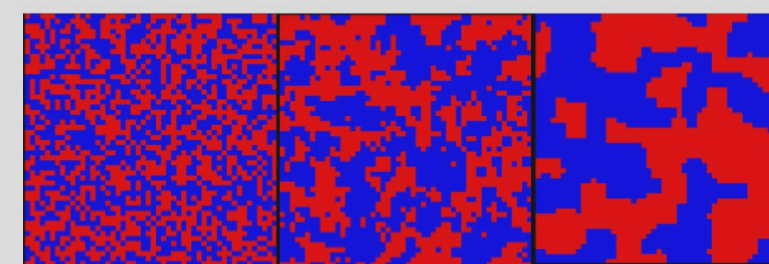


Schwarzschild megoldás



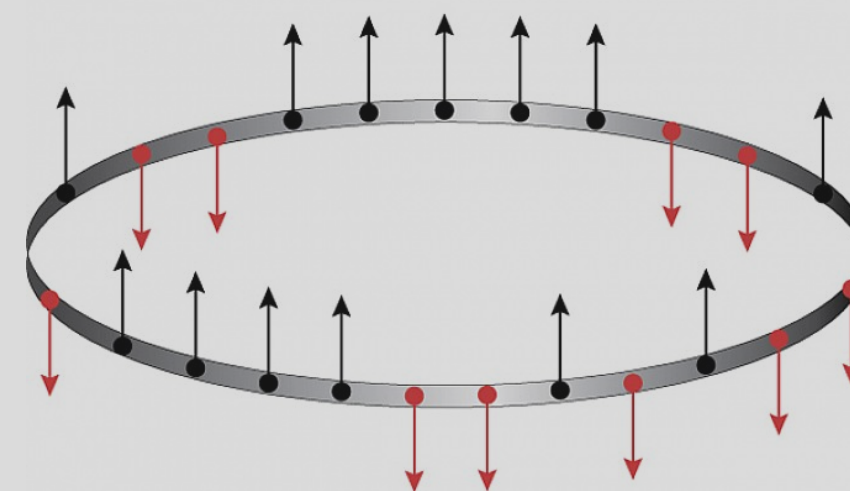
Általános relativitás elmélet tesztelése

Ising modell

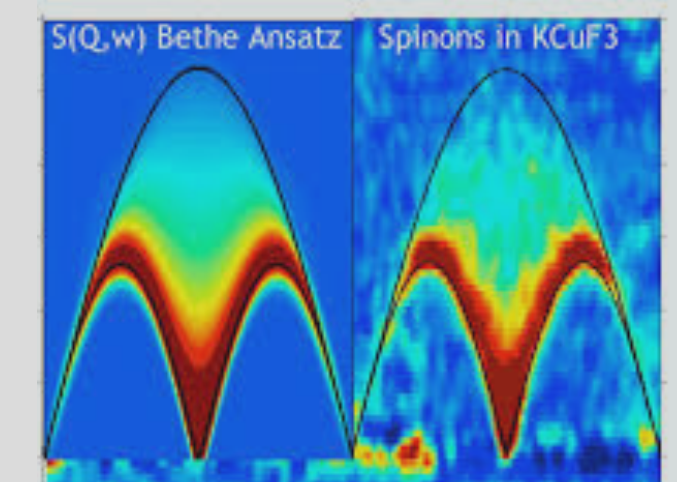


Másodrendű fázisátalakulások,
2 dimenziós konform térelméletek
statisztikus térelméletek

Heisenberg spin lánc



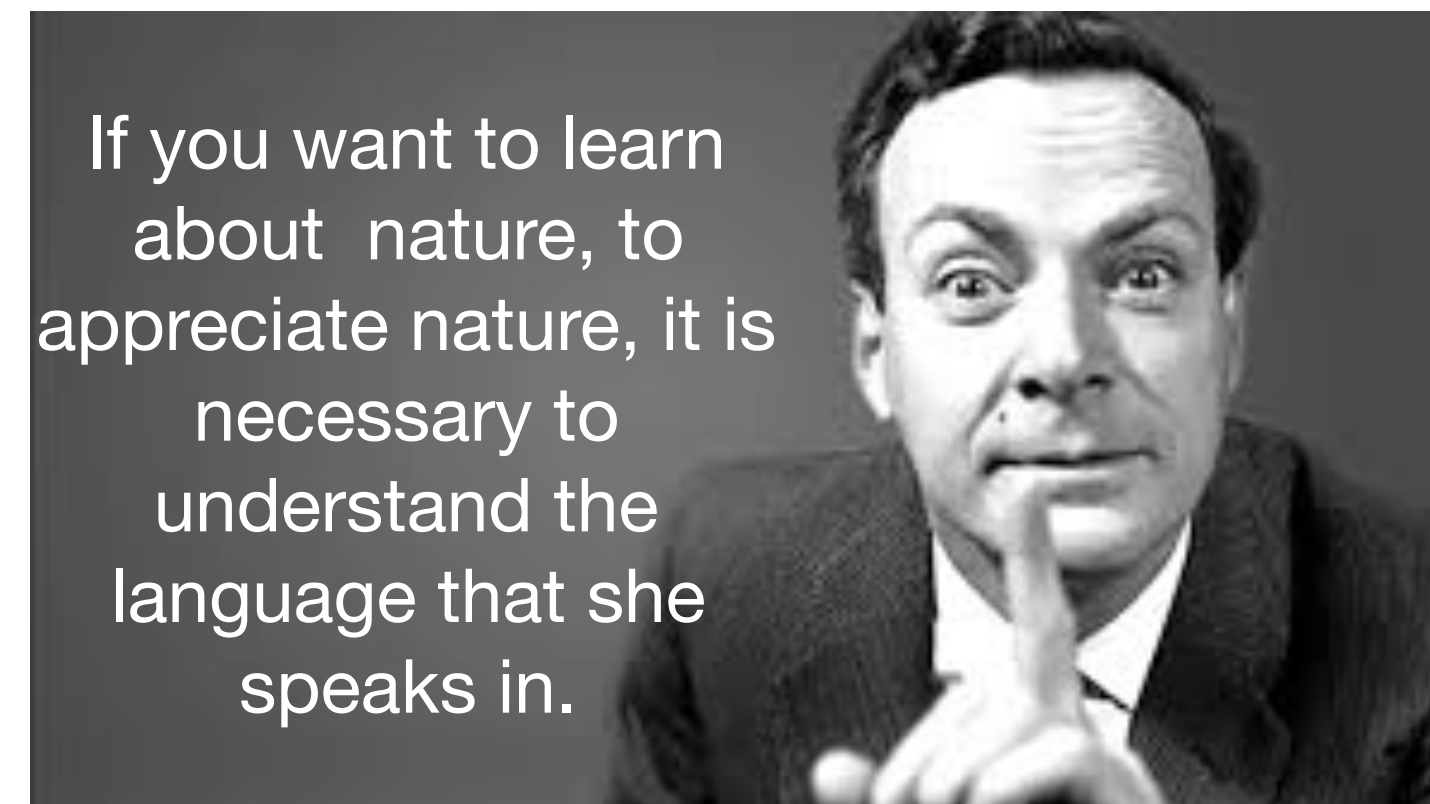
Integrálható spinláncok
erősen anizotróp szilárd testek



Maximálisan szimmetrikus
3+1 dimenziós mértékelmélet

A természet nyelve: mértékelmélet

[Feynman]



Mértékcsoport

Lie csoport **U(1)** **SU(N)** **N=2,3,...**

Részecsketartalom

mértékbozon



A

anyagrészecskék, fermionok



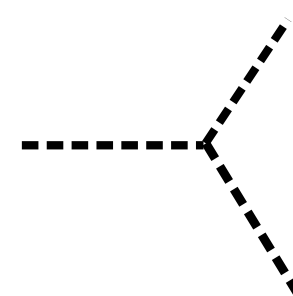
Ψ

skalárrészecskék

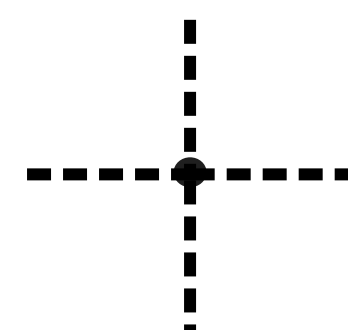


Φ

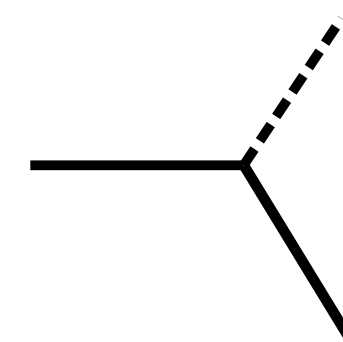
Kölcsönhatások



Φ^3

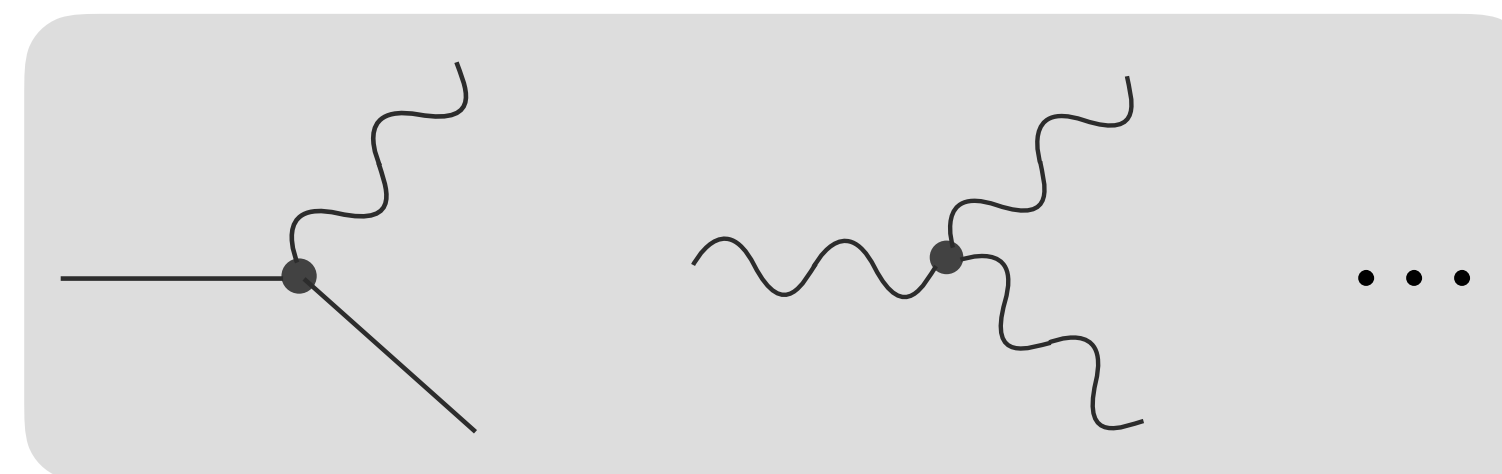


Φ^4



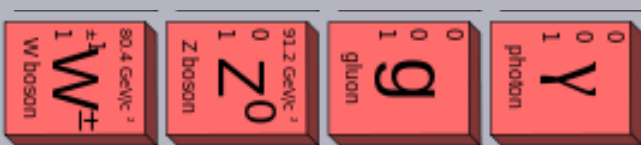

$\Psi\Phi\Psi$

mértékszimmétria rögzíti



$\Psi A \Psi$

Természet alapvető kölcsönhatásai

Lie csoport	Vektor bozon	Fermion	skalár	
U(1)	1	elektron pozitron	0	elektrodinamika
U(1)xSU(2)	1+3	6 kvark 3 neutrínó elektron,müon,tau	2 Higgs	elektrogyenge
U(1)xSU(2)xSU(3)	 1+3+8		2 Higgs	standard modell
SU(N)	$N^2 - 1$	$8(N^2 - 1)$	$6(N^2 - 1)$	Legszimmetrikusabb elmélet [Brink,Schwartz]

Maximálisan szimmetrikus mértékelmélet

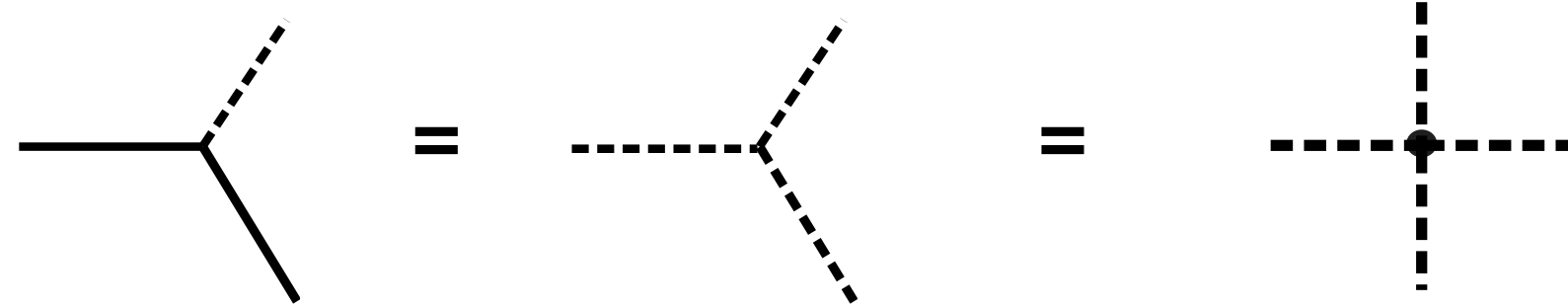
Szimmetria **SU(N)**

1 mértékbozon

8 fermion

6 skalár

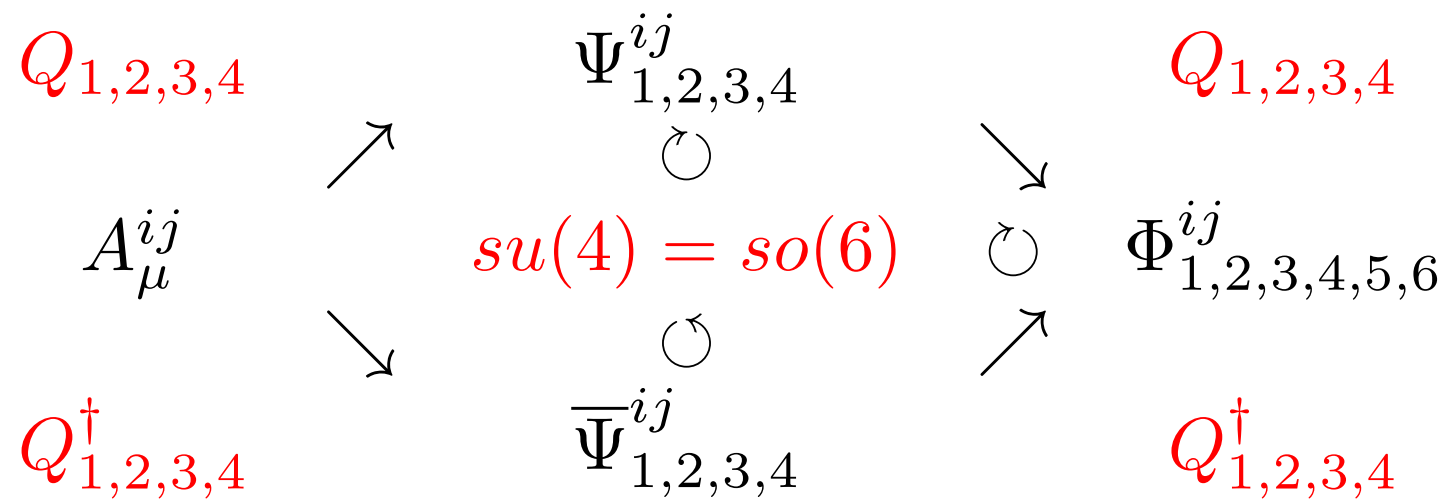
Kölcsönhatások



$$\mathcal{L} = \frac{2}{g_{YM}^2} \int d^4x \text{Tr} \left[-\frac{1}{4} F^2 - \frac{1}{2} (D\Phi)^2 + i\bar{\Psi} \not{D} \Psi + V \right]$$

$$V(\Phi, \Psi) = \frac{1}{4} [\Phi, \Phi]^2 + \bar{\Psi}[\Phi, \Psi]$$

Maximálisan szuperszimmetrikus elmélet 3+1 dimenzióban



perturbációszámítás

$$E = E_0 + \lambda E_1 + \dots$$

$$\lambda = g_{YM}^2 N$$

Standard modell

$$\mathcal{L}_{SM} = \mathcal{L}_{Dirac} + \mathcal{L}_{mass} + \mathcal{L}_{gauge} + \mathcal{L}_{gauge/\psi} . \quad (1)$$

