Integrable Models and Applications

École Normale Supérieure de Lyon, France, 11 - 15 September 2006

Recent developments in Boundary Quantum Field Theories

Z. Bajnok

Institute for Theoretical Physics, Eötvös University, Budapest

in collaboration with Changrim Ahn, G. Böhm, A. George, R. Nepomechie, L. Palla, Chaiho Rim, L. Samaj, G. Takács, Alyosha Zamolodchikov

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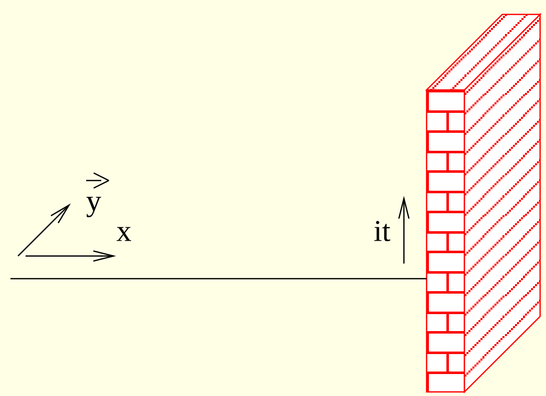
Relativistic Boundary QFT in 1+D (not only integrable)



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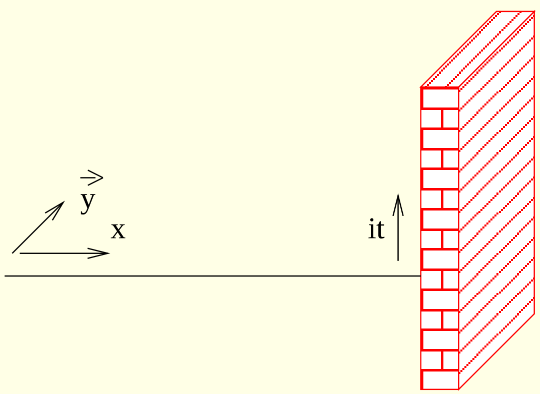
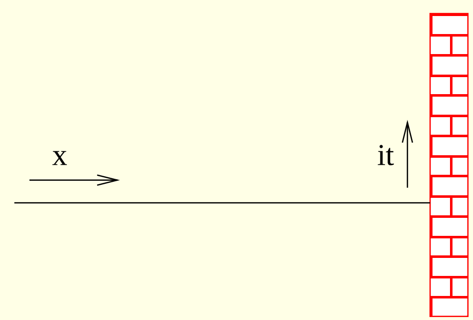
Relativistic Boundary QFT in 1+D (easy presentation) 1+1





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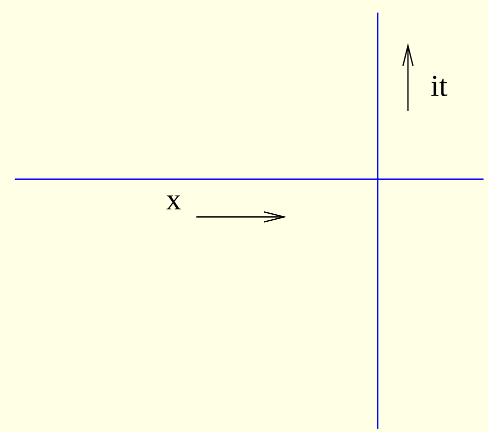
Relativistic Bulk QFT in 1+1



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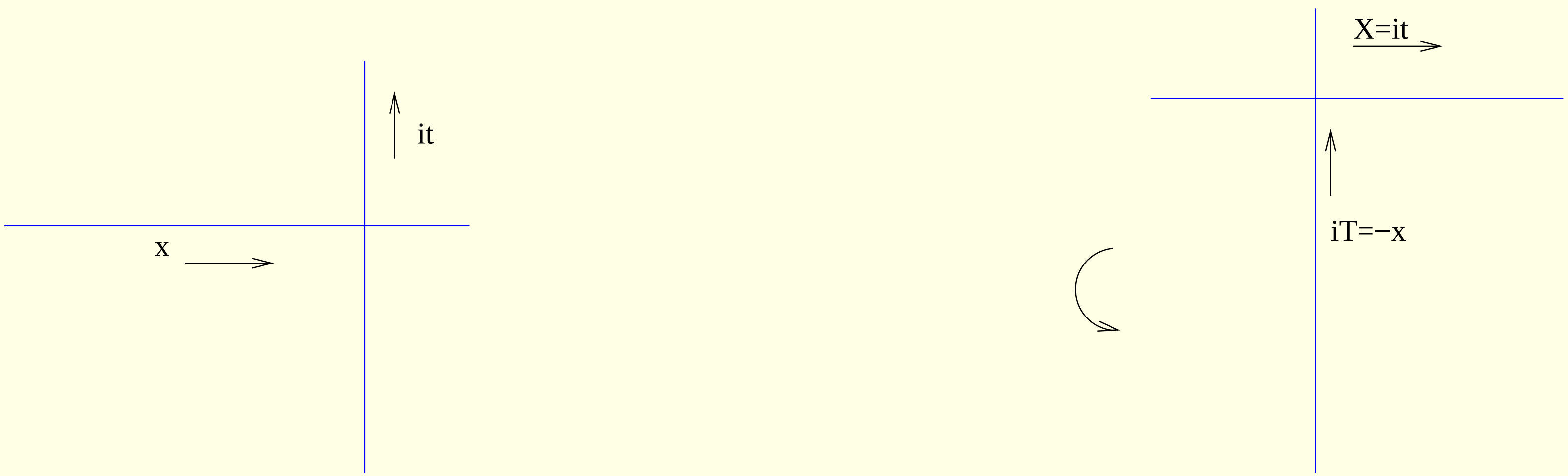
Relativistic Bulk QFT $(x, it) \leftrightarrow (-iT, X)$



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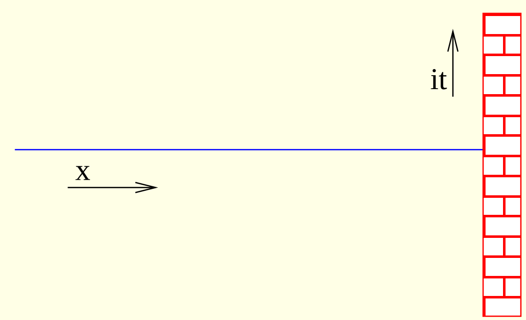
Boundary introduced in space



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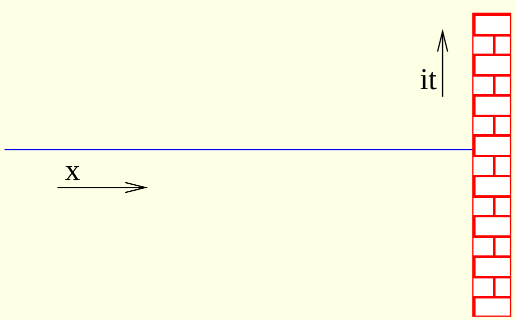
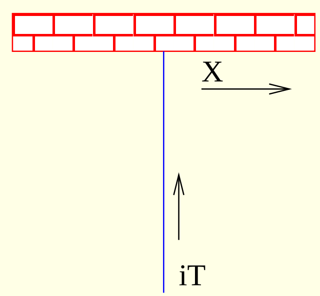
Boundary introduced in space — in time





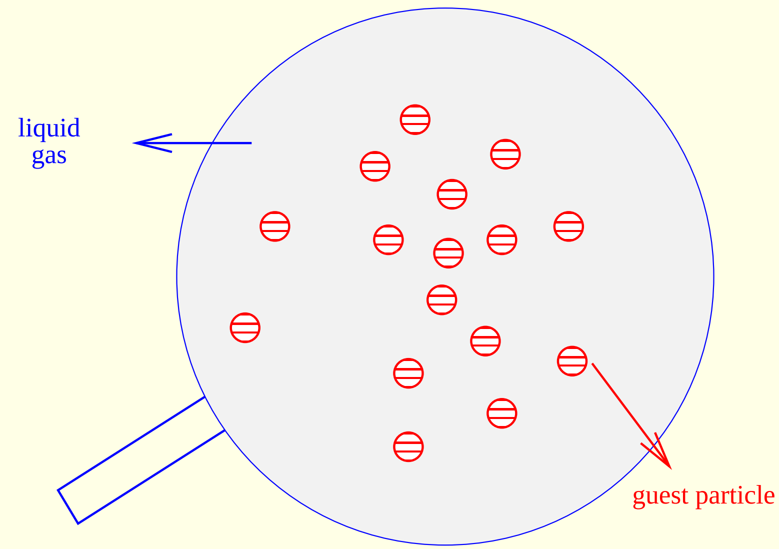
Motivation: Colloidal solution

Motivation: Colloidal solution

Colloidal solution:

charged or neutral particles

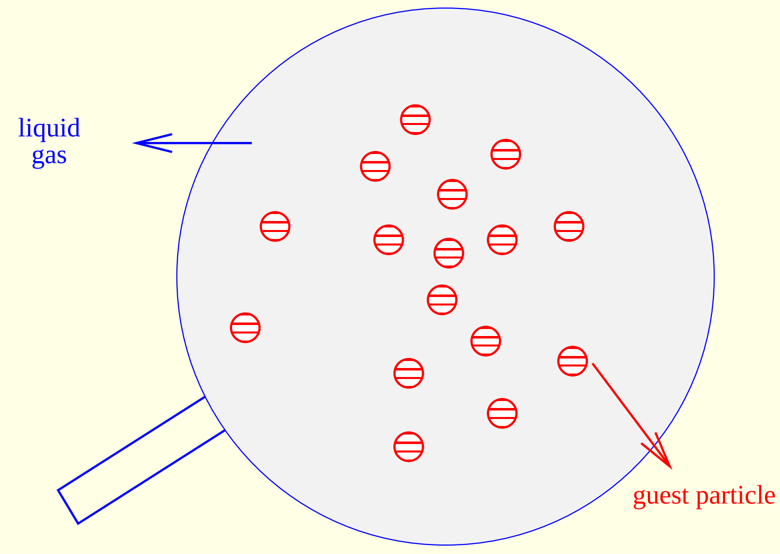
in liquid or in gas



Motivation: Colloidal solution

Colloidal solution:
charged or neutral particles
in liquid or in gas

Problem: Calculate the
effective interaction (Hendrik Casimir 1948)



Motivation: Colloidal solution

Colloidal solution:
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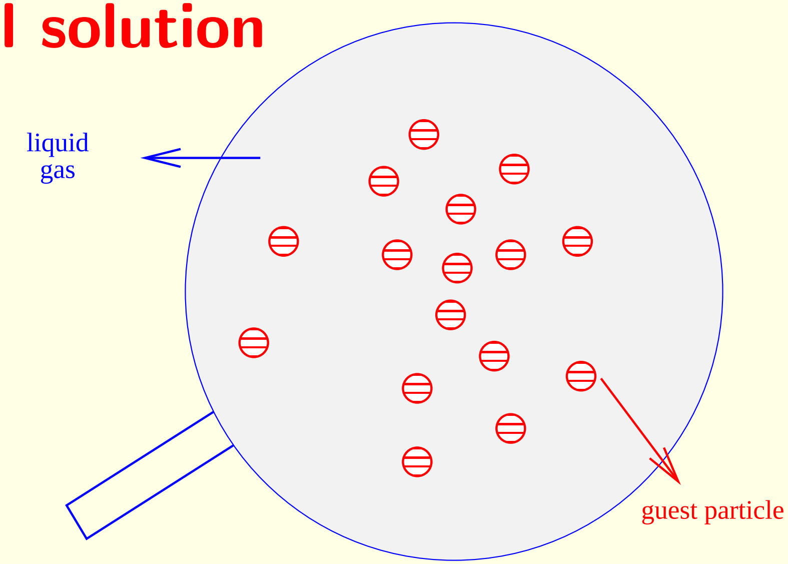
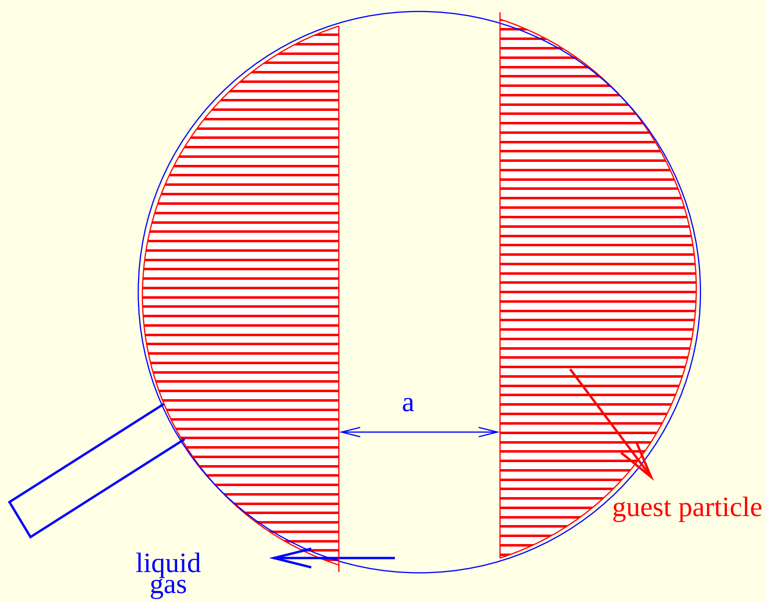


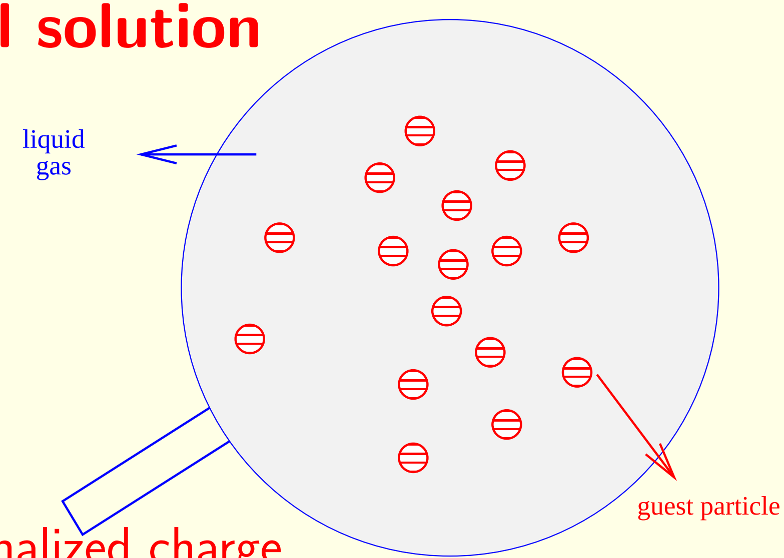
Problem: Calculate the
effective interaction (Hendrik Casimir 1948)



$$F \propto a^{-4}$$

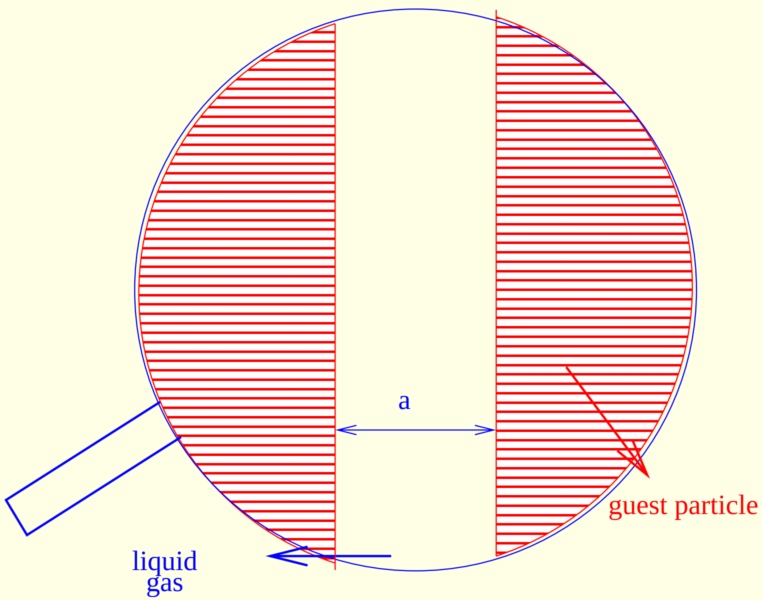
Motivation: Colloidal solution

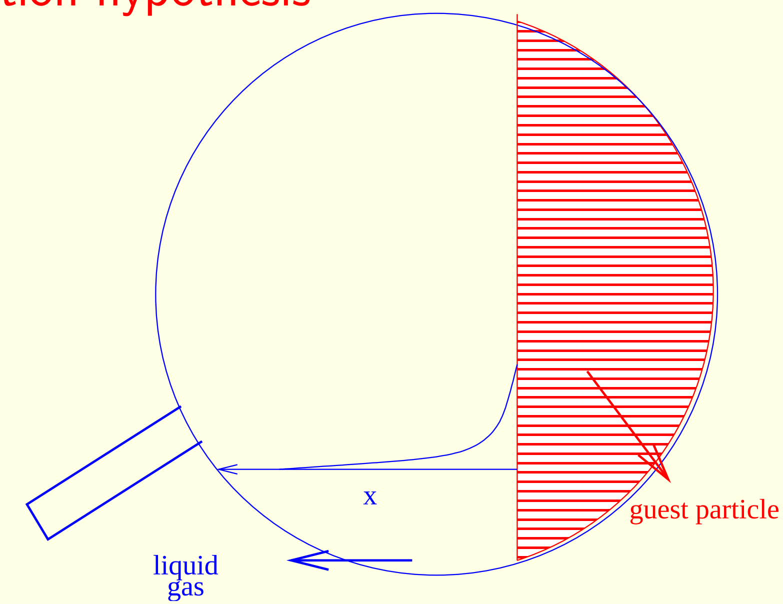
Colloidal solution:
charged or neutral particles
in liquid or in gas



Problem: Calculate the
effective interaction (Hendrik Casimir 1948)

renormalized charge
saturation hypothesis





$$F \propto a^{-4}$$

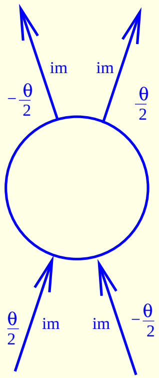
$$\phi \propto \langle \Phi(x) \rangle = Q(\varphi_0) e^{-mx} \quad ; Q(\infty) < \infty$$

Plan

Plan

Scattering

$^{out}\langle\theta_1^{'},\theta_2^{'}|\theta_1,\theta_2\rangle^{in}$



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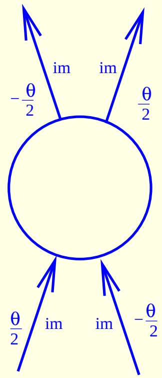
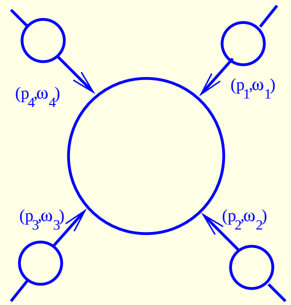
Scattering

$$^{out}\langle\theta_1^{'},\theta_2^{'}|\theta_1,\theta_2\rangle^{in}$$



Green function

$$G^4(p_i, i\omega_i)$$



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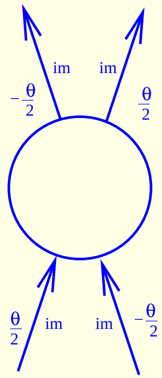
Scattering

$$^{out}\langle\theta_1^{'},\theta_2^{'}|\theta_1,\theta_2\rangle^{in}$$



Green function

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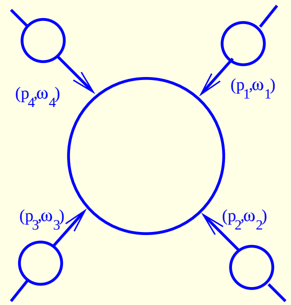
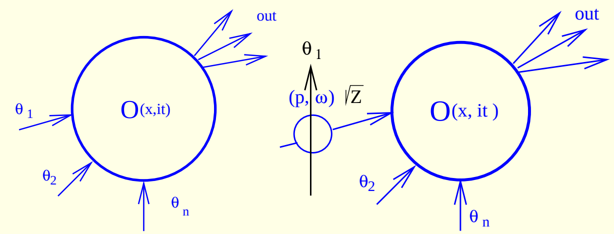


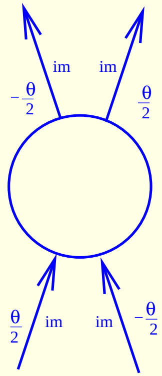
Reduction formula



Plan

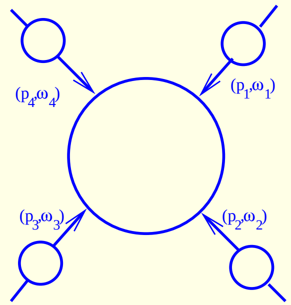
Scattering

$^{out}\langle\theta_1^{'},\theta_2^{'}|\theta_1,\theta_2\rangle^{in}$

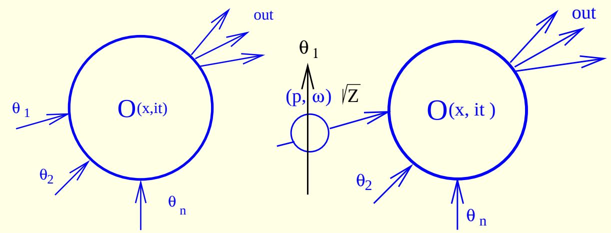


Green function

$G^4(p_i, i\omega_i)$

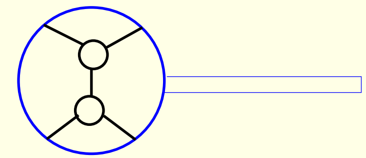


Reduction formula



S bootstrap

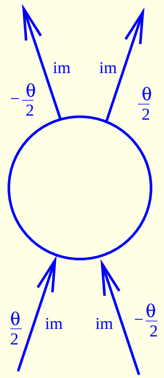
FF bootstrap



Plan

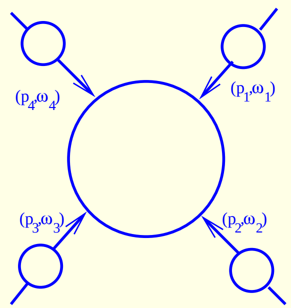
Scattering

$^{out}\langle \theta_1', \theta_2' | \theta_1, \theta_2 \rangle^{in}$

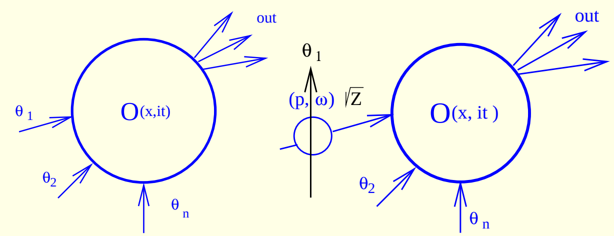


Green function

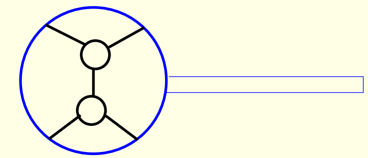
$G^4(p_i, i\omega_i)$



Reduction formula

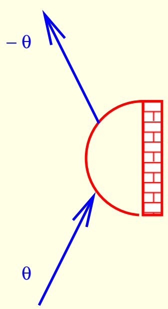


S bootstrap
FF bootstrap



Reflection

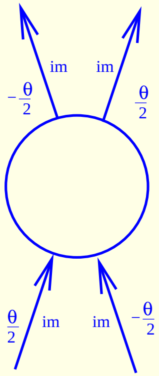
$^{out}\langle \theta' | \theta \rangle^{in}$



Plan

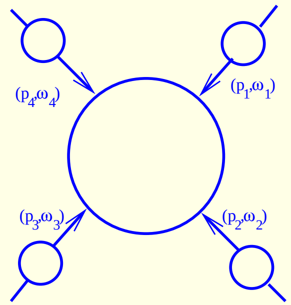
Scattering

$^{out}\langle\theta_1',\theta_2'|\theta_1,\theta_2\rangle^{in}$

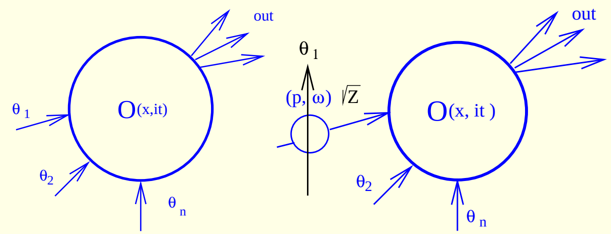


Green function

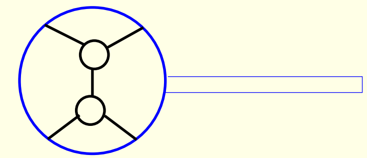
$G^4(p_i,i\omega_i)$



Reduction formula

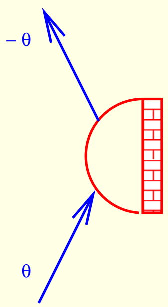


S bootstrap
FF bootstrap



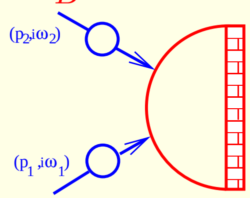
Reflection

$^{out}\langle\theta'|\theta\rangle^{in}$



BGreen function

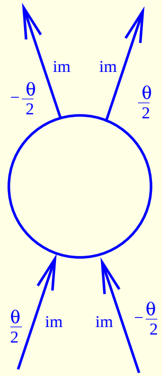
$G^2_B(p_i,i\omega_i)$



Plan

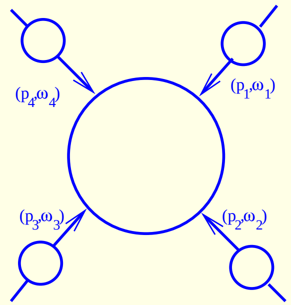
Scattering

$^{out}\langle \theta_1', \theta_2' | \theta_1, \theta_2 \rangle^{in}$

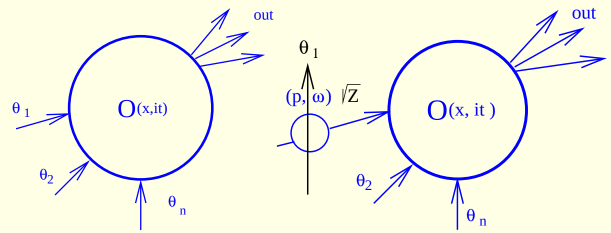


Green function

$G^4(p_i, i\omega_i)$



Reduction formula



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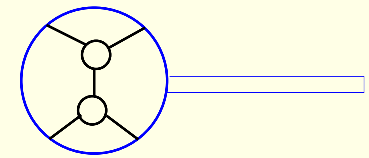
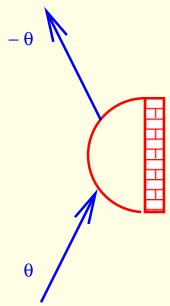


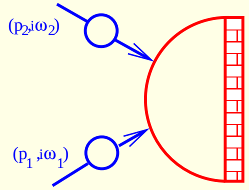
Reflection

$^{out}\langle \theta' | \theta \rangle^{in}$

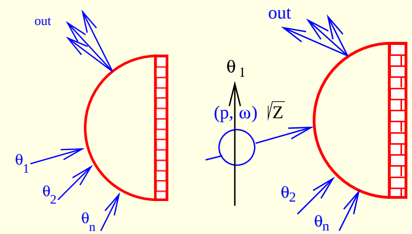


BGreen function

$G_B^2(p_i, i\omega_i)$



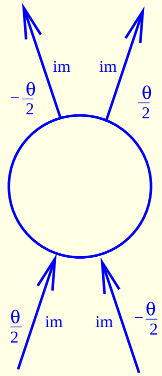
BReduction formula



Plan

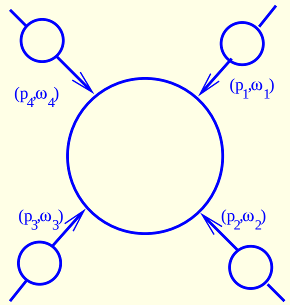
Scattering

$^{out}\langle\theta_1',\theta_2'|\theta_1,\theta_2\rangle^{in}$

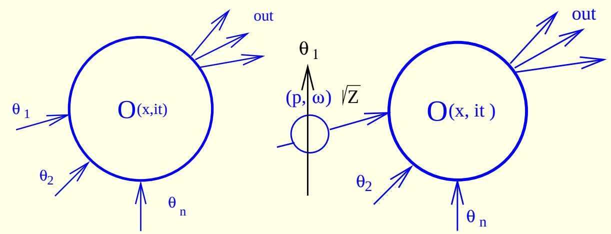


Green function

$G^4(p_i,i\omega_i)$



Reduction formula



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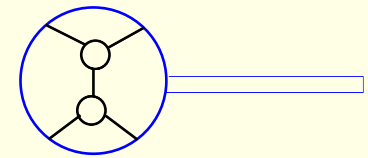
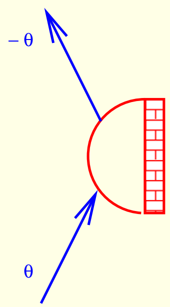
FF bootstrap



