ENTANGLEMENT AND THE INFRARED

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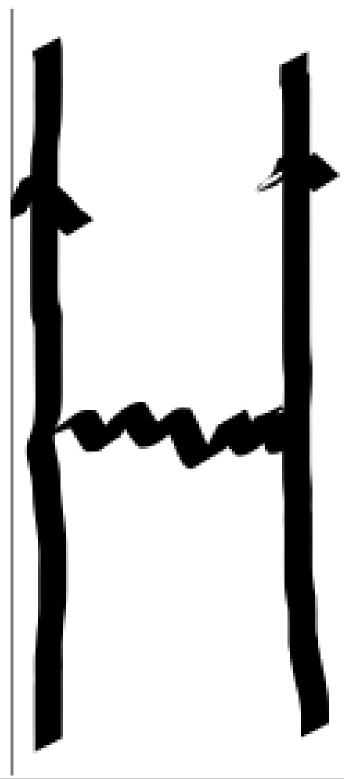
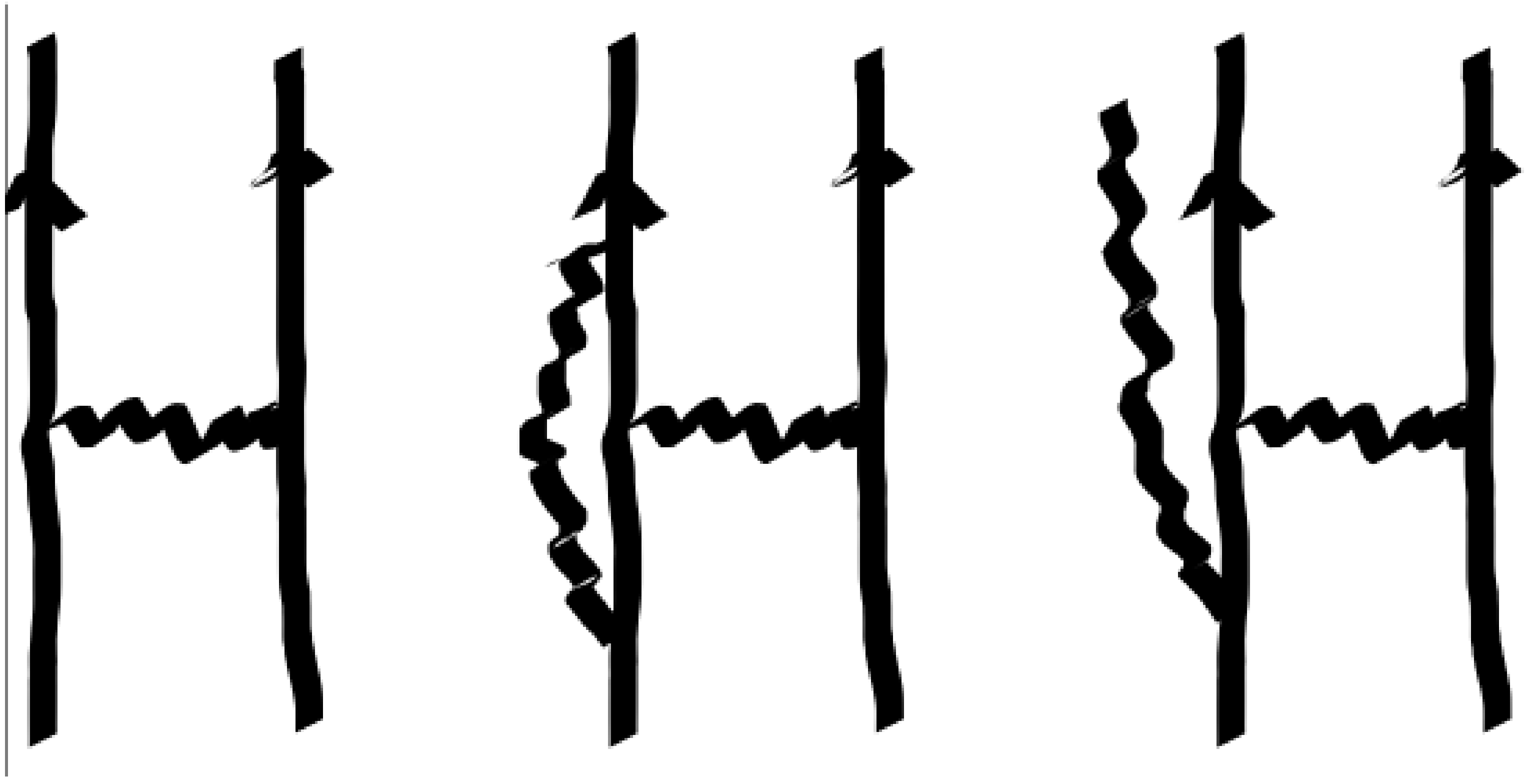
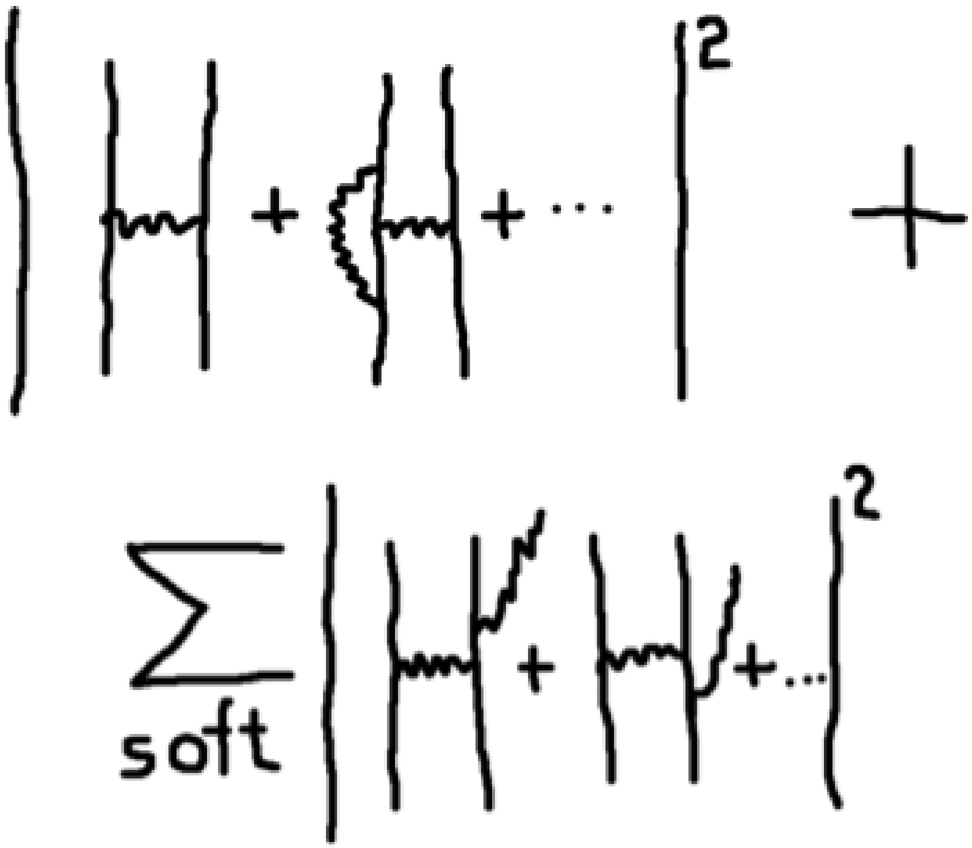
Quantum electrodynamics is, for all practical purposes, exactly solvable by perturbation theory.
Renormalized perturbation theory is an asymptotic expansions in the fine structure constant $\alpha \sim 1/137$ which converges rapidly.
The amplitude for Moeller scattering, to one percent accuracy, is given by