Here,

$$\mathcal{L}_{Dirac} = i\bar{e}_L^i \not{\partial} e_L^i + i\bar{\nu}_L^i \not{\partial} \nu_L^i + i\bar{e}_R^i \not{\partial} e_R^i + i\bar{u}_L^i \not{\partial} u_L^i + i\bar{d}_L^i \not{\partial} d_L^i + i\bar{u}_R^i \not{\partial} u_R^i + i\bar{d}_R^i \not{\partial} d_R^i ; \quad (2)$$

$$\mathcal{L}_{mass} = -v \left(\lambda_e^i \bar{e}_L^i e_R^i + \lambda_u^i \bar{u}_L^i u_R^i + \lambda_d^i \bar{d}_L^i d_R^i + \text{h.c.} \right) - M_W^2 W_\mu^+ W^{-\mu} - \frac{M_W^2}{2 \cos^2 \theta_W} Z_\mu Z^\mu ; \quad (3)$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} (G_{\mu\nu}^a)^2 - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{WZA} , \quad (4)$$

where

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c \\ W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned} \quad (5)$$

and

$$\begin{aligned} \mathcal{L}_{WZA} &= ig_2 \cos \theta_W \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu Z^\nu + W_{\mu\nu}^+ W^{-\mu\nu} Z^\nu - W_{\mu\nu}^- W^{+\mu\nu} Z^\nu \right] \\ &+ ie \left[(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+) \partial^\mu A^\nu + W_{\mu\nu}^+ W^{-\mu\nu} A^\nu - W_{\mu\nu}^- W^{+\mu\nu} A^\nu \right] \\ &+ g_2^2 \cos^2 \theta_W \left(W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu\nu} Z_\nu Z^\nu \right) \\ &+ g_2^2 \left(W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu\nu} A_\nu A^\nu \right) \\ &+ g_2 e \cos \theta_W \left[W_\mu^+ W_\nu^- (Z^\mu A^\nu + Z^\nu A^\mu) - 2W_\mu^+ W^{-\mu\nu} Z_\nu A^\nu \right] \\ &+ \frac{1}{2} g_2^2 \left(W_\mu^+ W_\nu^- \right) \left(W^{+\mu\nu} W^{-\nu\mu} - W^{+\nu\mu} W^{-\mu\nu} \right) ; \end{aligned} \quad (6)$$

and

$$\mathcal{L}_{gauge/\psi} = -g_3 A_\mu^a J_{(3)}^{\mu a} - g_2 \left(W_\mu^+ J_{W^+}^\mu + W_\mu^- J_{W^-}^\mu + Z_\mu J_Z^\mu \right) - e A_\mu J_A^\mu , \quad (7)$$

where

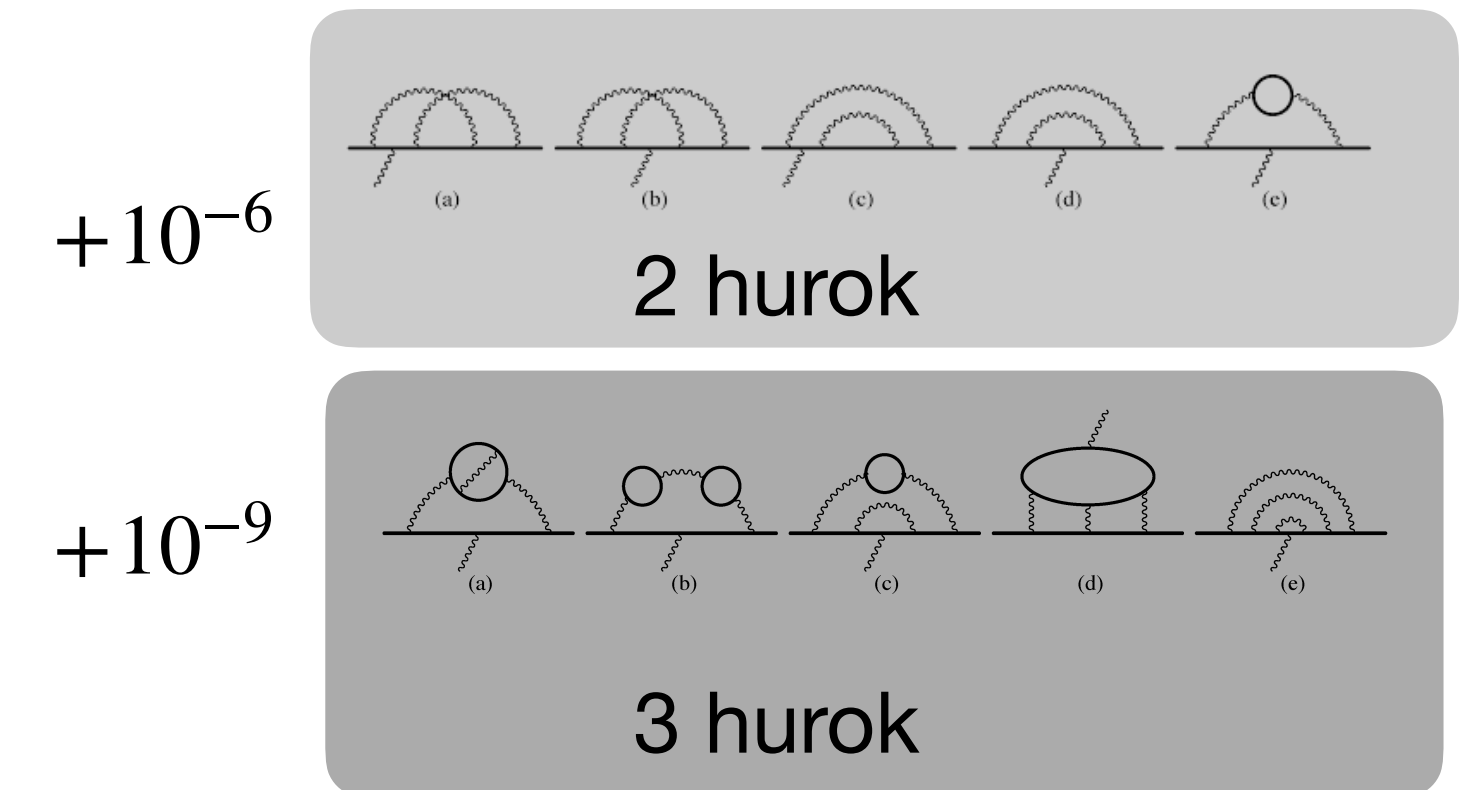
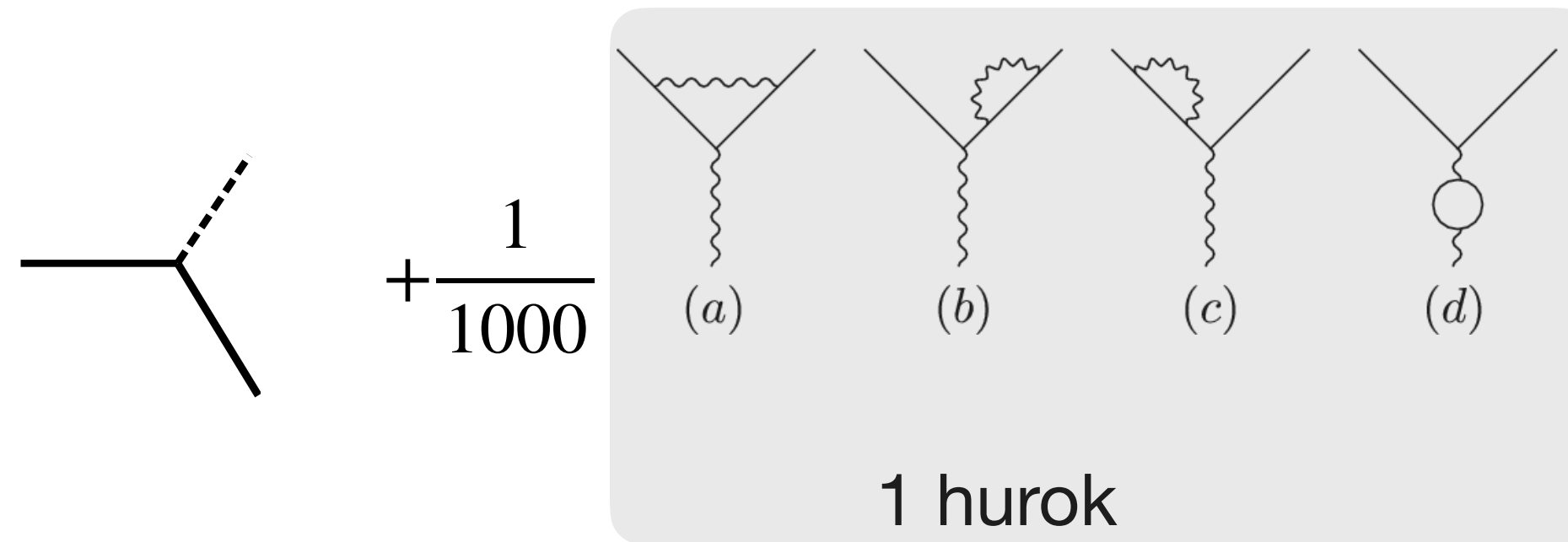
$$\begin{aligned} J_{(3)}^{\mu a} &= \bar{u}^i \gamma^\mu T_{(3)}^a u^i + \bar{d}^i \gamma^\mu T_{(3)}^a d^i \\ J_{W^+}^\mu &= \frac{1}{\sqrt{2}} \left(\bar{\nu}_L^i \gamma^\mu e_L^i + V^{ij} \bar{u}_L^i \gamma^\mu d_L^j \right) \\ J_{W^-}^\mu &= (J_{W^+}^\mu)^* \\ J_Z^\mu &= \frac{1}{\cos \theta_W} \left[\frac{1}{2} \bar{\nu}_L^i \gamma^\mu \nu_L^i + \left(-\frac{1}{2} + \sin^2 \theta_W \right) \bar{e}_L^i \gamma^\mu e_L^i + (\sin^2 \theta_W) \bar{e}_R^i \gamma^\mu e_R^i \right. \\ &\quad + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \bar{u}_L^i \gamma^\mu u_L^i + \left(-\frac{2}{3} \sin^2 \theta_W \right) \bar{u}_R^i \gamma^\mu u_R^i \\ &\quad \left. + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) \bar{d}_L^i \gamma^\mu d_L^i + \left(\frac{1}{3} \sin^2 \theta_W \right) \bar{d}_R^i \gamma^\mu d_R^i \right] \\ J_A^\mu &= (-1) \bar{e}^i \gamma^\mu e^i + \left(\frac{2}{3} \right) \bar{u}^i \gamma^\mu u^i + \left(-\frac{1}{3} \right) \bar{d}^i \gamma^\mu d^i . \end{aligned} \quad (8)$$