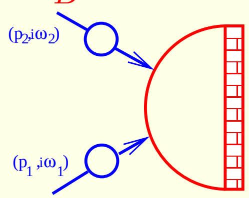
Reflection

$^{out}\langle\theta'|\theta\rangle^{in}$

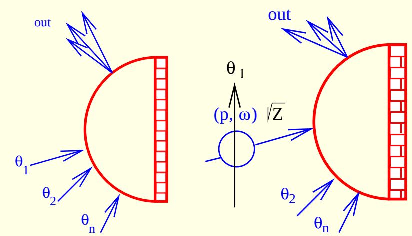


BGreen function

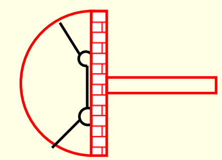
$G_B^2(p_i,i\omega_i)$



BReduction formula



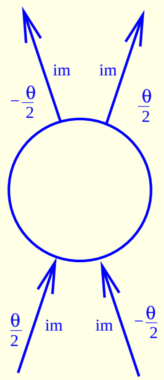
R bootstrap
BFF bootstrap



Plan

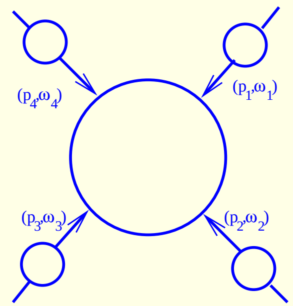
Scattering

$^{out}\langle\theta_1',\theta_2'|\theta_1,\theta_2\rangle^{in}$

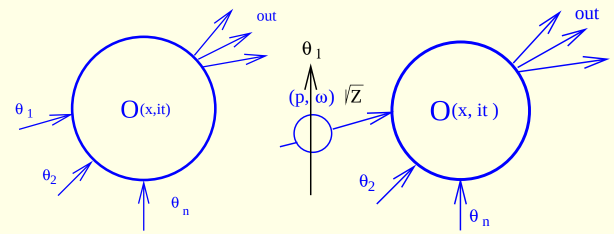


Green function

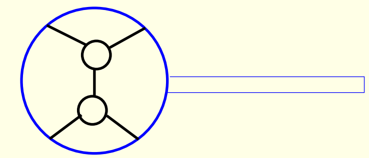
$G^4(p_i, i\omega_i)$



Reduction formula

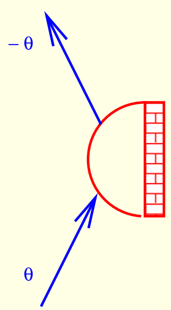


S bootstrap
FF bootstrap



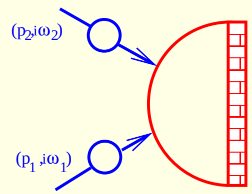
Reflection

$^{out}\langle\theta'|\theta\rangle^{in}$

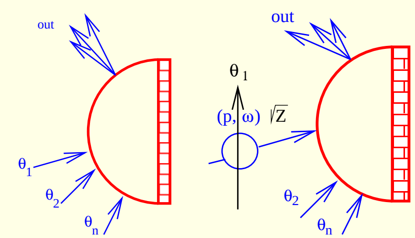


BGreen function

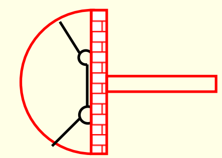
$G^2_B(p_i, i\omega_i)$



BReduction formula

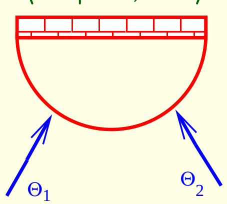


R bootstrap
BFF bootstrap

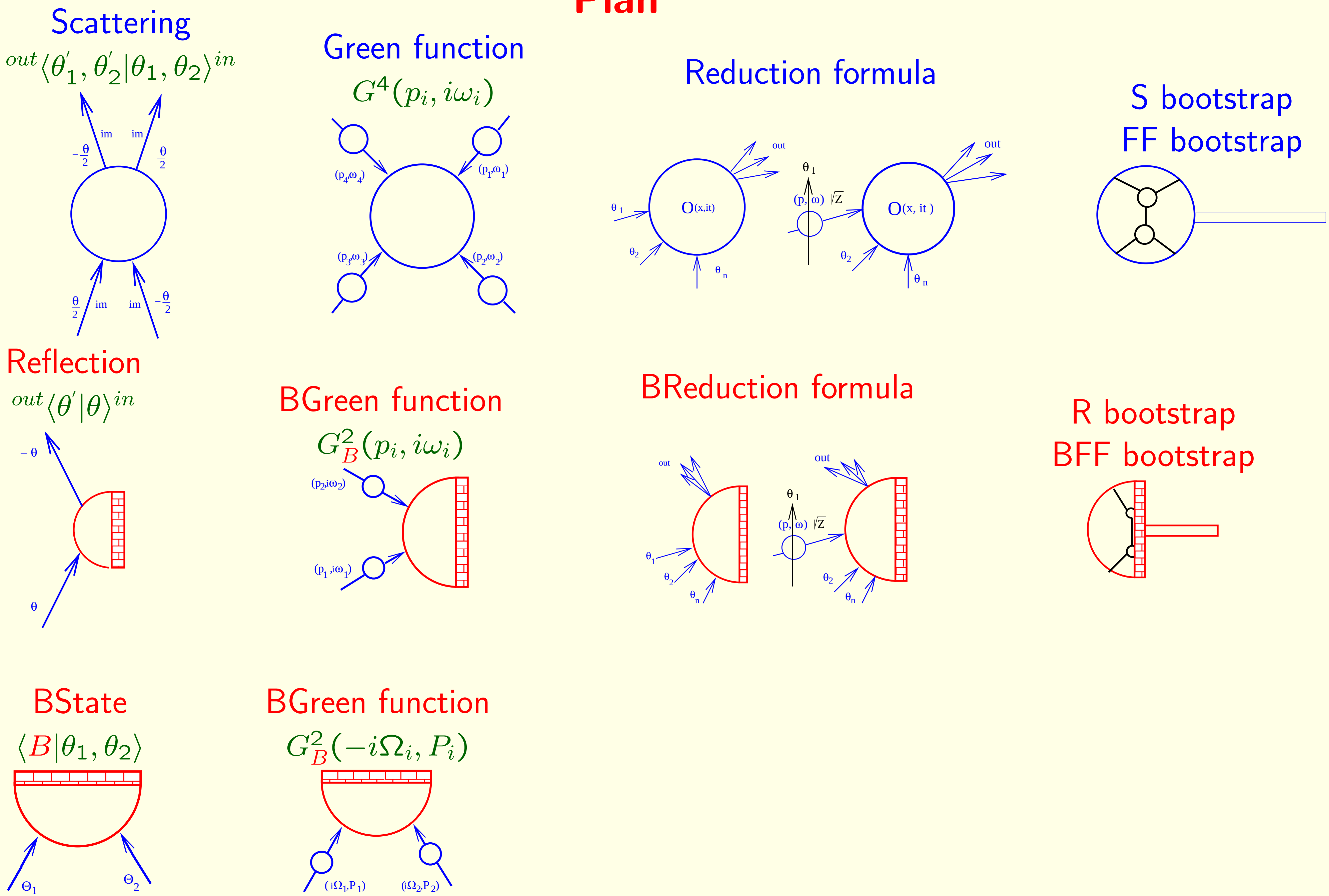


BState

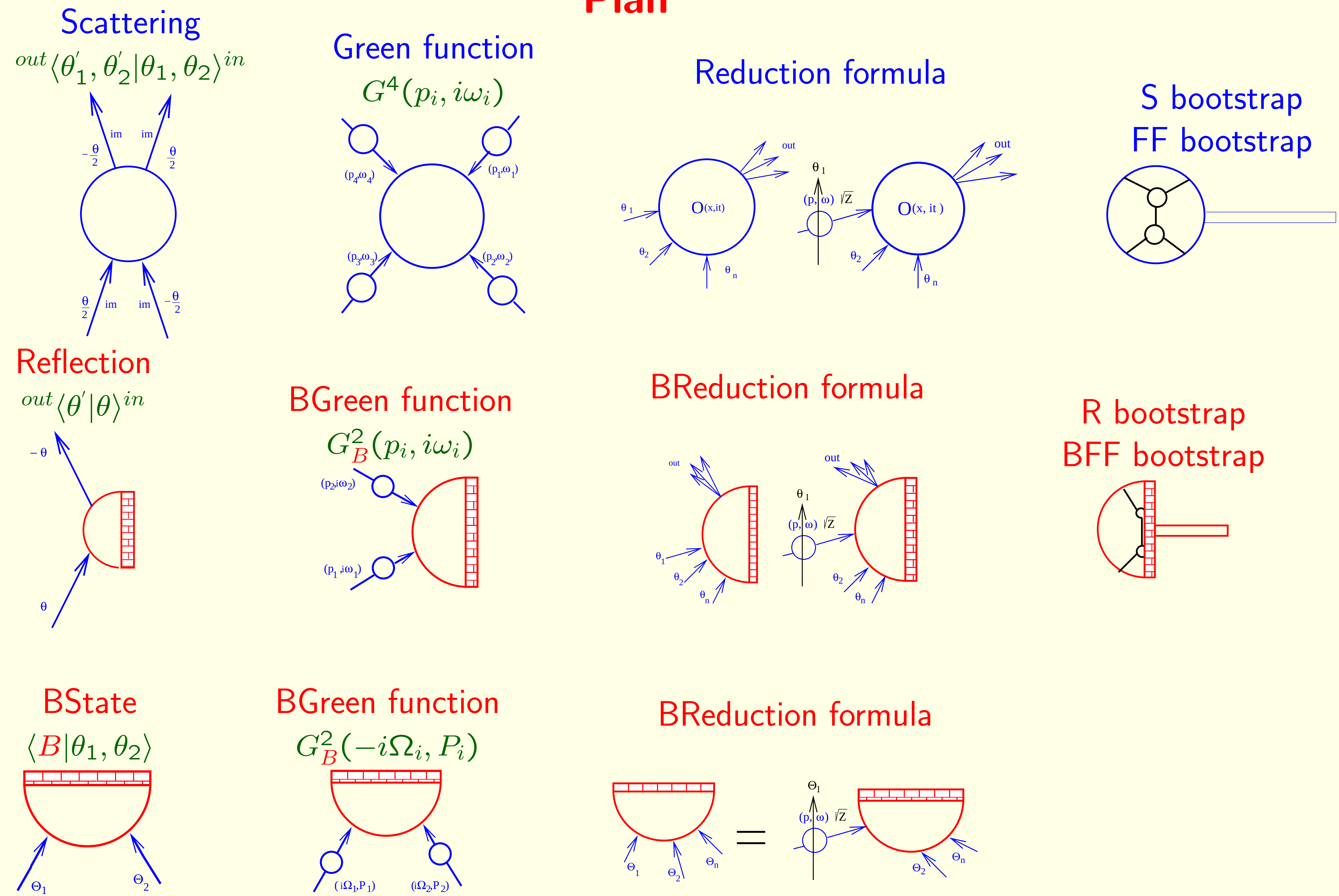
$\langle B|\theta_1,\theta_2\rangle$



Plan

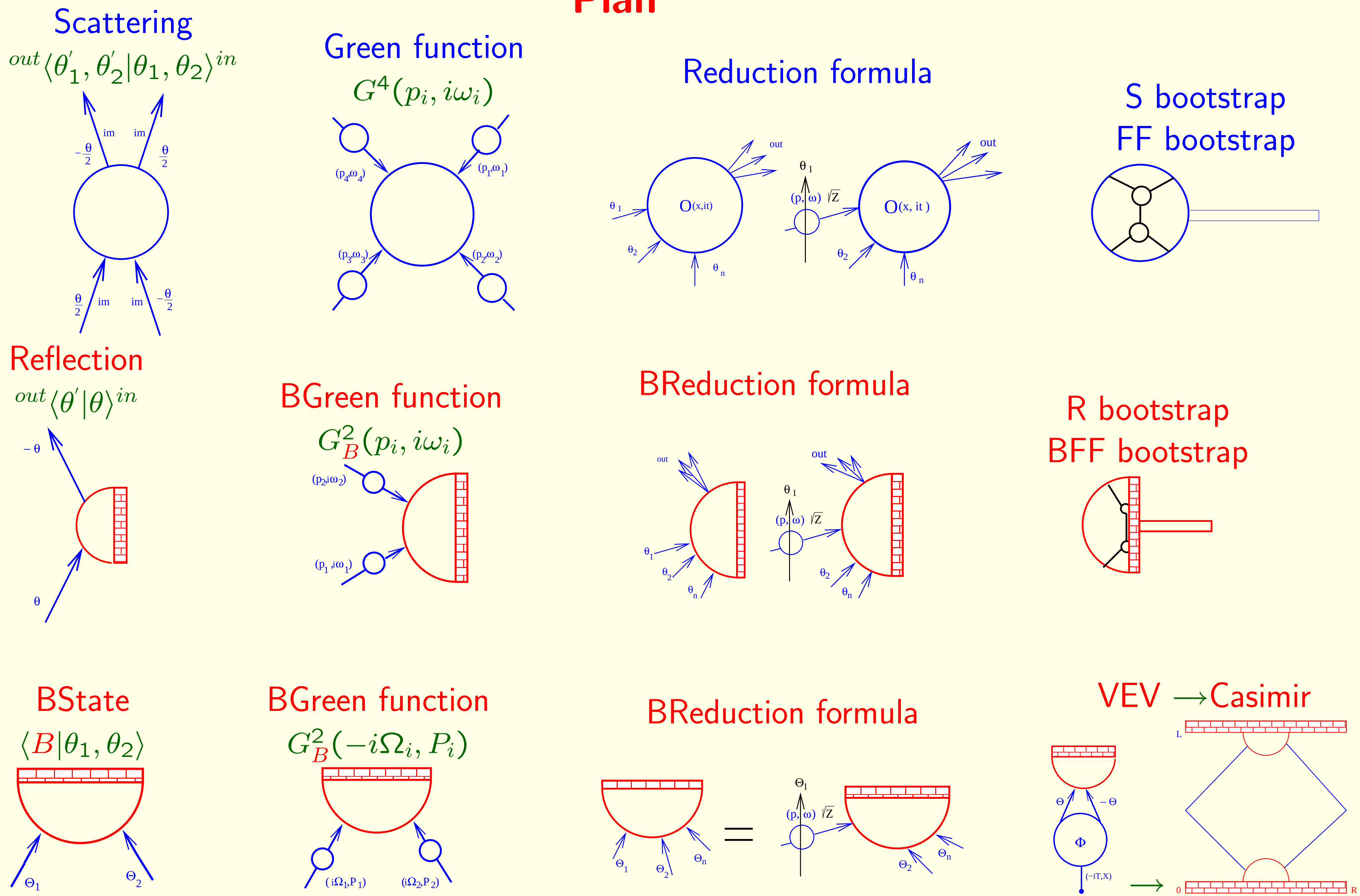

Scattering
$^{out}\langle\theta'_1,\theta'_2|\theta_1,\theta_2\rangle^{in}$
Green function
$G^4(p_i,i\omega_i)$
Reduction formula
O(x,it)
O(x, it)
out
(p, ω)
√Z
S bootstrap
FF bootstrap
Reflection
$^{out}\langle\theta'|\theta\rangle^{in}$
BGreen function
$G^2_B(p_i,i\omega_i)$
BReduction formula
out
(p, ω)
√Z
R bootstrap
BFF bootstrap
BState
$\langle B|\theta_1,\theta_2\rangle$
BGreen function
$G^2_B(-i\Omega_i,P_i)$

Plan



Scattering
$^{out}\langle \theta_1', \theta_2' | \theta_1, \theta_2 \rangle^{in}$
Green function
$G^4(p_i, i\omega_i)$
Reduction formula
S bootstrap
FF bootstrap
Reflection
$^{out}\langle \theta' | \theta \rangle^{in}$
BGreen function
$G_B^2(p_i, i\omega_i)$
BReduction formula
R bootstrap
BFF bootstrap
BState
$\langle B | \theta_1, \theta_2 \rangle$
BGreen function
$G_B^2(-i\Omega_i, P_i)$
BReduction formula

Plan



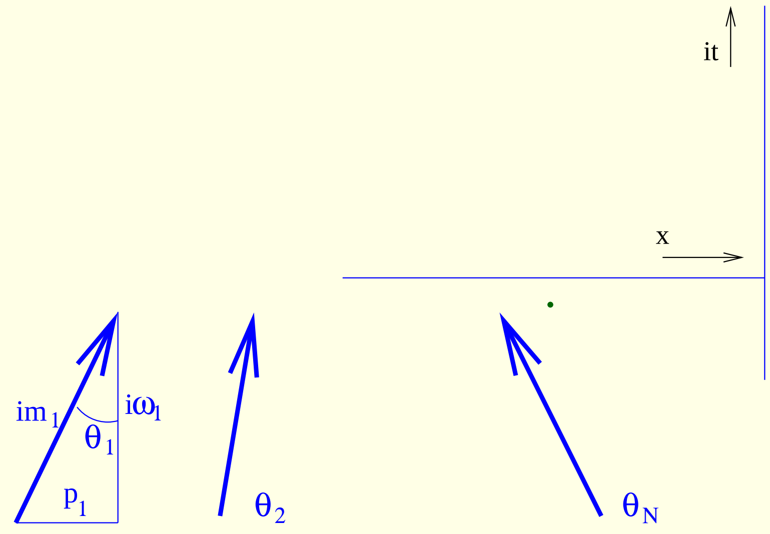
Scattering
$^{out}\langle\theta_1',\theta_2'|\theta_1,\theta_2\rangle^{in}$
im
im
$-\frac{\theta}{2}$
$\frac{\theta}{2}$
$\frac{\theta}{2}$
$-\frac{\theta}{2}$
Green function
$G^4(p_i,i\omega_i)$
(p_4,ω_4)
(p_1,ω_1)
(p_3,ω_3)
(p_2,ω_2)
Reduction formula
O(x,it)
O(x, it)
out
θ_1
θ_2
θ_n
(p, ω) $\sqrt{Z}$
S bootstrap
FF bootstrap
Reflection
$^{out}\langle\theta'|\theta\rangle^{in}$
$-\theta$
θ
BGreen function
$G_B^2(p_i,i\omega_i)$
$(p_2,i\omega_2)$
$(p_1,i\omega_1)$
BReduction formula
out
θ_1
θ_2
θ_n
(p, ω) $\sqrt{Z}$
R bootstrap
BFF bootstrap
BState
$\langle B|\theta_1,\theta_2\rangle$
Θ_1
Θ_2
BGreen function
$G_B^2(-i\Omega_i,P_i)$
$(i\Omega_1,P_1)$
$(i\Omega_2,P_2)$
BReduction formula
Θ_1
Θ_2
Θ_n
(p, ω) $\sqrt{Z}$
VEV $\longrightarrow$ Casimir
Θ
$-\Theta$
Φ
$(-iT,X)$
L
0
R

Bulk Hilbert space

Bulk initial state

(energy $i\omega = im\cosh\theta$, momentum $p = m\sinh\theta$)

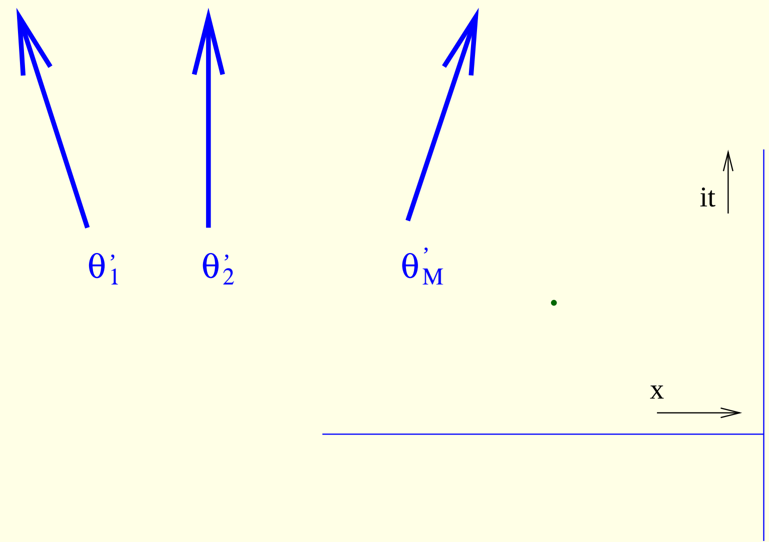
$$|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N$$



Bulk Hilbert space

Bulk final state

$$|\theta_1', \theta_2', \ldots, \theta_M'\rangle^{out}$$
$$\theta_1' < \theta_2' < \ldots < \theta_M'$$



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$$|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$
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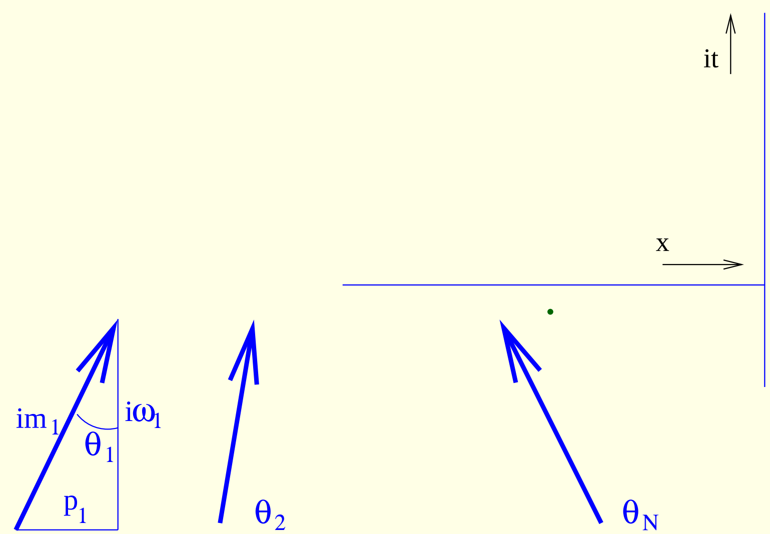


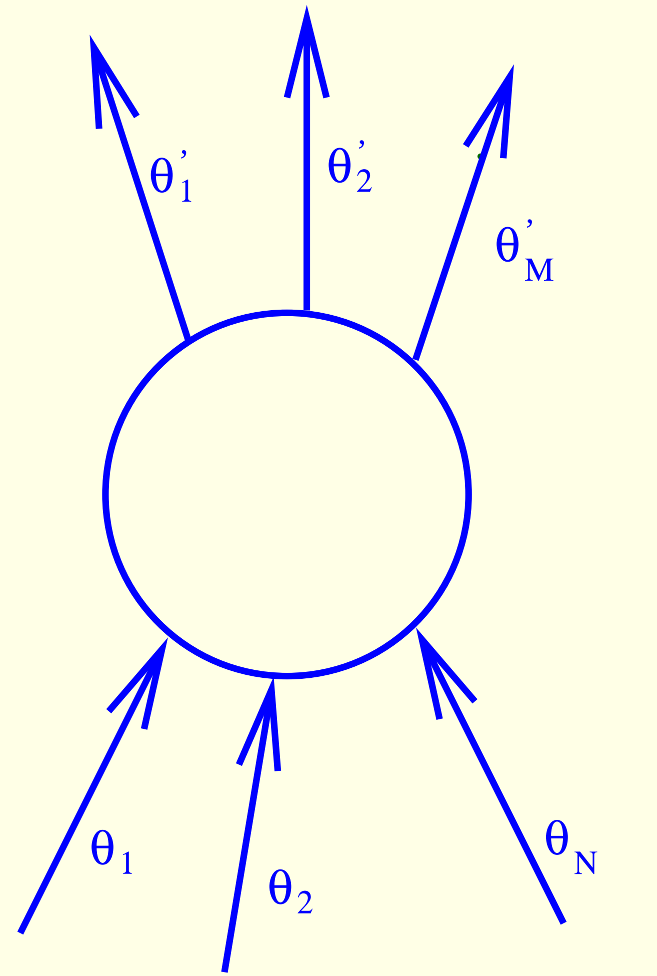
Scattering matrix

Bulk final state

$$|\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}\rangle^{out}$$
$$\theta_1^{'} < \theta_2^{'} < \ldots < \theta_M^{'}$$

Scattering matrix

$$S_N^M(\{\theta\}, \{\theta^{'}\}) =$$
$$^{out}\langle\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$



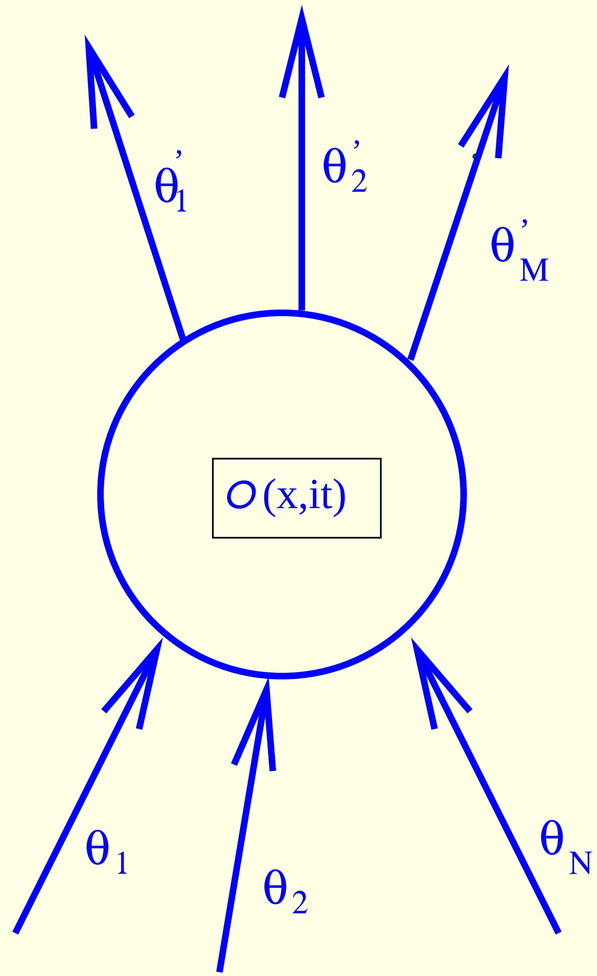
Bulk initial state

$$|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N$$

Form factors

Bulk final state

$$|\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}\rangle^{out}$$
$$\theta_1^{'} < \theta_2^{'} < \ldots < \theta_M^{'}$$



Form factors of local operators

$$F_{NM}^{\mathcal{O}}(\{\theta\}, \{\theta^{'}\}) =$$
$${}^{out}\langle\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}|\mathcal{O}(x, it)|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$

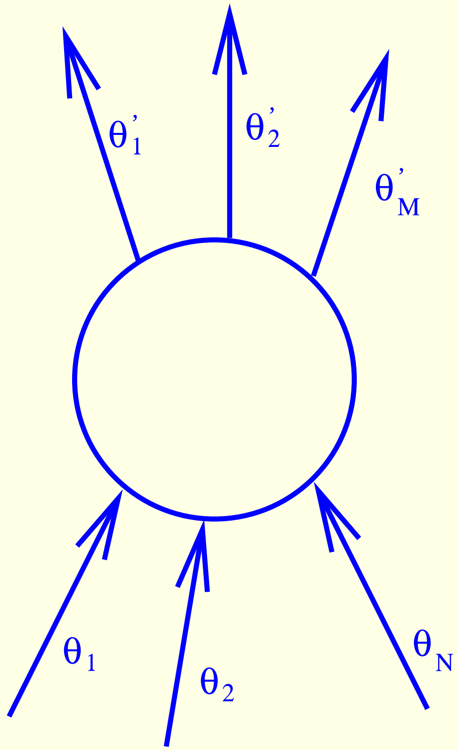
Bulk initial state

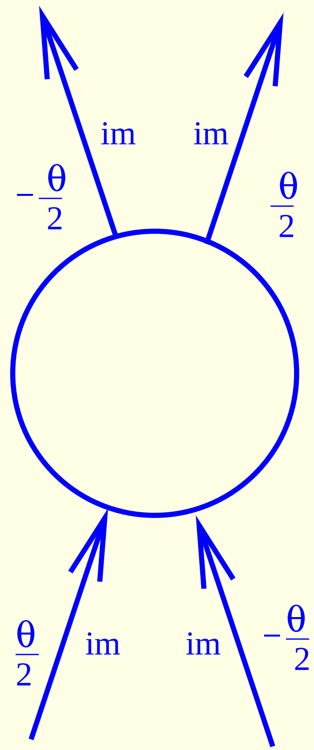
$$|\theta_1, \theta_2, \ldots, \theta_N\rangle^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N$$

Scattering matrix: properties (Lorentz invariance)

Scattering matrix: properties (Lorentz invariance)

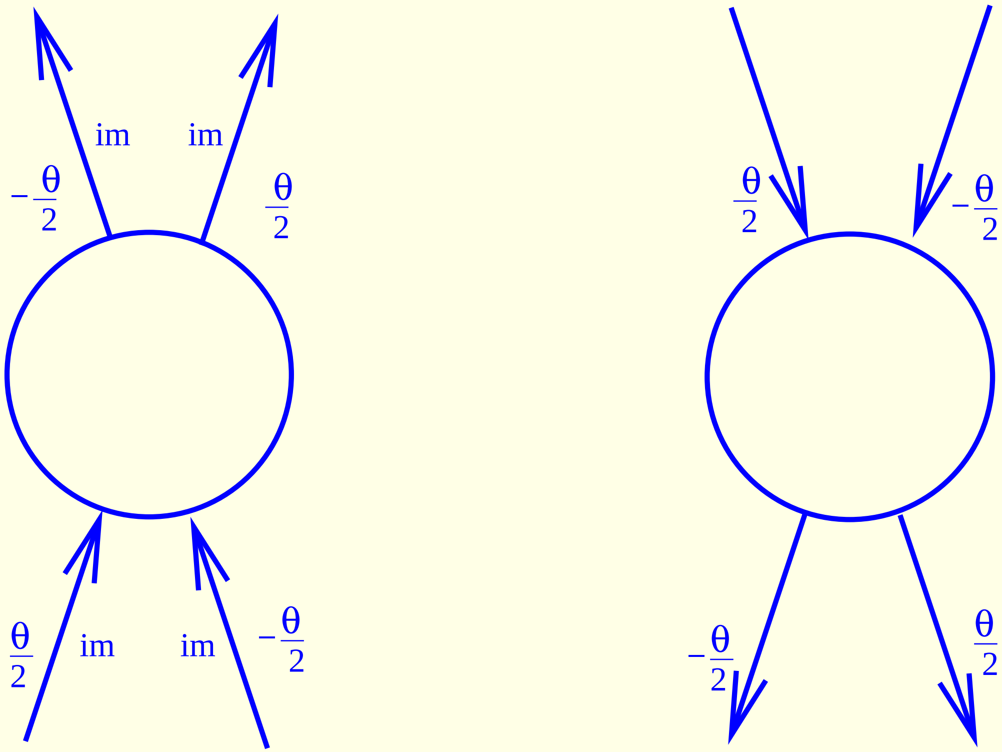
Simplest nontrivial scattering matrix $S_2^2(|\theta_1 - \theta_2|) = S(\theta)$ $\boxed{\theta > 0}$





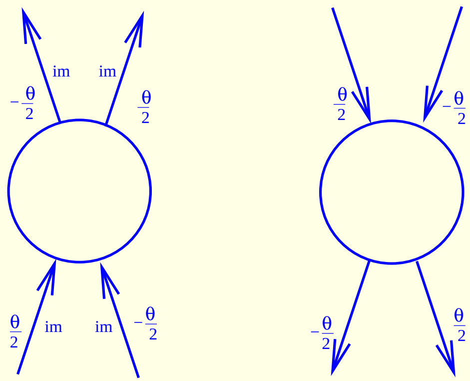
Scattering matrix: properties (Lorentz invariance)

Unitarity



$$S(\theta)^* = S(-\theta)$$

Scattering matrix: properties (Lorentz invariance)



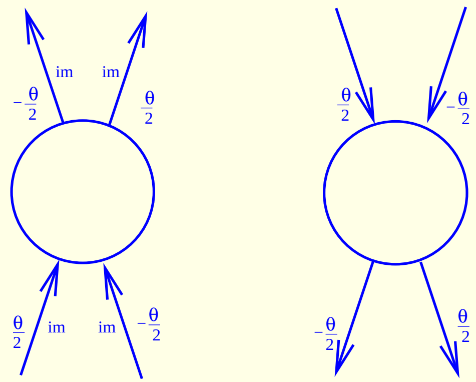
$$S(\theta)^* = S(-\theta)$$

Unitarity

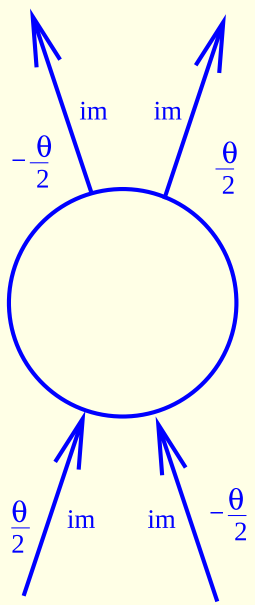
Scattering matrix: properties (Lorentz invariance)

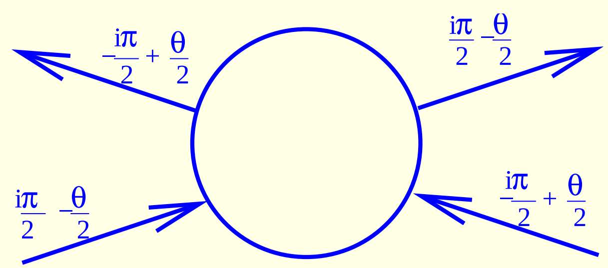
Unitarity

Crossing



$$S(\theta)^* = S(-\theta)$$

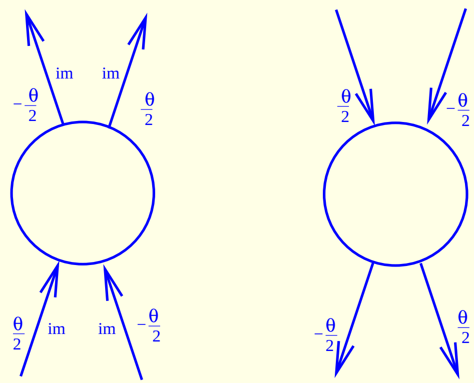




$$S(\theta) = S(i\pi - \theta)$$

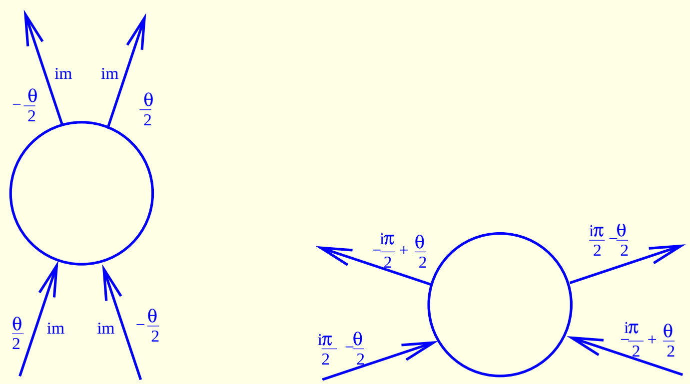
Scattering matrix: properties (Lorentz invariance)

Unitarity



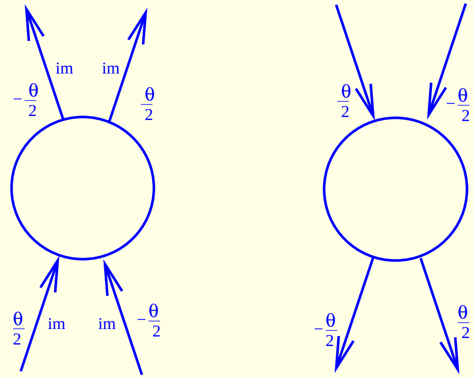
$$S(\theta)^* = S(-\theta)$$

Crossing



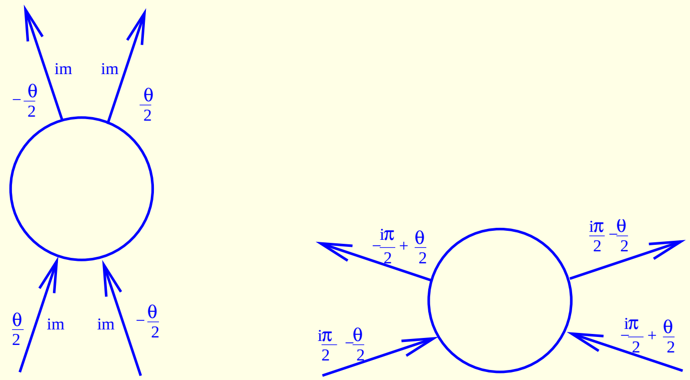
$$S(\theta) = S(i\pi - \theta)$$

Scattering matrix: properties (Lorentz invariance)



$$S(\theta)^* = S(-\theta)$$

Unitarity



$$S(\theta) = S(i\pi - \theta)$$

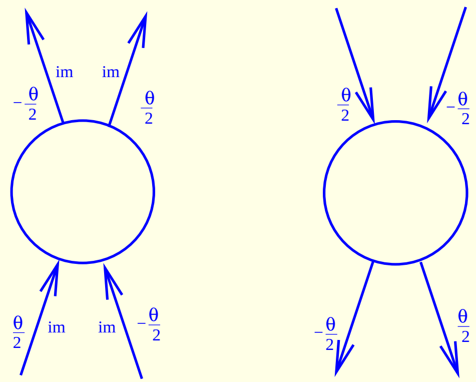
Crossing

Need for analytical continuation in θ

Need for singularity structure in θ

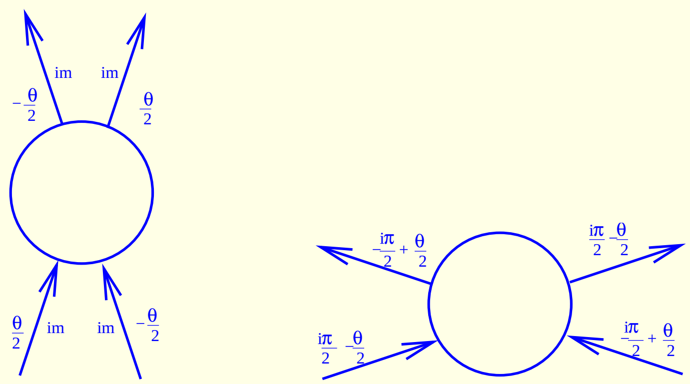
Scattering matrix: properties (Lorentz invariance)

Unitarity



$$S(\theta)^* = S(-\theta)$$

Crossing



$$S(\theta) = S(i\pi - \theta)$$

Need for analytical continuation in θ

Need for singularity structure in θ

Reduction formula, Landau equations

Analytic structure: reduction formula

Analytic structure: reduction formula

need: analytic structure of		reduction formula		known: analytic structure of
S matrix	$\longleftrightarrow$		$\longleftrightarrow$	correlators

Analytic structure: reduction formula

need: analytic structure of S matrix	$\leftrightarrow$	reduction formula	$\leftrightarrow$	known: analytic structure of correlators

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n > = 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n >$$

Analytic structure: reduction formula

need: analytic structure of S matrix ↔ reduction formula ↔ known: analytic structure of correlators

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n > \; = \; 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n >$$

$$-Z^{-1/2}\int_{-\infty}^{\infty} d(it')\, e^{-i(i\omega(\theta_1))(it')}\int_{-\infty}^{\infty} dx'\, e^{-ip(\theta_1)x'}\{-\partial^2_{it'}-\partial^2_{x'}+m^2\}$$

Analytic structure: reduction formula

need: analytic structure of S matrix $\leftrightarrow$ reduction formula $\leftrightarrow$ known: analytic structure of correlators

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n> = 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n>$$

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$$< out|T(\Phi(x^{'},it^{'})\mathcal{O}(x,it))|\theta_2,\ldots,\theta_n>$$

Analytic structure: reduction formula

need: analytic structure of	↔	reduction formula	↔	known: analytic structure of
S matrix				correlators

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n> \;=\; 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n>$$

$$-Z^{-1/2}\int_{-\infty}^{\infty} d(it')\, e^{-i(i\omega(\theta_1))(it')}\int_{-\infty}^{\infty} dx'\, e^{-ip(\theta_1)x'}\{-\partial_{it'}^2-\partial_{x'}^2+m^2\}$$

$$< out|T(\Phi(x',it')\mathcal{O}(x,it))|\theta_2,\ldots,\theta_n>$$

Analytic continuation in θ_1:

(time reversal) $it\rightarrow -it$ continuation: $\boxed{\theta\rightarrow i\pi-\theta}$

Crossing $(x,it)\rightarrow(-iT,X)\leftrightarrow(\omega,ip)\rightarrow(-iP,\Omega)$ continuation $\boxed{\theta\rightarrow \frac{i\pi}{2}-\theta}$

Analytic structure: reduction formula

need: analytic structure of S matrix $\leftrightarrow$ reduction formula $\leftrightarrow$ known: analytic structure of correlators

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n> = 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n>$$

$$-Z^{-1/2}\int_{-\infty}^{\infty} d(it^{'})\, e^{-i(i\omega(\theta_1))(it^{'})}\int_{-\infty}^{\infty} dx^{'} e^{-ip(\theta_1)x^{'}}\{-\partial^2_{it^{'}}-\partial^2_{x^{'}}+m^2\}$$

$$< out|T(\Phi(x^{'},it^{'})\mathcal{O}(x,it))|\theta_2,\ldots,\theta_n>$$

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(time reversal) $it\longrightarrow -it$ continuation: $\theta\longrightarrow i\pi-\theta$

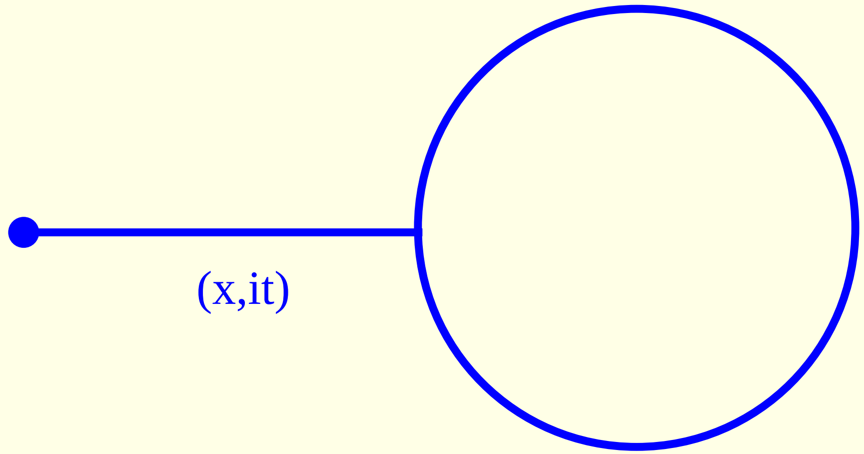
Crossing $(x,it)\longrightarrow(-iT,X)\leftrightarrow(\omega,ip)\longrightarrow(-iP,\Omega)$ continuation $\theta\longrightarrow\frac{i\pi}{2}-\theta$