However, there is a subtlety due to infrared divergences:



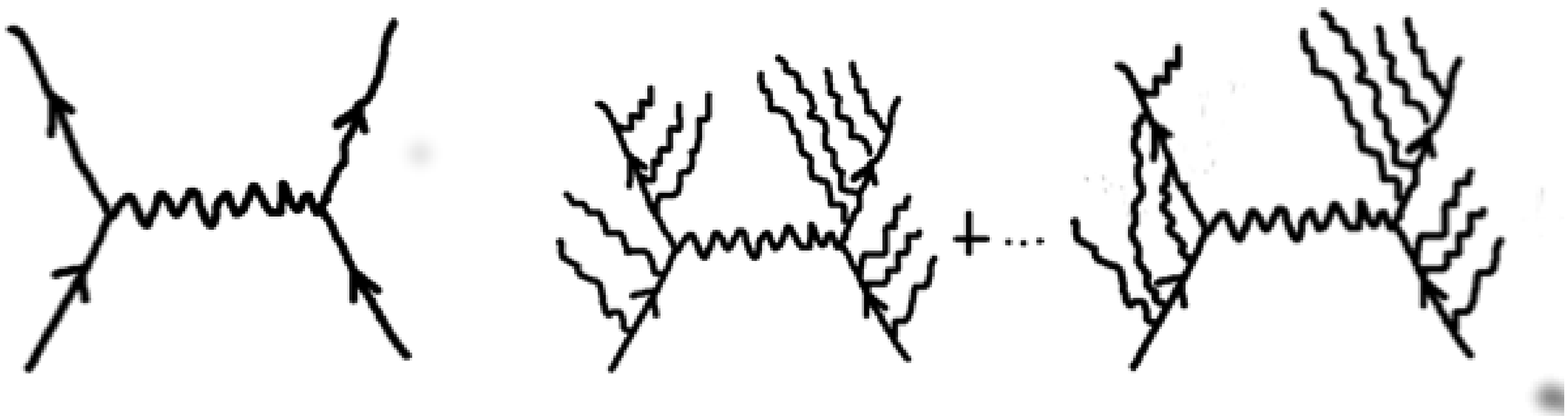
Inclusive probability



= finite

Cancellation of infrared divergences is guaranteed by unitarity of infrared cutoff S-matrix.

Any scattering of charged particles is accompanied by the emission of an infinite number of soft photons

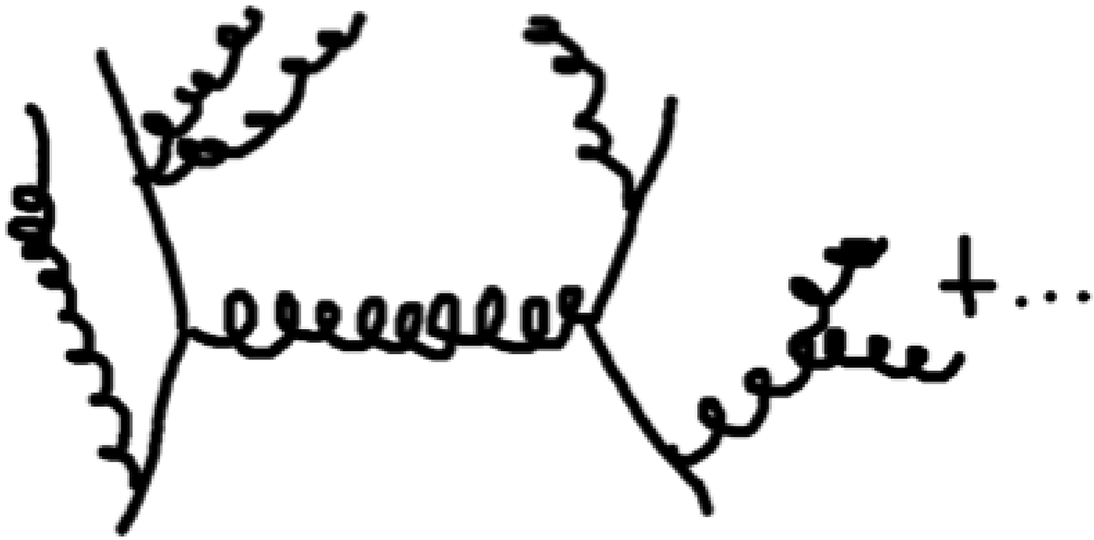


F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937)