Perturbációs számítás

Kvantumelektrodinamika

Elektron mágneses momentuma

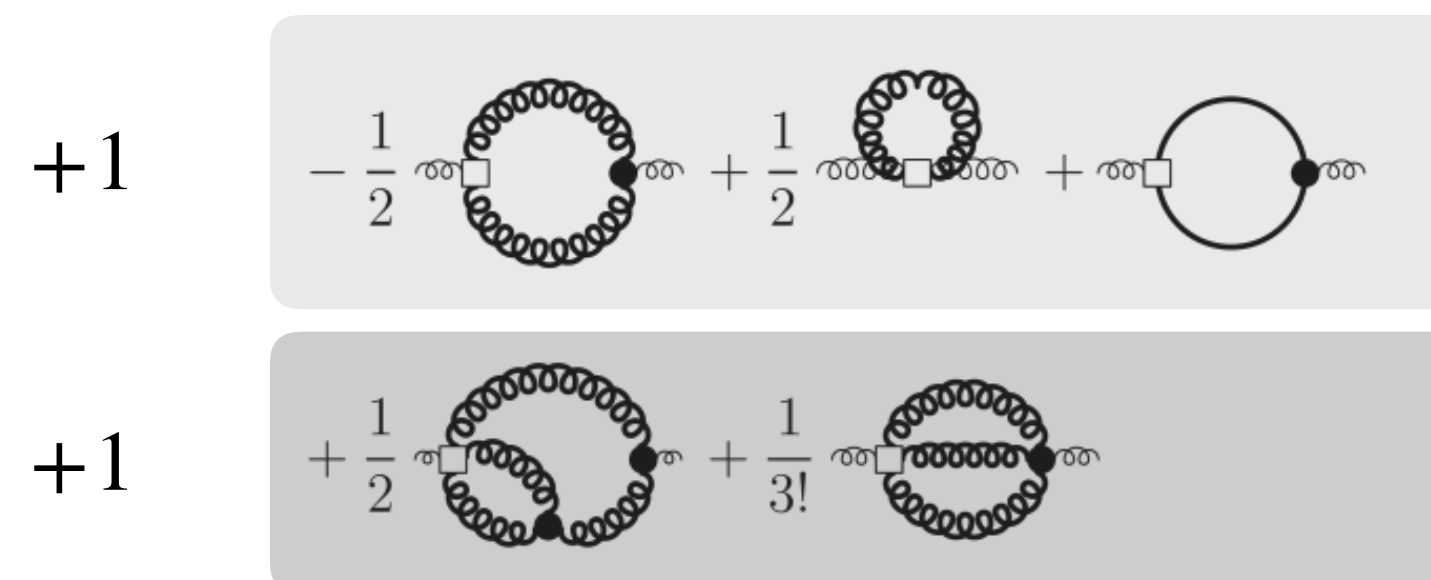
10 jegyre egyezik a kísérlettel



QCD: erős kölcsönhatás

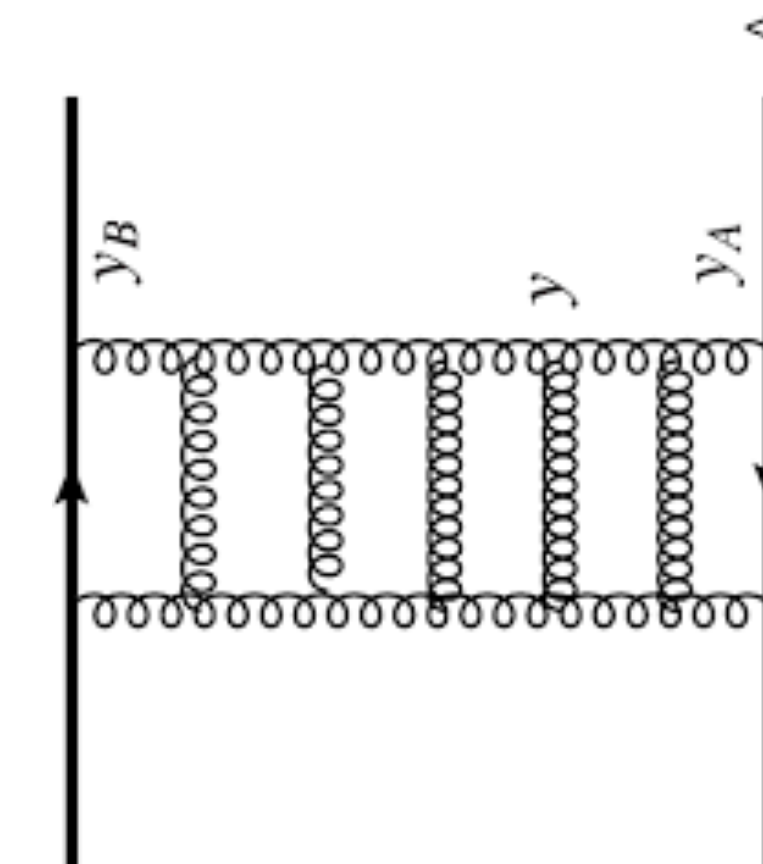
gluon propagátor

$$\text{gluon propagator}^{-1} = \text{tree-level} + \text{loop}$$



Gluonlabda tömege?

Millenium díj: 1 millió dollár



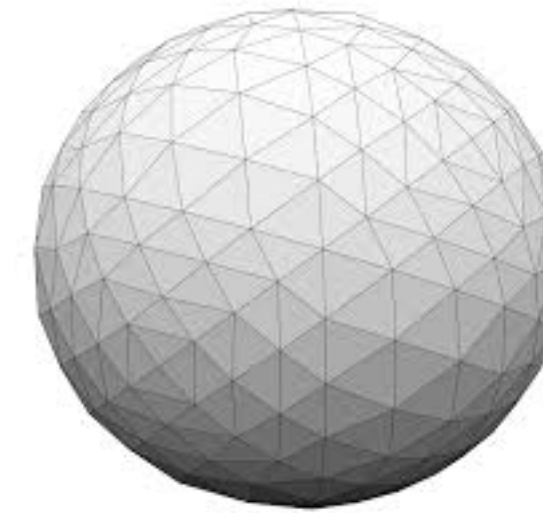
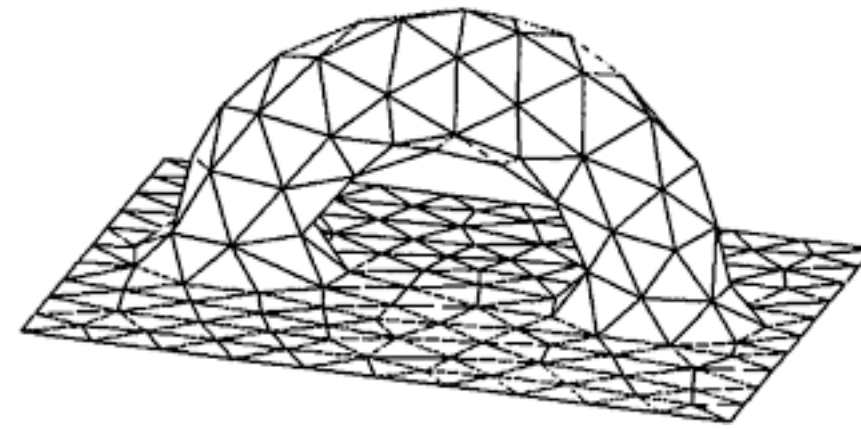
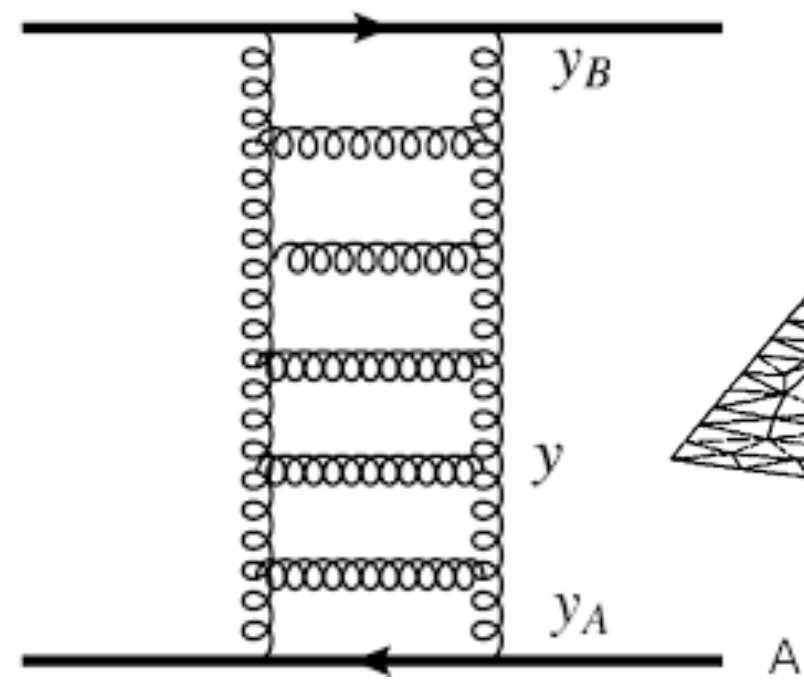
Mezonok tömege

Erős csatolású határeset

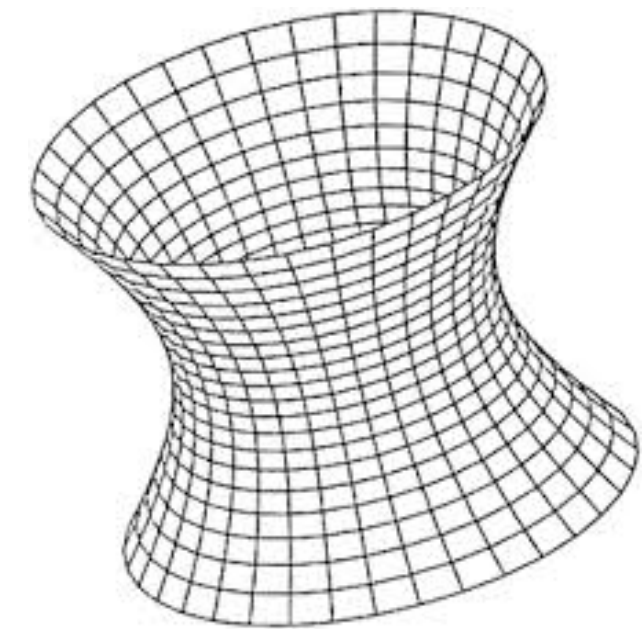
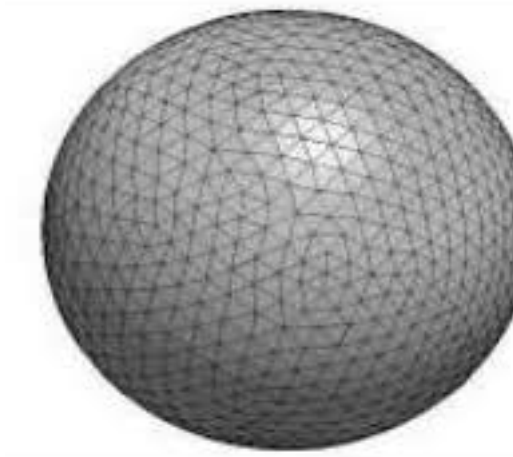
kvark-antikvark kölcsönhatás

A sűrű Feynmann gráfok egy felületet rajzolnak ki

[t Hooft]



$$X_1^2 + X_2^2 + \dots + X_6^2 = R^2$$



$$-Y_1^2 - Y_2^2 + \dots + Y_5^2 = -R^2$$

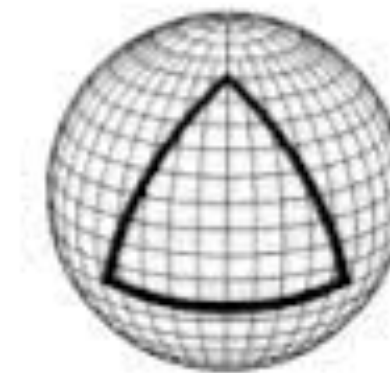
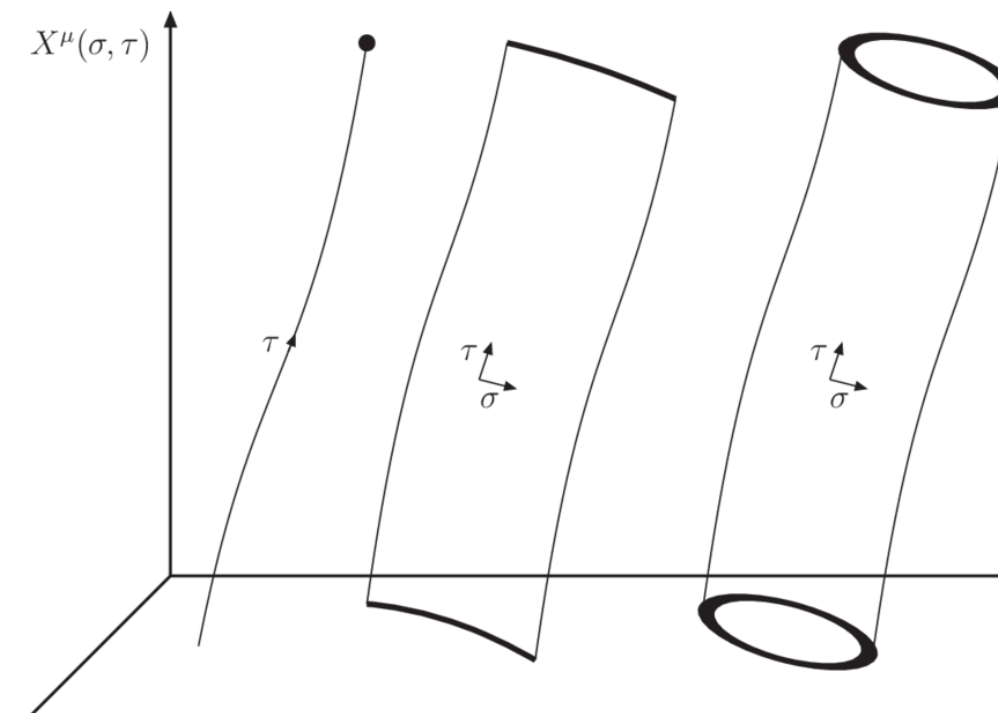
A téridőfelület a húr által kisöpört világlepedő

húrhatás

$$\sqrt{\lambda} \int d\tau d\sigma \text{Area}(\tau, \sigma)$$

szemiklasszikus kifejtés

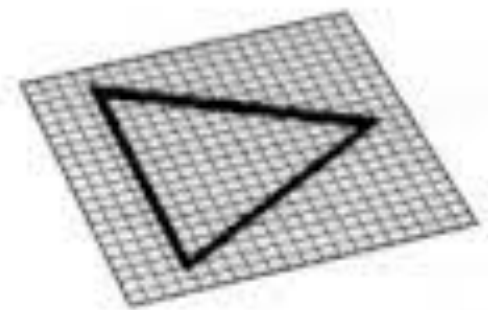
$$E = E_\infty + \frac{1}{\sqrt{\lambda}} E_{\frac{1}{2}} + \dots$$