Analytical properties of the correlators: Landau equations

Correlation functions

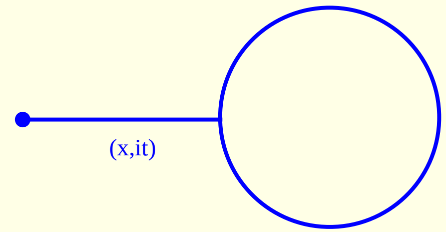
Correlation functions

One point function $\langle 0|\Phi(x,it)|0\rangle = G^1(x,it) = G^1$



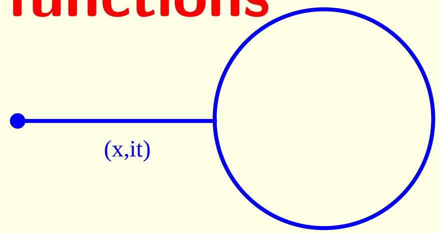
Correlation functions

One point function $\langle 0|\Phi(x,it)|0\rangle$



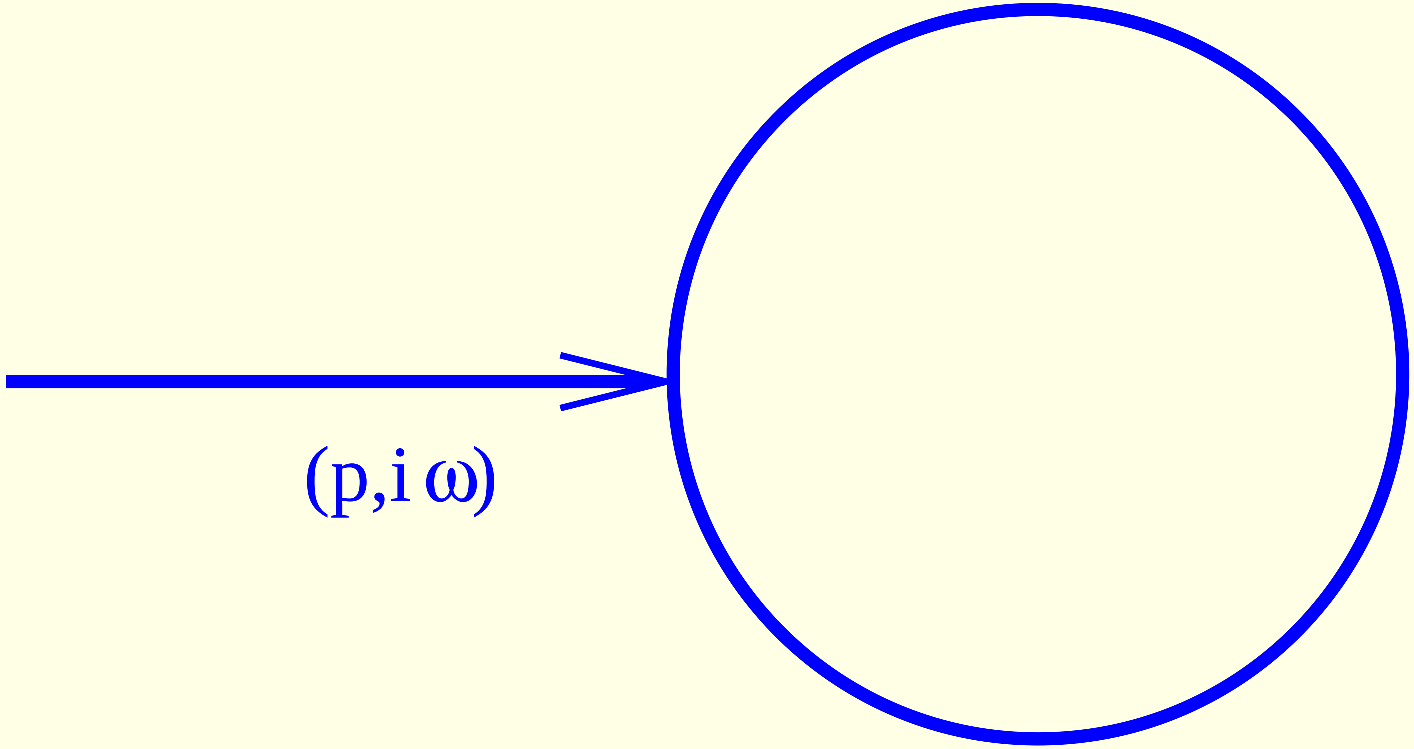
Correlation functions

One point function $\langle 0|\Phi(x,it)|0\rangle$



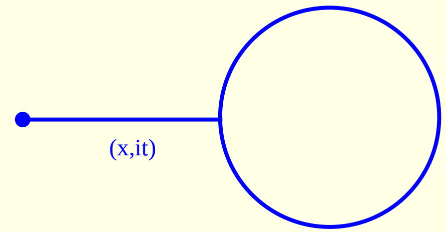
Momentum space formulation $\langle 0|\Phi(x,it)|0\rangle = \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p,i\omega)$

$G^1(p,i\omega) = (2\pi)^2 \delta(p)\delta(i\omega) G^1$

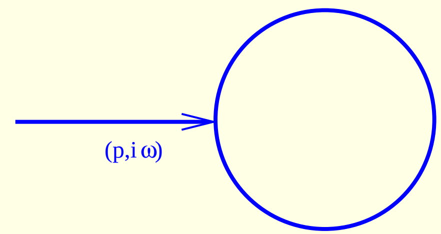


Correlation functions

One point function $\langle 0|\Phi(x, it)|0\rangle$



$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p, i\omega)$$



Correlation functions

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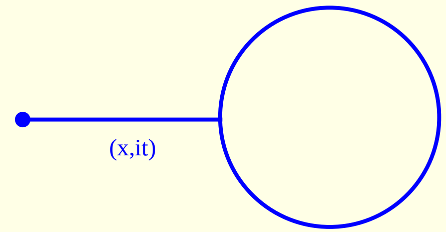

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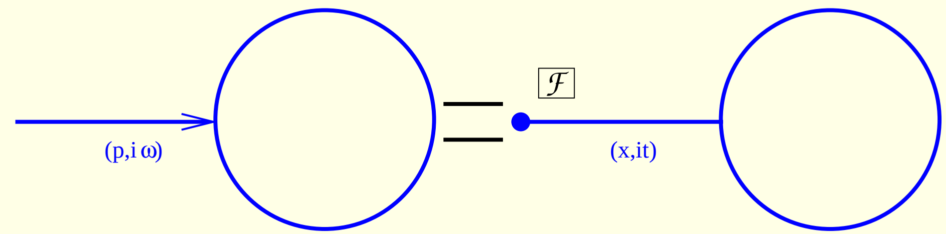

Correlation functions

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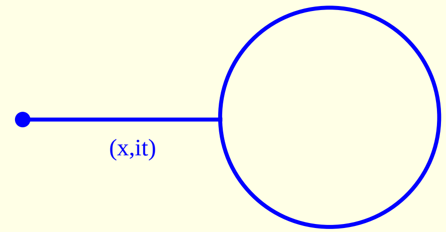

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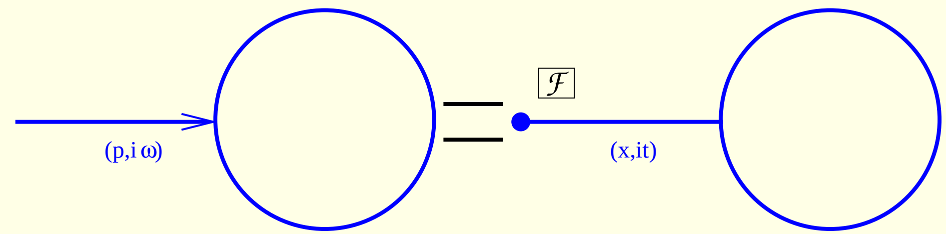
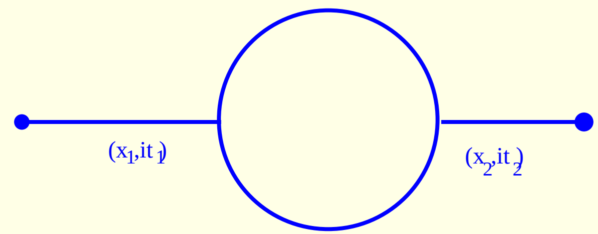


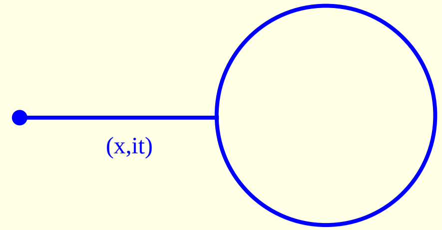
Two point function



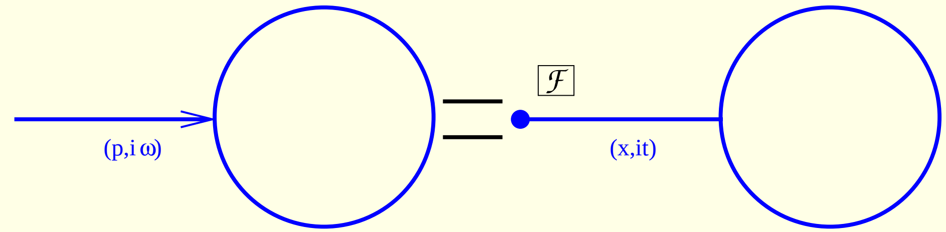
$$\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle = G^2(x_1-x_2,it_1-it_2)$$

Correlation functions

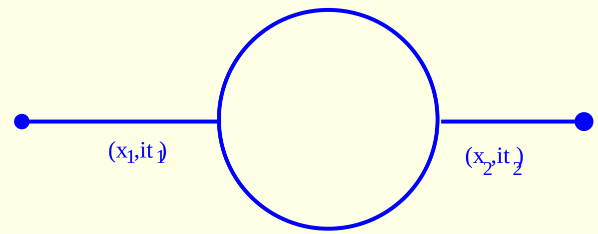
One point function $\langle 0|\Phi(x,it)|0\rangle$



$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p,i\omega)$$

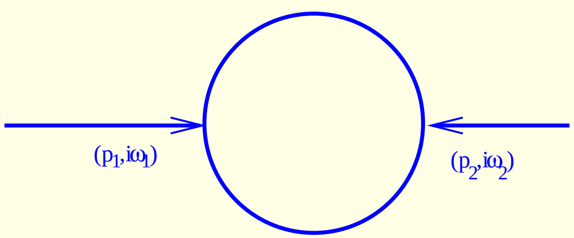


Two point function



$$\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle = G^2(x_1-x_2,it_1-it_2)$$

$$G^2(p_1,i\omega_1,p_2,i\omega_2) = (2\pi)^2\delta(p_1+p_2)\delta(\omega_1+\omega_2)G^2(p,i\omega)$$



Correlation functions

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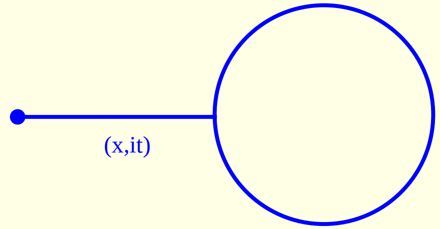
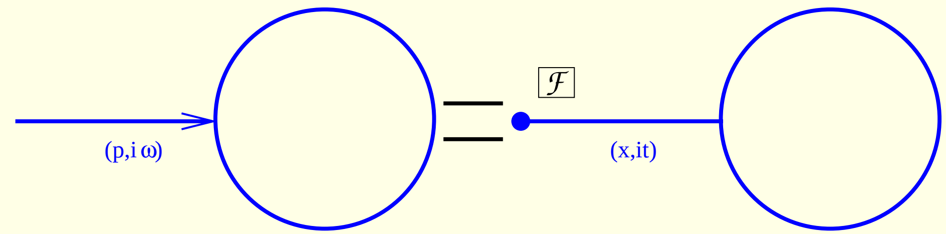
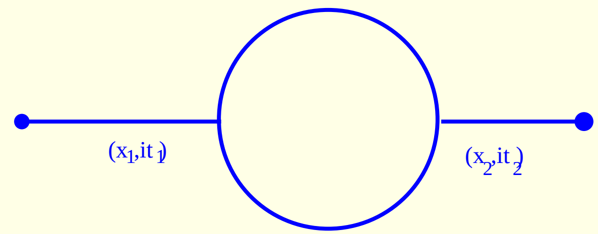


$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p,i\omega)$$

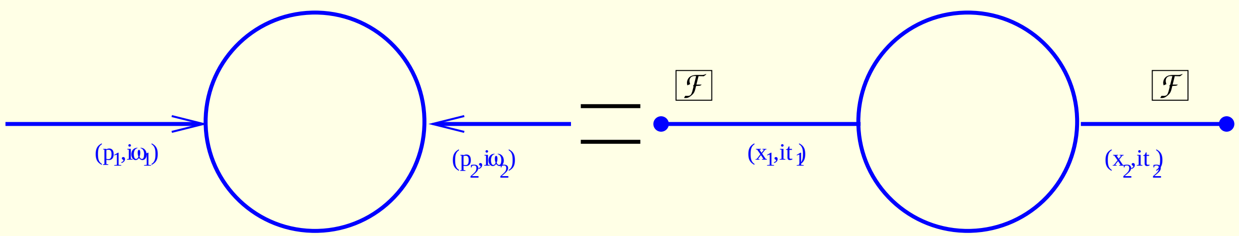


Two point function



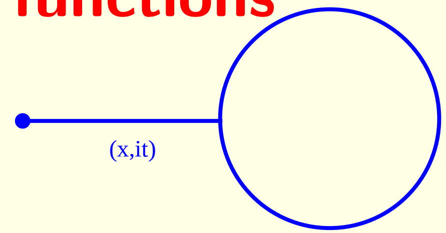
$$\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle = G^2(x_1-x_2, it_1-it_2)$$

$$G^2(p_1,i\omega_1,p_2,i\omega_2) = (2\pi)^2\delta(p_1+p_2)\delta(\omega_1+\omega_2)G^2(p,i\omega)$$

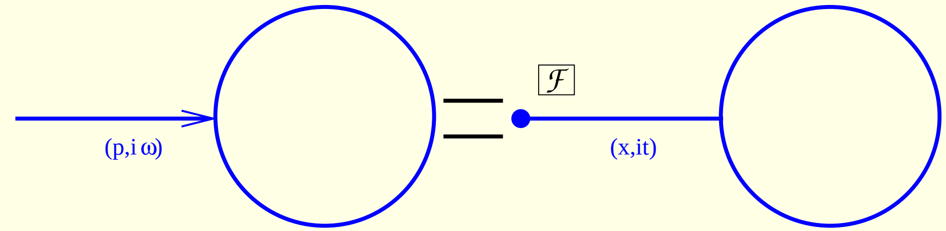


Correlation functions

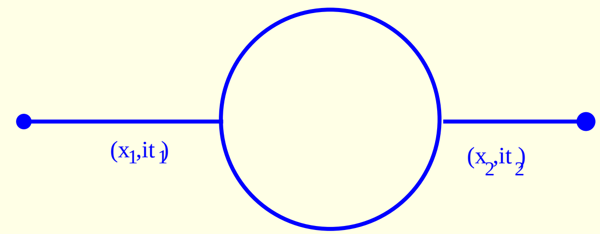
One point function $\langle 0|\Phi(x,it)|0\rangle$



$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p,i\omega)$$



Two point function



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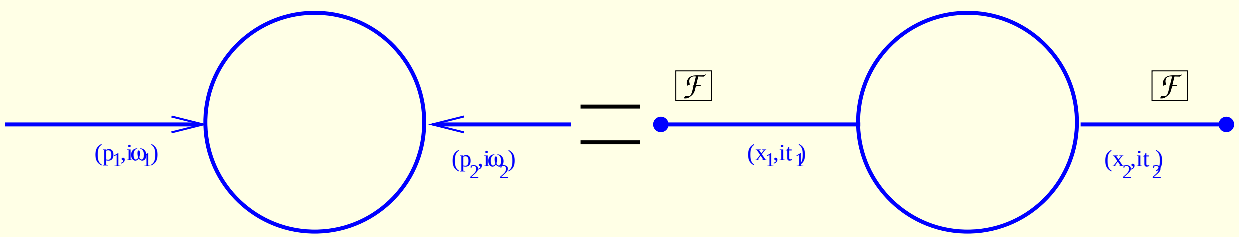

$$\boxed{G^2(p,i\omega) =}$$

Correlation functions

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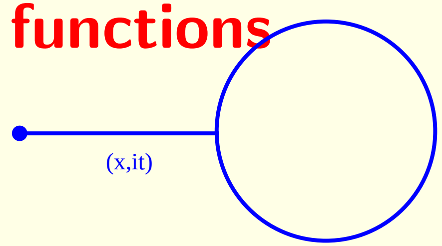

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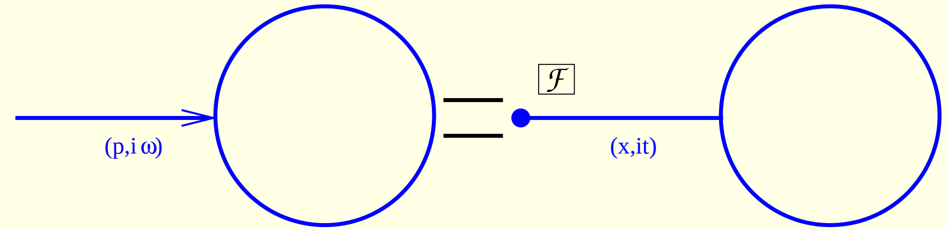
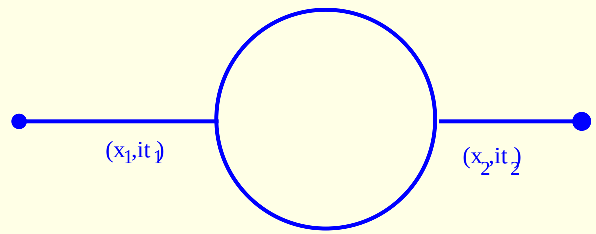


Two point function



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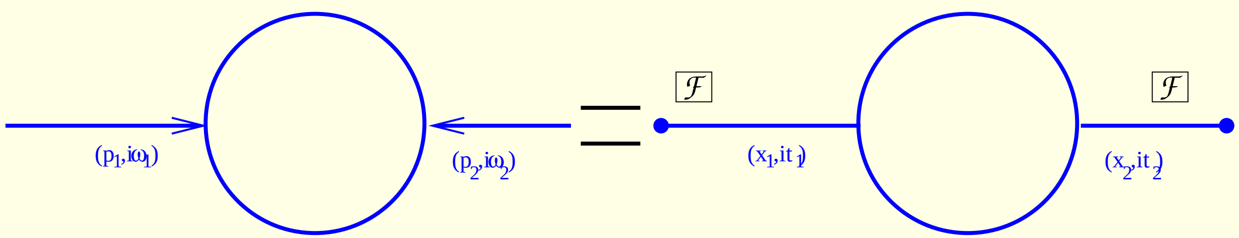

$$\boxed{G^2(p,i\omega) =} \quad \boxed{\frac{Z}{-(i\omega)^2 - p^2 - m^2 + i\epsilon}}$$

Correlation functions

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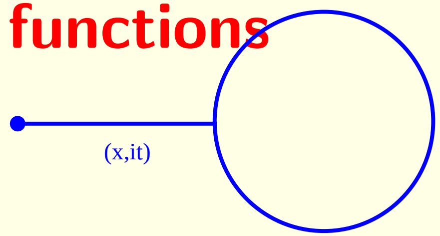
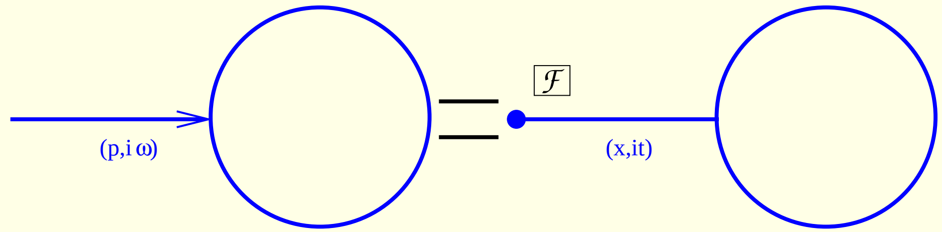
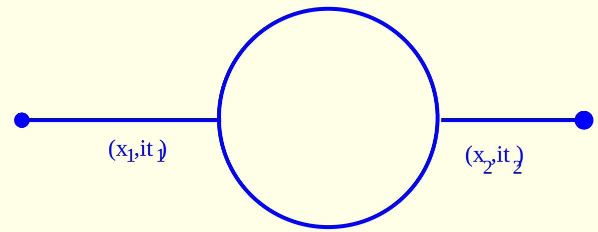


$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} d(it) e^{i(i\omega)(it)} G^1(p,i\omega)$$



Two point function



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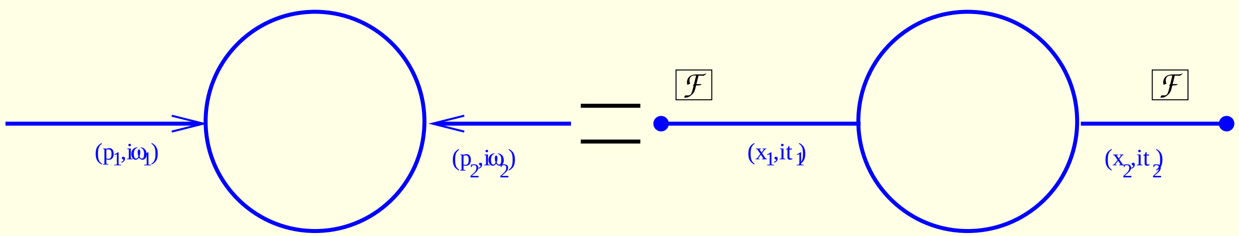
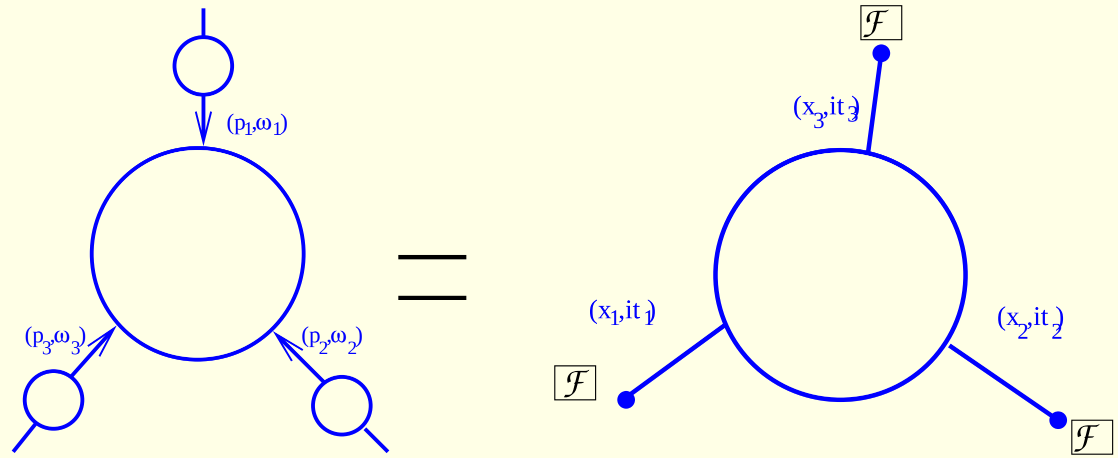


$$G^2(p,i\omega) = \frac{Z}{-(i\omega)^2-p^2-m^2+i\epsilon} + \int_{4m^2}^{\infty} dm'^2 \frac{\sigma(m'^2)}{-(i\omega)^2-p^2-m'^2+i\epsilon}$$

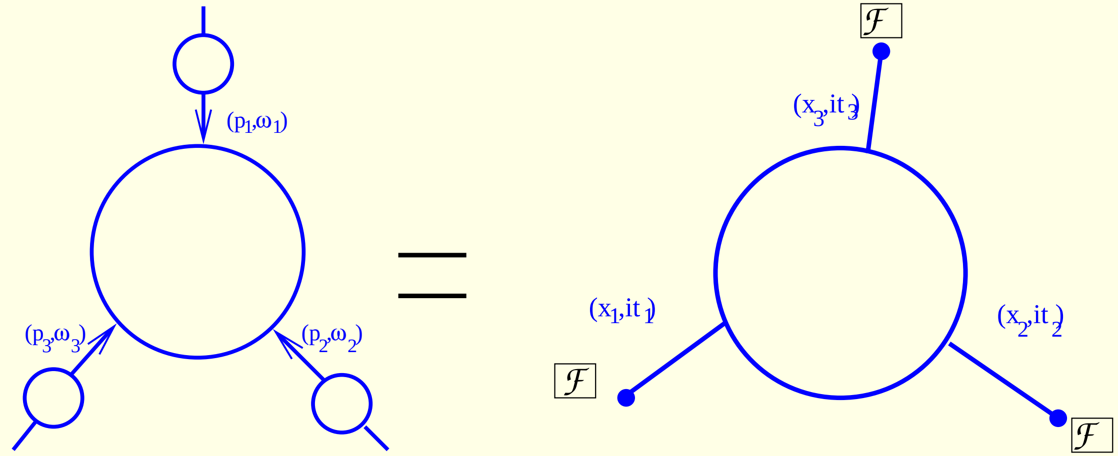
Correlation functions: higher

Correlation functions: higher

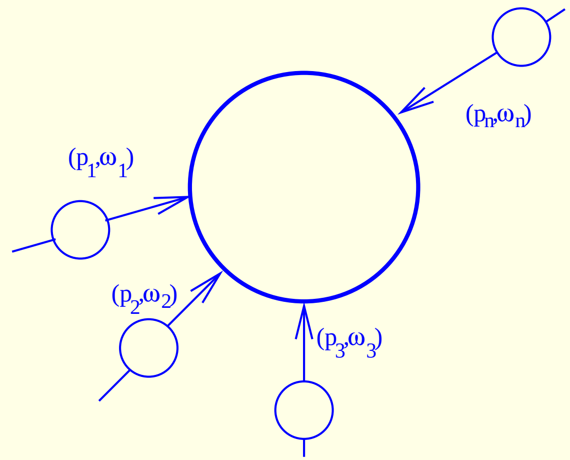
Three point function



Correlation functions: higher



Three point function



n- point function

Reduction formula revisited

Reduction formula revisited

$$\boxed{< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n >} = \boxed{2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n >}$$

Reduction formula revisited

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n > \; = \; 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n >$$

$$-Z^{-1/2}\int_{-\infty}^{\infty} d(it^{'})\, e^{-i(i\omega(\theta_1))(it^{'})}\int_{-\infty}^{\infty} dx^{'} e^{-ip(\theta_1)x^{'}}\{-\partial^2_{it^{'}}-\partial^2_{x^{'}}+m^2\}$$

$$< out|T(\Phi(x^{'},it^{'})\mathcal{O}(x,it))|\theta_2,\ldots,\theta_n >$$

Reduction formula revisited

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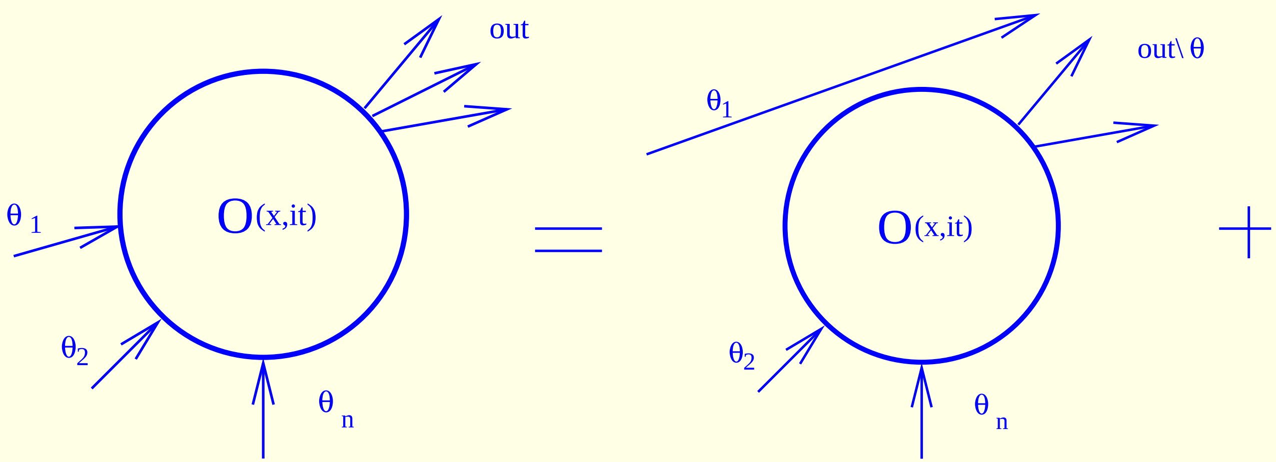
$$< out|T(\Phi(x',it')\mathcal{O}(x,it))|\theta_2,\ldots,\theta_n >$$

Diagrammatically



Reduction formula revisited

$$< out|\mathcal{O}(x,it)|\theta_1,\theta_2,\ldots,\theta_n > \; = \; 2\pi\delta(\theta_1-\theta)\langle out\backslash\theta|\mathcal{O}(x,it)|\theta_2,\ldots,\theta_n >$$

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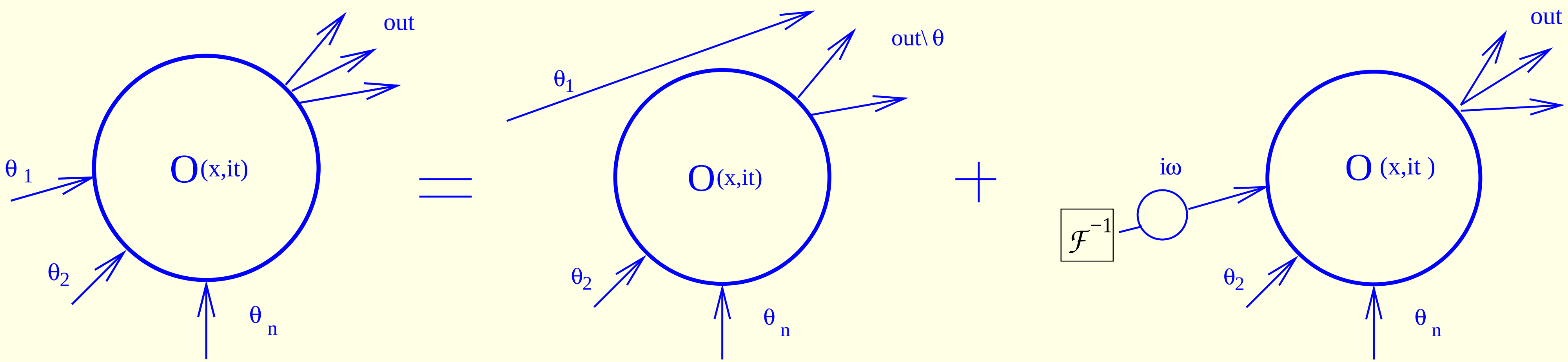
Diagrammatically



Reduction formula revisited

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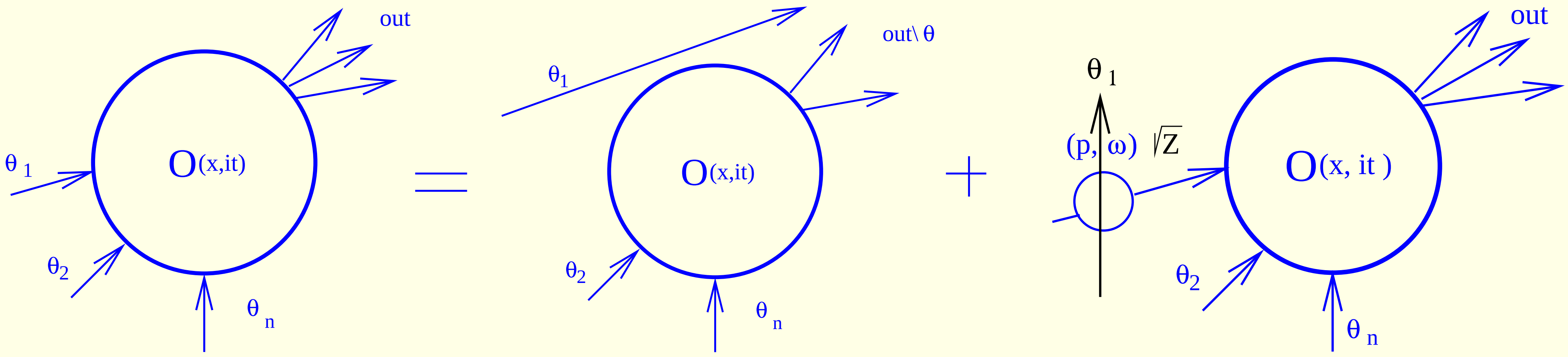
Diagrammatically



Singularity structure: Coleman-Thun mechanism

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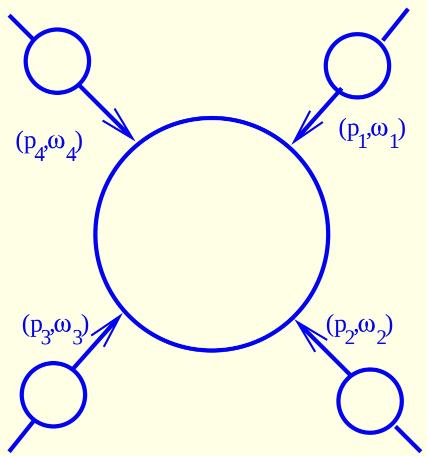
Singularity in correlation function = on mass shell particles



Singularity structure: Coleman-Thun mechanism

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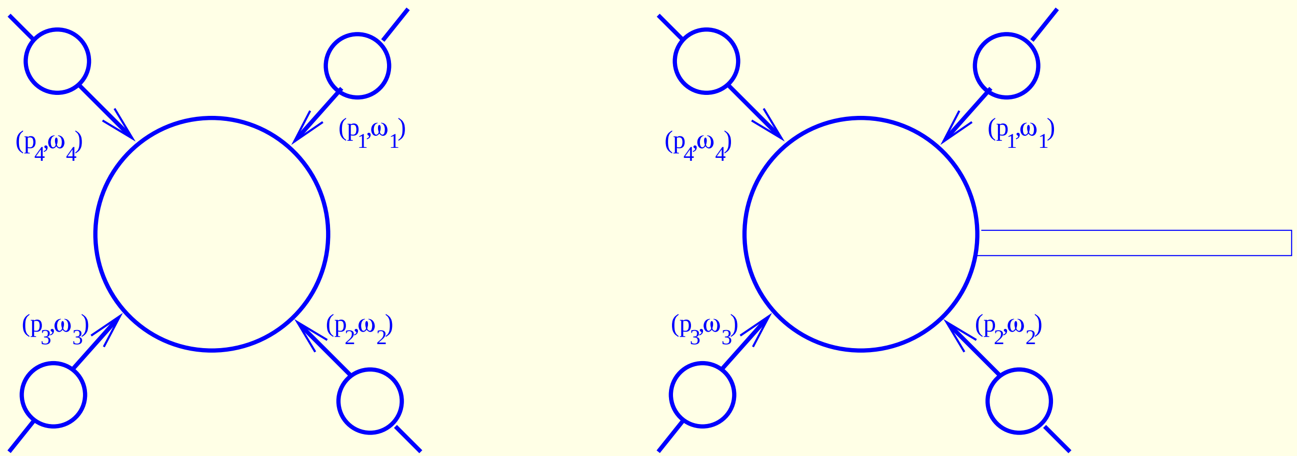