D. R. Yennie, S. C. Frautschi, H. Suura, Ann. Phys. 13, 379 (1961)

soft photon theorems

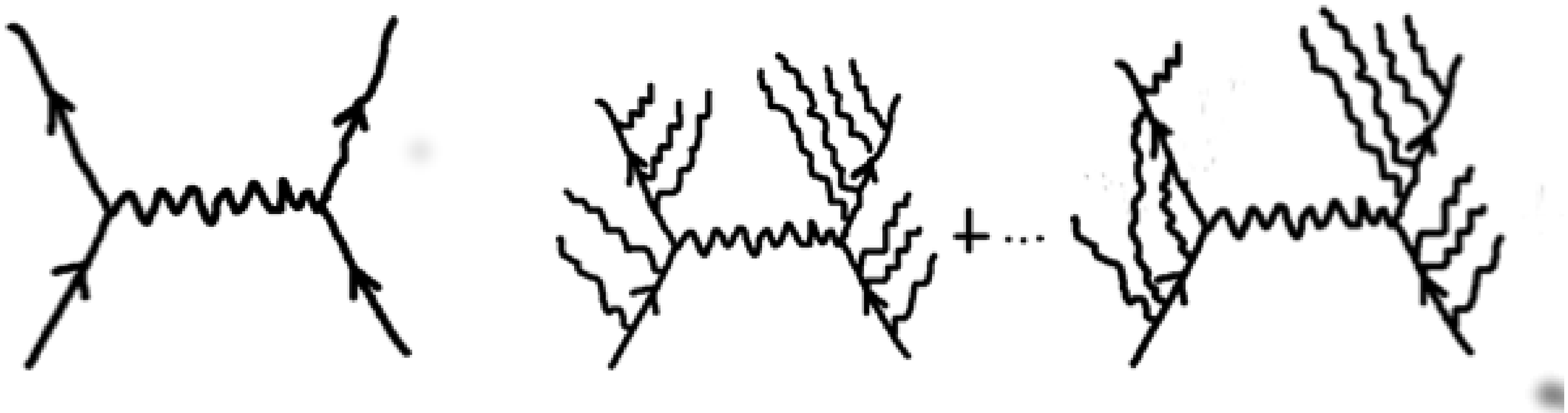
The infrared problem in perturbative quantum gravity mirrors that in quantum electrodynamics, with the additional fact that all particles, including the gravitons themselves carry gravitational charge.



S. Weinberg, Phys. Rev. 140, B516 (1965)

soft graviton theorem

The soft photons which escape detection have polarizations and directions of propagation.



How much information do they carry away?

G.Grignani,GWS, Phys. Lett. B 772 (2017) 699.
D.Carney,L.Chaurette,D.Neuenfeld, GWS,
Phys.Rev.Lett.119(2017)no.18,180502
Phys.Rev. D97 (2018) no.2, 025007
arXiv:1803.02370

Information loss due to entanglement:

Composite system of two qubits: $|\ >_1 \otimes |\ >_2$

If subsystem $|\ >_2$ becomes inaccessible, how much information about $|\ >_1$ do we lose?

Unentangled state: $|\psi >= |\uparrow>_1 \otimes |\downarrow>_2$

Entangled state: $|\psi >= \frac{1}{\sqrt{2}} |\uparrow>_1 \otimes |\downarrow>_2 + \frac{1}{\sqrt{2}} |\downarrow>_1 \otimes |\uparrow>_2$

Reduced density matrix: $\rho = \mathrm{Tr}_2 |\psi >< \psi|$

Unentangled state: $\rightarrow \rho = |\uparrow>_1 <\uparrow|$

Entangled state: $\rightarrow \rho = \frac{1}{2} |\uparrow>_1 <\uparrow| + \frac{1}{2} |\downarrow>_1 <\downarrow| = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

Entanglement entropy: $S = -\mathrm{Tr}_2 \rho \ln \rho$

Unentangled state $S = 0;$

Entangled state $S = 2 \ln 2$

Scattering: in-states evolve to a superposition of in-states, with coefficients the S-matrix elements

$$|\alpha> \;\; \rightarrow \;\; \sum_{\beta,\gamma} S^{\dagger}_{\alpha,\beta\gamma} \; |\beta\gamma>$$

where γ are soft photons.

$$|\alpha><\alpha| \; \rightarrow \; \sum_{\beta\gamma} S^{\dagger}_{\alpha,\beta\gamma} \; |\beta\gamma> \sum_{\tilde{\beta}\tilde{\gamma}} <\tilde{\beta},\tilde{\gamma}| \; S_{\tilde{\beta}\tilde{\gamma},\alpha}$$