Positive Curvature



Negative Curvature

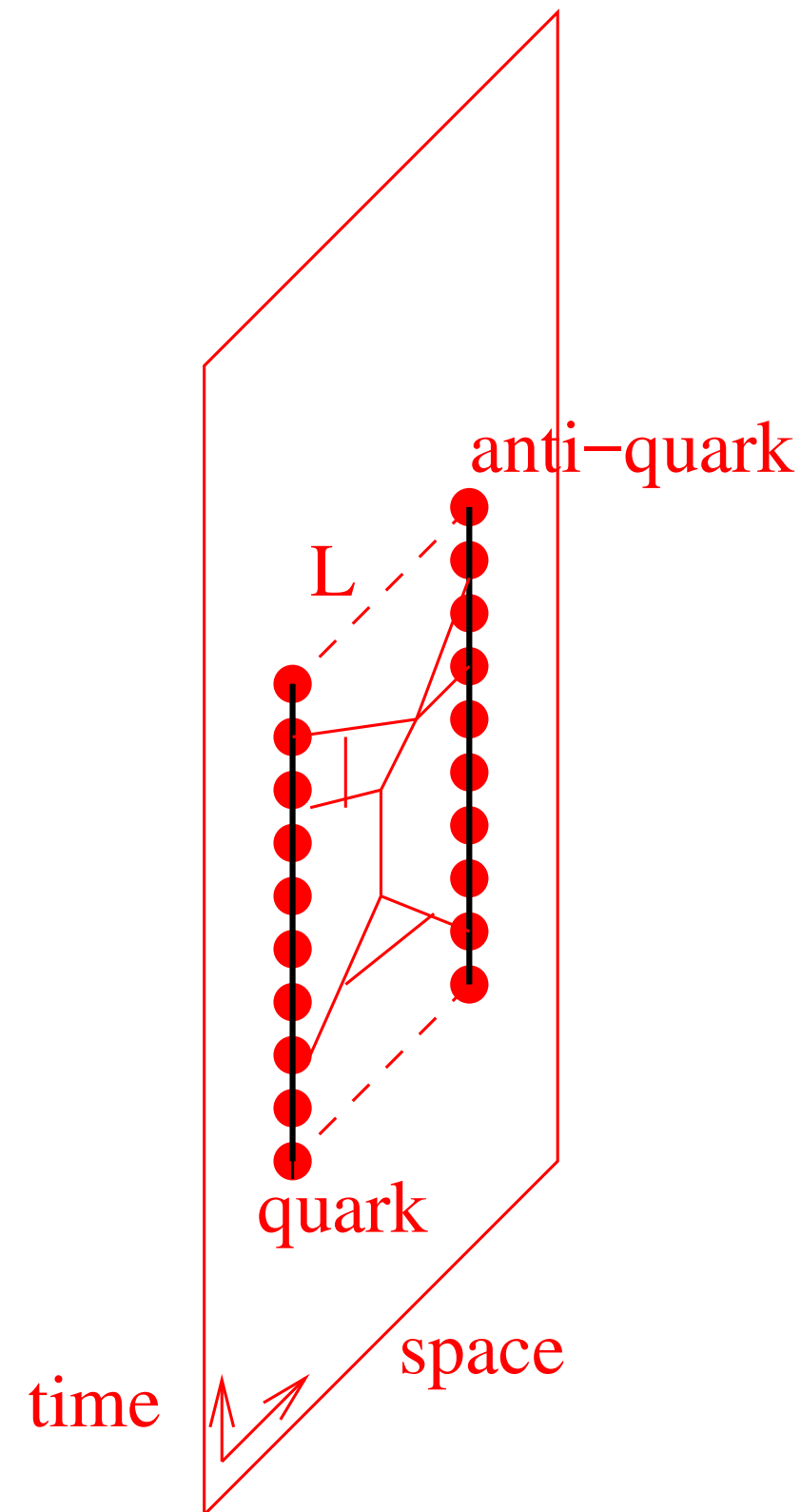


Flat Curvature

Holografikus dualitási sejtés

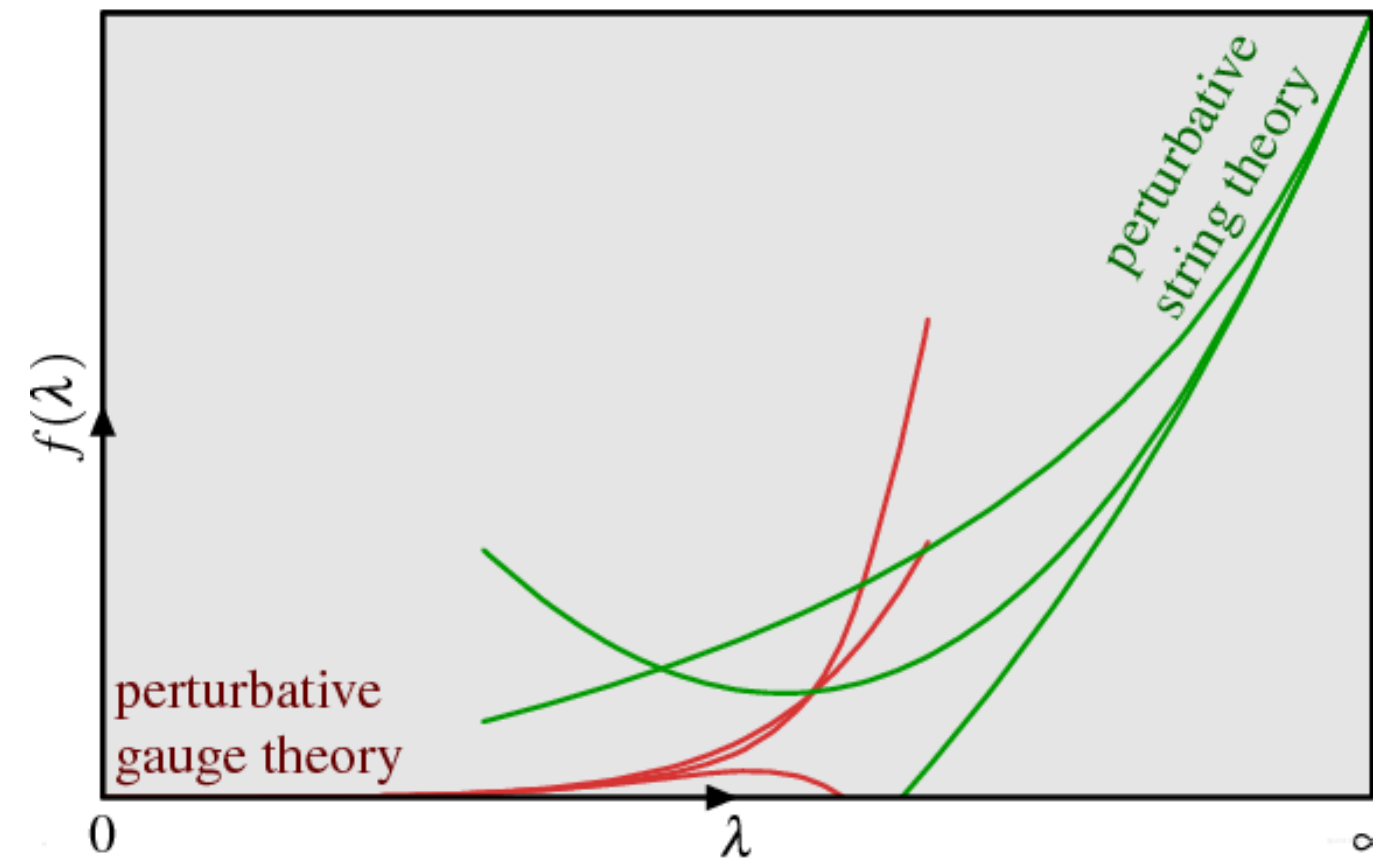
[Maldacena]

3+1 dimenziós mértékelmélet



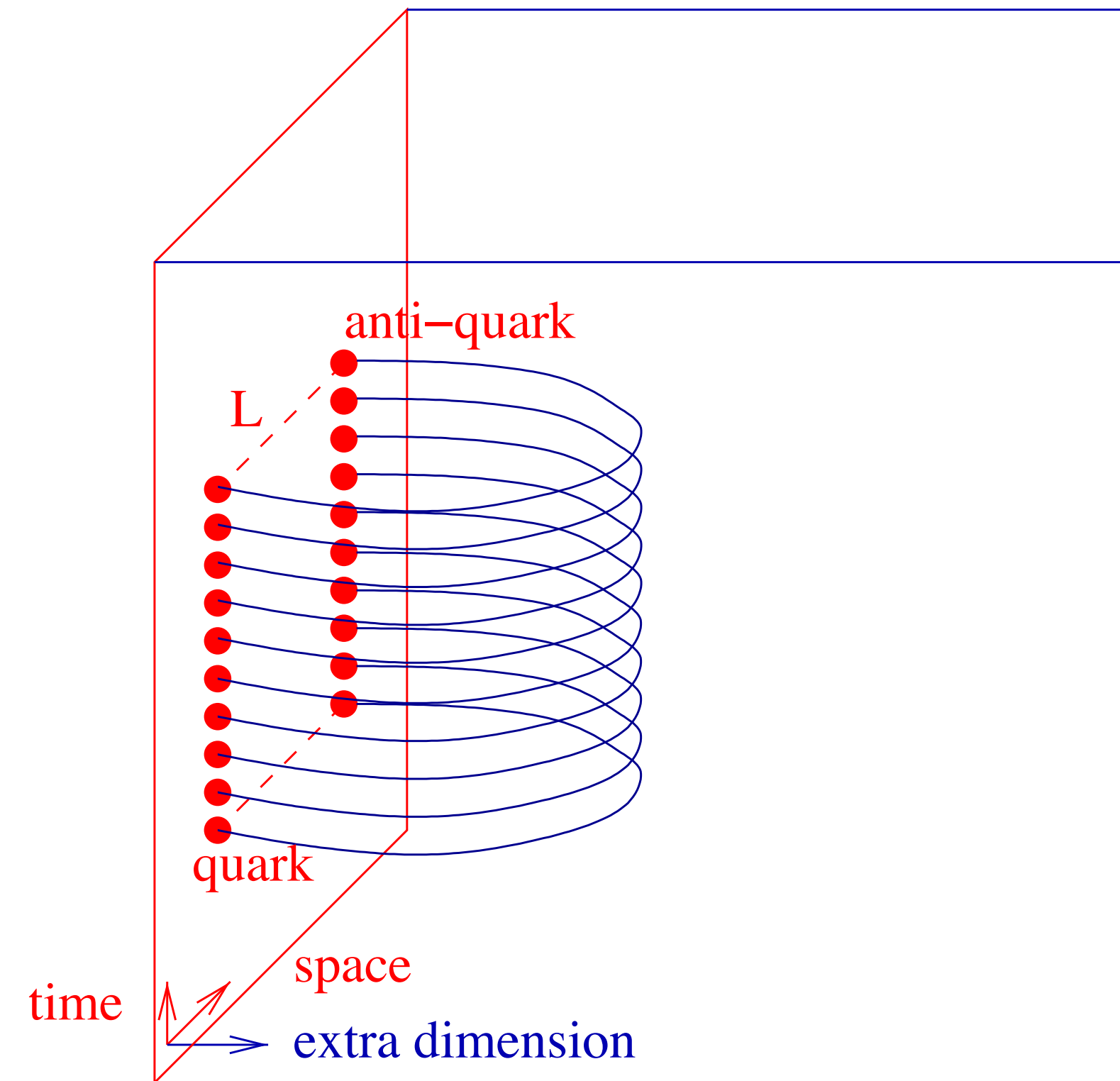
$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$

hurokkifejtés



$$V(L) = \frac{f(\lambda)}{L}$$

9+1 dimenziós húrelmélet



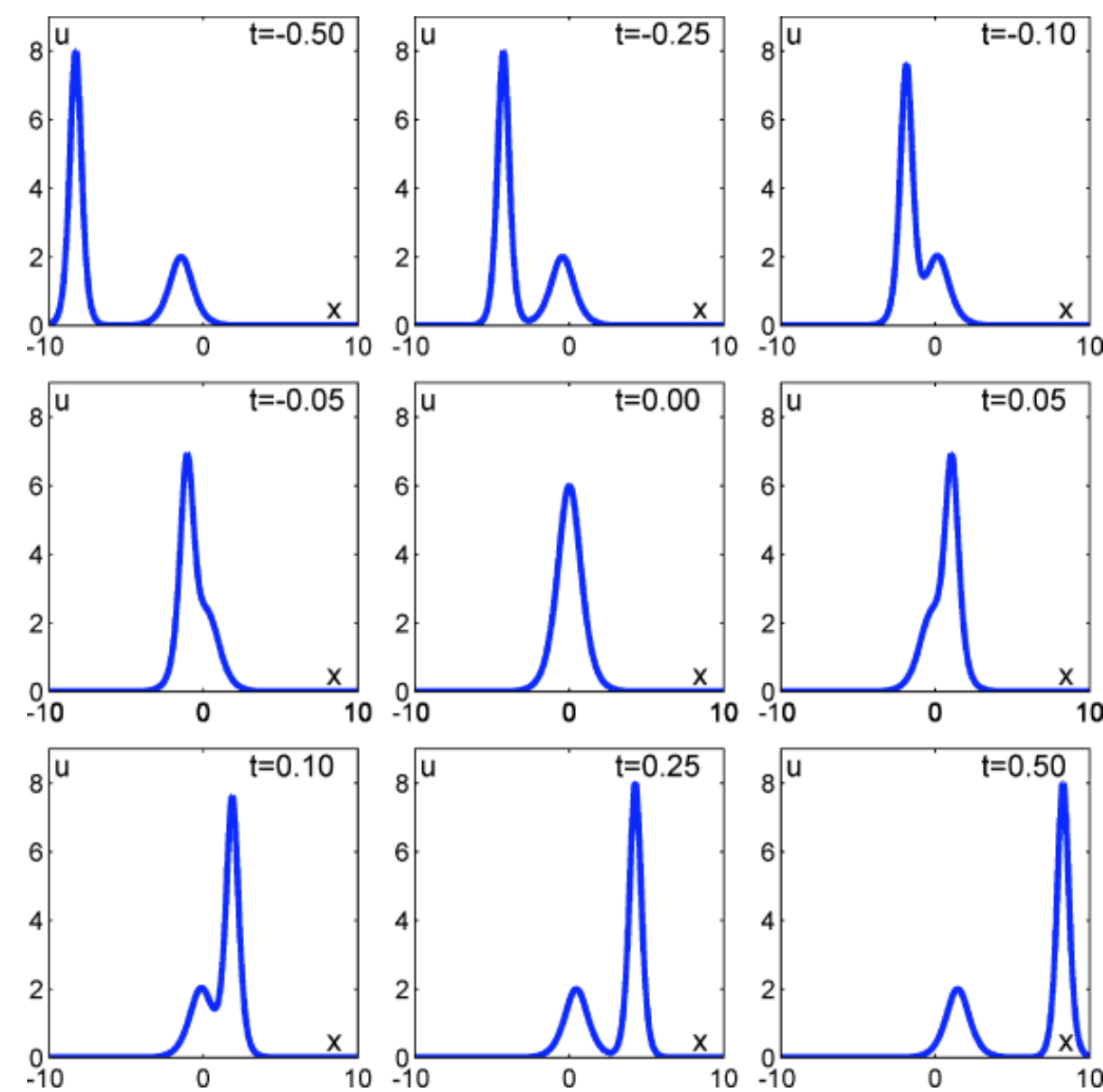
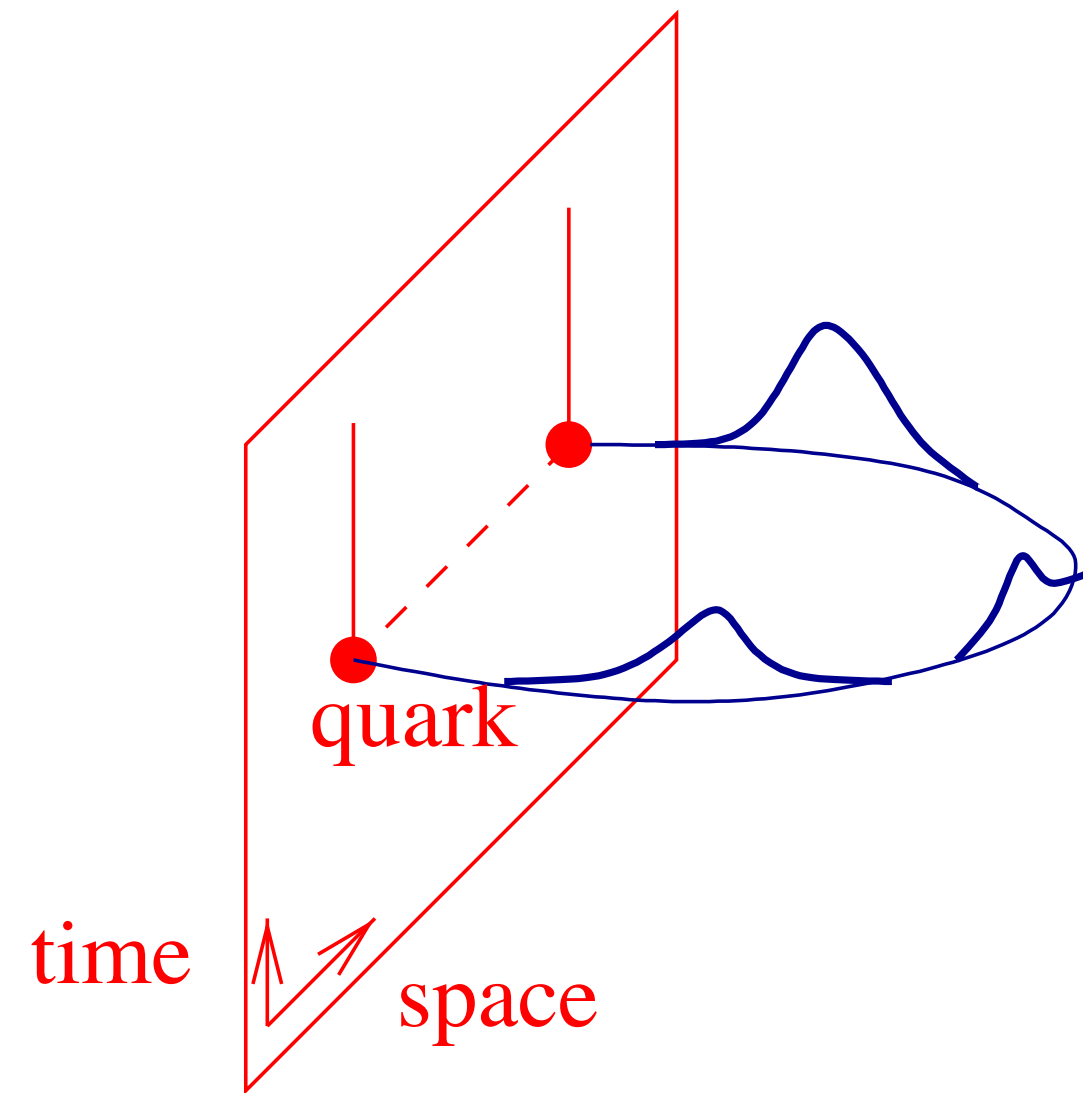
$$V(L) = -\frac{4\pi^2 \sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimális felület