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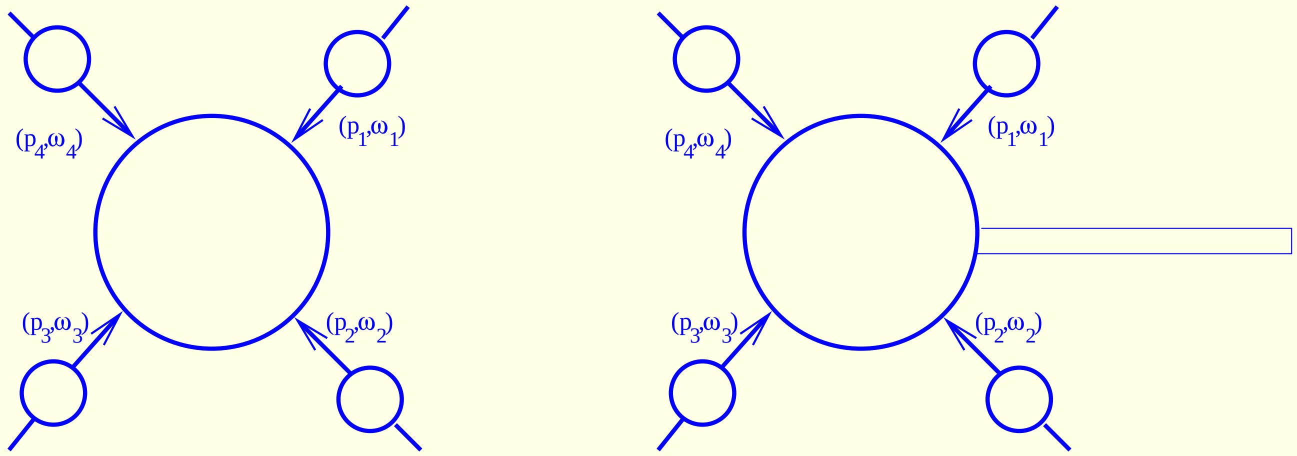

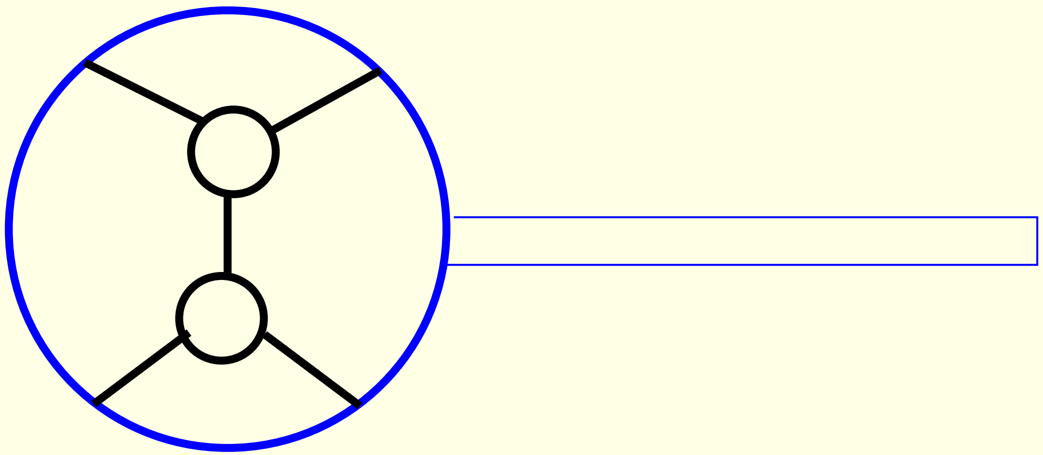

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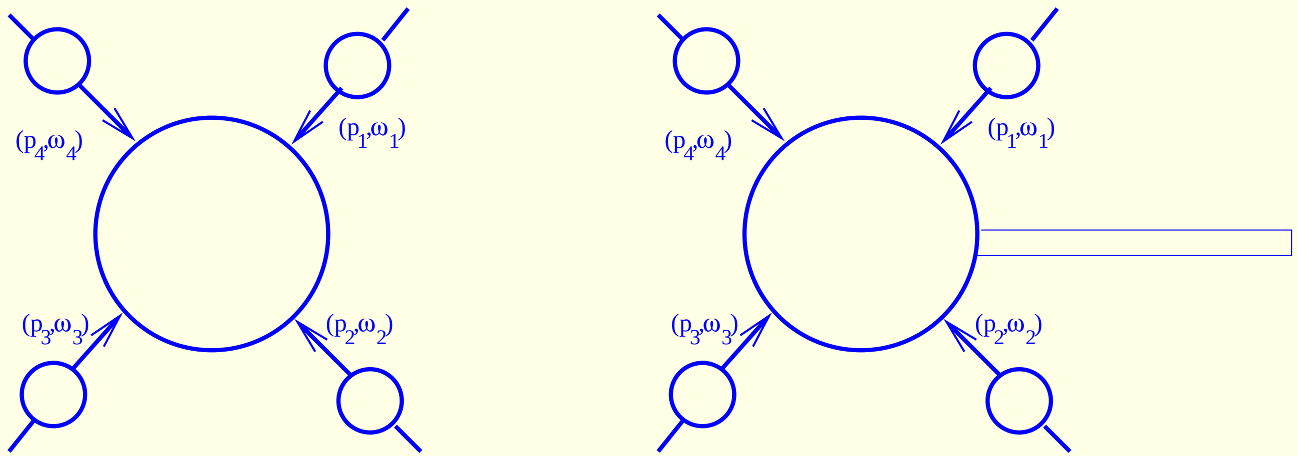
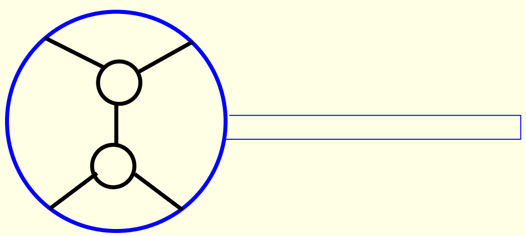


Coleman-Norton interpretation



Singularity structure: Coleman-Thun mechanism

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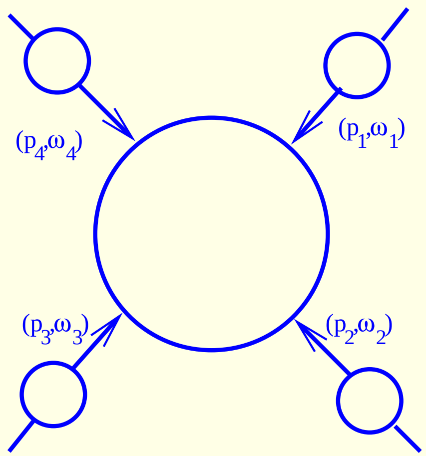

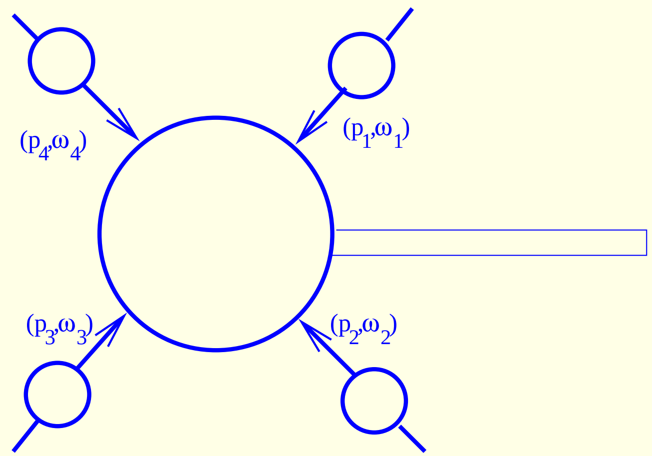

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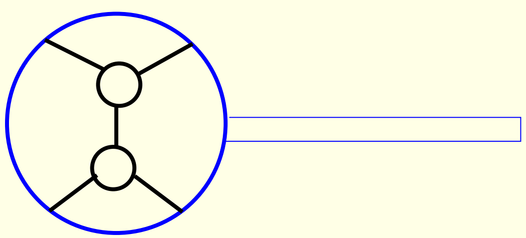

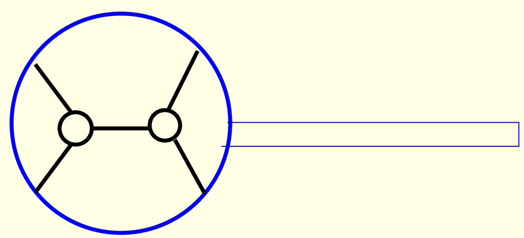

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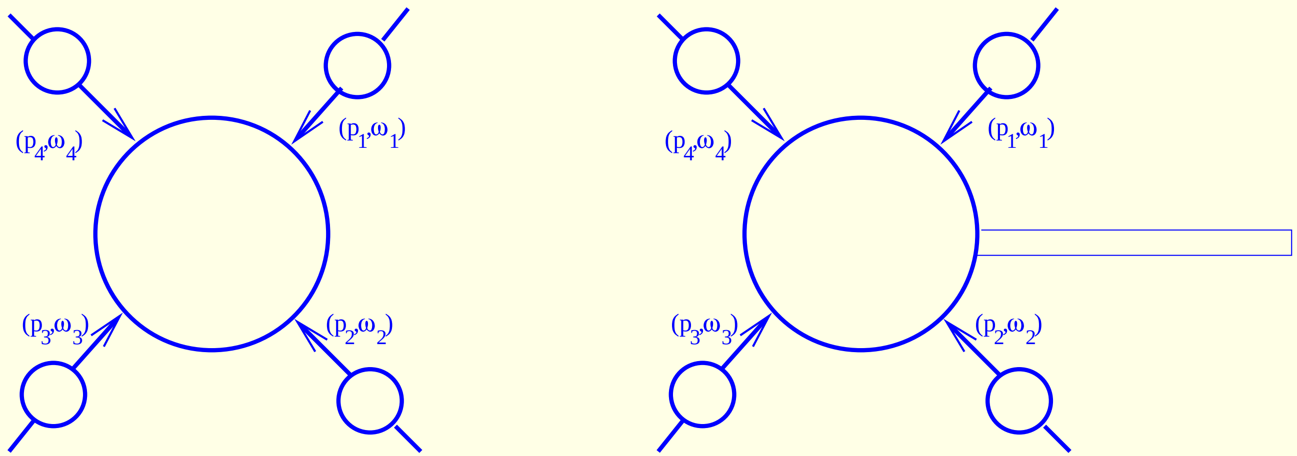

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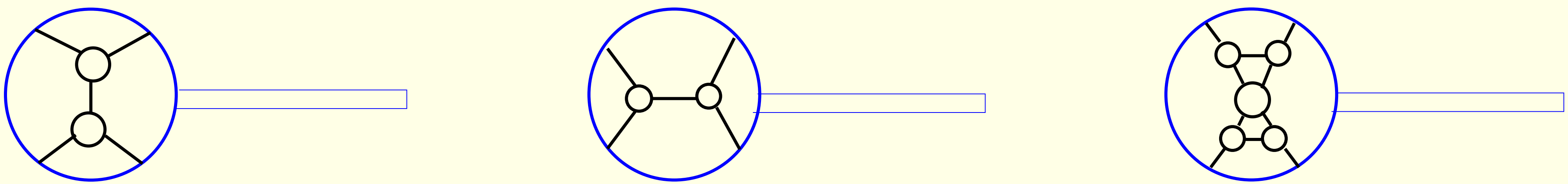

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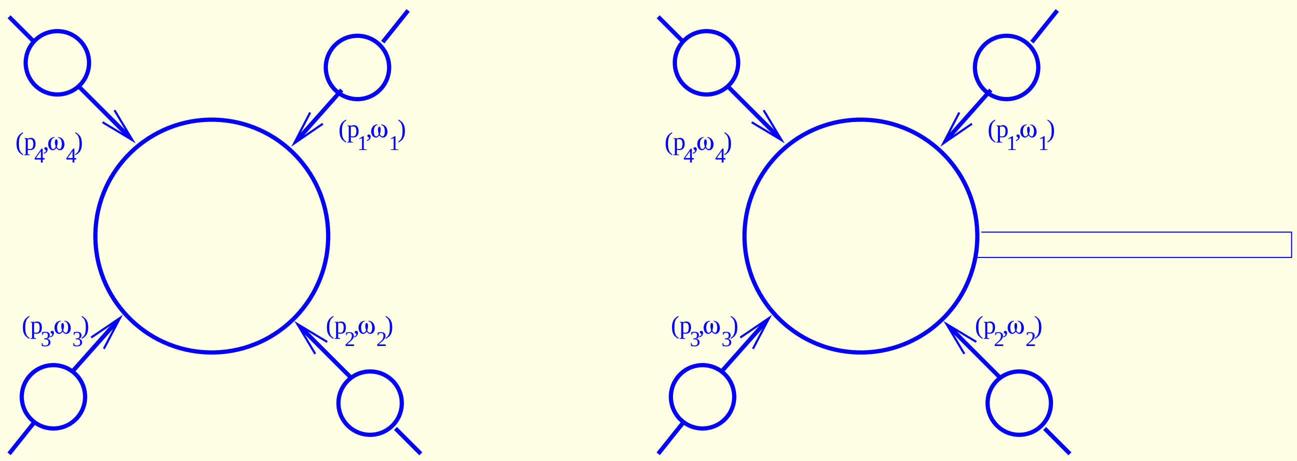
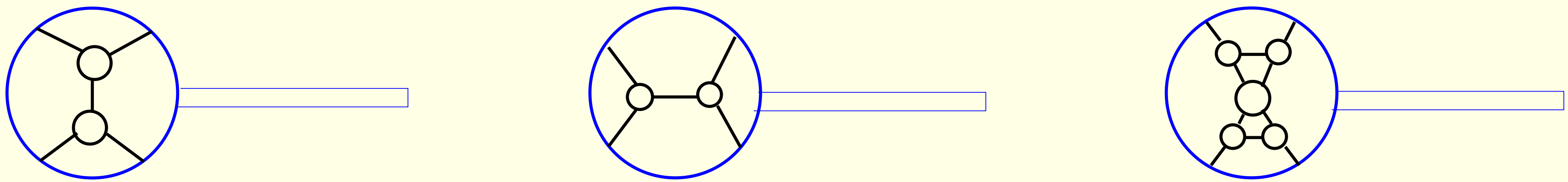


Coleman-Norton interpretation

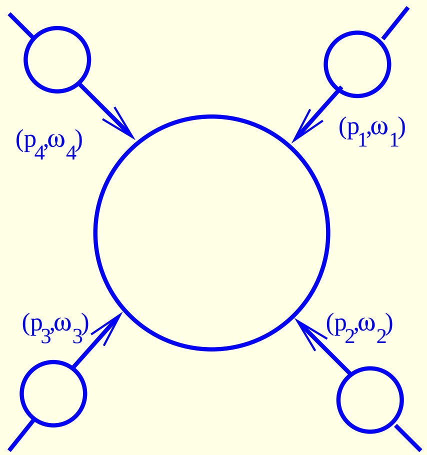


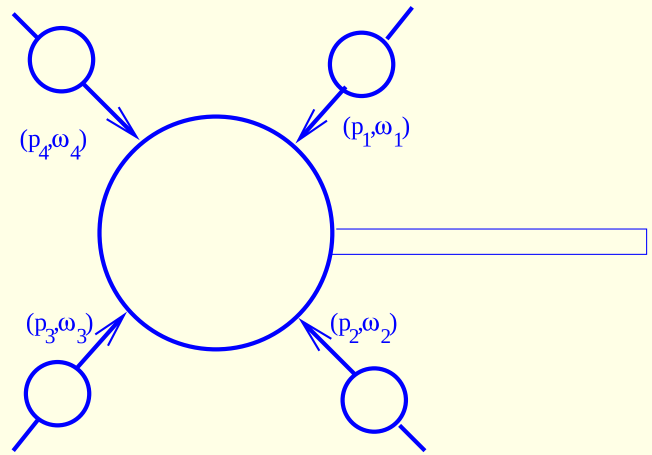
Cutkosky rules

$$(G^3(u_1))^2$$

Singularity structure: Coleman-Thun mechanism

Singularity in correlation function$=$ on mass shell particles





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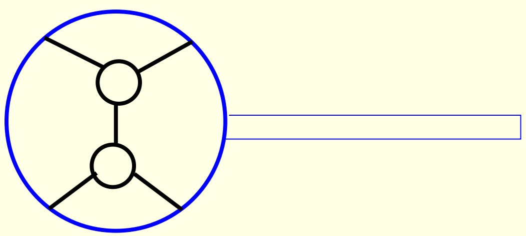

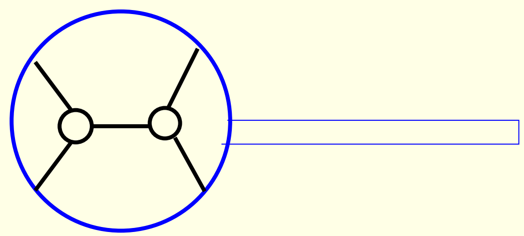

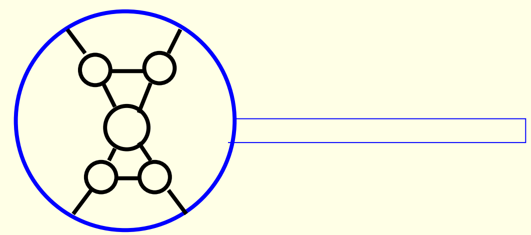

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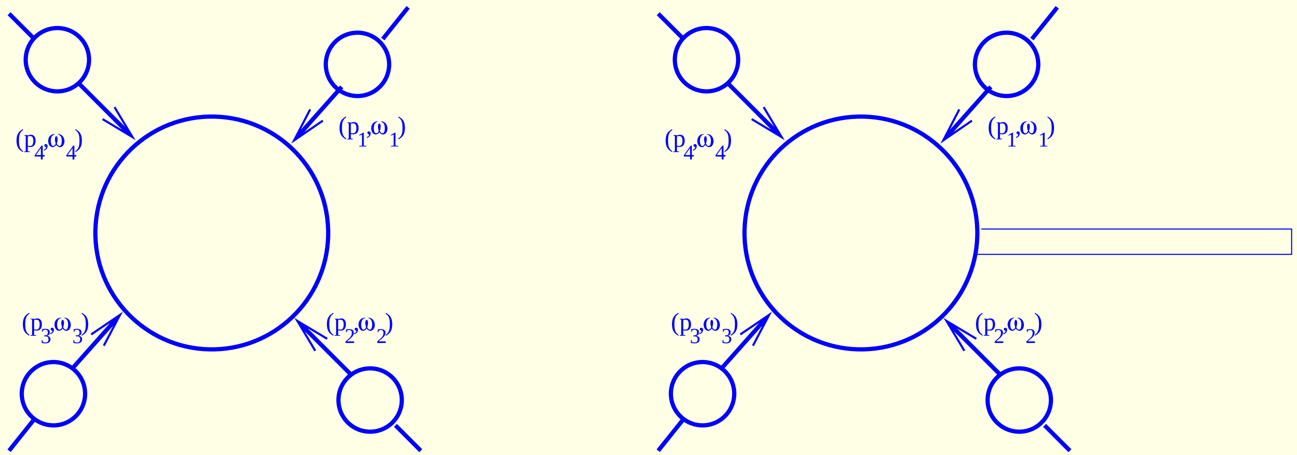

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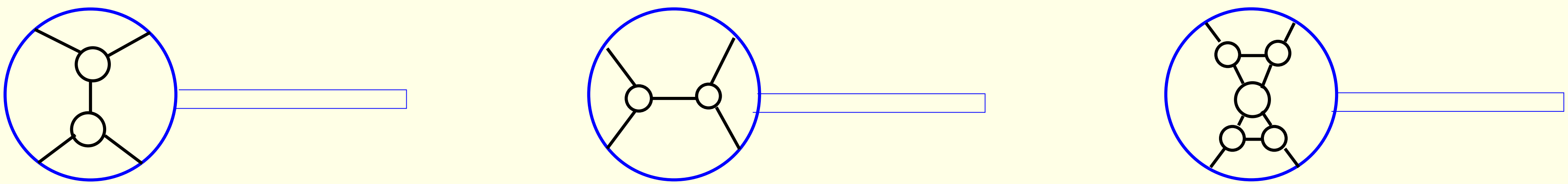

Cutkosky rules

$(G^3(u_1))^2$ $\qquad$ $(G^3(u_1))^2$ $\qquad$ $(G^3(u_2))^4 G^4(u_3)$

Integrable aspects: S-matrix bootsrap

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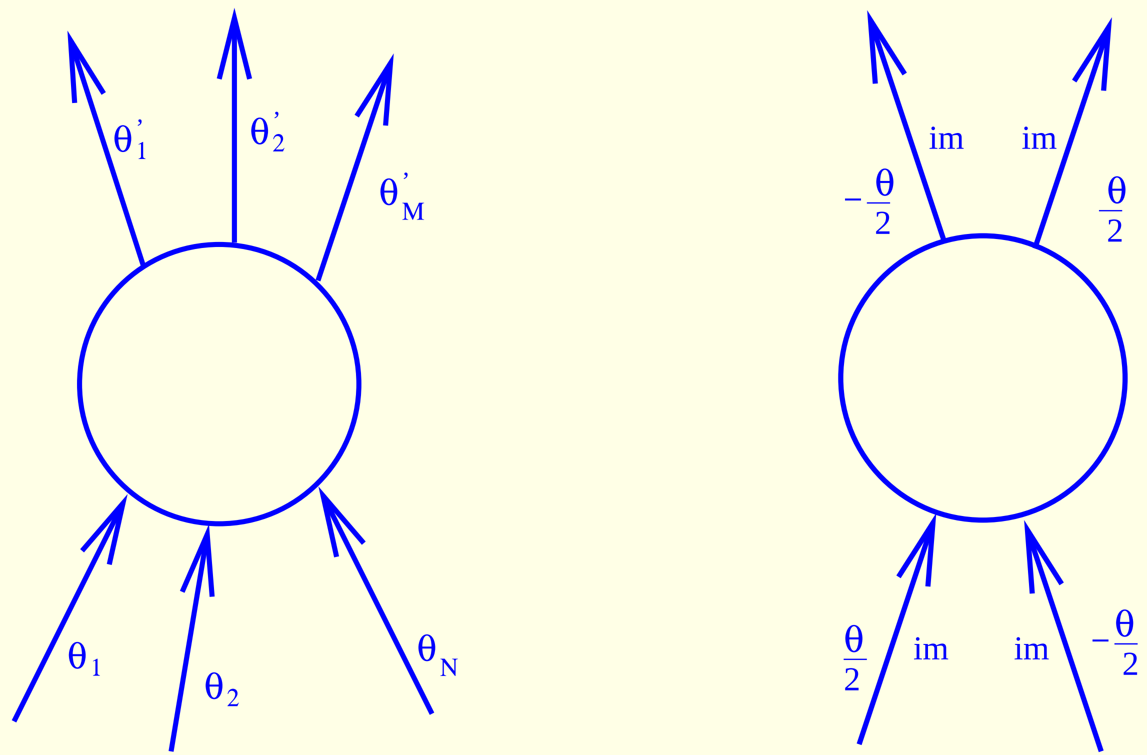
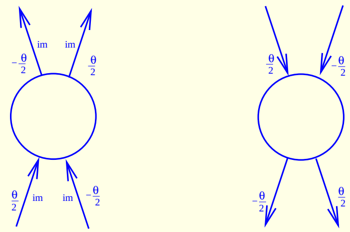
Integrability: shifting the trajectories →factorization + boostrap

The only nontrivial scattering matrix $S_2^2(|\theta_1-\theta_2|)=S(\theta)$ $\boxed{\theta>0}$



Integrable aspects: S-matrix bootsrap

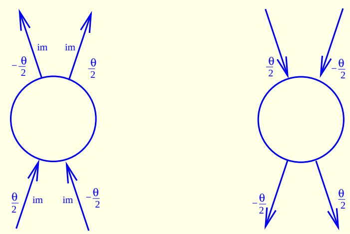
Unitarity



$$S(\theta)^{-1} = S(-\theta)$$

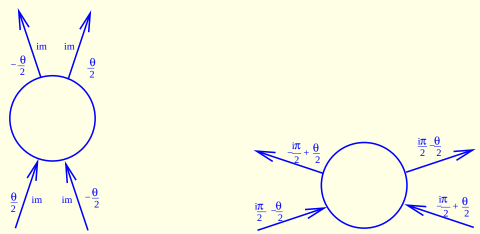
Integrable aspects: S-matrix bootsrap

Unitarity



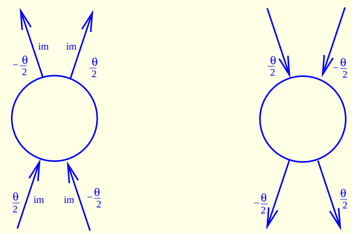
$$S(\theta)^{-1} = S(-\theta)$$

Crossing



$$S(\theta) = S(i\pi - \theta)$$

Integrable aspects: S-matrix bootsrap



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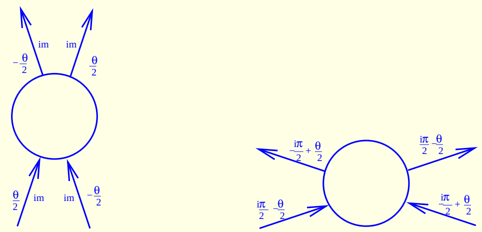
Unitarity



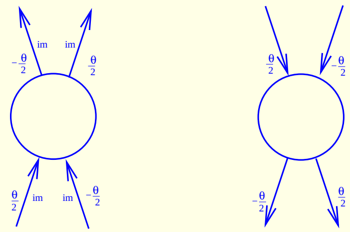
$$S(\theta) = S(i\pi - \theta)$$

Crossing

Simplest nontrivial solution $S(\theta) = \frac{\sinh\theta - i\sin p}{\sinh\theta + i\sin p}$

Integrable aspects: S-matrix bootsrap

Unitarity



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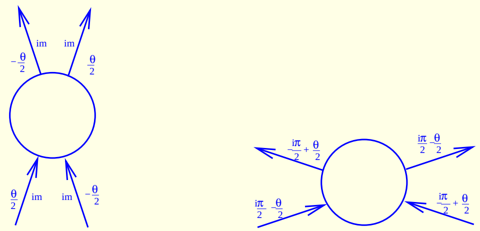
Crossing

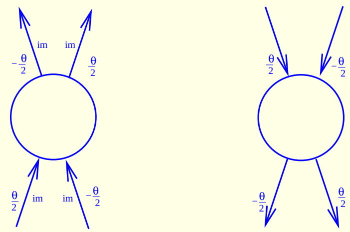


$$S(\theta) = S(i\pi - \theta)$$

Simplest nontrivial solution $S(\theta) = \frac{\sinh\theta - i\sin p}{\sinh\theta + i\sin p}$

Minimality: all singularity has physical origin: $p > 0$ end of the story (sinh-Gordon):

Integrable aspects: S-matrix bootsrap



Unitarity

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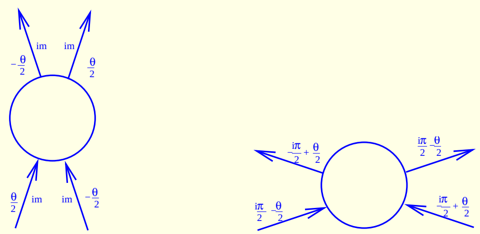

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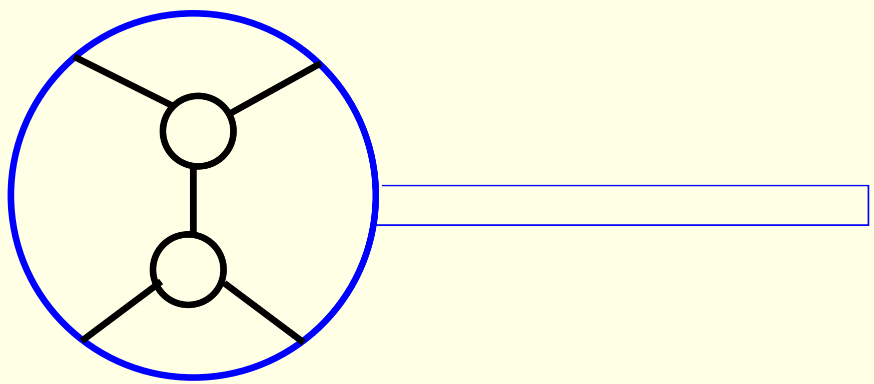
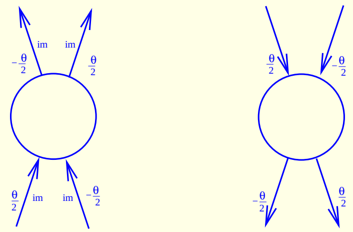


$p < 0$ boundstate

Integrable aspects: S-matrix bootsrap

Unitarity



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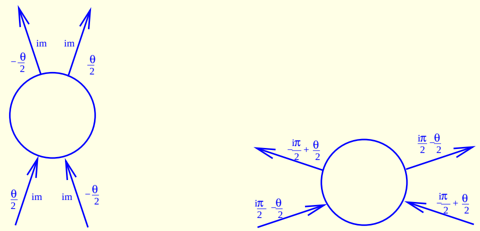
Crossing



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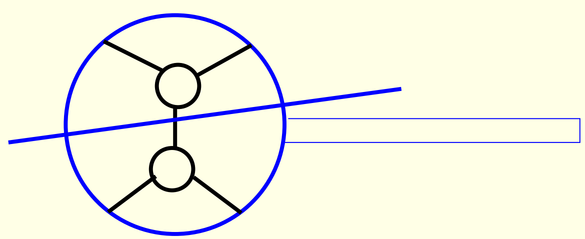
Simplest nontrivial solution $S(\theta) = \frac{\sinh\theta - i\sin p}{\sinh\theta + i\sin p}$

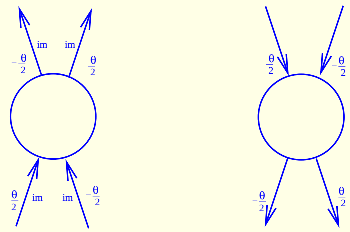
Minimality: all singularity has physical origin: $p > 0$ end of the story (sinh-Gordon):

Bootstrap :



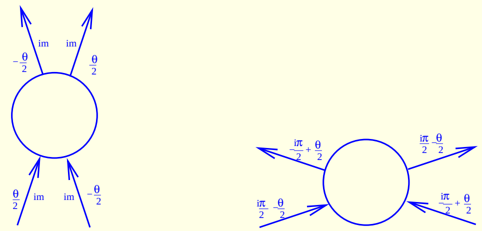
Integrable aspects: S-matrix bootsrap

Unitarity



$$S(\theta)^{-1} = S(-\theta)$$

Crossing



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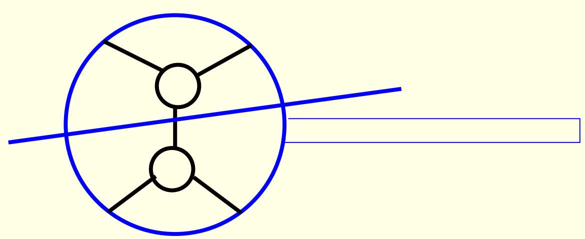

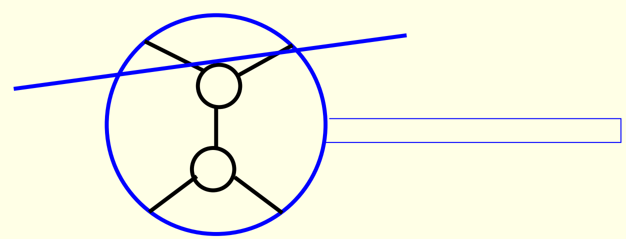

Integrable aspects: S-matrix bootsrap

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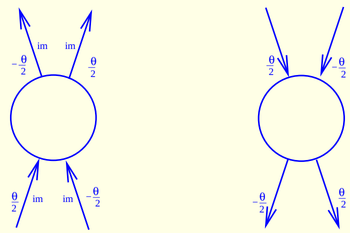

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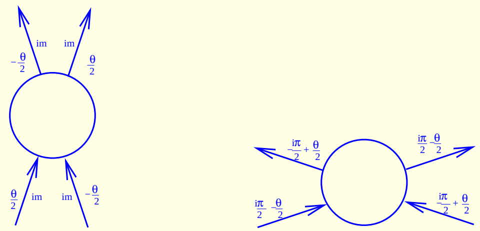

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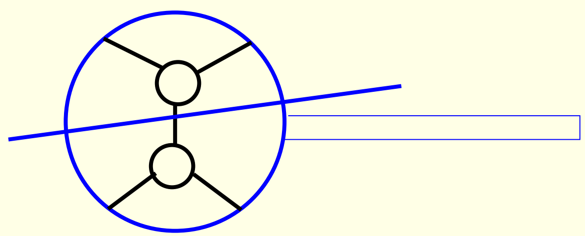
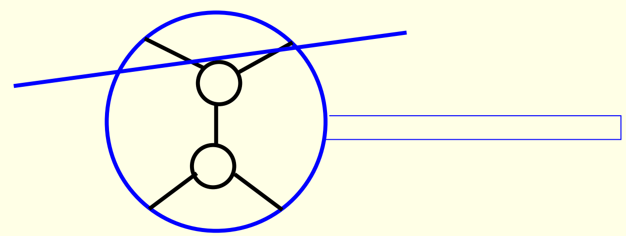
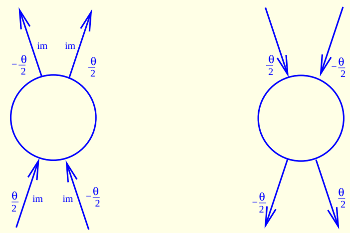


$S_{new}(\theta)$



Integrable aspects: S-matrix bootsrap

Unitarity



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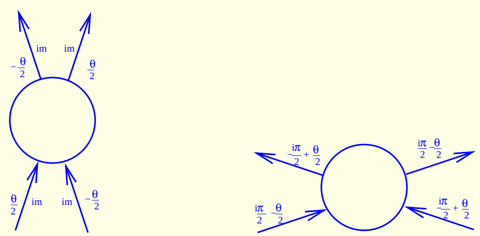

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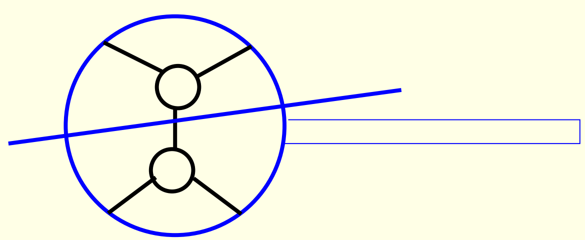

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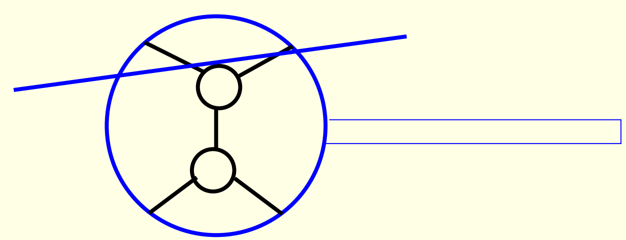
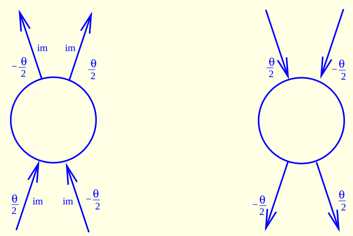


$S_{old}(\theta + iu)S_{old}(\theta - iu)$

Integrable aspects: S-matrix bootsrap

Unitarity



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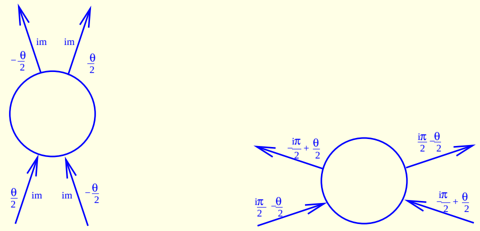