The S-matrix is infrared divergent.
Infrared divergences cancel from inclusive transition probabilities, i.e. from the **diagonal** elements of the reduced density matrix

$$\rho = \sum_{\hat{\gamma}} <\hat{\gamma}| \left[\sum_{\beta\gamma} S^{\dagger}_{\alpha,\beta\gamma} \; |\beta\gamma> \sum_{\tilde{\beta}\tilde{\gamma}} <\tilde{\beta},\tilde{\gamma}| \; S_{\tilde{\beta}\tilde{\gamma},\alpha} \right] |\hat{\gamma}>$$

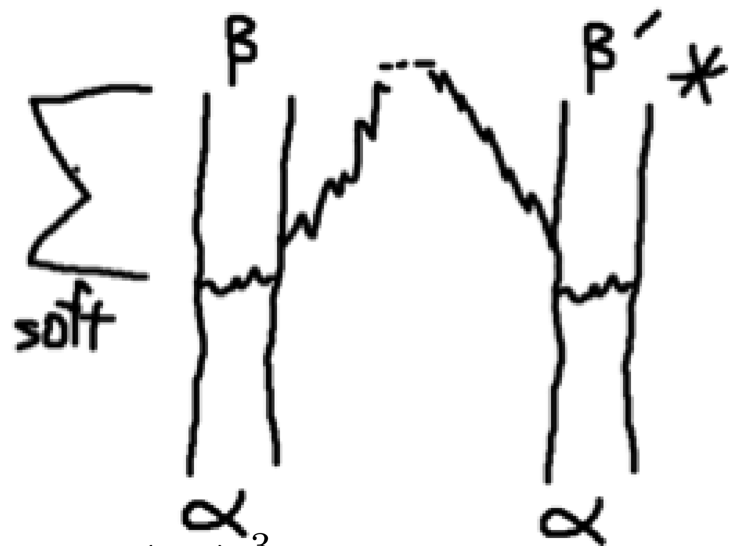
What about off-diagonal matrix elements of ρ?

Entanglement entropy:

$$S = -\mathrm{Tr}\rho \ln \rho = -\sum_i \rho_i \ln \rho_i$$

Density matrix = pure state + trace...

$\rho = \left[\ \ S^\dagger |\alpha >< \alpha| S \ \ \right]_{\beta\beta'} +$



Eigenvalues 0 and 1 perturbed by $\left(\frac{e^2}{4\pi}\right)^3 \ln \frac{\Lambda}{m_{\mathrm{ph}}}$
The density matrix eigenvalues, ρ_i, are logarithmically infrared divergent at order e^6.

Soft photon theorem applied to the density matrix:

Cutoffs:

$m_{\rm ph}$ photon mass as fundamental infrared cutoff

Λ_1 =infrared cutoff in internal loops containing photon lines;
Λ_2 =detector resolution

E_T=total energy of soft photons

We need the hierarchy

$$\alpha\beta\tilde{\beta} >> \Lambda_1, \Lambda_2, E_T >> m_{\rm ph}$$

The S-matrix is infrared divergent and it must be defined with a fundamental cutoff $m_{\text{ph.}}$ on the momenta of certain loops in Feynman diagrams.

We can use soft photon theorem to show

$$\sum_{\gamma} \Theta(E_T - \sum E_i) \prod_i \Theta(\Lambda_2 - |k_i|) S^{m_{\text{ph.}}\dagger}_{\beta\gamma,\alpha} S^{m_{\text{ph.}}}_{\alpha,\tilde{\beta}\gamma}$$

$$= S^{m_{\text{ph.}}\dagger}_{\beta,\alpha} S^{m_{\text{ph.}}}_{\alpha,\tilde{\beta}} \left(\frac{\Lambda_2}{m_{\text{ph}}}\right)^{\tilde{A}_{\alpha\beta,\alpha\tilde{\beta}}} F(E_T)$$

$$F(\infty) = 1$$

where

$$A_{X,Y} = -\sum_{n\in X, m'\in Y} \frac{e_n e_{n'} \eta_n \eta'_n}{8\pi \beta_{nn'}} \ln\left[\frac{1+\beta_{nn'}}{1-\beta_{nn'}}\right]$$

$$\beta_{nn'} = \text{relative relativistic velocity}$$

The S-matrix is infrared divergent and it must be defined with a fundamental cutoff $m_{\mathrm{ph.}}$ on the momenta of certain loops in Feynman diagrams.