kvadratis fluktuációk

Kétdimenziós integrálható leírás

1+1 dimenziós húrelmélet
integrálható
nincs részecskekeltés

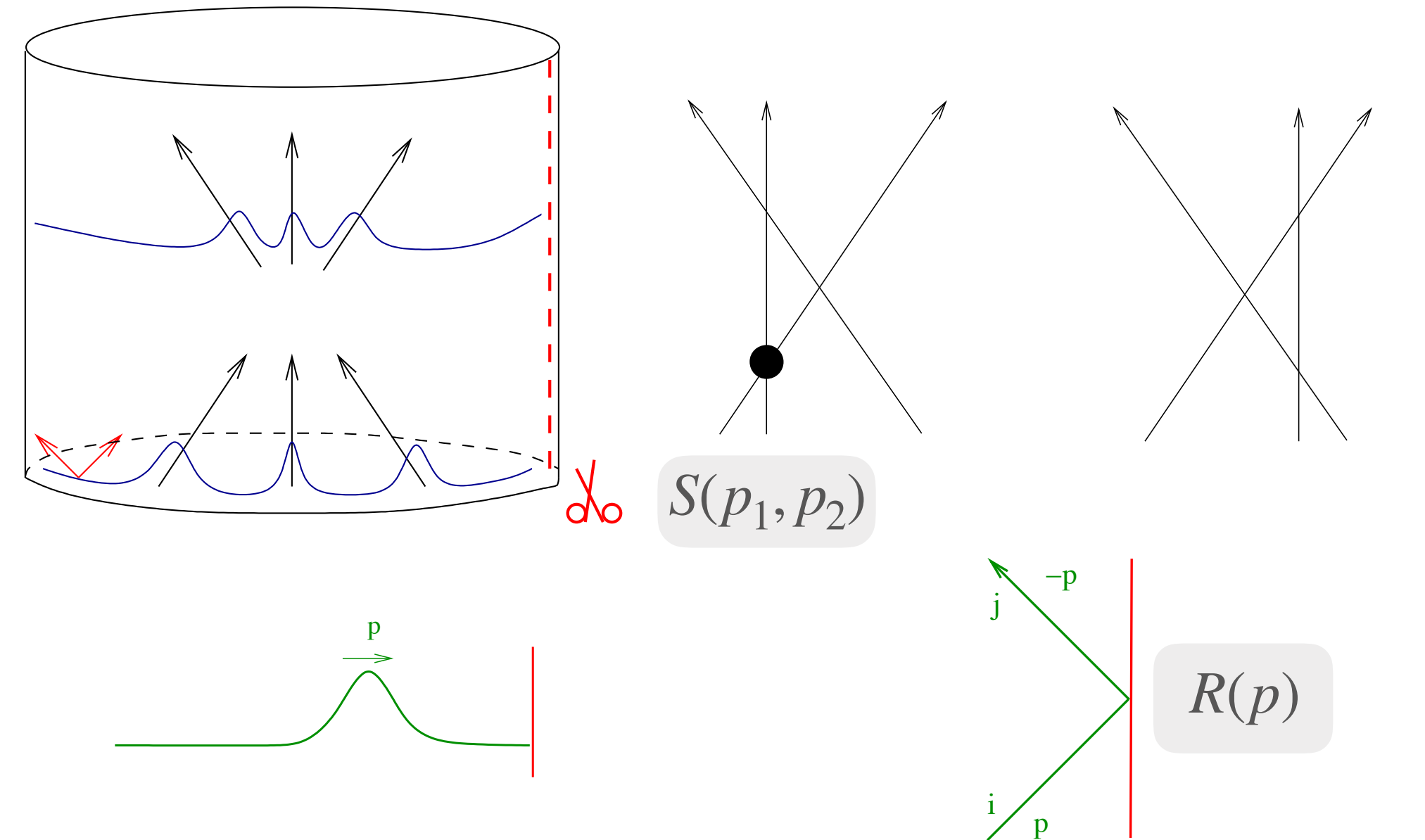


[Bajnok, Palla, Takacs]

Kvark antikvark potenciál= 1+1 dimenziós Casimir effektus
alapállapotú energia az intervallumon

$$E_0 = \int dp \log(1 - R(p)\bar{R}(p)e^{-\epsilon})$$

téridő diagram



szórás, reflexió,
diszperziós reláció
egzaktul számolható

$$E(p) = \sqrt{1 + \lambda \sin^2 \frac{p}{2}}$$

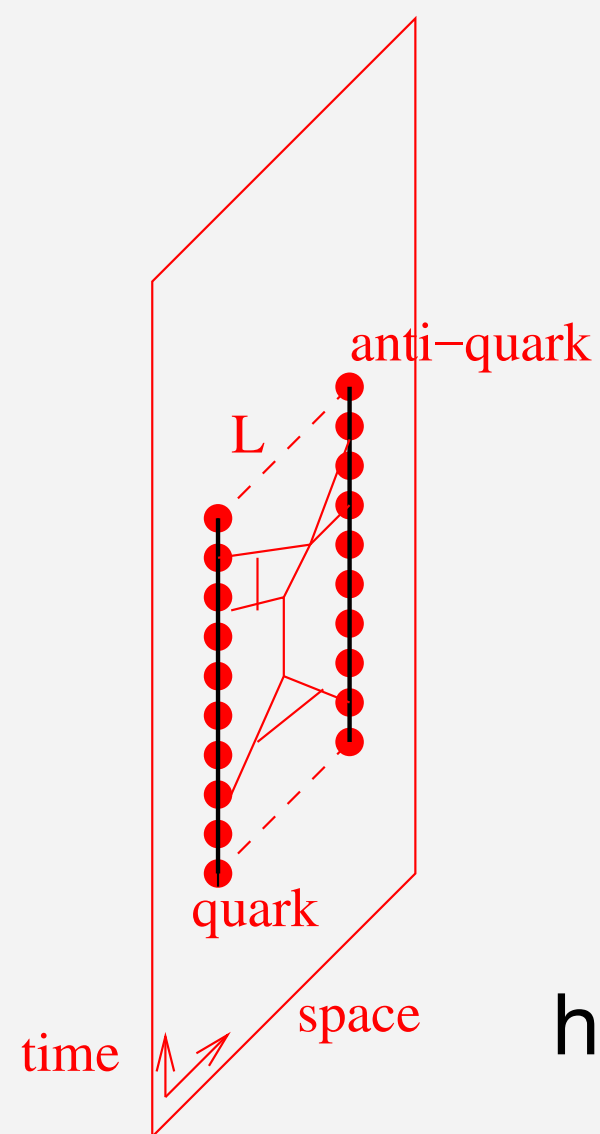
[Correa, Maldacena, Sever][Drukker]

[Bajnok, Balog et al]