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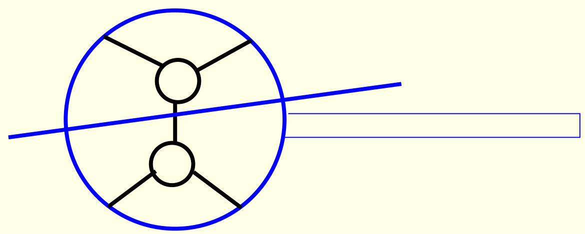
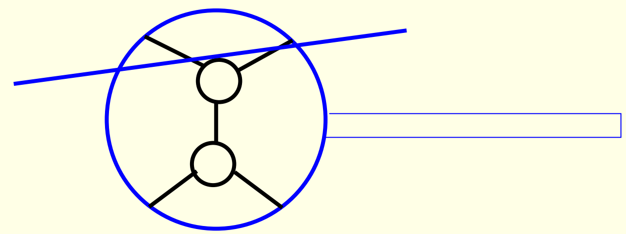


$S_{new}(\theta)$



$S_{old}(\theta + iu)S_{old}(\theta - iu)$

for $p = -\frac{2}{3}$ self-fusion: Lee-Yang,

for generic p sine-Gordon $B_2, B_3, \ldots, B_n, s, \bar{s}$

Form factor boostrap

Permutation

$$F_n^{\mathcal{O}}(\theta_1, \ldots \theta_i, \theta_{i+1}, \ldots, \theta_n) =$$
$$S(\theta_i - \theta_{i+1}) F_n^{\mathcal{O}}(\theta_1, \ldots \theta_{i+1}, \theta_i, \ldots, \theta_n)$$

Form factor boostrap

Permutation

$$F_n^{\mathcal{O}}(\theta_1, \ldots \theta_i, \theta_{i+1}, \ldots, \theta_n) =$$
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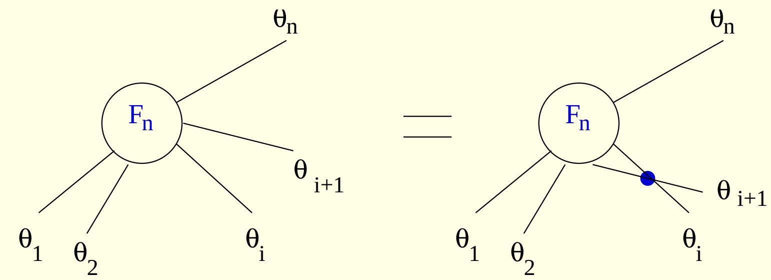

Form factor boostrap

Permutation

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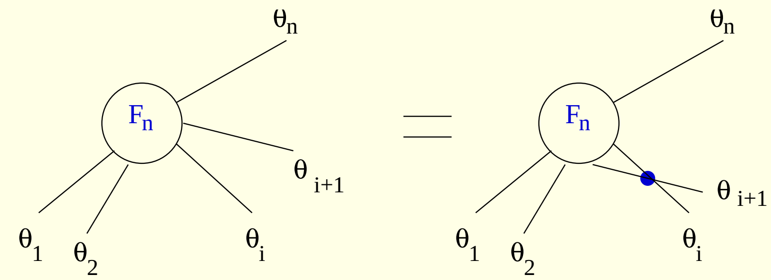

Periodicity

$$F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}, \theta_n) = F_n^{\mathcal{O}}(\theta_n + 2\pi i, \theta_1, \ldots, \theta_{n-1})$$

Form factor boostrap

Permutation

$$F_n^{\mathcal{O}}(\theta_1, \ldots \theta_i, \theta_{i+1}, \ldots, \theta_n) =$$
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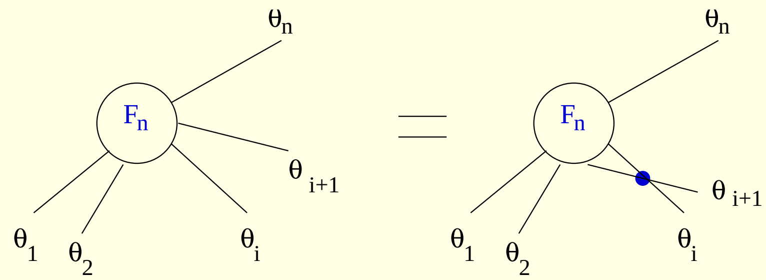

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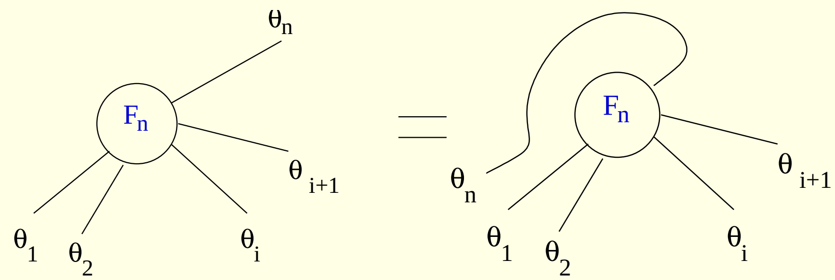

Form factor boostrap

Permutation

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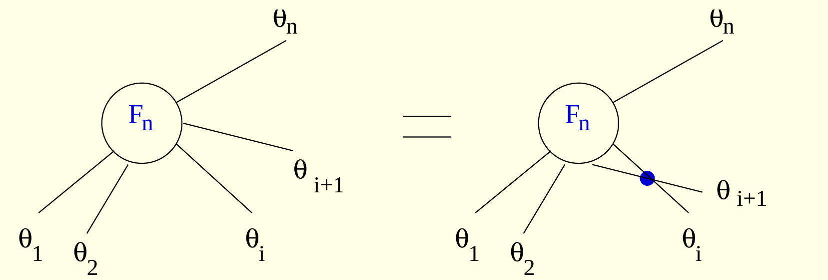

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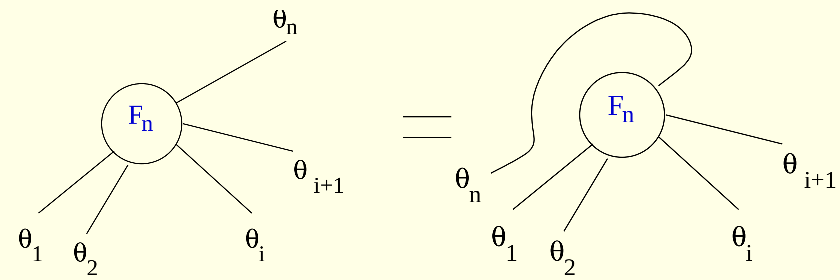


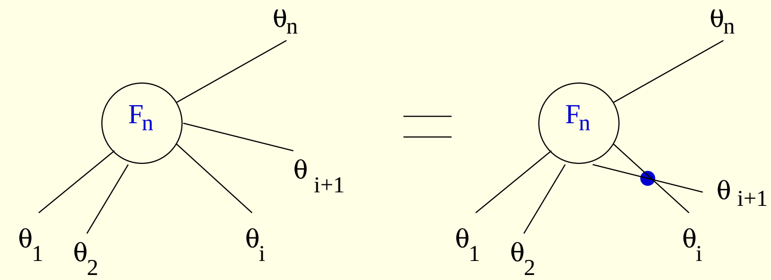
Kinematical singularities

$$i\,\mathrm{res}_{\theta=\theta'}F_{n+2}^{\mathcal{O}}(\theta+i\pi,\theta',\theta_1,\ldots,\theta_n) =$$
$$= \left(1-\prod_{i=1}^{n}S(\theta-\theta_i)\right)F_n^{\mathcal{O}}(\theta_1,\ldots,\theta_n)$$

Form factor boostrap

Permutation

$$F_n^{\mathcal{O}}(\theta_1,\dots\theta_i,\theta_{i+1},\dots,\theta_n) = S(\theta_i-\theta_{i+1})F_n^{\mathcal{O}}(\theta_1,\dots\theta_{i+1},\theta_i,\dots,\theta_n)$$



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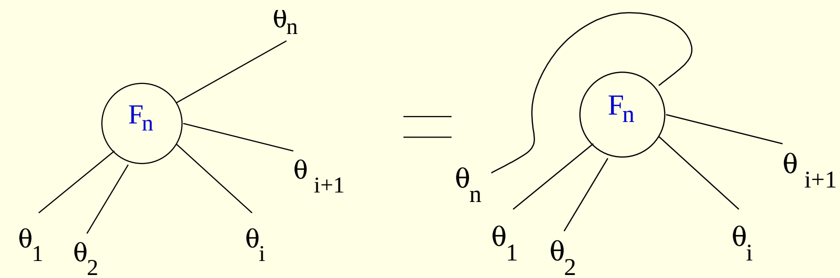

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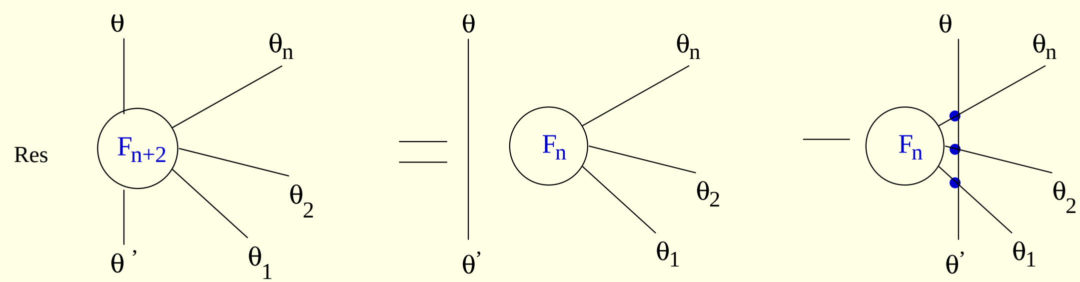

Form factor boostrap

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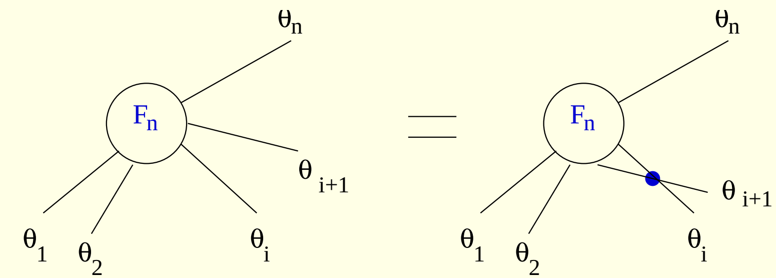

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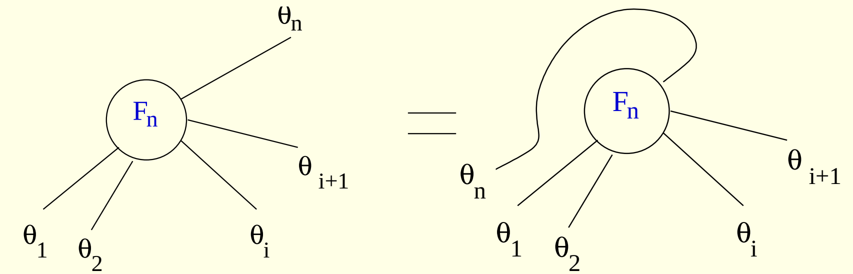

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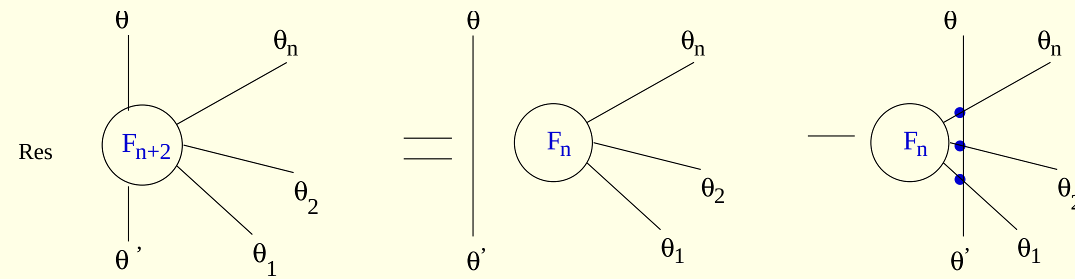


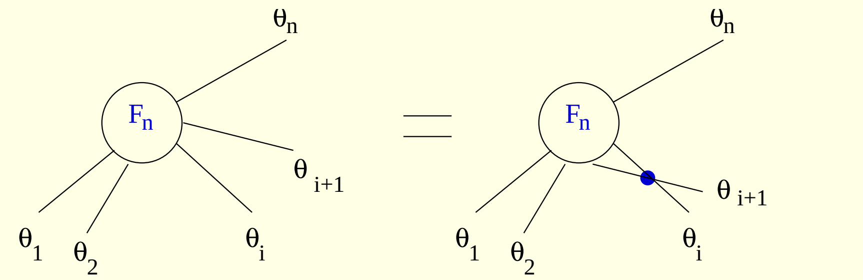
Dynamical singularities

$$i\mathrm{res}_{\theta=\theta'}F_{n+2}^{\mathcal{O}}(\theta+iu,\theta'-iu,\theta_1,\ldots,\theta_n) = \Gamma F_{n+1}^{\mathcal{O}}(\theta,\theta_1,\ldots,\theta_n)$$

Form factor boostrap

Permutation

$$F_n^{\mathcal{O}}(\theta_1,\dots\theta_i,\theta_{i+1},\dots,\theta_n) =$$
$$S(\theta_i-\theta_{i+1})F_n^{\mathcal{O}}(\theta_1,\dots\theta_{i+1},\theta_i,\dots,\theta_n)$$



Periodicity

$$F_n^{\mathcal{O}}(\theta_1,\dots,\theta_{n-1},\theta_n) =$$
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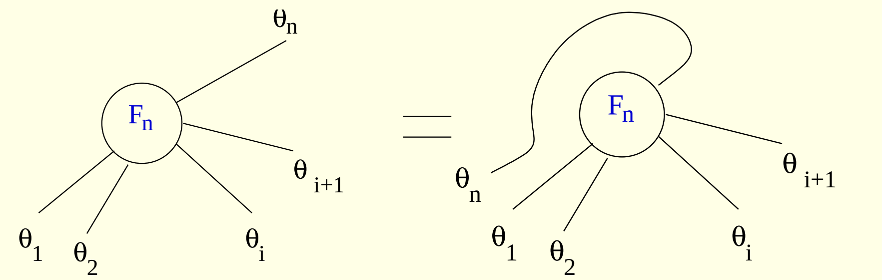

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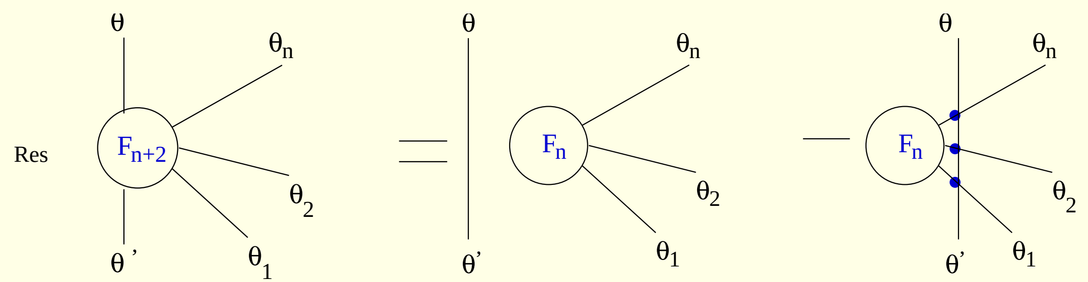

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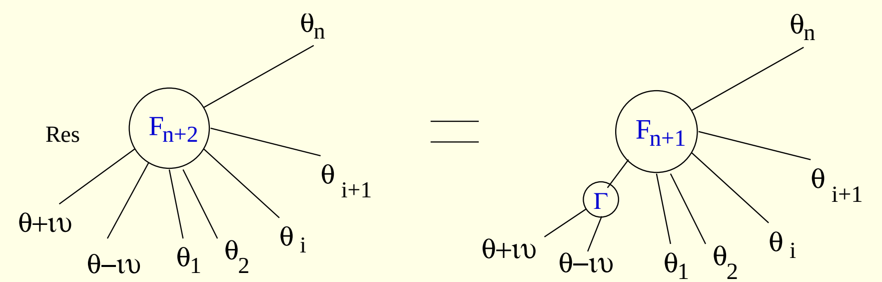

Form factor boostrap

Permutation

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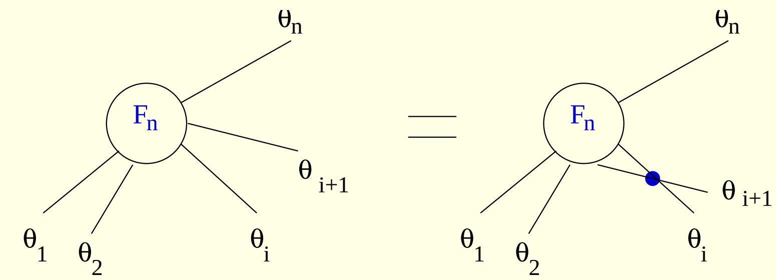

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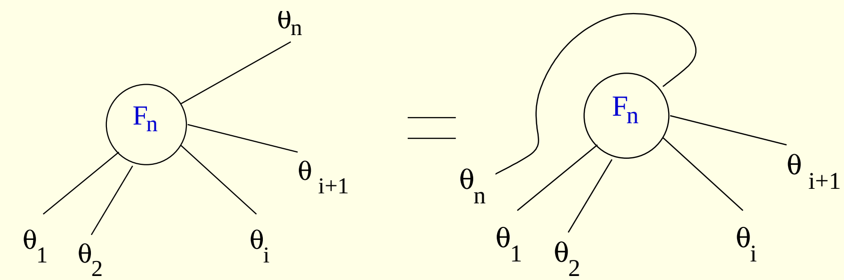

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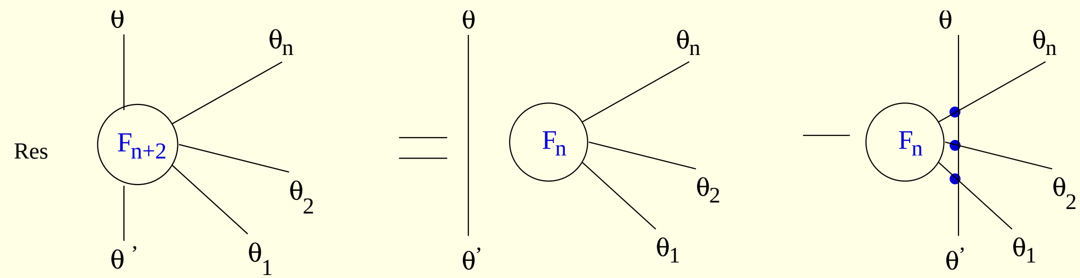

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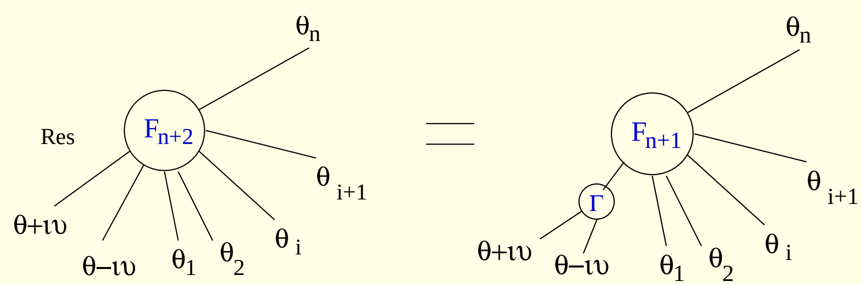
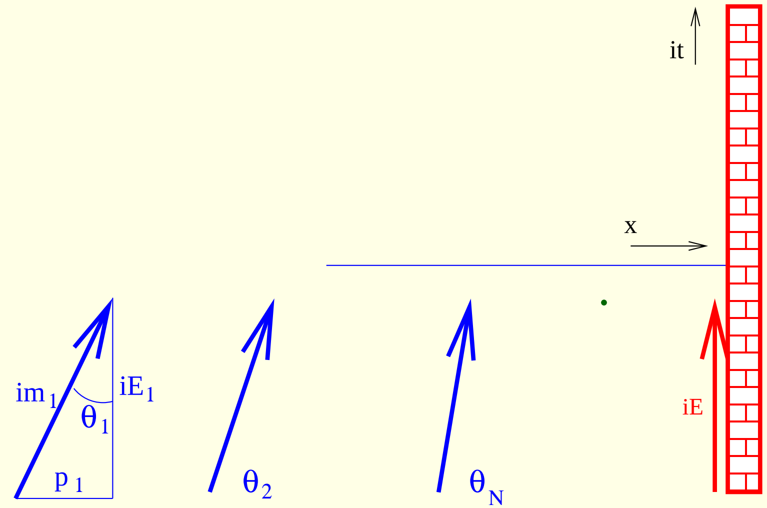


Form factor solutions for generic p: sinh-Gordon, Lee-Yang, B_1form factor in sine-Gordon

Boundary Hilbert space

Boundary initial state

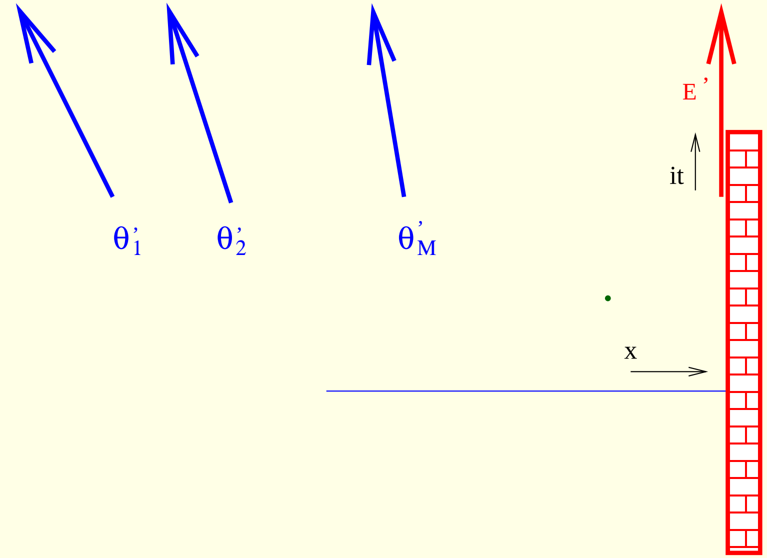
$$|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N > 0$$



Boundary Hilbert space

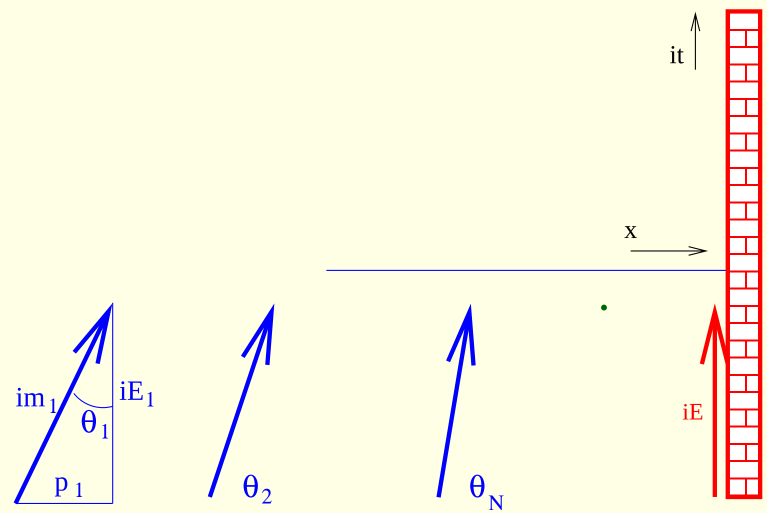
Boundary final state

$$|\theta_1', \theta_2', \ldots, \theta_M'; E'\rangle_B^{out}$$
$$\theta_1' < \theta_2' < \ldots < \theta_M' < 0$$



Boundary initial state

$$|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N > 0$$



Reflection matrix

Boundary final state

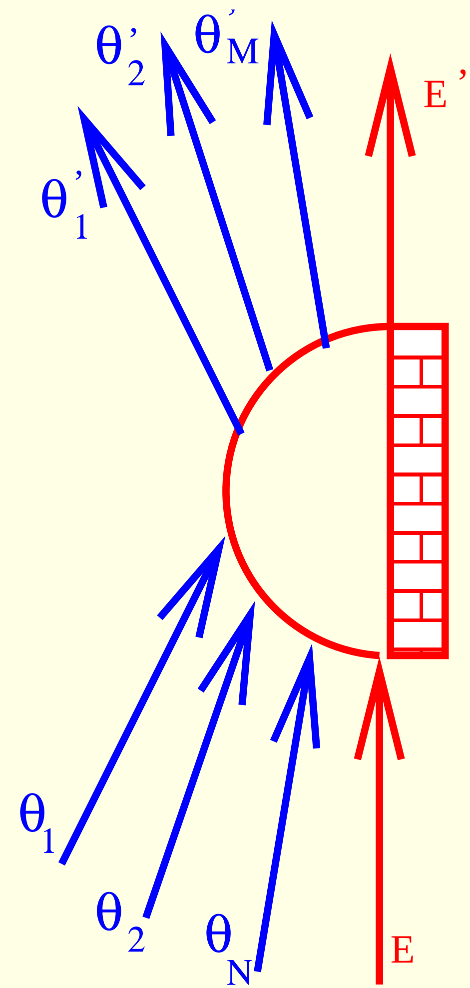
$$|\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}; E^{'}\rangle_B^{out}$$
$$\theta_1^{'} < \theta_2^{'} < \ldots < \theta_M^{'} < 0$$

Reflection matrix

$$R_N^M(\{\theta; E\}, \{\theta^{'}; E^{'}\}) =$$
$${}_B^{out}\langle\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}; E^{'}|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$

Boundary initial state

$$|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N > 0$$

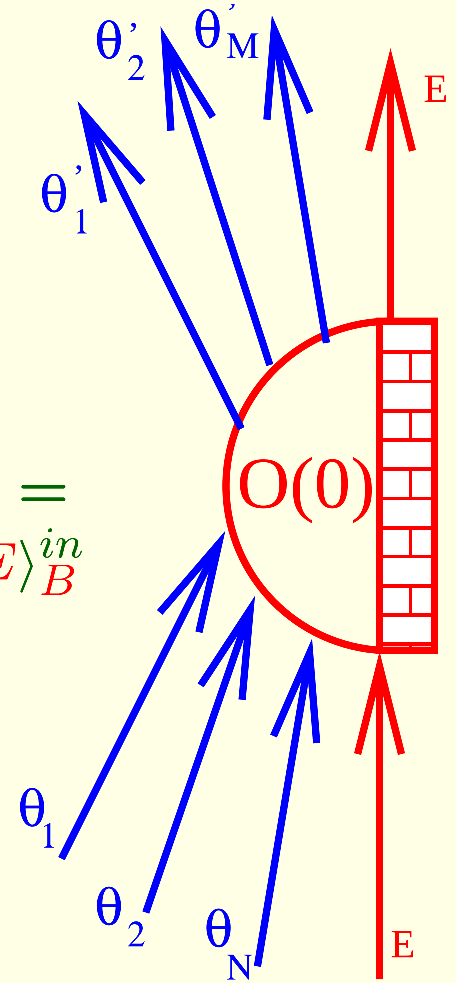


Analytic structure?

Boundary form factors

Boundary final state

$$|\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}; E^{'}\rangle_B^{out}$$
$$\theta_1^{'} < \theta_2^{'} < \ldots < \theta_M^{'} < 0$$



Boundary form factors

$$R_N^M(\{\theta; E\}, \{\theta^{'}; E^{'}\}) =$$
$${}_B^{out}\langle\theta_1^{'}, \theta_2^{'}, \ldots, \theta_M^{'}; E^{'}|\mathcal{O}(0,0)|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$

Boundary initial state

$$|\theta_1, \theta_2, \ldots, \theta_N; E\rangle_B^{in}$$
$$\theta_1 > \theta_2 > \ldots > \theta_N > 0$$

Analytic structure?

Analytic structure: boundary reduction formula

Analytic structure: boundary reduction formula

need: analytic structure of	↔	boundary reduction formula	↔	analytic structure of
R matrix				boundary correlators

Analytic structure: boundary reduction formula

need: analytic structure of R matrix ↔ boundary reduction formula ↔ analytic structure of boundary correlators

$$ {}_B < out|\mathcal{O}(it)|\theta_1, \theta_2, \ldots, \theta_n >_B = 2\pi\delta(\theta_1 - \theta)\ {}_B < out \setminus \theta|\mathcal{O}(it)|\theta_2, \ldots, \theta_n >_B $$

Analytic structure: boundary reduction formula

need: analytic structure of	↔	boundary reduction formula	↔	analytic structure of
R matrix				boundary correlators

$${}_B < out|\mathcal{O}(it)|\theta_1, \theta_2, \ldots, \theta_n >_B = 2\pi\delta(\theta_1 - \theta)\ {}_B < out \setminus \theta|\mathcal{O}(it)|\theta_2, \ldots, \theta_n >_B$$

$$-2Z^{-1/2} \int_{-\infty}^{\infty} d(it') \, e^{i(i\omega(\theta_1))(it')} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x')\{-\partial^2_{it'} - \partial^2_{x'} + m^2 + \delta(x')\partial_{x'}\}$$

Analytic structure: boundary reduction formula

need: analytic structure of R matrix $\leftrightarrow$ boundary reduction formula $\leftrightarrow$ analytic structure of boundary correlators

$$ {}_B < out|\mathcal{O}(it)|\theta_1,\theta_2,\dots,\theta_n >_B = 2\pi\delta(\theta_1-\theta)\ {}_B < out\setminus\theta|\mathcal{O}(it)|\theta_2,\dots,\theta_n >_B $$

$$ -2Z^{-1/2}\int_{-\infty}^{\infty} d(it')\, e^{i(i\omega(\theta_1))(it')}\int_{-\infty}^{0} dx' \cos(p(\theta_1)x')\{-\partial^2_{it'}-\partial^2_{x'}+m^2+\delta(x')\partial_{x'}\} $$

$$ {}_B < out|T(\Phi(x',it')\mathcal{O}(it))|\theta_2,\dots,\theta_n >_B $$

Analytic structure: boundary reduction formula

need: analytic structure of R matrix $\leftrightarrow$ boundary reduction formula $\leftrightarrow$ analytic structure of boundary correlators

$$_B < out|\mathcal{O}(it)|\theta_1, \theta_2, \dots, \theta_n >_B = 2\pi\delta(\theta_1 - \theta)\ _B < out \setminus \theta|\mathcal{O}(it)|\theta_2, \dots, \theta_n >_B$$

$$-2Z^{-1/2} \int_{-\infty}^{\infty} d(it')\, e^{i(i\omega(\theta_1))(it')} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x')\{-\partial_{it'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'}\}$$

$$_B < out|T(\Phi(x', it')\mathcal{O}(it))|\theta_2, \dots, \theta_n >_B$$

Analytic continuation in θ_1:

(left/right matrix element) $\longrightarrow$ continuation: $\theta \to i\pi + \theta$

Crossing ?

Analytic structure: boundary reduction formula

need: analytic structure of R matrix	$\leftrightarrow$	boundary reduction formula	$\leftrightarrow$	analytic structure of boundary correlators

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Analytic continuation in θ_1:

(left/right matrix element) $\longrightarrow$ continuation: $\theta \rightarrow i\pi+\theta$

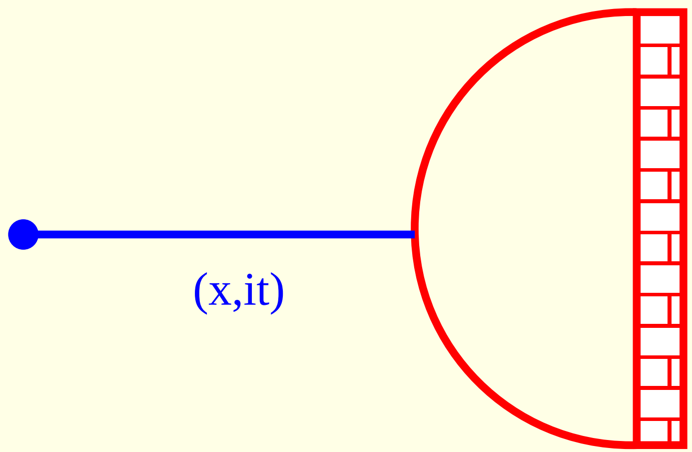
Crossing ?