We can use soft photon theorem to show

$$S^{m_{\mathrm{ph.}}}_{\alpha,\tilde{\beta}} = S^{\Lambda_1}_{\alpha,\tilde{\beta}} \left(\frac{m_{\mathrm{ph}}}{\Lambda_1}\right)^{\frac{1}{2}A_{\alpha\tilde{\beta},\alpha\tilde{\beta}}}$$

where

$$A_{X,Y} = -\sum_{n\in X, m'\in Y} \frac{e_n e_{n'} \eta_n \eta'_n}{8\pi\beta_{nn'}} \ln\left[\frac{1+\beta_{nn'}}{1-\beta_{nn'}}\right]$$

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Soft photon theorem applied to the density matrix:

$m_{\rm ph}$ photon mass as fundamental infrared cutoff

Λ_1 =infrared cutoff in internal loops; Λ_2 =detector resolution

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$\alpha\beta\tilde{\beta} >> \Lambda_1, \Lambda_2, E_T >> m_{\rm ph}$

We can use soft photon theorem to show

$$\rho_{\beta\tilde{\beta}} = S^{\dagger}_{\beta\alpha} S_{\alpha\tilde{\beta}} \left(\frac{m_{\rm ph}}{\Lambda_1}\right)^{\frac{1}{2}A_{\alpha\beta,\alpha\beta}} \left(\frac{m_{\rm ph}}{\Lambda_1}\right)^{\frac{1}{2}A_{\alpha\tilde{\beta},\alpha\tilde{\beta}}} \left(\frac{\Lambda_2}{m_{\rm ph}}\right)^{\tilde{A}_{\alpha\beta,\alpha\tilde{\beta}}} F(E_T)$$

$$\sim m_{\rm ph}^{\Delta A}\ , \ \Delta A = \tfrac{1}{2}A_{\alpha\beta,\alpha\beta} + \tfrac{1}{2}A_{\alpha\tilde{\beta},\alpha\tilde{\beta}} - A_{\alpha\beta,\alpha\tilde{\beta}} \geq 0$$

$$A_{X,Y} = - \sum_{n\in X, m'\in Y} \frac{e_n e_{n'} \eta_n \eta'_n}{8\pi\beta_{nn'}} \ln\left[\frac{1+\beta_{nn'}}{1-\beta_{nn'}}\right]$$

$\beta_{nn'}$ =relative relativistic velocity

- A generic density matrix element is proportional $\sim m_{\mathrm{ph}}^{\Delta A}$, where $\Delta A \geq 0$ and depends on incoming and outgoing four-momenta.
- $\Delta A = 0$ for diagonal elements of the density matrix (transition probabilities)
- Generically, $\Delta A > 0$ for off-diagonal elements
- The inequality is saturated, $\Delta A = 0$, and density matrix element nonzero only when the set of outgoing currents match:

$$\beta = \left\{ \frac{e_1 p_1^\mu}{2\omega(p_1)}, ..., \frac{e_n p_n^\mu}{2\omega(p_n)} \right\}$$

equals

$$\tilde{\beta} = \left\{ \frac{\tilde{e}_1 \tilde{p}_1^\mu}{2\omega(\tilde{p}_1)}, ..., \frac{\tilde{e}_{\tilde{n}} \tilde{p}_{\tilde{n}}^\mu}{2\omega(\tilde{p}_{\tilde{n}})} \right\}$$

- **decoherence** momentum eigenstates are pointer basis

Example: Compton scattering

$\rho_{k',q';\tilde{k}',\tilde{q}'} = m_{\rm ph}^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]}$, β =relative electron velocity

Exponent ≥ 0. Exponent $= 0$ only when $\beta = 0$.

As $m_{\rm ph} \to 0$, $\rho_{k',q';\tilde{k}',\tilde{q}'} = 0$ **unless** $k'_\mu = \tilde{k}'_\mu$.