ϵ : egzakt TBA integrálegyenlet

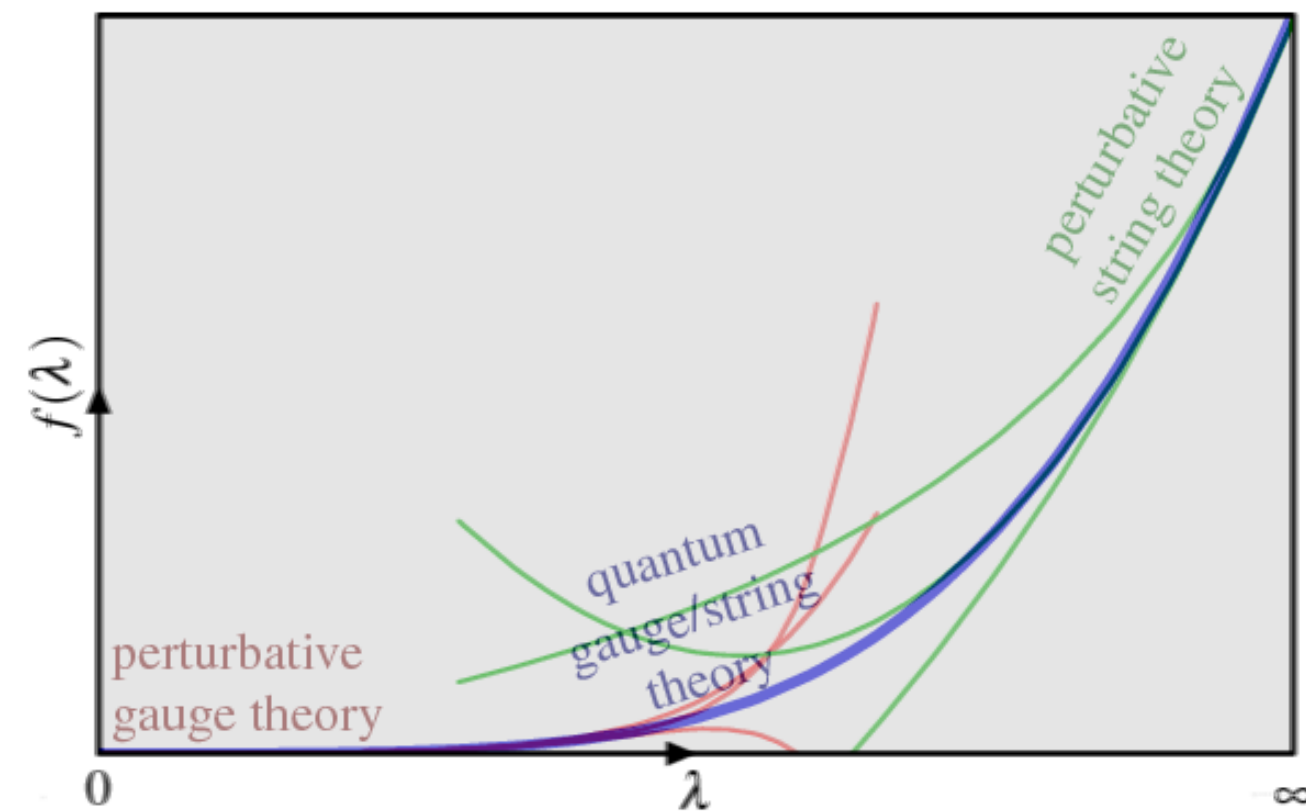
Trialitás

3+1 dimenziós mértékelmélet

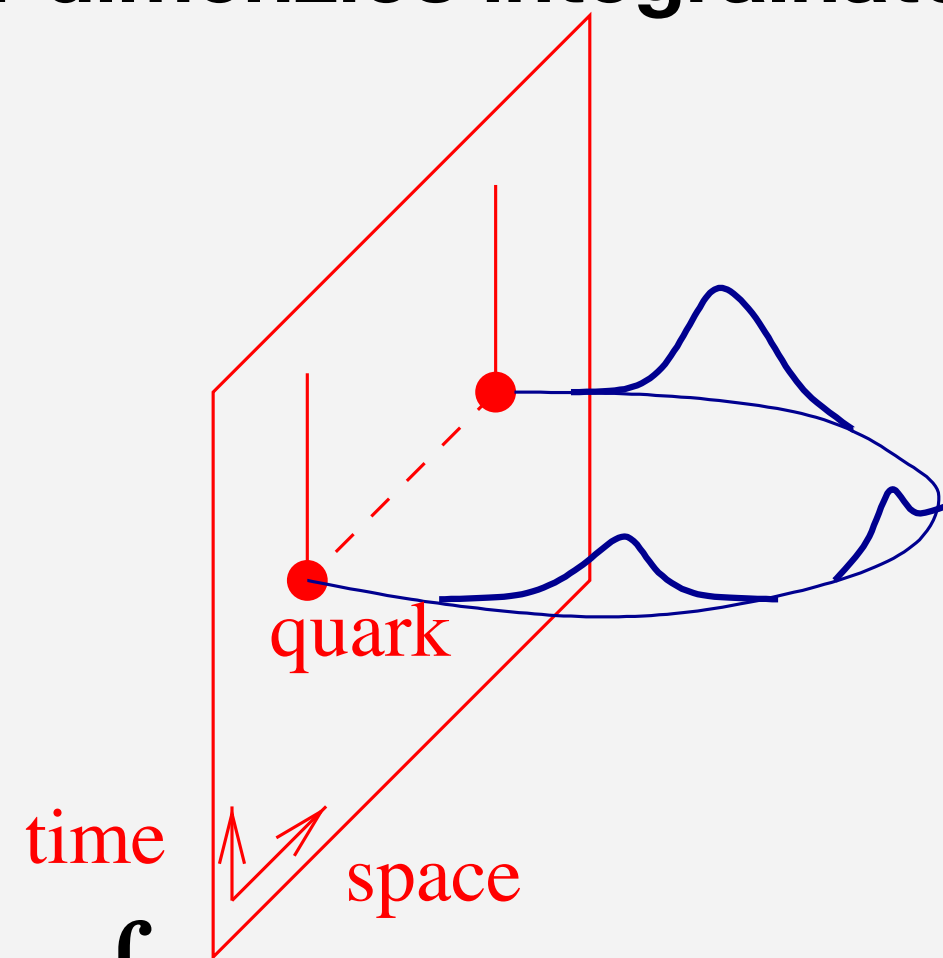


hurokkifejtés

$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$



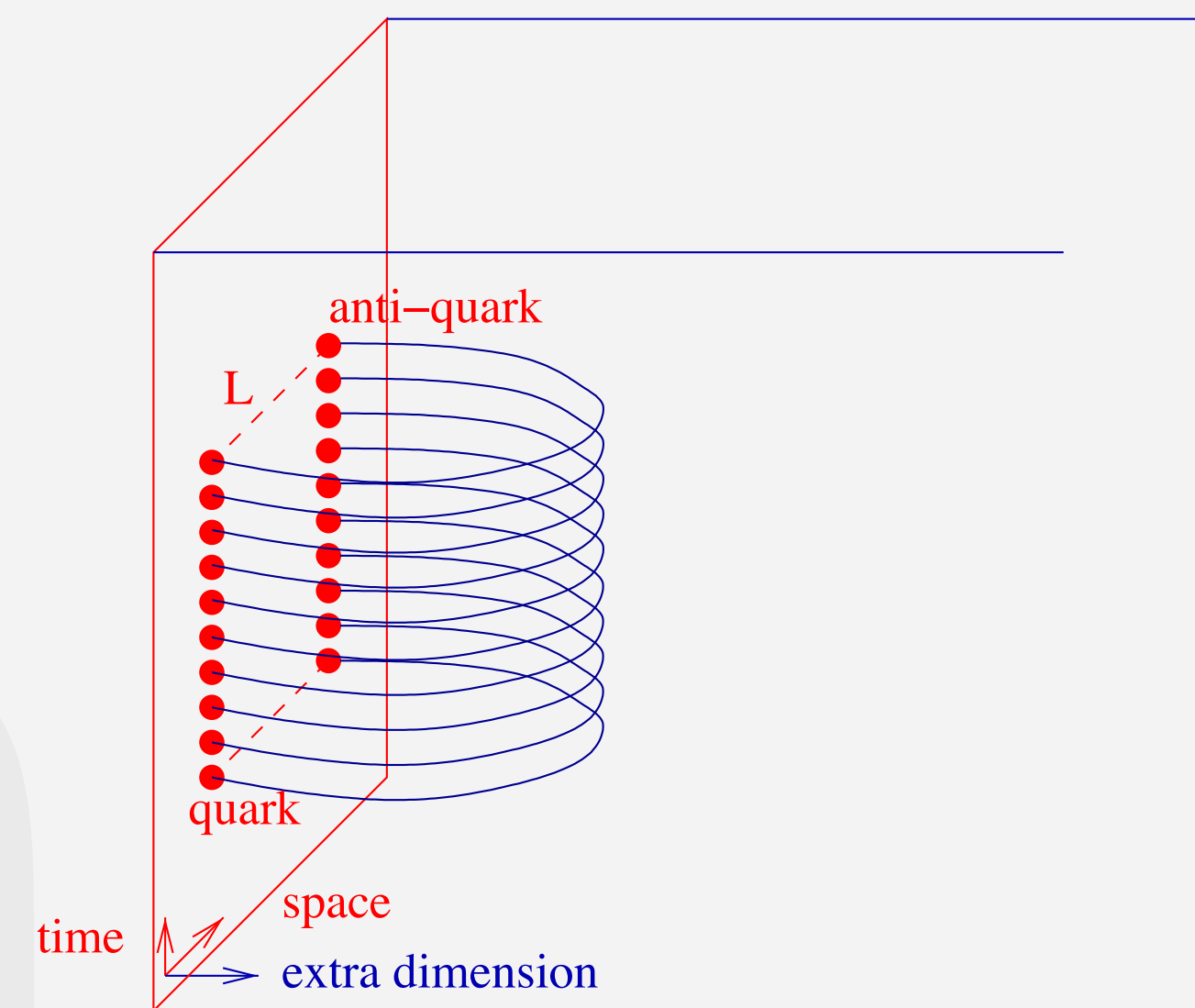
1+1 dimenziós integrálható elmélet



$$V(L) = L^{-1} \int dp \log(1 - R(p)\bar{R}(p)e^{-\epsilon(p)})$$

Egzakt alapállapot energiája

1+9 dimenziós húrelmélet



$$V(L) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4} \frac{1}{L} \left(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots\right)$$

minimális felület

fluktuációk

Spektrum

3+1 dimenziós mértékelmélet

Konform térelmélet

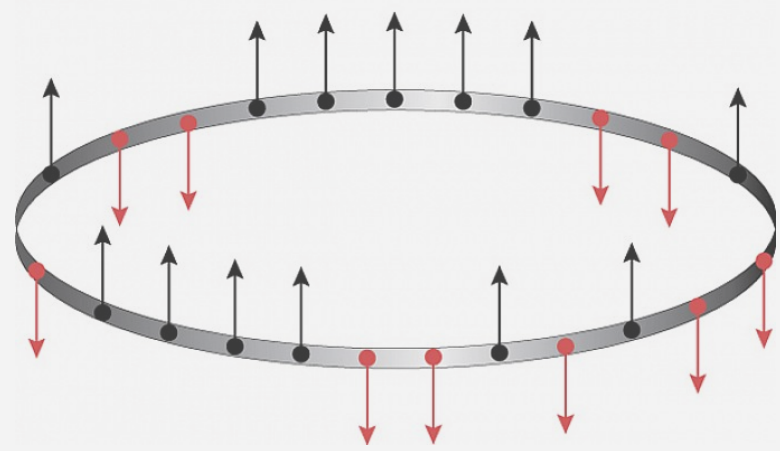
$$\langle \mathcal{O}_i(x) \mathcal{O}_j(0) \rangle = \frac{\delta_{ij}}{x^{2\Delta(\lambda)}}$$

$$\mathcal{O} = \text{Tr}(Z^J)$$

$$Z = \Phi_1 + i\Phi_2$$

$$X = \Phi_3 + i\Phi_4$$

$$\mathcal{O} = \text{Tr}(Z^{J-M} X^M) \quad |\uparrow\uparrow \cdot \downarrow\downarrow\rangle$$

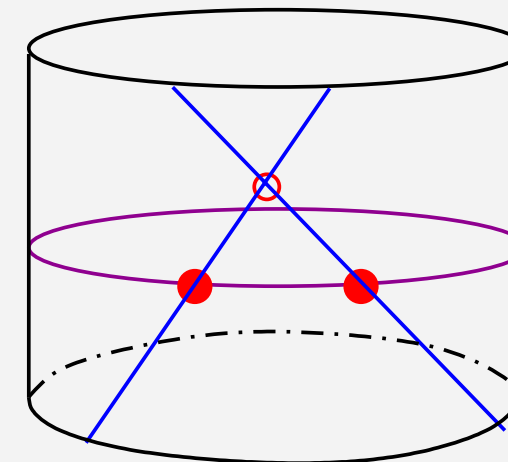


Integrálható spin lánc

1+1 dimenziós integrálható elmélet

multirészecskeállapotok
végesméret-spektruma

$$E(J) = 2E(p) + \delta E(p)$$



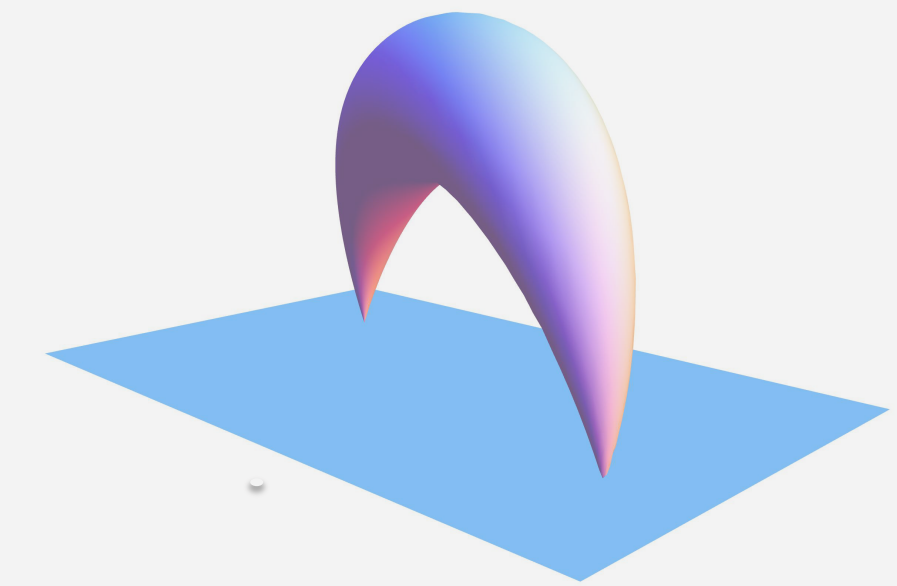
Bethe ansatz $e^{ipJ} S(p, -p) = 1$

vákuum polarizáció

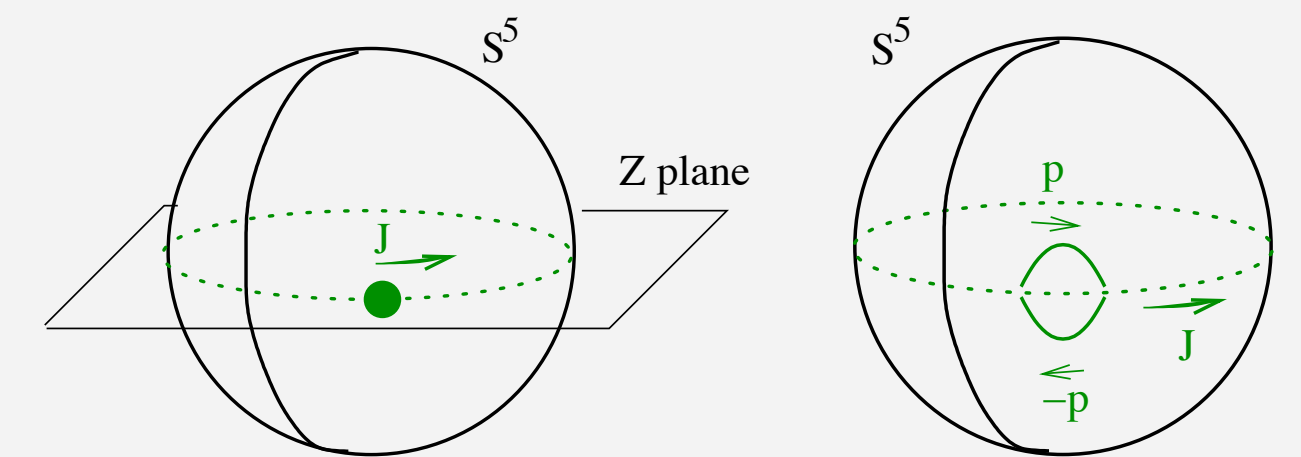
[Bajnok, Janik]