Analytical properties of the correlators: Landau equations

Boundary Correlation functions

Boundary Correlation functions

One point function ${}_B\langle 0|\Phi(x,it)|0\rangle_B = G_B^1(x,it) = G_B^1(x)$



Boundary Correlation functions

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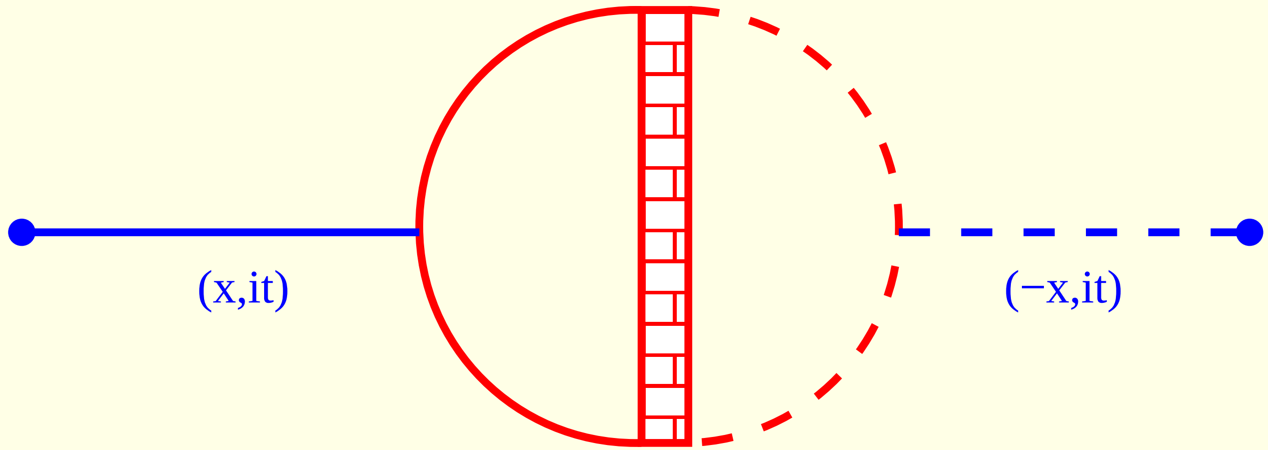

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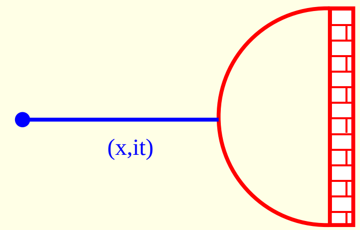

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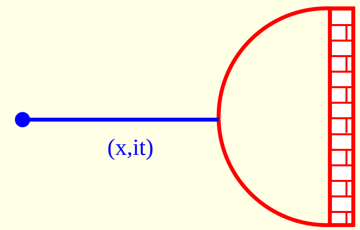
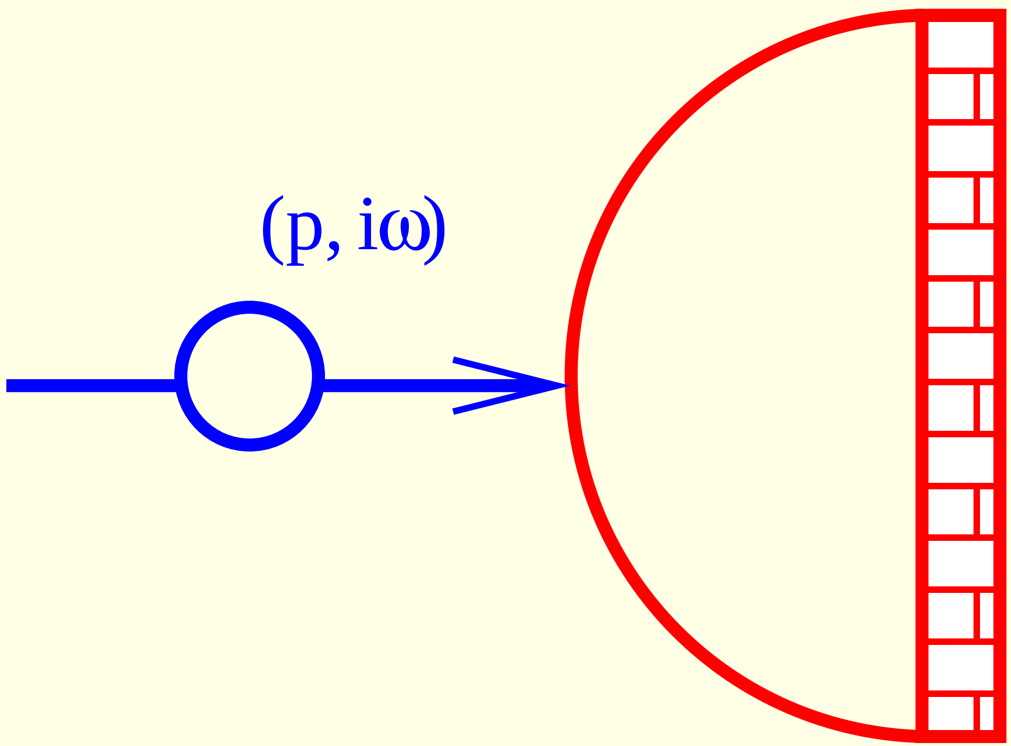


Momentum space formulation ${}_B\langle 0|\Phi(x,it)|0\rangle_B = \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} dt e^{i(i\omega)(it)} G_B^1(p,i\omega)$

$$G_B^1(p,i\omega) = (2\pi)\delta(i\omega)G^2(p,i\omega)G_B^1(p)$$



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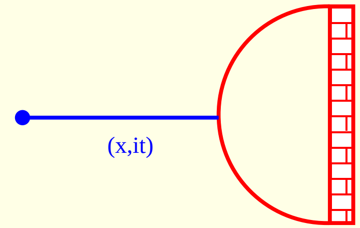
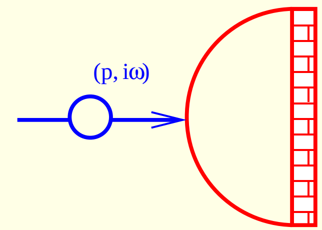


$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} dt e^{i(i\omega)(it)} G_B^1(p, i\omega)$$



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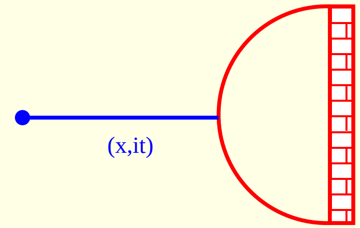

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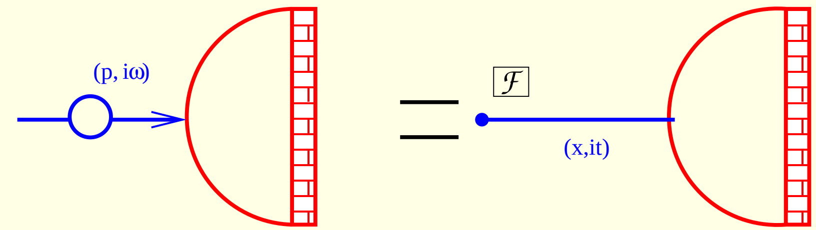

Boundary Correlation functions

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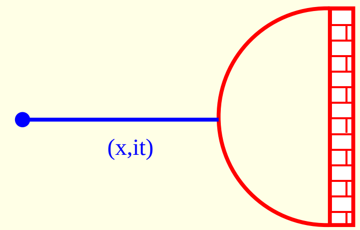

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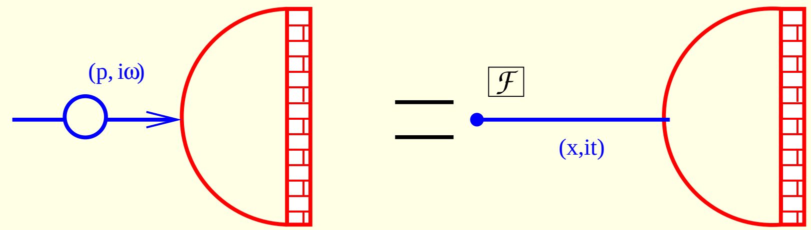
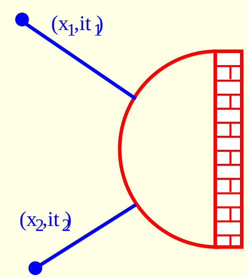


Two point function

$$_B\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle_B = G_B^2(x_1,x_2,it_1-it_2)$$



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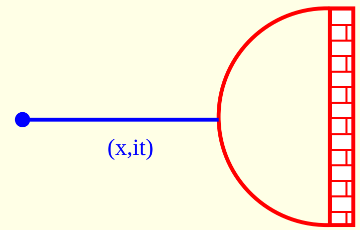
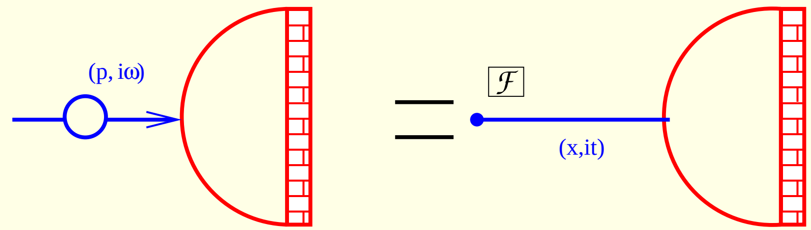
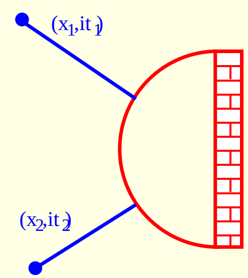


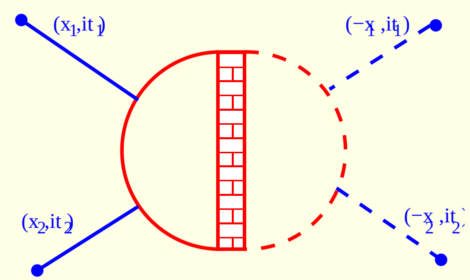
$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} dt e^{i(i\omega)(it)} G^1_B(p,i\omega)$$



Two point function

$${}_B\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle_B = G^2_B(x_1,x_2,it_1-it_2)$$





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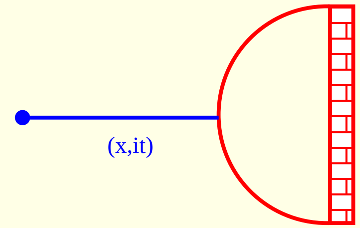
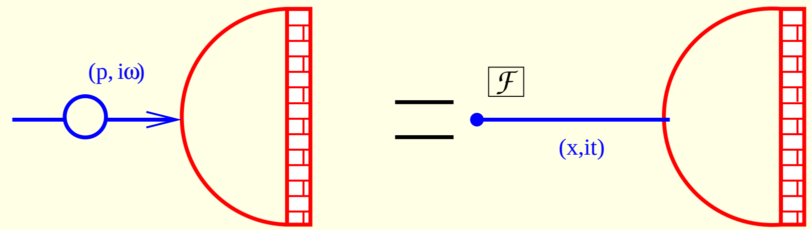


$$= \int_{-\infty}^{\infty} dx e^{ipx} \int_{-\infty}^{\infty} dt e^{i(i\omega)(it)} G_B^1(p,i\omega)$$



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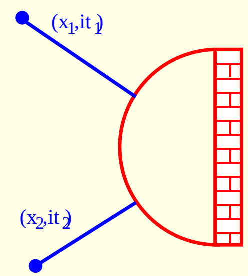

$${}_B\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle_B = G_B^2(x_1,x_2,it_1-it_2)$$

$$G_B^2(p_1,i\omega_1,p_2,i\omega_2) = (2\pi)\delta(i\omega_1+i\omega_2)G_B^2(p_1,p_2,i\omega_1)$$

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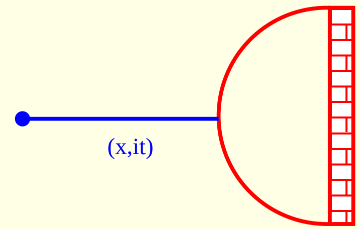

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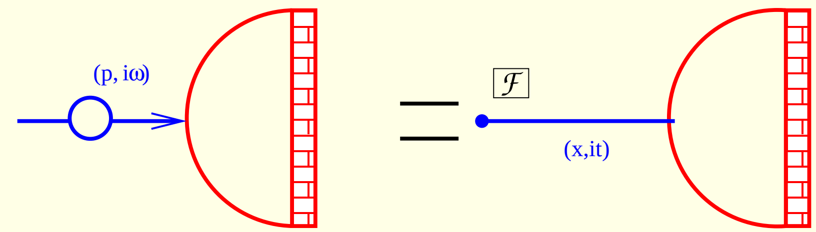

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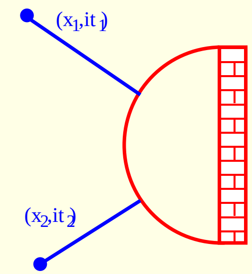

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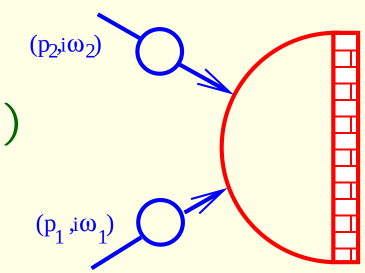

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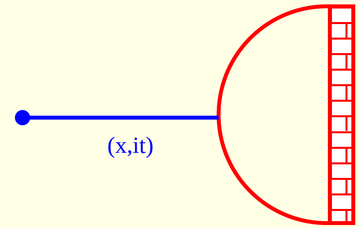

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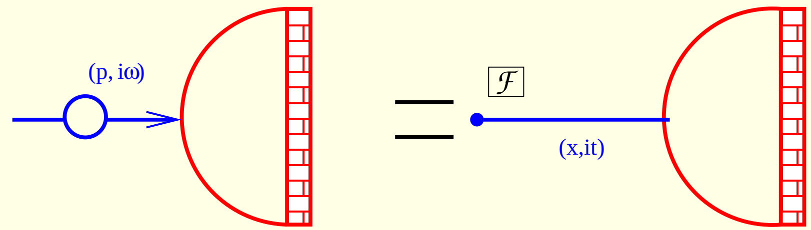

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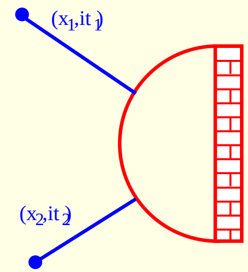

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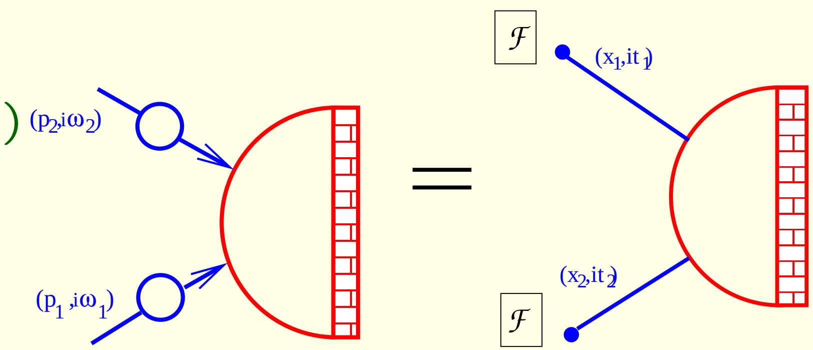

Boundary reduction formula revisited

Boundary reduction formula revisited

$$\boxed{{}_B < \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B} = \boxed{\delta(\theta_1 - \theta)\, {}_B < \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B}$$

Boundary reduction formula revisited

$$ {}_B< \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta)\, {}_B< \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B $$

$$ i2Z^{-1/2} \int_{-\infty}^{\infty} dt' \, e^{-i\omega(\theta_1)t'} \int_{-\infty}^{0} dx' \cos(p(\theta_1)x') \{ \partial_{t'}^2 - \partial_{x'}^2 + m^2 + \delta(x')\partial_{x'} \} $$

Boundary reduction formula revisited

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$$ {}_B< \theta_2, \ldots, \theta_n | T(\Phi(x^{'}, t^{'})\mathcal{O}(t)) | in >_B $$

Boundary reduction formula revisited

$$_B < \theta_1, \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in >_B = \delta(\theta_1 - \theta) \, _B < \theta_2, \ldots, \theta_n | \mathcal{O}(t) | in \setminus \theta >_B$$

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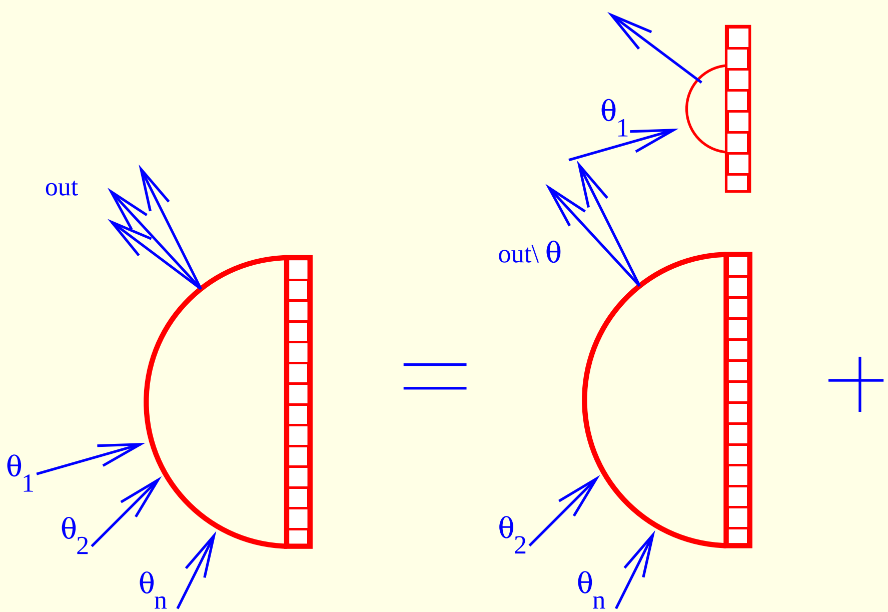
Diagrammatically



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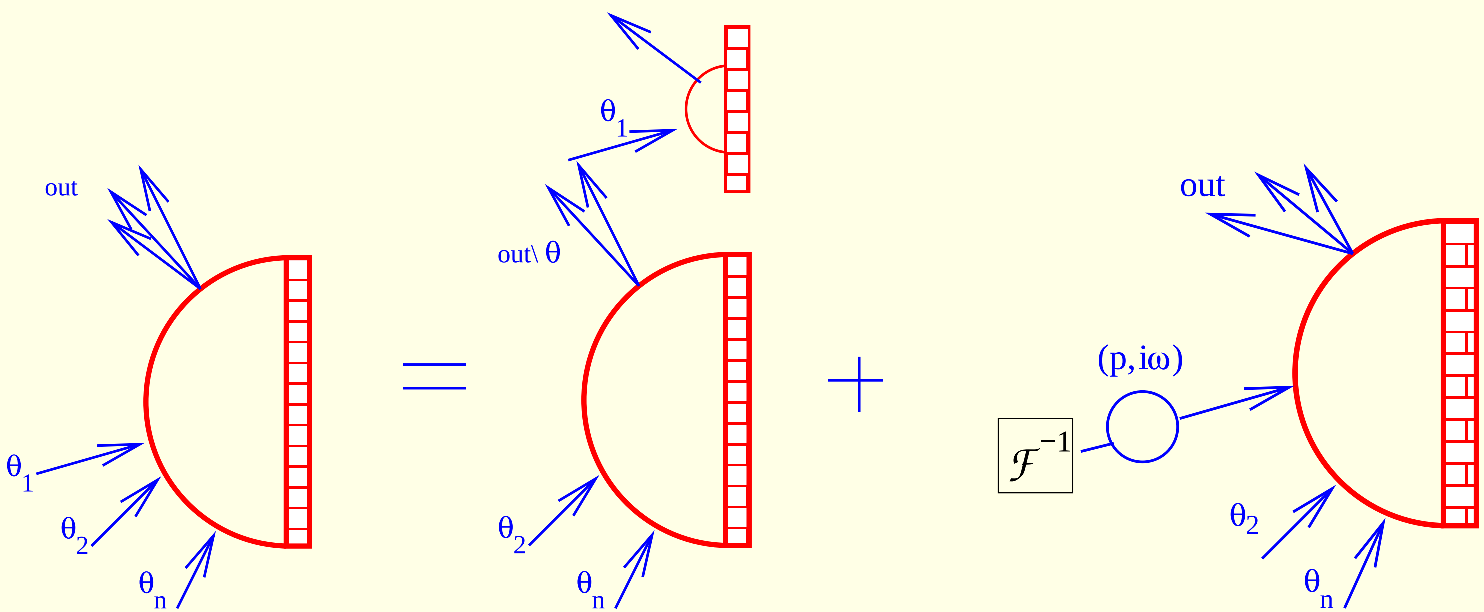

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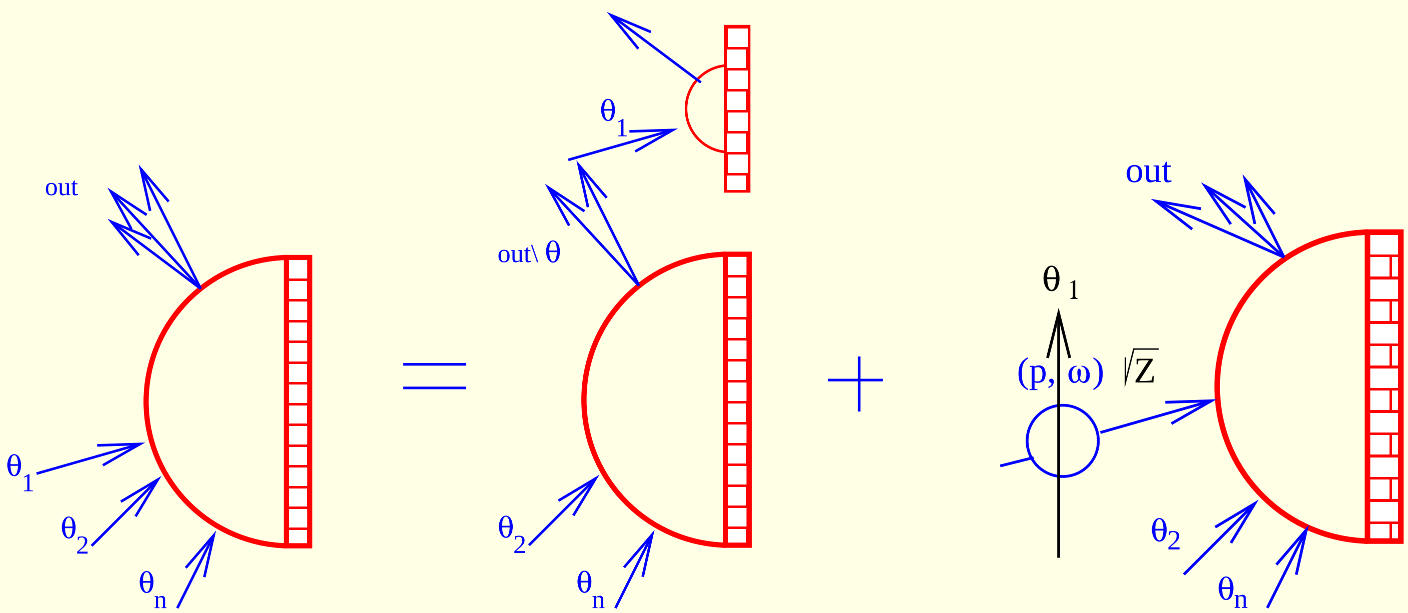
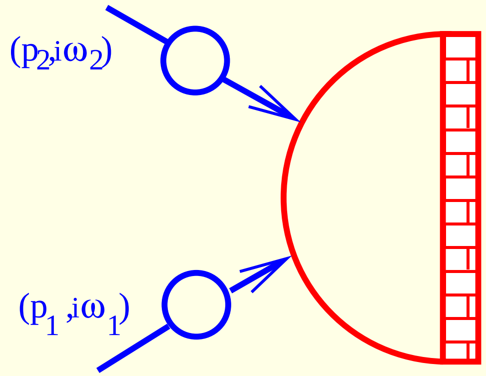


Singularity structure: Boundary Coleman-Thun mechanism

Singularity structure: Boundary Coleman-Thun mechanism

Singularity in correlation function $=$ on mass shell particles



Singularity structure: Boundary Coleman-Thun mechanism

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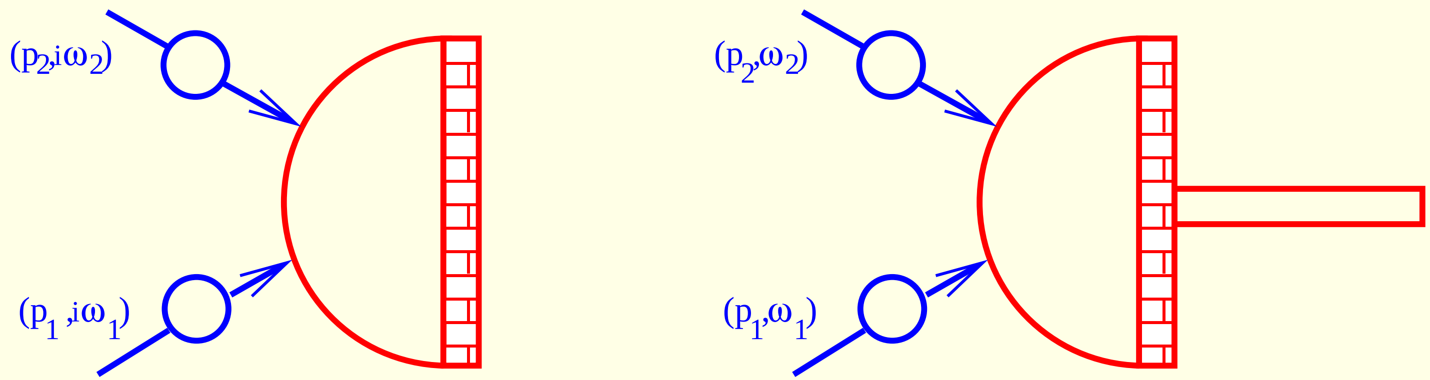

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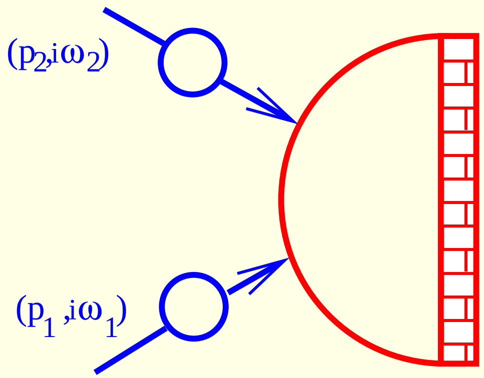

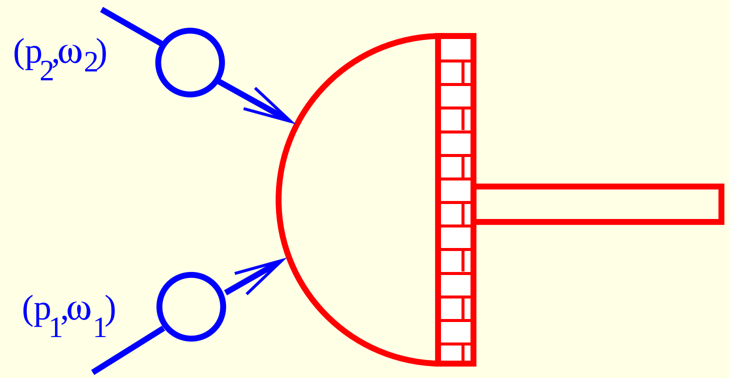

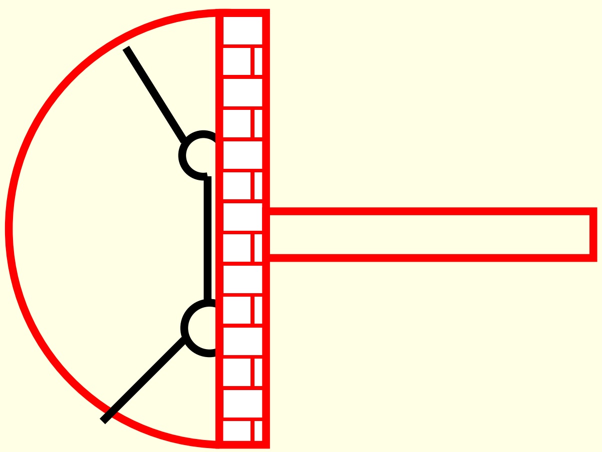

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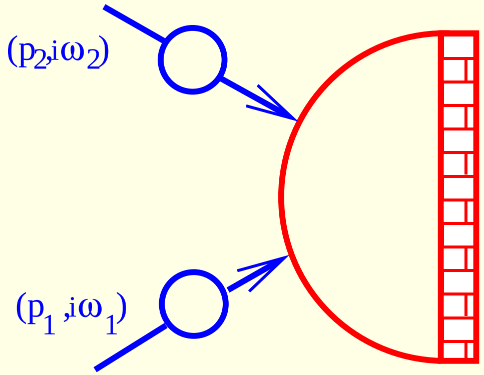
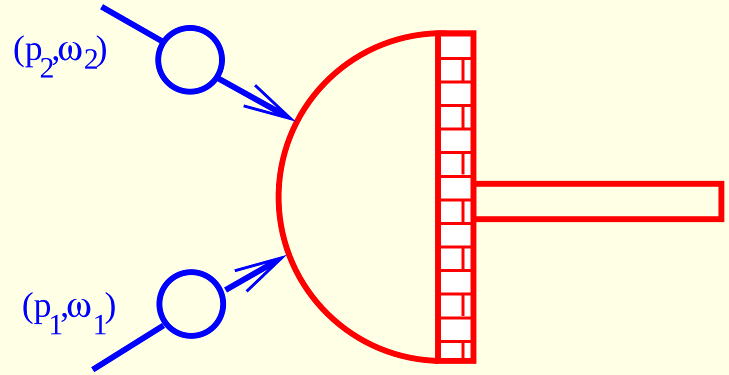
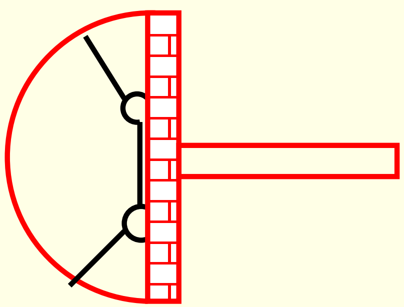


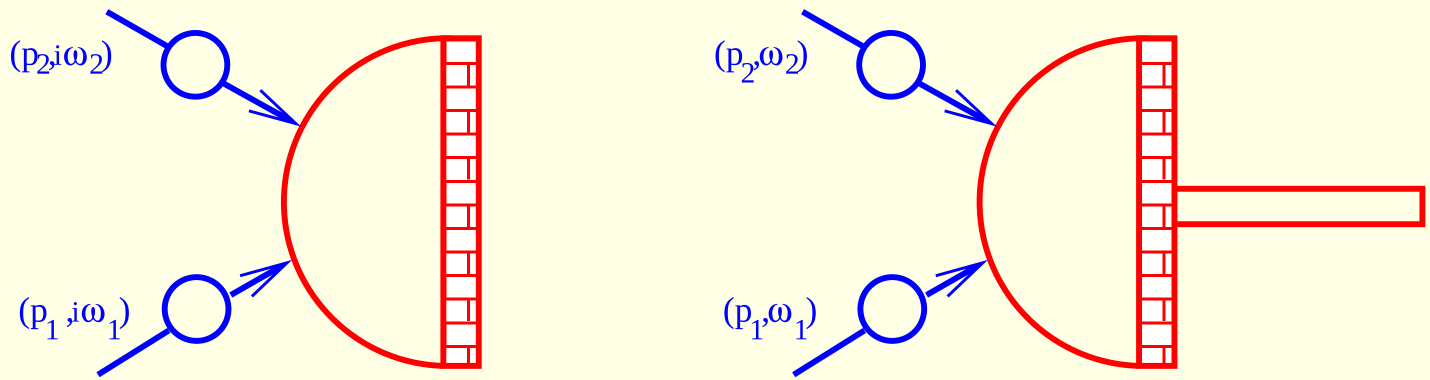


Coleman-Norton type interpretation



Singularity structure: Boundary Coleman-Thun mechanism

Singularity in correlation function $=$ on mass shell particles



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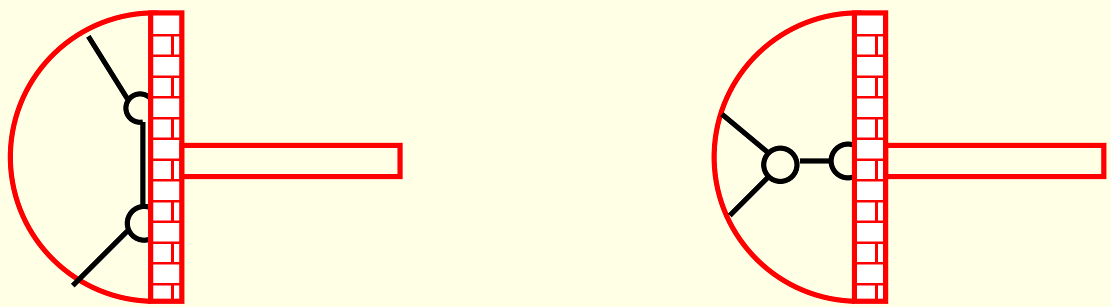
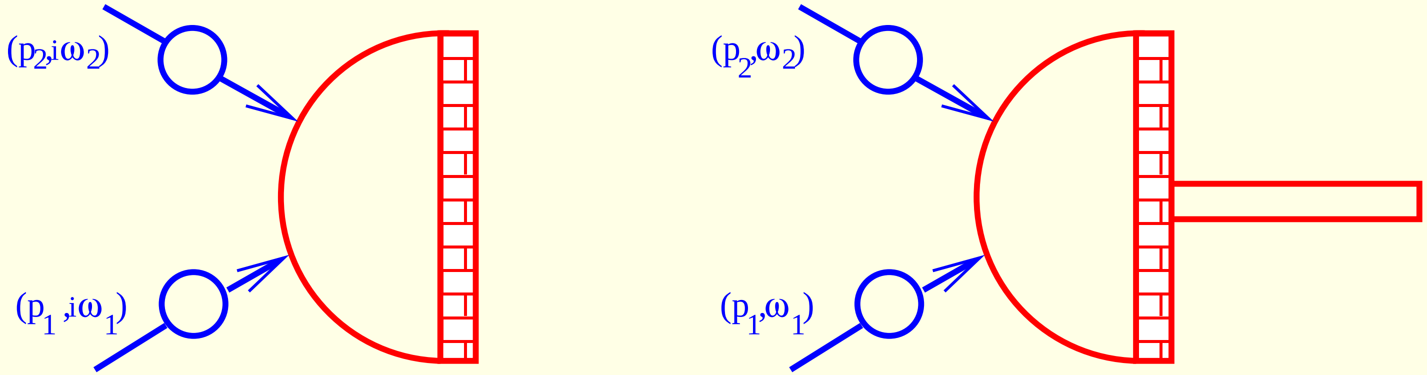
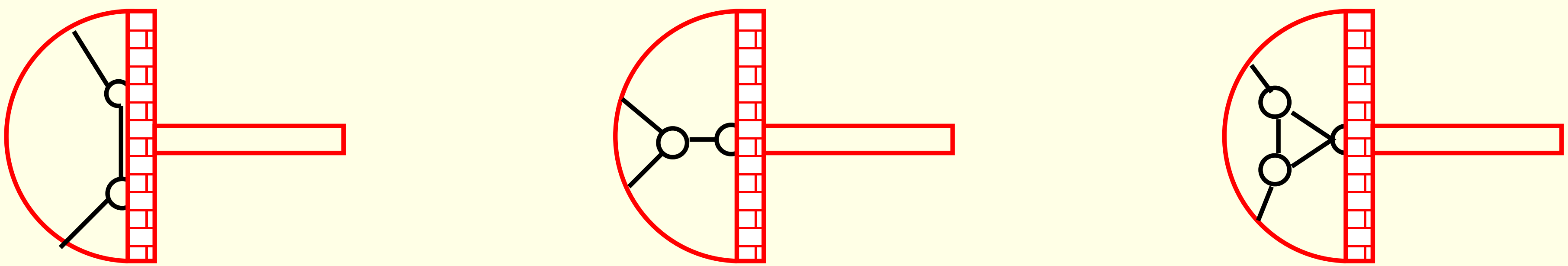