Implication: *Diagonal elements* of the density matrix are the transition probabilities for QED processes.

$\rho_{k',q';k',q'}$ **= Probability of** $|k,q> \rightarrow |k'q'>$

Off-diagonal elements vanish $\rho_{k',q';\tilde{k}',\tilde{q}'} = 0, \; k \neq \tilde{k}'$

Probability $|k,q> \rightarrow \frac{1}{\sqrt{2}}|k'_1,q'_1> + \frac{1}{\sqrt{2}}|k'_2,q'_2>$

equals

$\frac{1}{2}\cdot$**Probability** $|k,q> \rightarrow |k'_1,q'_1>$

$+$

$\frac{1}{2}\cdot$**Probability** $|k,q> \rightarrow |k'_2,q'_2>$

Infrared safe "dressed states"

For each charged particle, add a coherent state of soft photons:

$$|p> \to |p>_D \equiv W(p)|p>$$

$$W(p) = \exp\left\{ \sum_{\ell} \int_0^{\Lambda} \frac{d^3k}{2\sqrt{\vec{k}^2 + m_{\rm ph}^2}} \left[\frac{p\cdot\epsilon_\ell(k)}{p\cdot k} a_\ell^\dagger(k) - \frac{p\cdot\epsilon_\ell^*(k)}{p\cdot k} a_\ell(k) \right] \right\}$$

$$m_{\rm ph} << \Lambda << p \quad k\cdot\epsilon_\ell(k) = 0$$

$\tilde{S}_{\alpha\beta} \equiv_D < \alpha|S|\beta >_D$ is infrared finite. Out-state can be a pure state

$$|\alpha >_D < \alpha| \;\to\; \tilde{\rho} = \sum_{\beta} \tilde{S}^\dagger_{\alpha,\beta} \; |\beta >_D \sum_{\tilde{\beta}} \; {}_D < \tilde{\beta}| \; \tilde{S}_{\tilde{\beta},\alpha}$$

$$\mathrm{Tr}_{\rm soft\ photons}\tilde{\rho} = \left(\frac{m_{\rm ph}}{\Lambda}\right)^{\Delta A}$$

Conclusions:

- The solution of the infrared problem in quantum electrodynamics (and in perturbative quantum gravity) leads to a fundamental decoherence of final states.
- There are other "infrared safe" approaches. **V.Chung, Phys.Rev.140, B1110 (1965); T.W.B.Kibble, J.Math.Phys.9, 315 (1968); P.P.Kulish, L.D.Faddeev, Theor.Math.Phys.4, 745 (1970); J.Ware, R.Saotome, R.Akhoury, JHEP10, 159 (2013), 1308.6285.** Same decoherence when in-coming state is "infrared safe" coherent state.
- Proper description of incoming wavepackets requires infrared safe incoming states. Decoherence remains.
- Could such a decoherence be observable?

What if the photon has a mass?

$$\rho_{k',q';\tilde{k}',\tilde{q}'} = (m_{\text{ph}})^{\frac{e^2}{4\pi^2}\left[\frac{1}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right]}$$

$m_{\text{ph}} \sim 10^{-32} m_{\text{el}}$

$$\sim e^{-0.1\beta^2} \;\; \beta << 1 \; , \;\; \sim \left(\frac{1-\beta}{2}\right)^{0.1} \;\; \beta \sim 1$$

Gravity is even more weakly coupled.

Black hole information paradox

In a theory of quantum gravity, the collision of two high-energy particles (i.e. gravitons) could produce a black hole which would the evaporate by emitting Hawking radiation.
Pure quantum state of two incoming particles evolves to thermal state of Hawking radiation.

$$|\psi>=\sum_E |E,\tilde{E}> \quad , \quad \rho=\sum_E e^{-\beta_H E}\ |E><E|$$

Strominger's idea: (**A.Strominger, arXiv:1706.07143**): soft gravitons purify the Hawking radiation

$$|\psi>=\sum_E |E,\mathrm{soft}> \ , \ \rho=\mathrm{Tr}_{\mathrm{soft}}|\psi><\psi|=\sum_E e^{-\beta_H E}|E><E|$$

But $|\psi>=\sum_E |E,\mathrm{soft},\tilde{E}>$. Monogamy of entanglement.