$$\delta E(p) = \int dq S(q, p) S(q, -p) e^{-\epsilon(q)}$$

9+1 dimenziós húrelmélet



Forgó hurok energiája



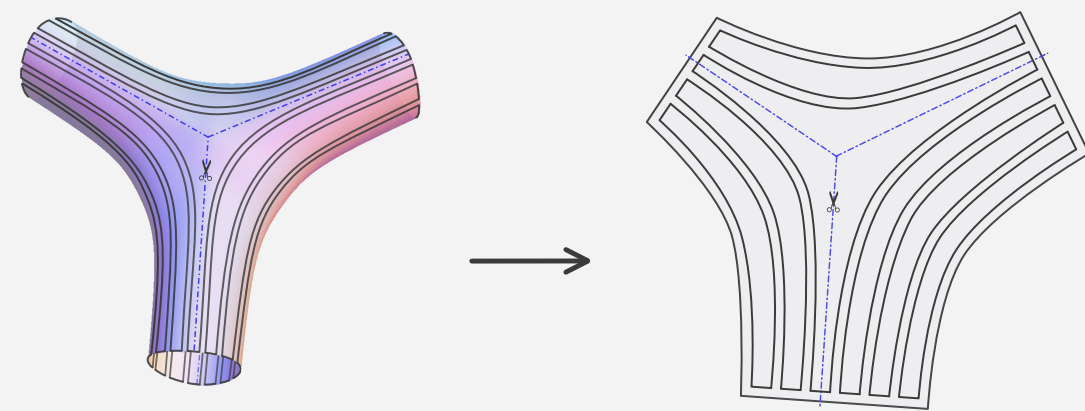
Kölcsönhatás

3+1 dimenziós mértékelmélet

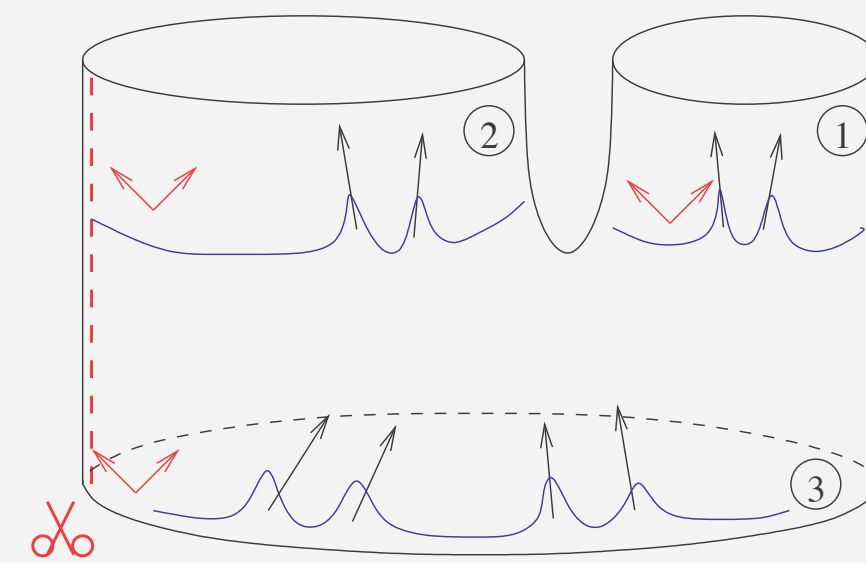
hárompontfüggvény

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle = C_{ijk}(\lambda)$$

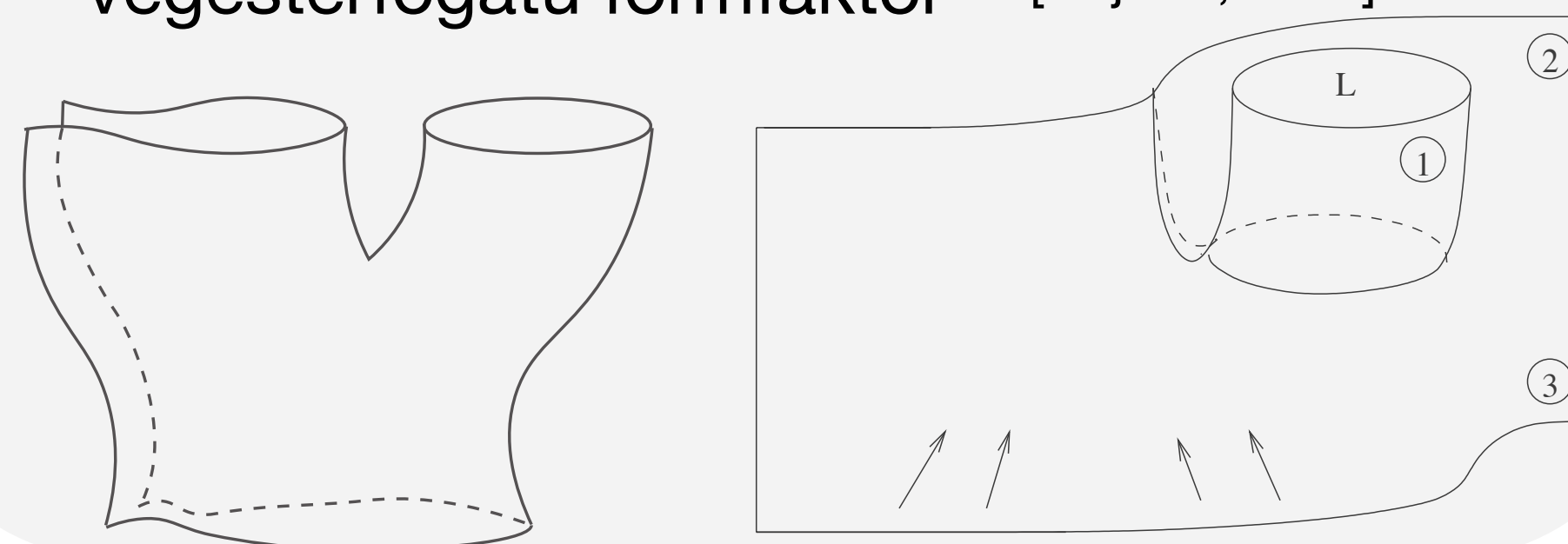
Spinláncátfedések



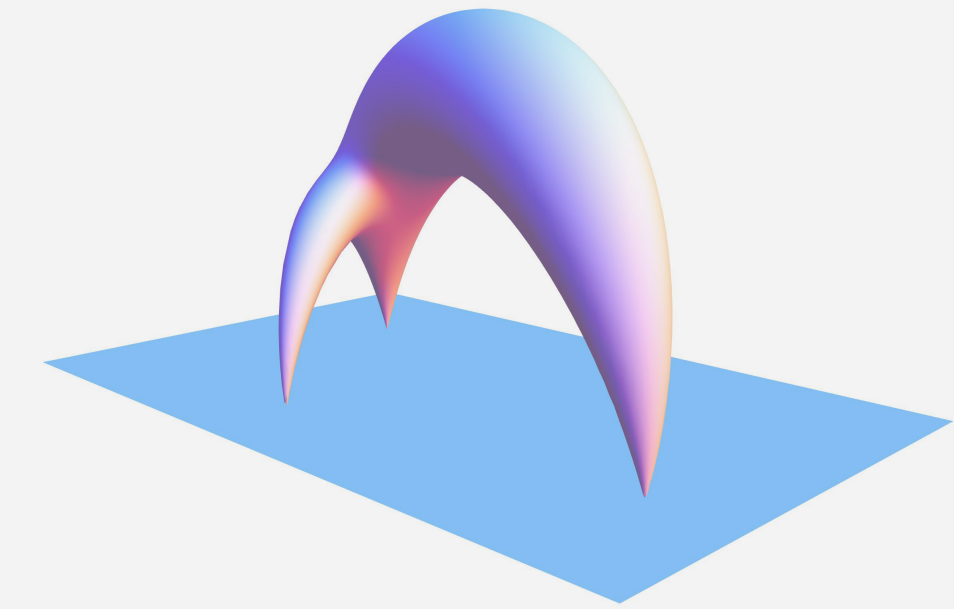
1+1 dimenziós integrálható elmélet



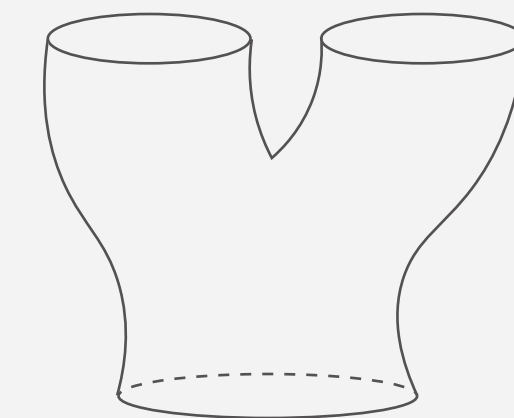
végestérfogatú formfaktor [Bajnok, Janik]



9+1 dimenziós húrelmélet

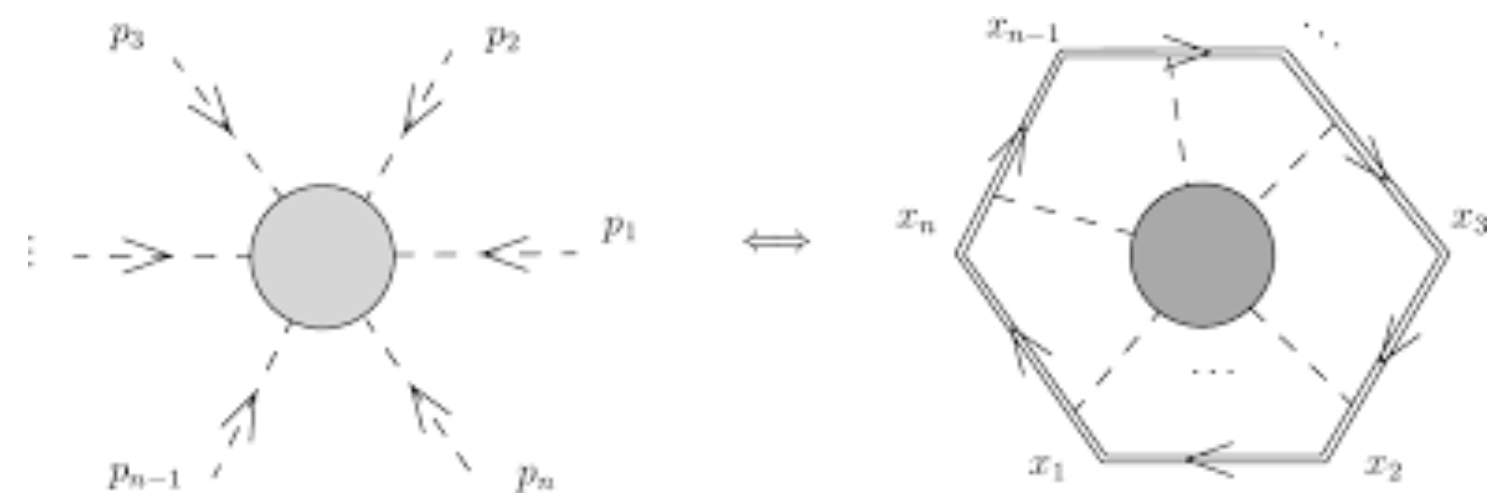


húrok kölcsönhatási vertexe

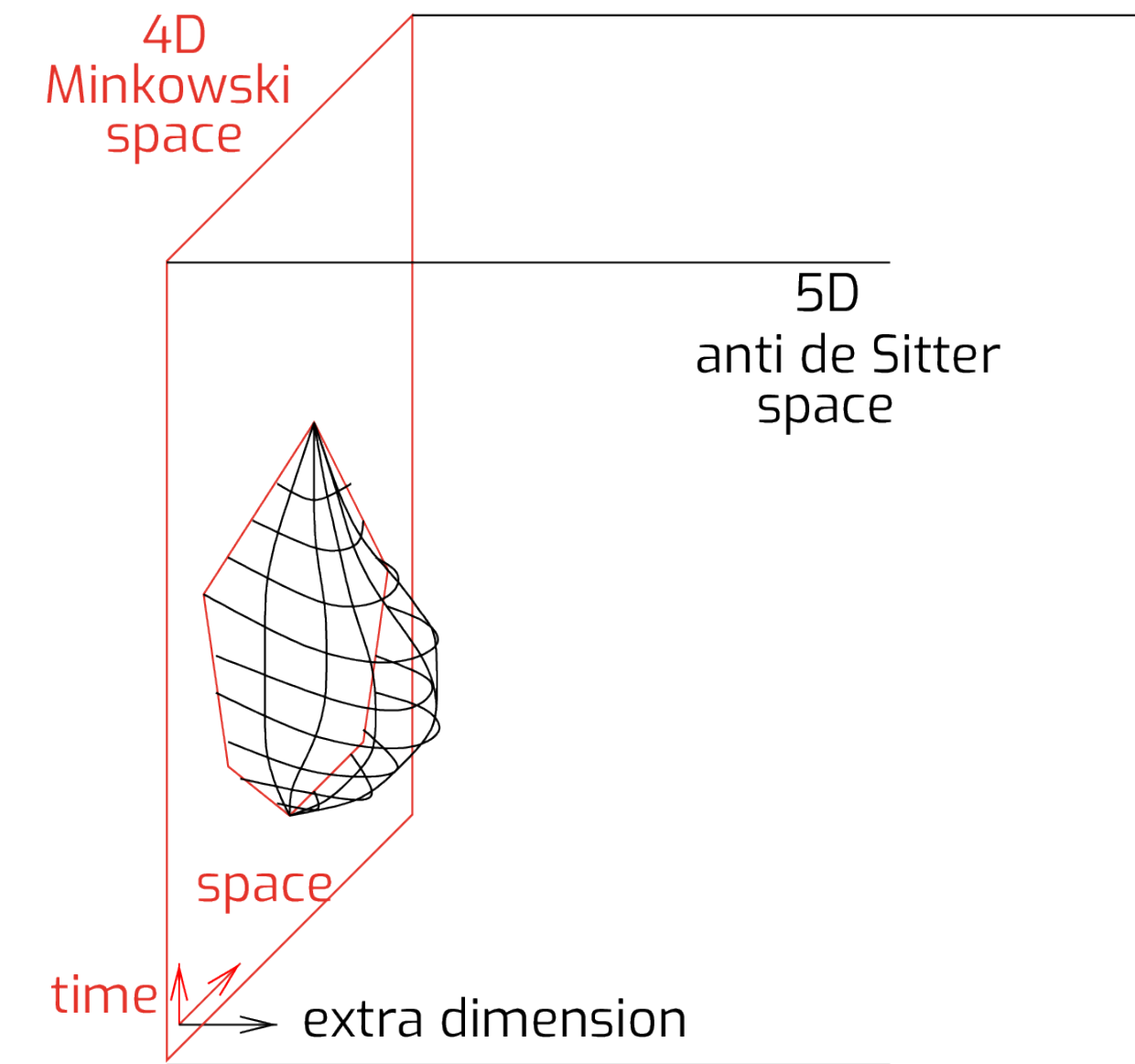
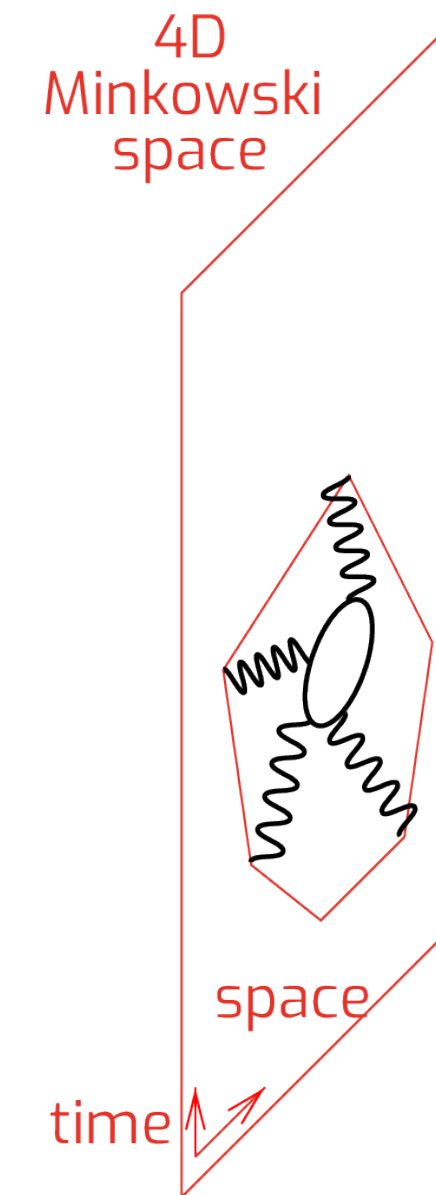