Singularity structure: Boundary Coleman-Thun mechanism

Singularity in correlation function = on mass shell particles

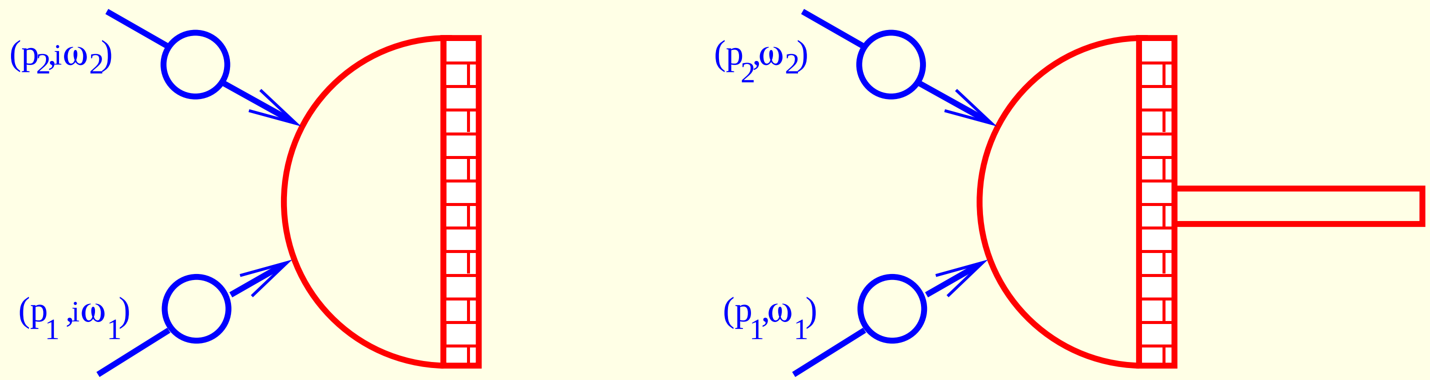


Coleman-Norton type interpretation

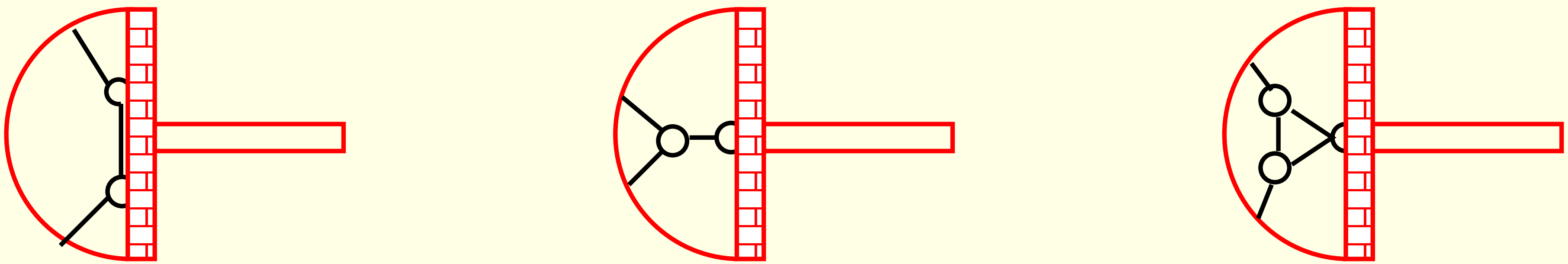


Singularity structure: Boundary Coleman-Thun mechanism

Singularity in correlation function= on mass shell particles



Coleman-Norton type interpretation

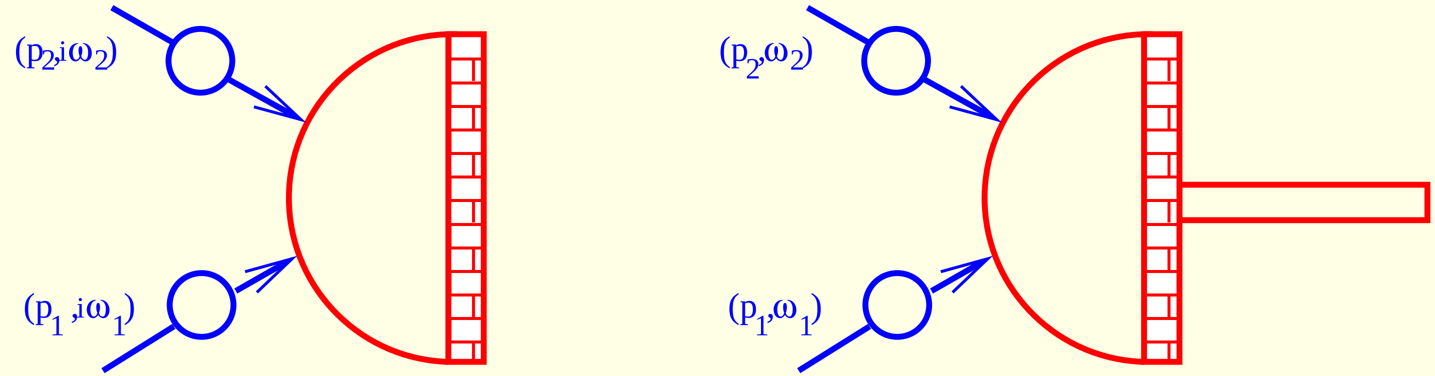


Cutkosky type rules

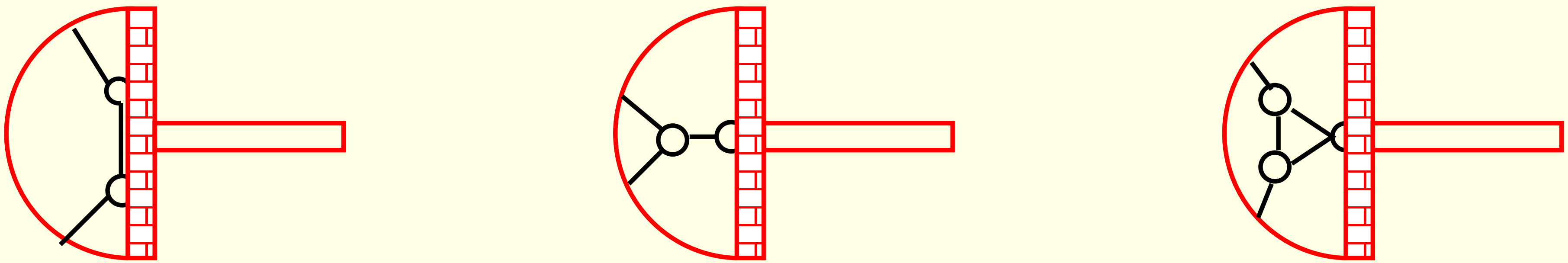
$(G_B^1(u_1))^2$

Singularity structure: Boundary Coleman-Thun mechanism

Singularity in correlation function= on mass shell particles



Coleman-Norton type interpretation



Cutkosky type rules

$(G^1_B(u_1))^2$ $\qquad$ $(G^3(u_1))G^1_B(0)$

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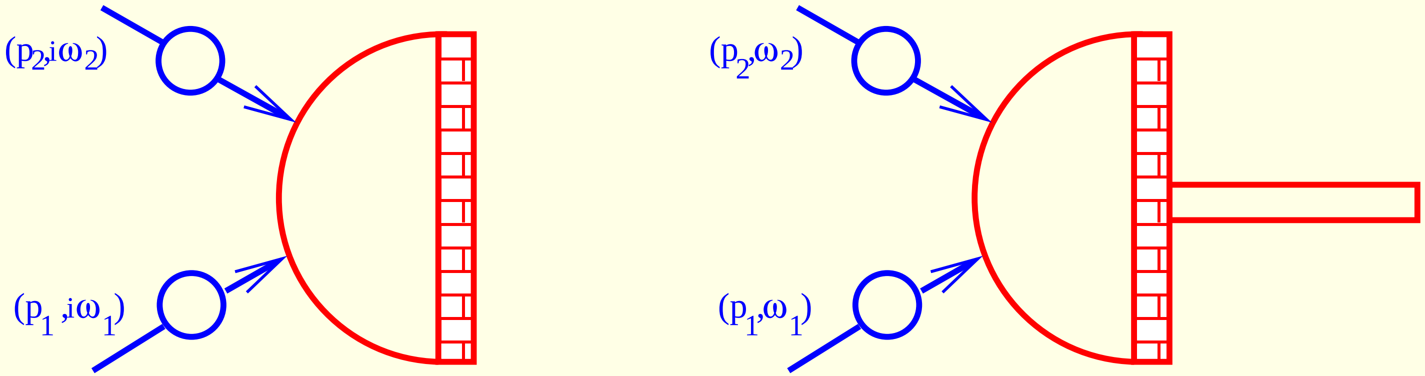

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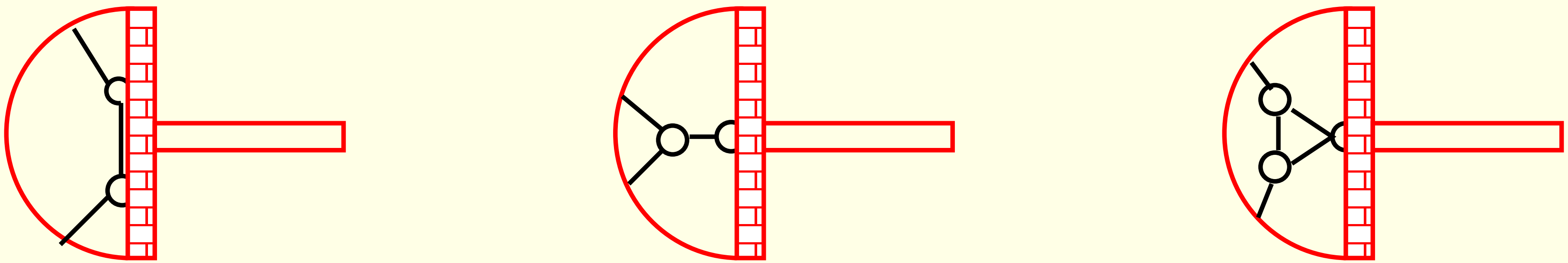

Cutkosky type rules

$(G^1_B(u_1))^2$ $(G^3(u_1))G^1_B(0)$ $(G^3(u_2))^2G^2_B(u_3)$

Integrable aspects: R-matrix bootsrap

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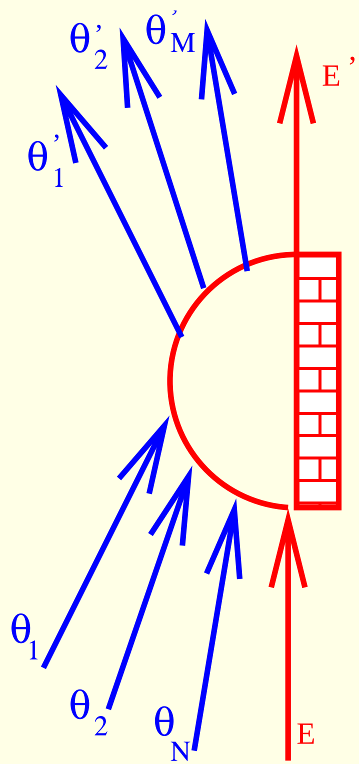
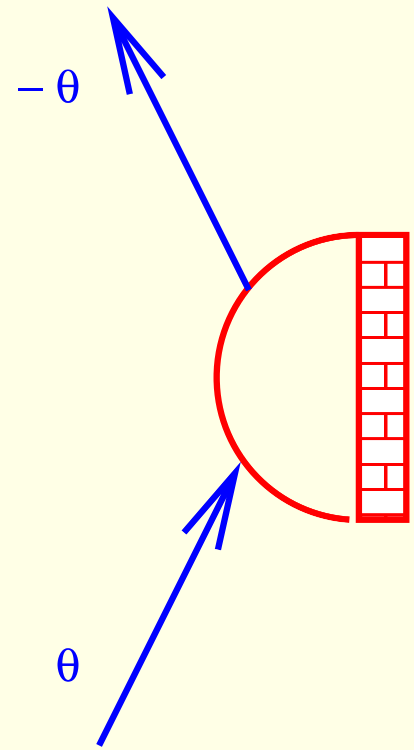
Integrability: shifting the trajectories $\rightarrow$factorization + boostrap

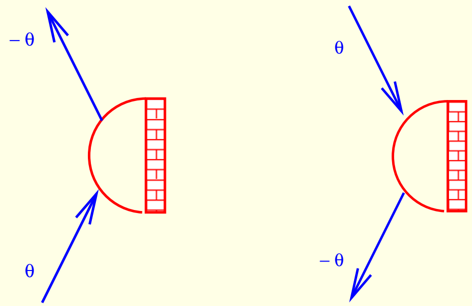
The only nontrivial is the 1pt reflection matrix $R(|\theta_1|) = R(\theta)$ $\boxed{\theta > 0}$





Integrable aspects: R-matrix bootsrap

Unitarity



$$R(\theta)^{-1} = R(-\theta)$$

Integrable aspects: R-matrix bootsrap

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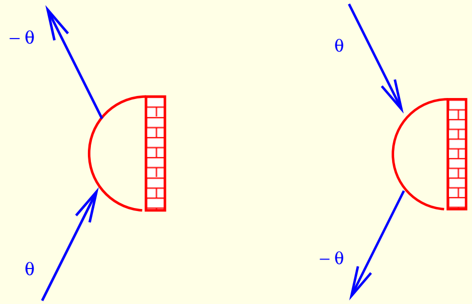

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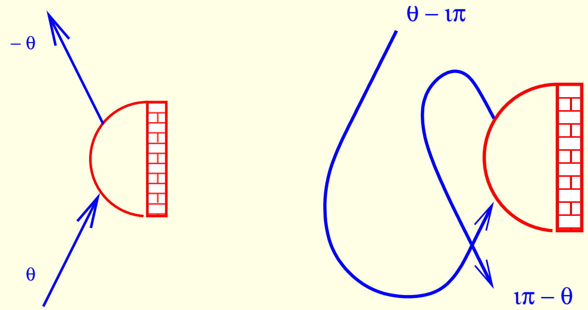
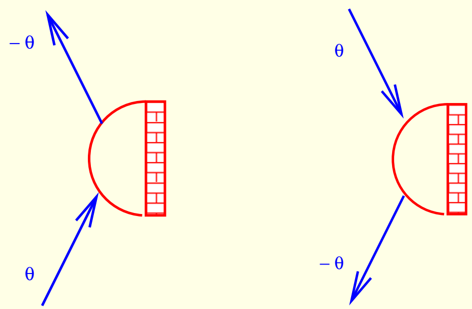
Crossing



$$R(\theta) = S(i\pi - 2\theta)R(i\pi - \theta)$$

Integrable aspects: R-matrix bootsrap

Unitarity



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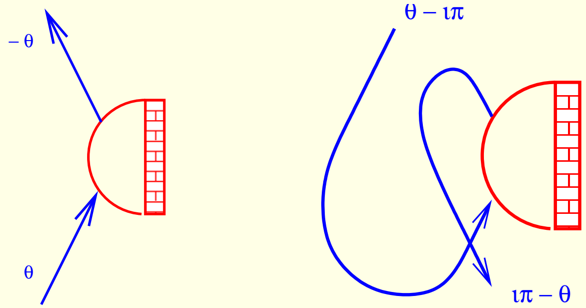

$$R(\theta) = S(i\pi - 2\theta)R(i\pi - \theta)$$

Simplest models: $S(\theta) = \frac{\sinh\theta - i\sin p}{\sinh\theta + i\sin p}$ solution $R(\theta) = \frac{\left(\frac{1}{2}\right)\left(1-\frac{p}{2}\right)}{\left(\frac{3}{2}-\frac{p}{2}\right)}$; $(x) = \frac{\sinh\left(\frac{\theta}{2}+\frac{i\pi x}{2}\right)}{\sinh\left(\frac{\theta}{2}-\frac{i\pi x}{2}\right)}$

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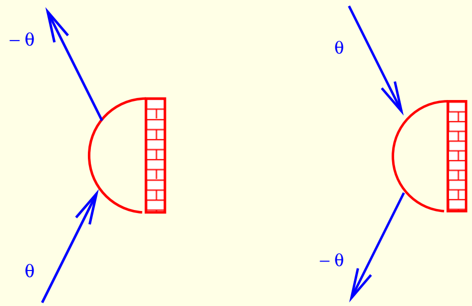

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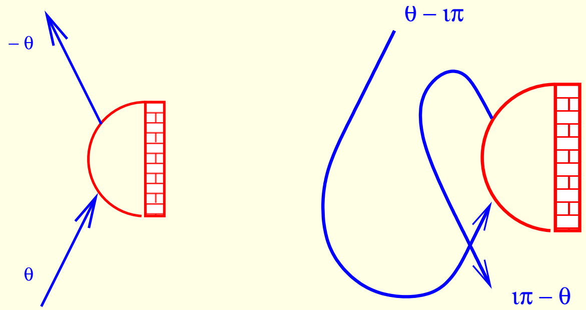

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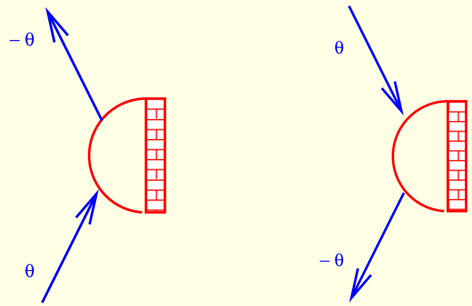

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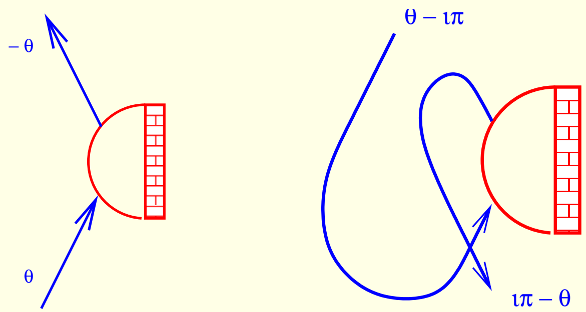

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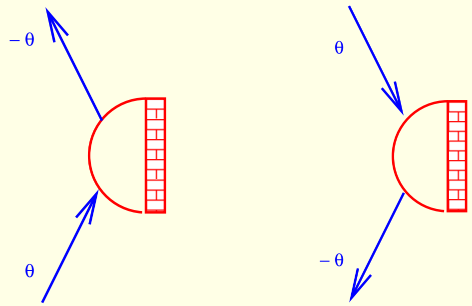
Minimality: all singularity has physical origin:

Integrable aspects: R-matrix bootsrap

Unitarity



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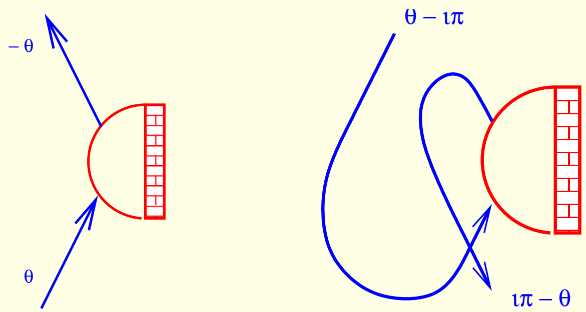

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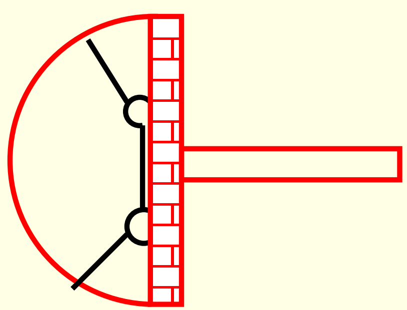
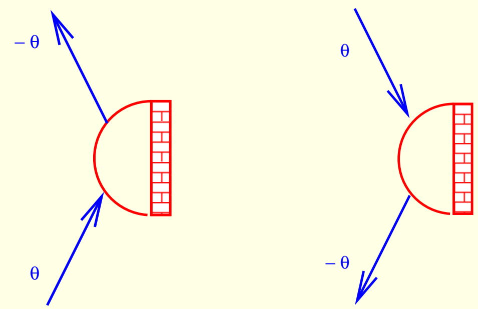
boundary boundstate



Integrable aspects: R-matrix bootstrap

Unitarity



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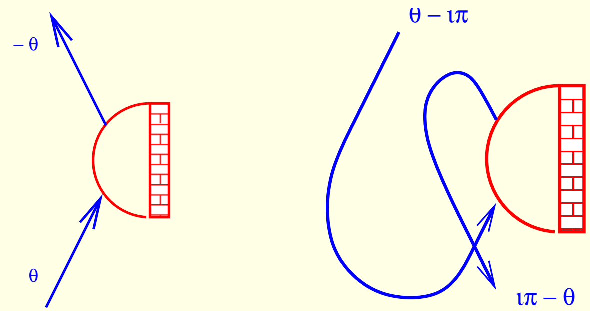

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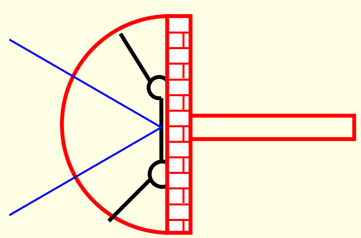

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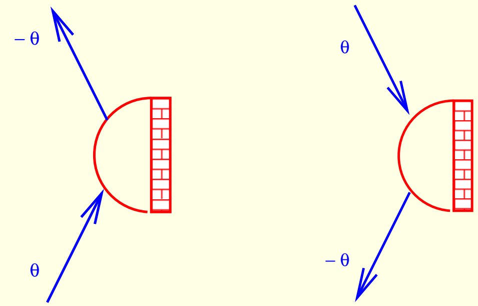

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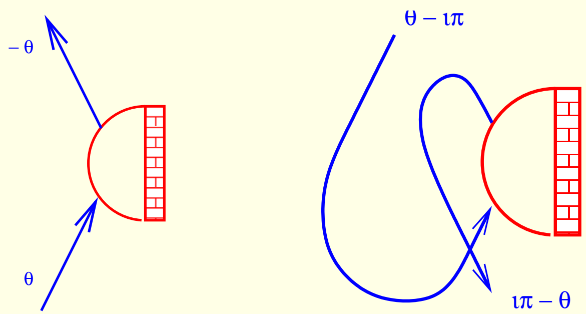

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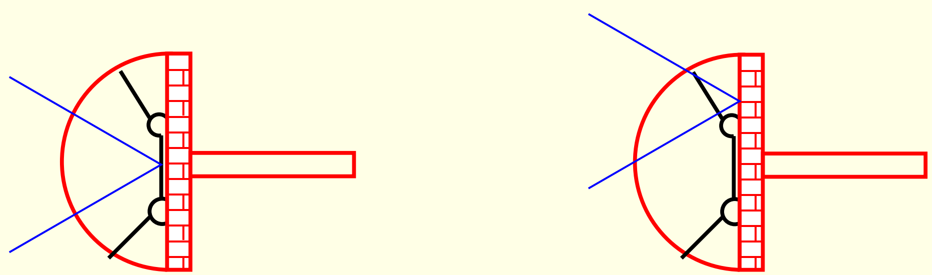

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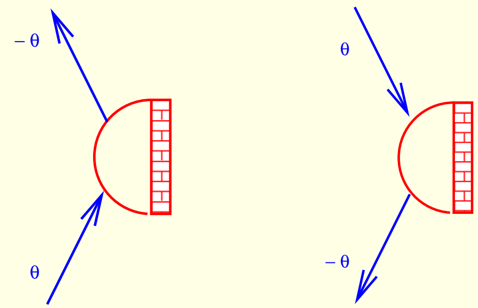

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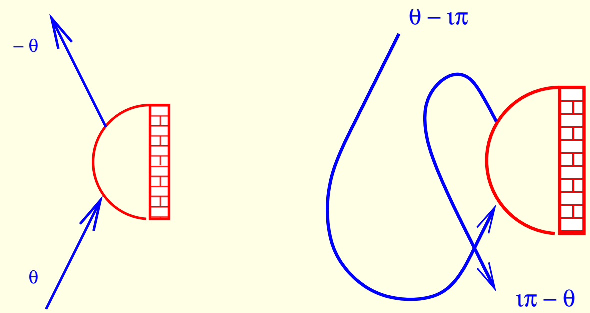

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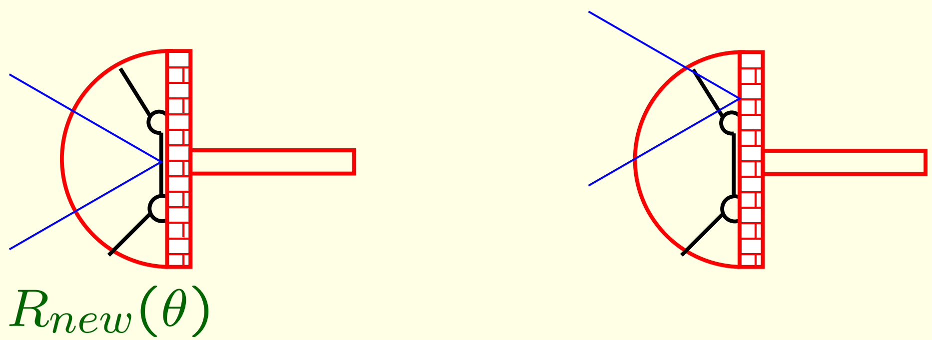

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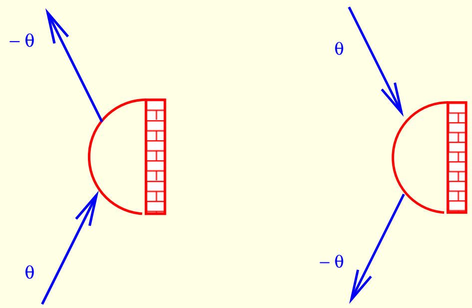

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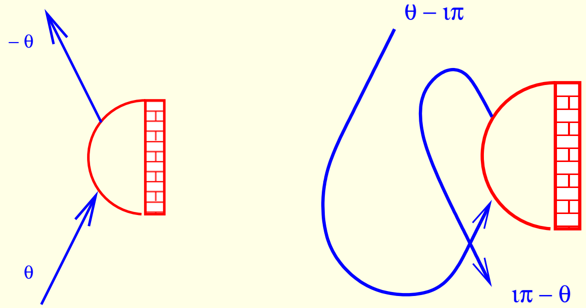

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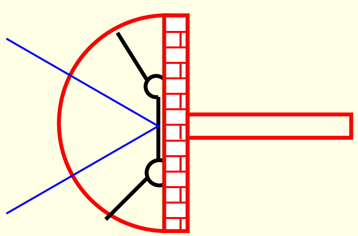
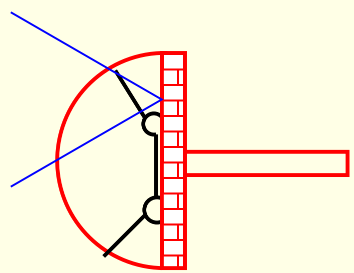


$R_{new}(\theta)$



$S(\theta + iu) R_{old}(\theta) S(\theta - iu)$

Boundary form factor axioms I

Reflection

$$F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}, \theta_n) =$$
$$R(\theta_n) F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_{n-1}, -\theta_n)$$

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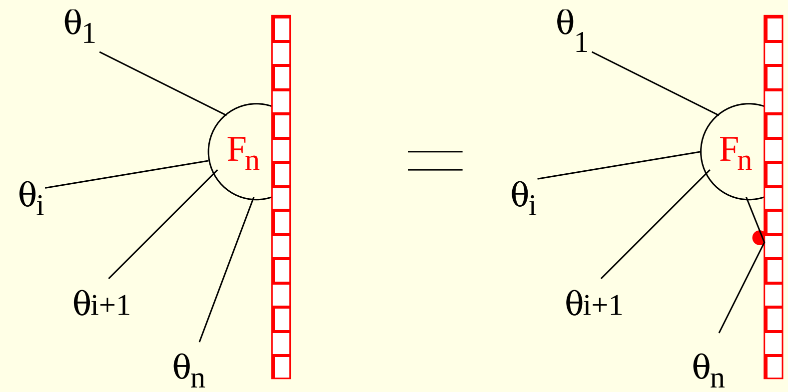

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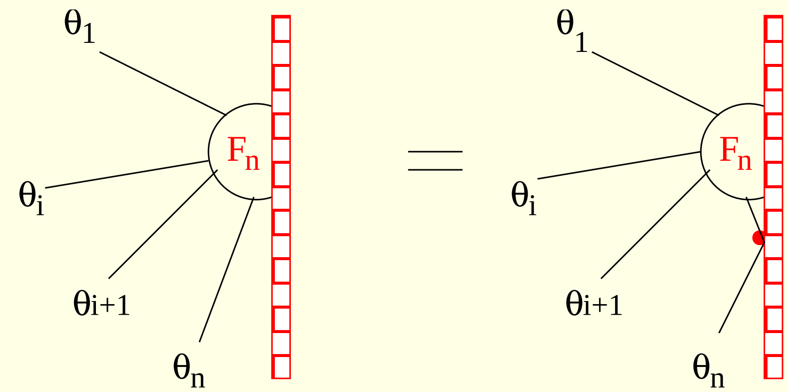

Permutation

$$F_n^{\mathcal{O}}(\theta_1,\ldots\theta_i,\theta_{i+1},\ldots,\theta_n) = S(\theta_i-\theta_{i+1})F_n^{\mathcal{O}}(\theta_1,\ldots\theta_{i+1},\theta_i,\ldots,\theta_n)$$

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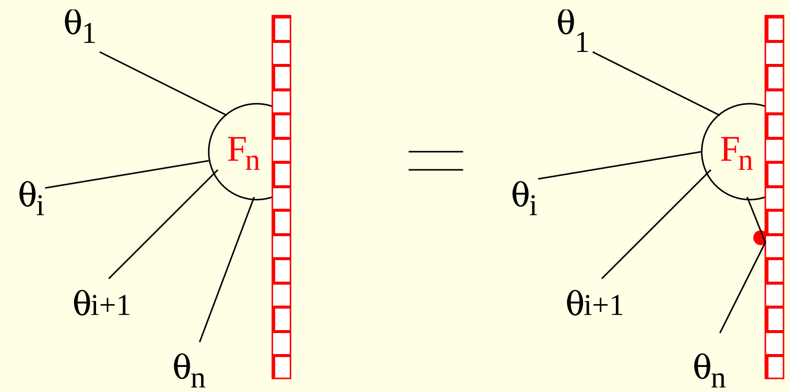

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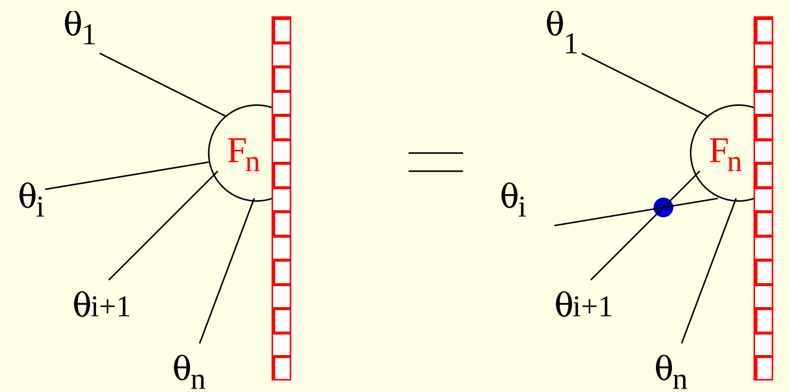

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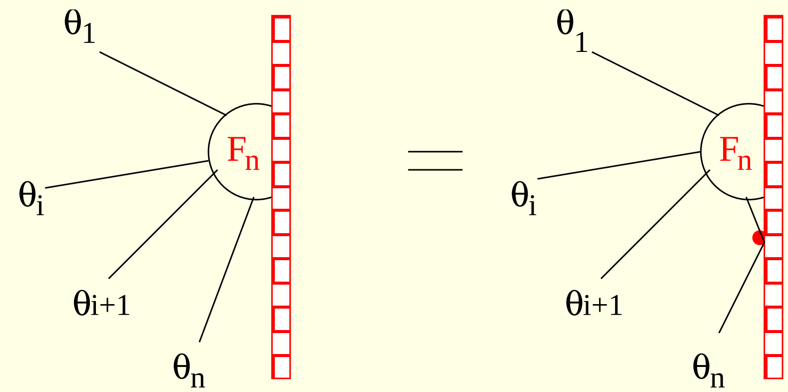

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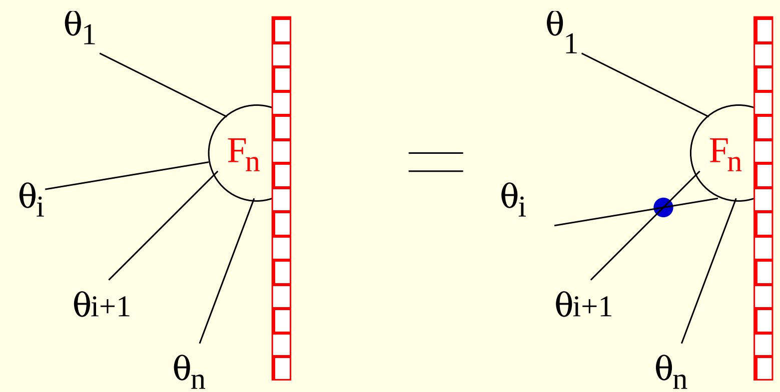

Boundary periodicity

$$F_n^{\mathcal{O}}(\theta_1,\theta_2,\ldots,\theta_n) = R(i\pi-\theta_1) F_n^{\mathcal{O}}(2i\pi-\theta_1,\theta_2,\ldots,\theta_n)$$

Boundary form factor axioms I

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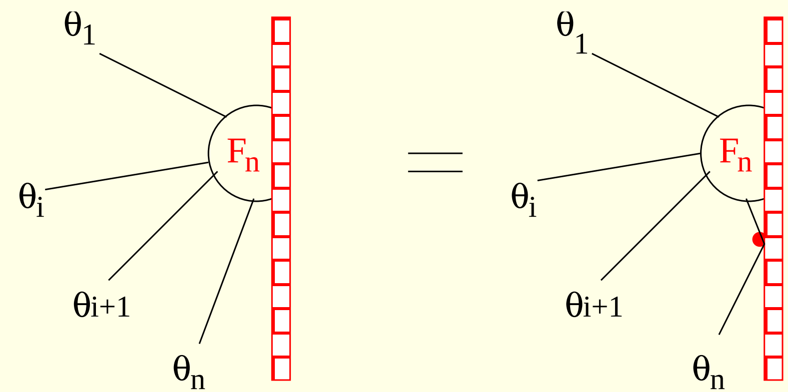

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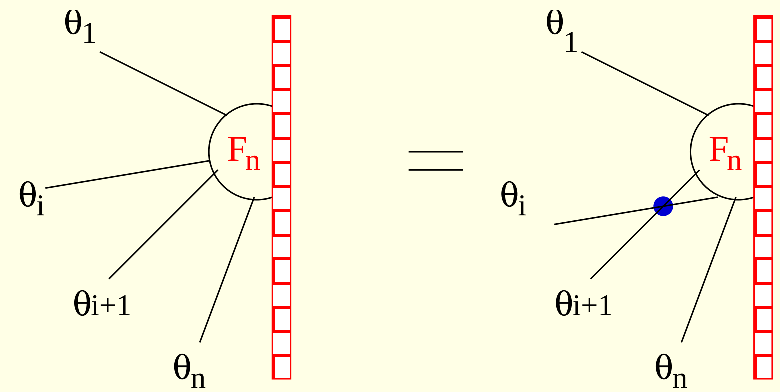

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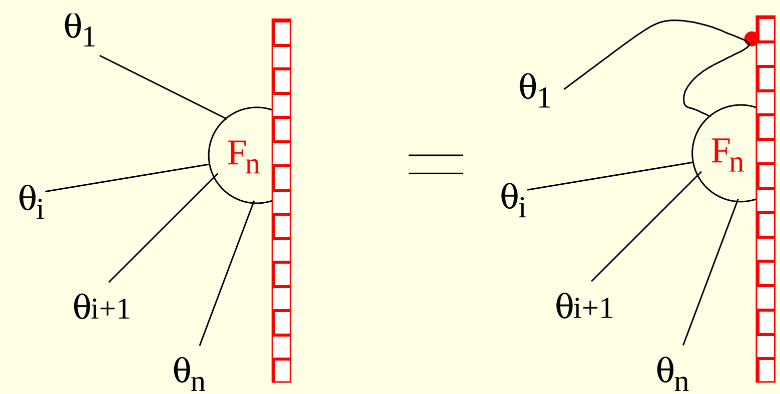

Boundary form factor axioms II: Singularities

Kinematical singularities
$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta+i\pi,\theta',\theta_1,\ldots,\theta_n) = \left(1-\prod_{i=1}^{n} S(\theta-\theta_i)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\ldots,\theta_n)$
$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(-\theta+i\pi,\theta',\theta_1,\ldots,\theta_n) = \left(R(\theta)-\prod_{i=1}^{n} S(\theta-\theta_i)R(\theta)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\ldots,\theta_n)$

Boundary form factor axioms II: Singularities

Kinematical singularities

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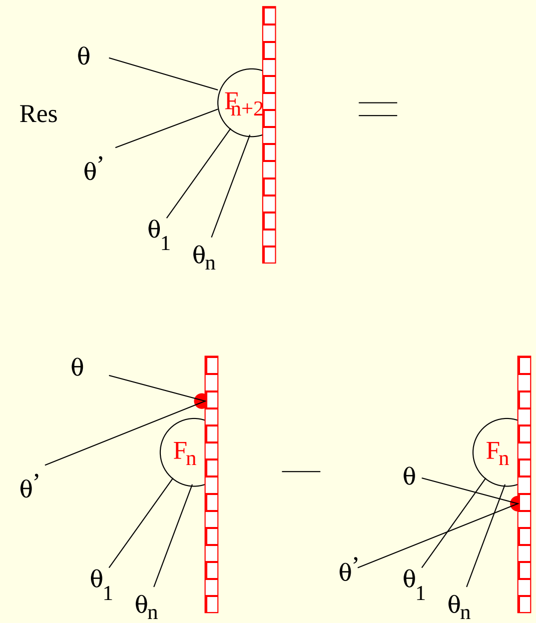
$$i\,\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(-\theta + i\pi, \theta', \theta_1, \dots, \theta_n) = \left(R(\theta) - \prod_{i=1}^{n} S(\theta - \theta_i) R(\theta) S(\theta + \theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1, \dots, \theta_n)$$



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Kinematical singularities

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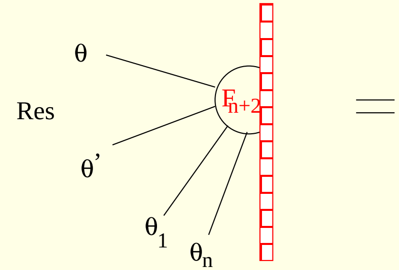
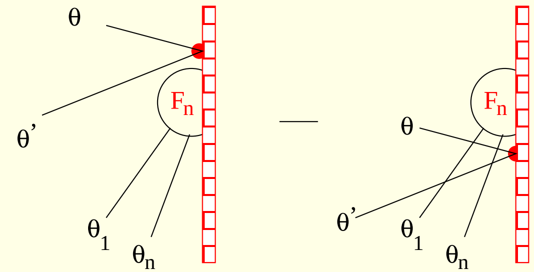
$$\left(1-\prod_{i=1}^{n} S(\theta-\theta_i)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\dots,\theta_n)$$

$$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(-\theta+i\pi,\theta',\theta_1,\dots,\theta_n) =$$
$$\left(R(\theta)-\prod_{i=1}^{n} S(\theta-\theta_i)R(\theta)S(\theta+\theta_i)\right) F^{\mathcal{O}}_{n}(\theta_1,\dots,\theta_n)$$