Érdekes jelenségek

Gluonszórési amplitudó = fényszerű Wilson hurok



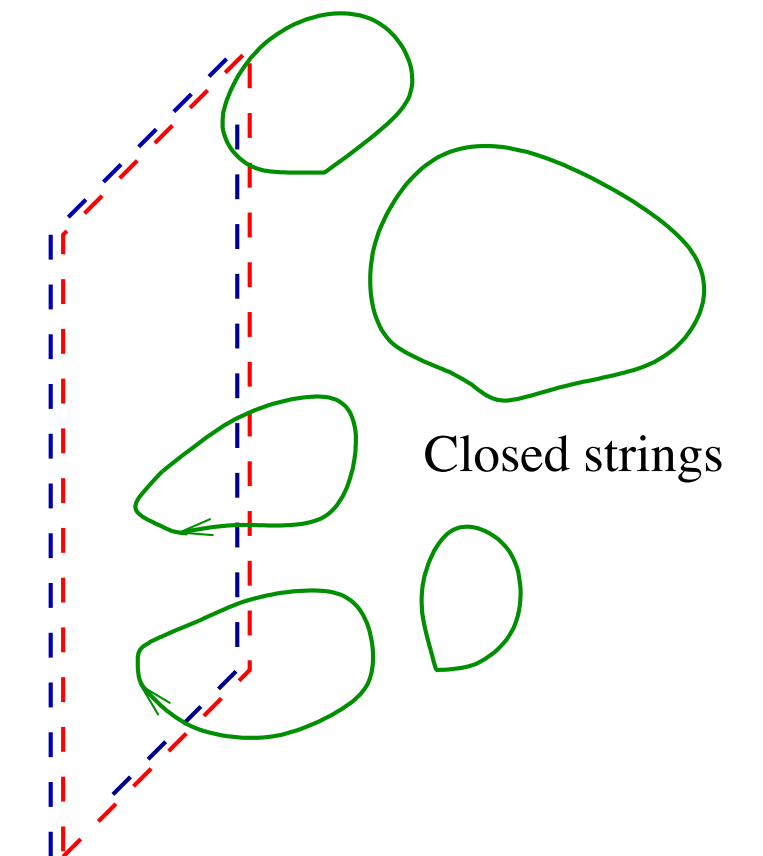
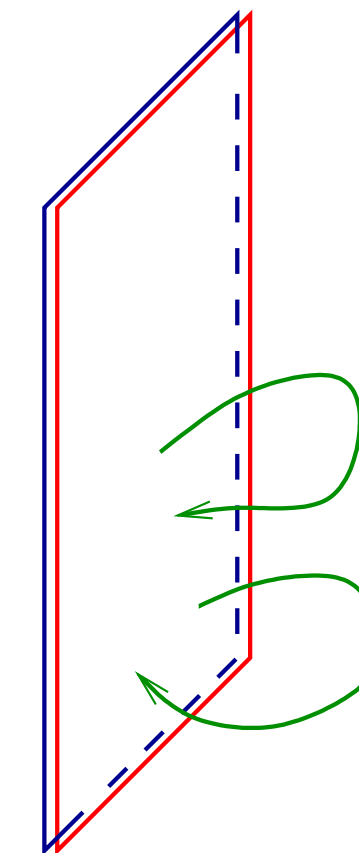
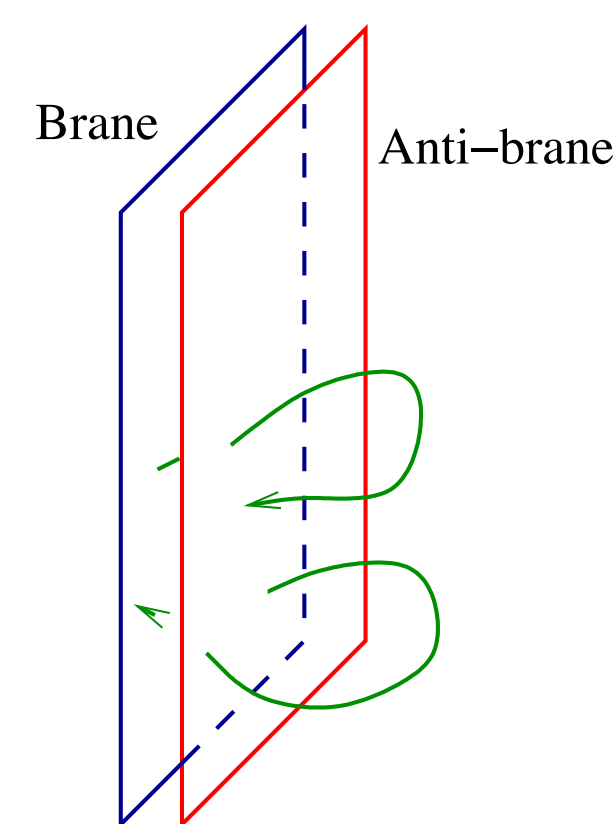
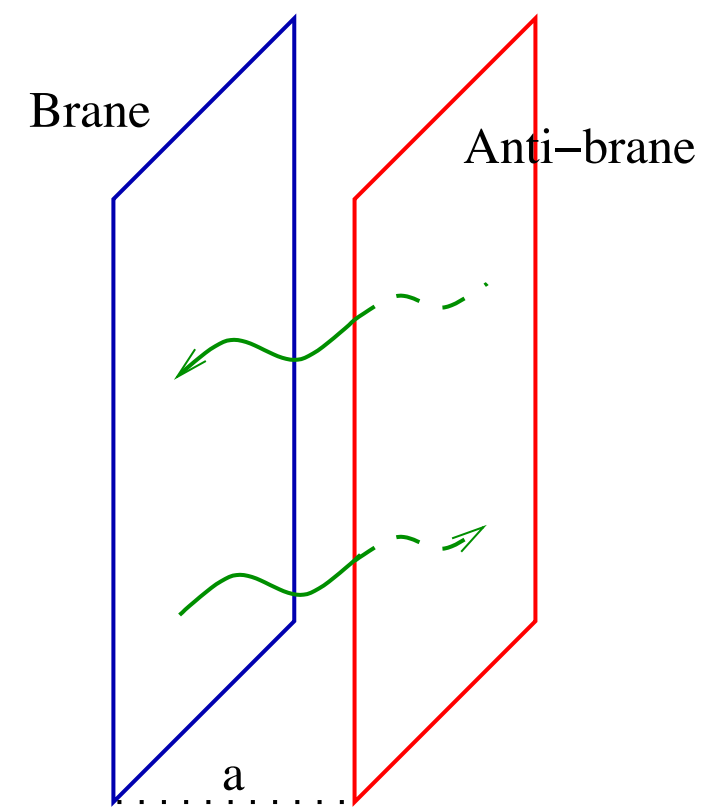
[Bajnok,Balog,Ito,Satoh,Toth]



Tachion kondenzáció

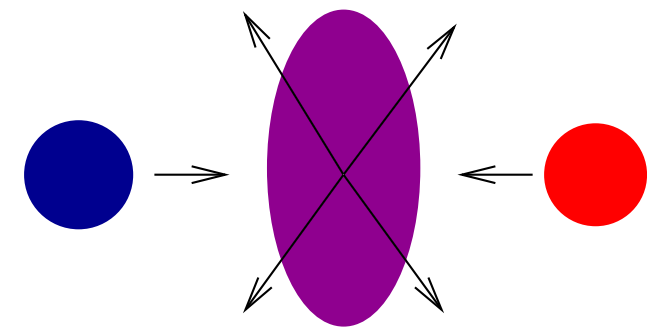
$$\mathcal{O} = \text{Det}(Z \dots ZXZ \dots ZX \dots Z)$$

[Bajnok,Drukker et al.]

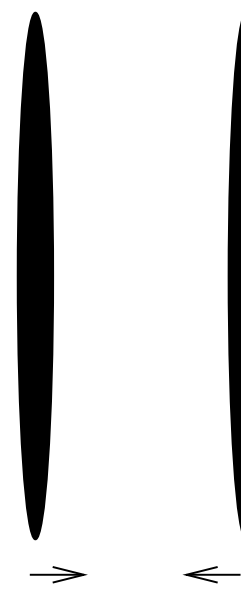


Kvark-gluon plazma

nehézion ütközés



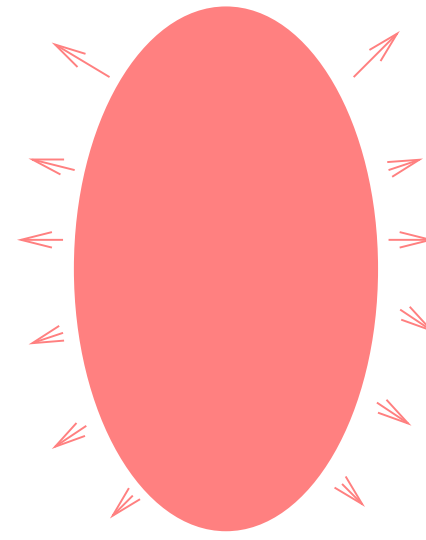
heavy ion collision



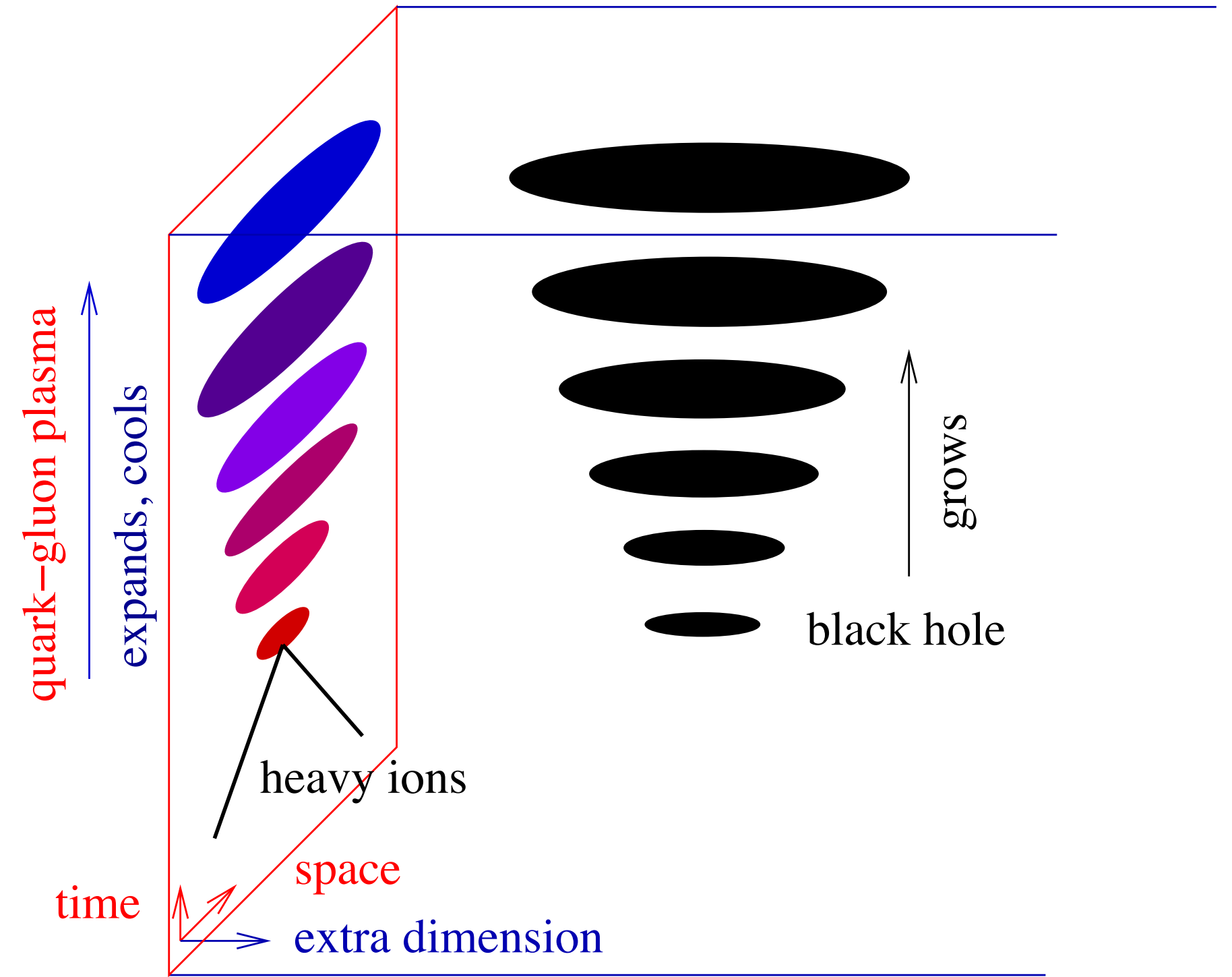
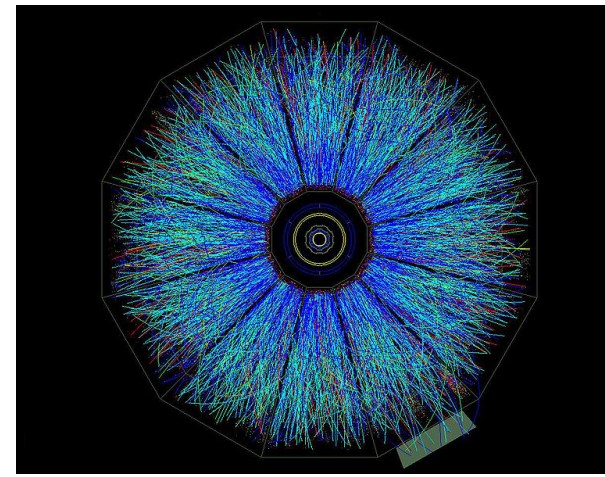
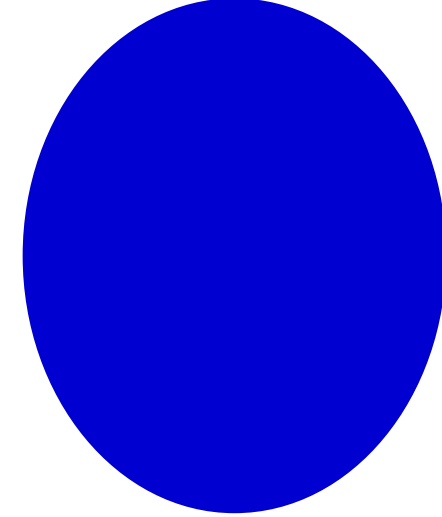
thermalization isotropization



Expansion Cooling



Hadronization



Mértékelmélet véges hőmérsékleten

Relativisztikus hidrodinamika

tökéletes folyadék $\frac{\eta}{s} = \frac{1}{4\pi}$

Húrelmélet alacsonyenergián = gravitáció

Einstein egyenletek

növekvő feketelyuk

[Janik et al]

Kvalitatíven hasonló jelenség a gyorsítókban az igazi kvark-gluon plazmában

Az egzakt megoldások mégsem olyan haszontalanok!