Bulk dynamical singularities

$$i\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta+iu,\theta'-iu,\theta_1,\dots,\theta_n) =$$
$$\Gamma F^{\mathcal{O}}_{n+1}(\theta,\theta_1,\dots,\theta_n)$$

Boundary form factor axioms II: Singularities

Kinematical singularities

$$i\,\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta+i\pi,\theta',\theta_1,\dots,\theta_n) = \left(1-\prod_{i=1}^{n} S(\theta-\theta_i)S(\theta+\theta_i)\right) F^{\mathcal{O}}_n(\theta_1,\dots,\theta_n)$$

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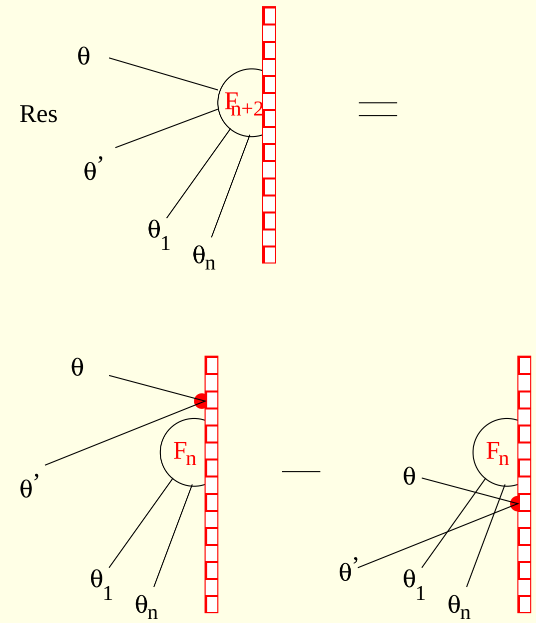

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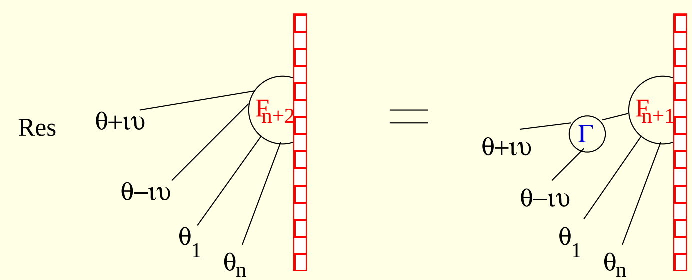

Boundary form factor axioms II: Singularities

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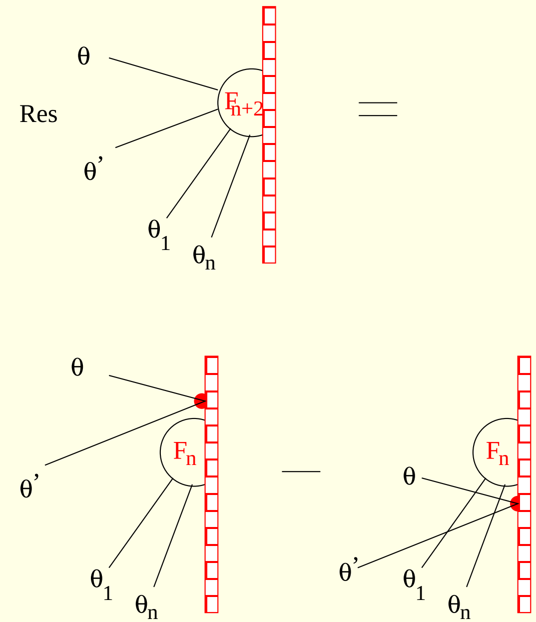
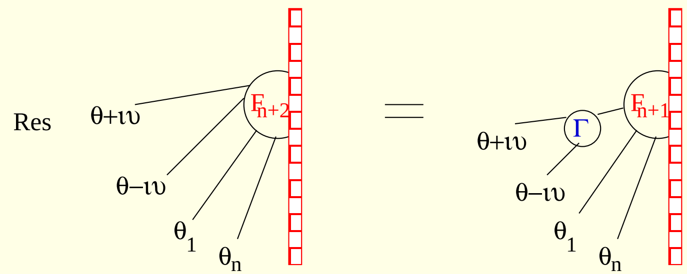


Bulk dynamical singularities

$$i\,\mathrm{res}_{\theta=\theta'} F^{\mathcal{O}}_{n+2}(\theta+iu,\theta'-iu,\theta_1,\ldots,\theta_n) = \Gamma F^{\mathcal{O}}_{n+1}(\theta,\theta_1,\ldots,\theta_n)$$



Boundary dynamical singularities

$$i\,\mathrm{res}_{\theta=iu} F^{\mathcal{O}}_{n+1}(\theta_1,\ldots,\theta_n,\theta) = g\tilde{F}^{\mathcal{O}}(\theta_1,\ldots,\theta_n)$$

Boundary form factor axioms II: Singularities

Kinematical singularities

$$i\mathrm{res}_{\theta=\theta'}F^{\mathcal{O}}_{n+2}(\theta+i\pi,\theta',\theta_1,\dots,\theta_n)=$$
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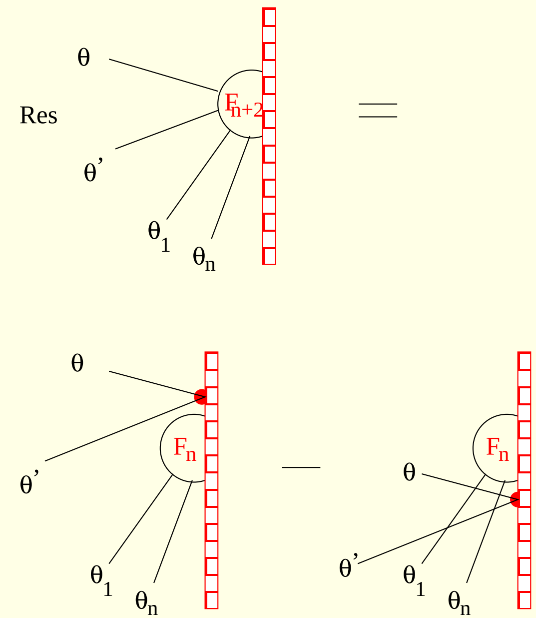

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$$i\mathrm{res}_{\theta=\theta'}F^{\mathcal{O}}_{n+2}(\theta+iu,\theta'-iu,\theta_1,\dots,\theta_n)=$$
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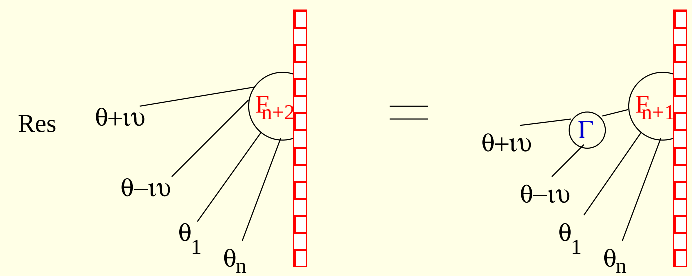

Boundary dynamical singularities

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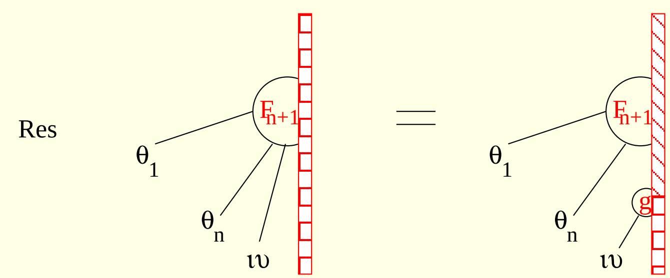
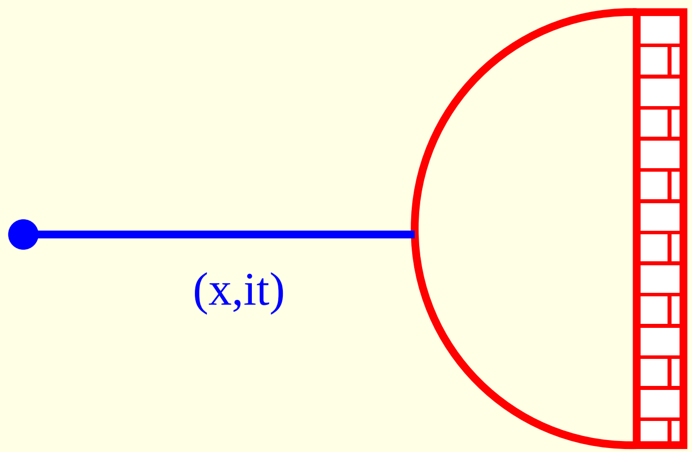


Crossing with a boundary

Crossing with a boundary

One point function ${}_B\langle 0|\Phi(x,it)|0\rangle_B = G^1_B(x,it) = G^1_B(x)$

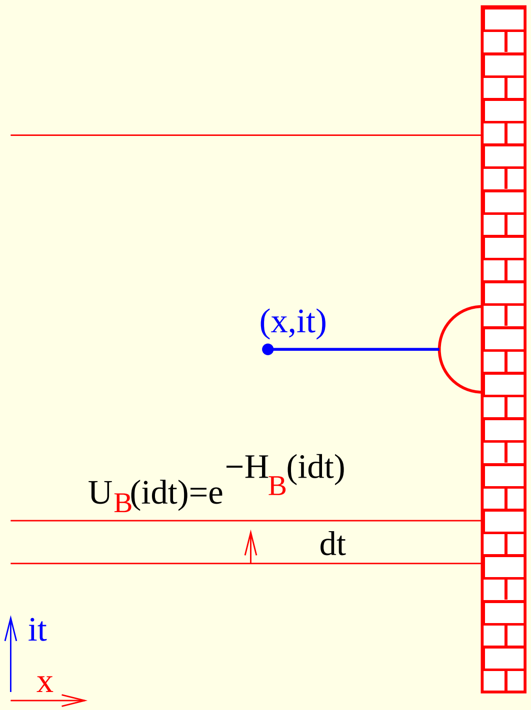


Crossing with a boundary

Path integral representation:

$$ {}_B\langle 0|\Phi(x,it)|0\rangle_B $$

$$ = Z^{-1}{}_B\langle 0|U_B(\infty,it)\Phi(x,it)U_B(it,-\infty)|0\rangle_B $$



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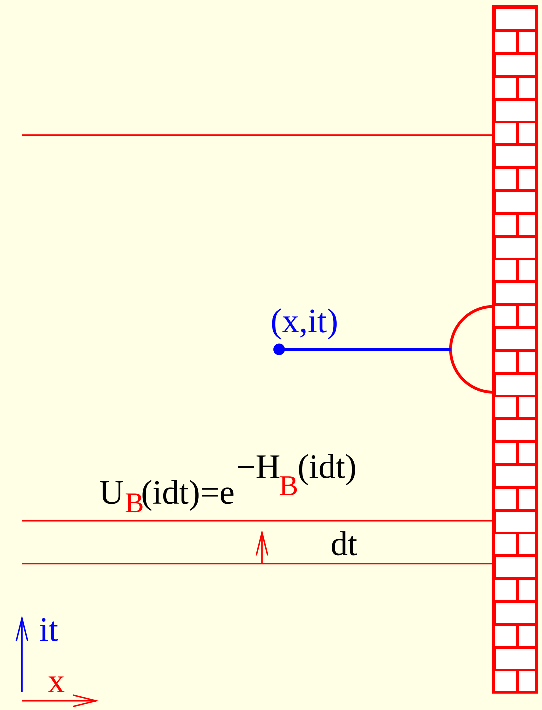
$$= \int \mathcal{D}\Phi\, \Phi(x,it)e^{-S[\Phi(x,it)]}$$



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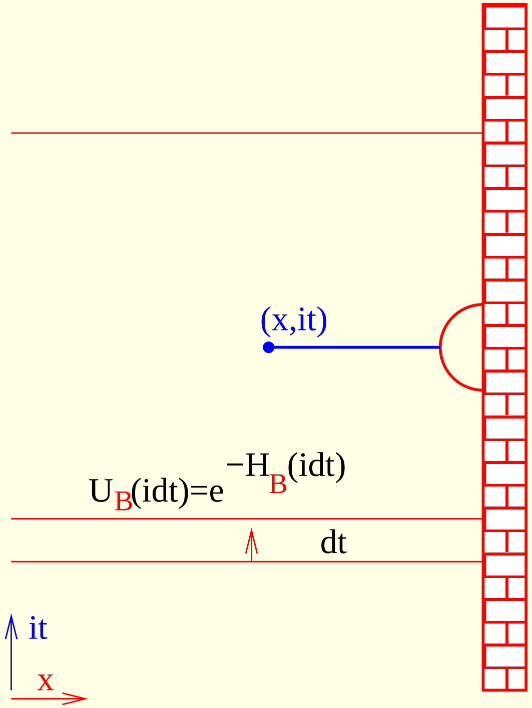
$$= \int \mathcal{D}\Phi\,\Phi(-iT,X)e^{-S[\Phi(-iT,X)]}$$



Crossing with a boundary

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$$_B\langle 0|\Phi(x,it)|0\rangle_B$$

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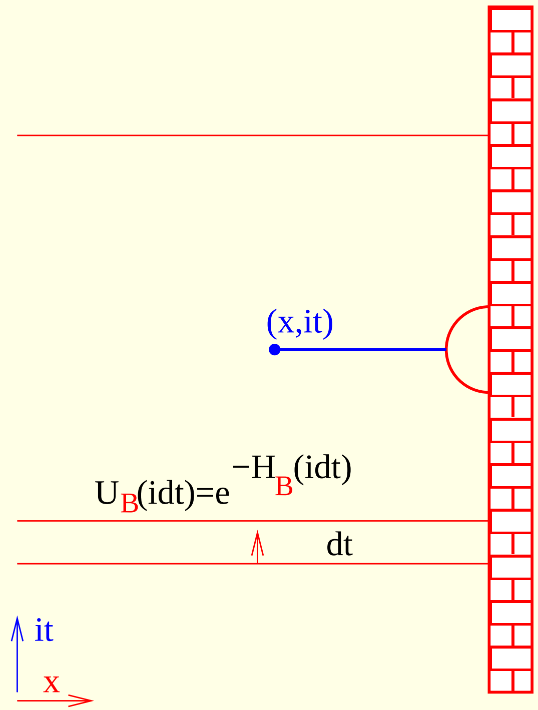
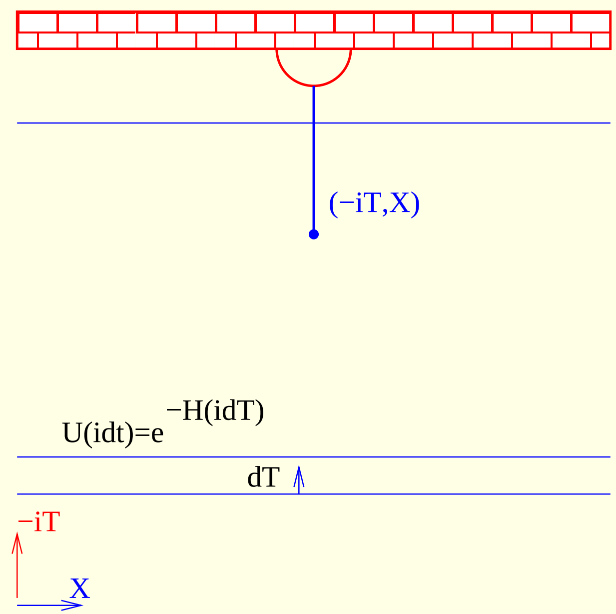
$$= \int \mathcal{D}\Phi\, \Phi(-iT,X)e^{-S[\Phi(-iT,X)]}$$

$$= Z^{-1}\langle B|U(\infty,iT)\Phi(-iT,X)U(-iT,-\infty)|0\rangle$$





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$${}_B\langle 0|\Phi(x,it)|0\rangle_B$$

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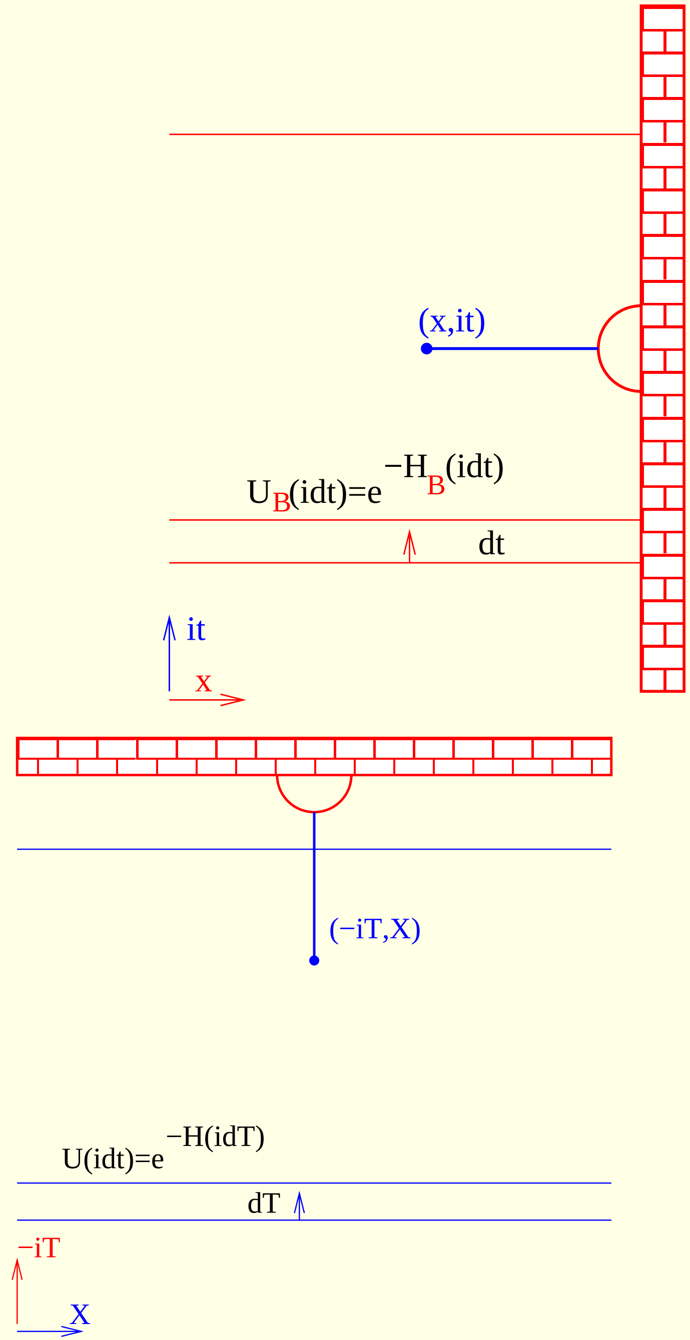
$$= \langle B|\Phi(-iT,X)|0\rangle$$



Boundary correlation functions: crossed channel

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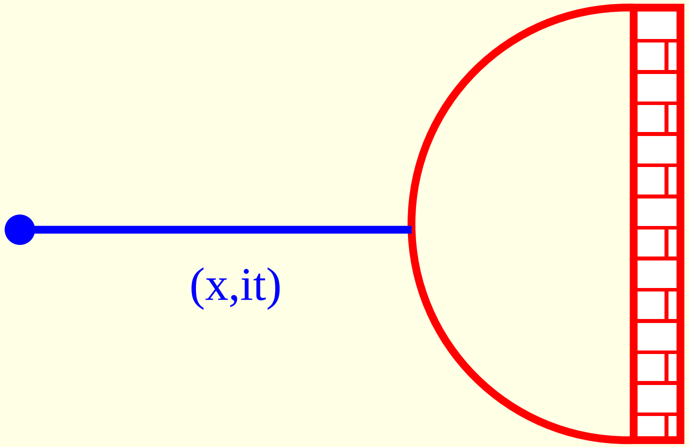
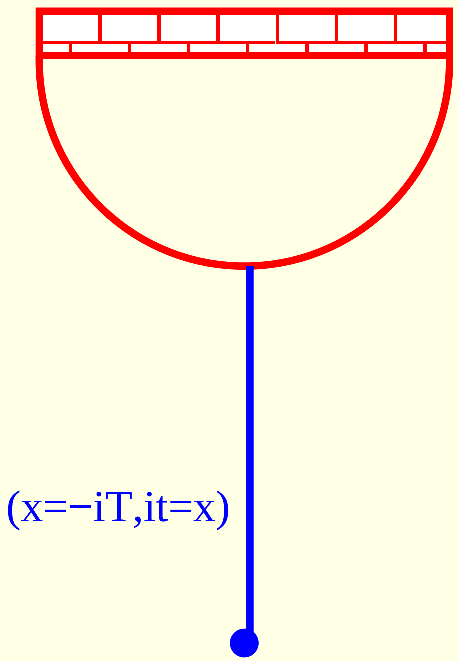
One point function ${}_B\langle 0|\Phi(x,it)|0\rangle_B = G_B^1(x,it) = G_B^1(x)$



Boundary correlation functions: crossed channel

One point function $\langle B|\Phi(-iT,X)|0\rangle^{in} = G^1_B(-iT,X) = G^1_B(-iT)$



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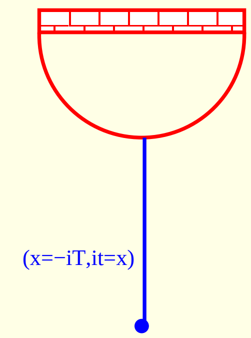

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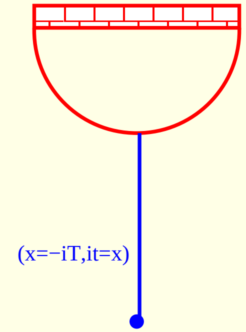
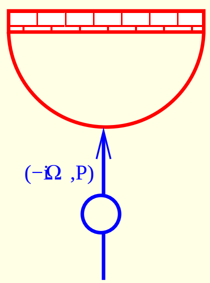


Momentum space: $\langle B|\Phi(-iT,X)|0\rangle = \int_{-\infty}^{\infty} dx e^{iPX} \int_{-\infty}^{\infty} d(iT) e^{i(-i\Omega)(-iT)} G^1_B(P,-i\Omega)$

$$G^1(P,-i\Omega) = (2\pi)\delta(P) G^2(P,-i\Omega) G^1_B(-i\Omega)$$



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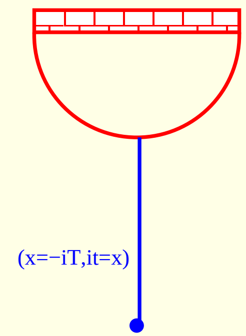
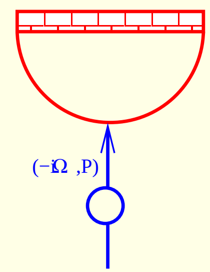


$$= \int_{-\infty}^{\infty} dX e^{iPX} \int_{-\infty}^{\infty} d(-iT) e^{i(-i\Omega)(-iT)} G^1_B(P,-i\Omega)$$



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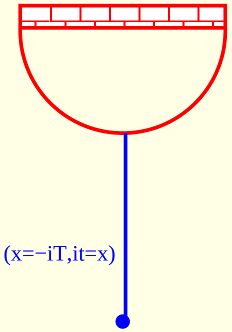
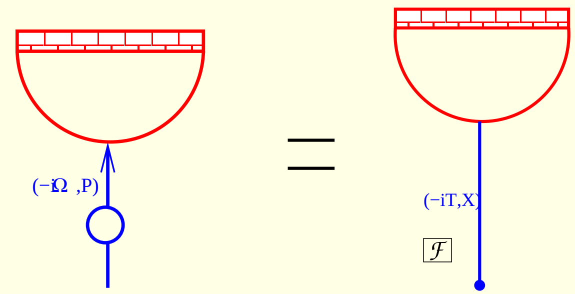


$$= \int_{-\infty}^{\infty} dX e^{iPX} \int_{-\infty}^{\infty} d(-iT) e^{i(-i\Omega)(-iT)} G^1_B(P,-i\Omega)$$



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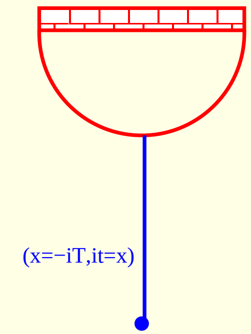

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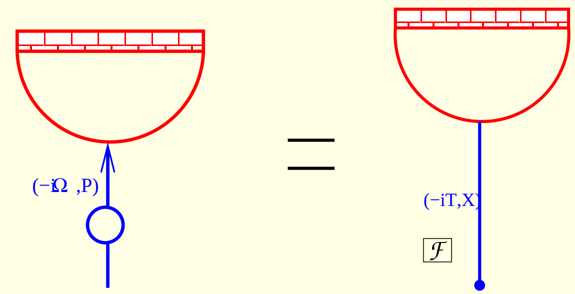
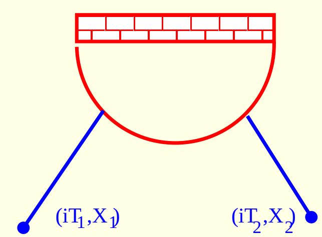


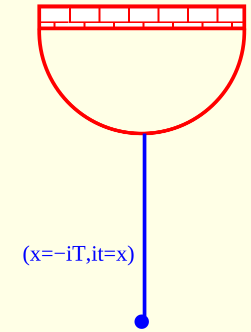
Two point function

$$\langle B|T(\Phi(-iT_1,X_1)\Phi(-iT_2,X_2))|0\rangle = G^2_B(-iT_1,-iT_2,X_1-X_2)$$

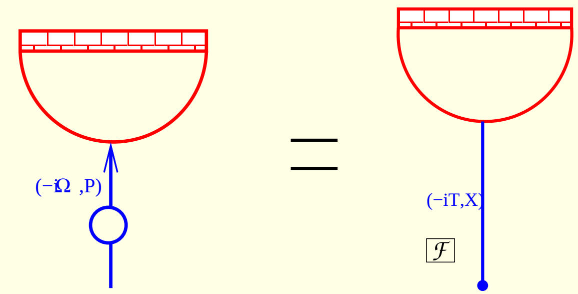


Boundary correlation functions: crossed channel

One point function $\langle B|\Phi(-iT,X)|0\rangle^{in} = G_B^1(-iT,X) = G_B^1(-iT)$

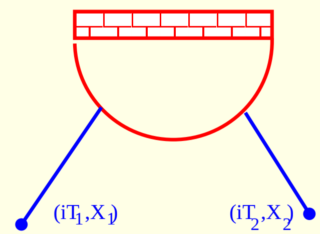


$$= \int_{-\infty}^{\infty} dX e^{iPX} \int_{-\infty}^{\infty} d(-iT) e^{i(-i\Omega)(-iT)} G_B^1(P,-i\Omega)$$

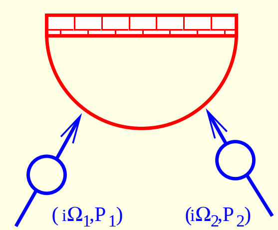


Two point function

$$\langle B|T(\Phi(-iT_1,X_1)\Phi(-iT_2,X_2))|0\rangle = G_B^2(-iT_1,-iT_2,X_1-X_2)$$



$$G_B^2(P_1,i\Omega_1,P_2,i\Omega_2) = (2\pi)\delta(P_1+P_2) G_B^2(P_1,i\Omega_1,i\Omega_2)$$



Boundary correlation functions: crossed channel

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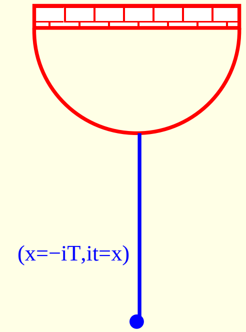
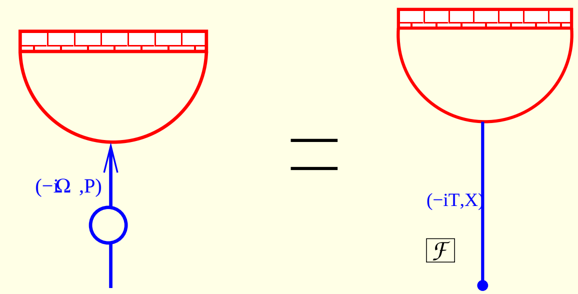
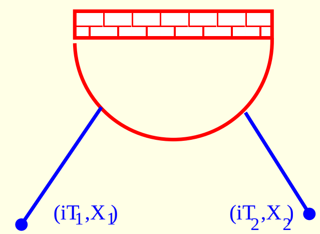


$$= \int_{-\infty}^{\infty} dX e^{iPX} \int_{-\infty}^{\infty} d(-iT) e^{i(-i\Omega)(-iT)} G^1_B(P,-i\Omega)$$



Two point function



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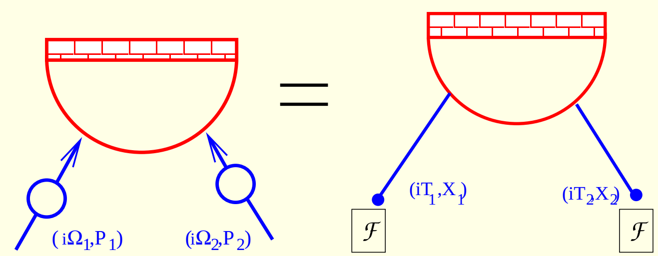
$$G^2_B(P_1,i\Omega_1,P_2,i\Omega_2) = (2\pi)\delta(P_1+P_2)G^2_B(P_1,i\Omega_1,i\Omega_2)$$



Boundary state: definition

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One point function

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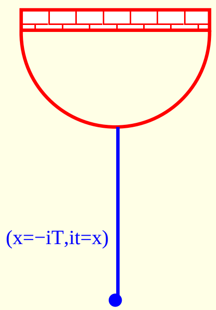

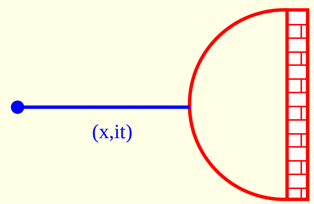

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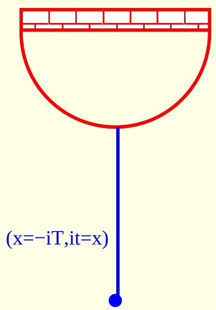

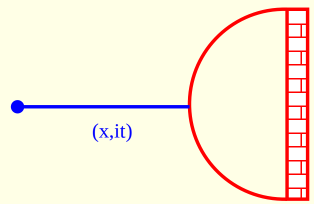

Two point function

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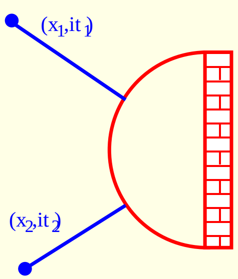
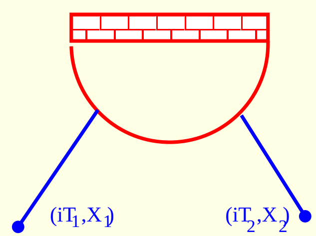


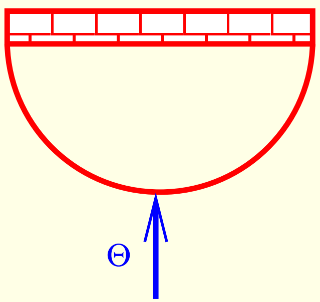


Boundary state: determination

Boundary state: determination

One particle contribution

$$\langle B|\Theta\rangle^{in} = K^1 2\pi\delta(P(\Theta))$$



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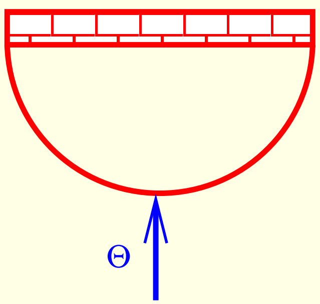
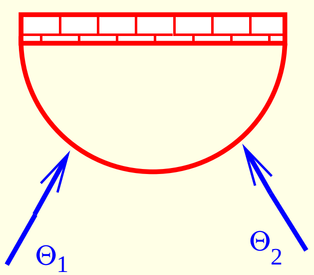


Two particle contribution

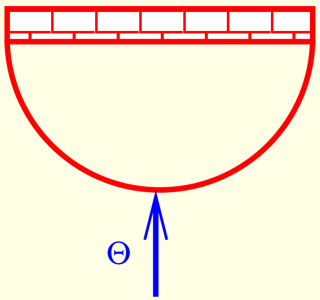
$$\langle B|\Theta_1,\Theta_2\rangle^{in} = K^2(\Theta_1)2\pi\delta(P(\Theta_1)+P(\Theta_2))$$



Boundary state: determination

One particle contribution

$$\langle B|\Theta\rangle^{in} = K^1 2\pi\delta(P(\Theta))$$



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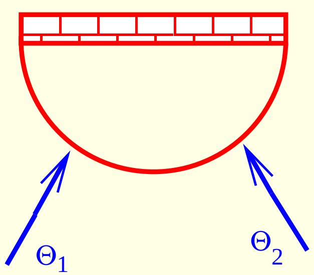
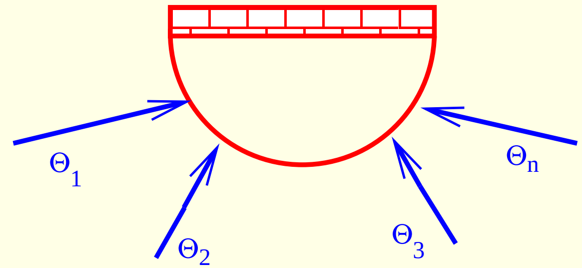


n particle contribution

$$\langle B|\Theta_1,\Theta_2,\ldots,\Theta_n\rangle^{in} =$$

$$K^n(\Theta_1,\ldots,\Theta_n)2\pi\delta(\textstyle\sum_i P(\Theta_i))$$



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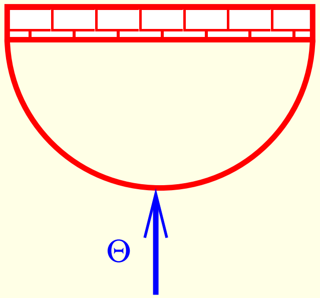

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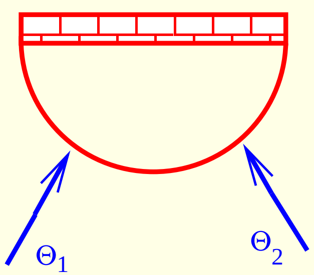

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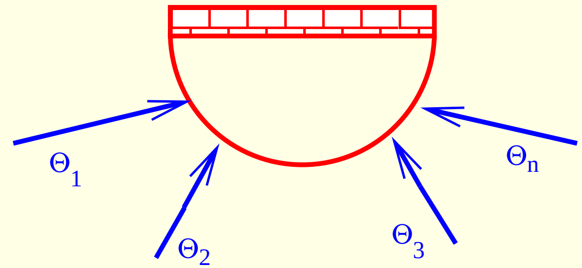

Connection to correlators?

Crossed channel reduction formula

Crossed channel boundary reduction formula

Crossed channel boundary reduction formula

$$\boxed{<\theta_1, \theta_2, \ldots, \theta_n|B>} =$$

Crossed channel boundary reduction formula

$$< \theta_1, \theta_2, \ldots, \theta_n | B > =$$

$$iZ^{-1/2} \int_{-\infty}^{\infty} dX' \int_{-\infty}^{0} dT' e^{-i\Omega(\theta_1)T' + iP(\theta_1)X'} \{ -\partial^2_{iT'} - \partial^2_{X'} + m^2 - \delta(iT')(\partial_{iT'} + i\Omega$$

Crossed channel boundary reduction formula

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$$< \theta_2, \ldots, \theta_n | \Phi(x', t') | B >$$

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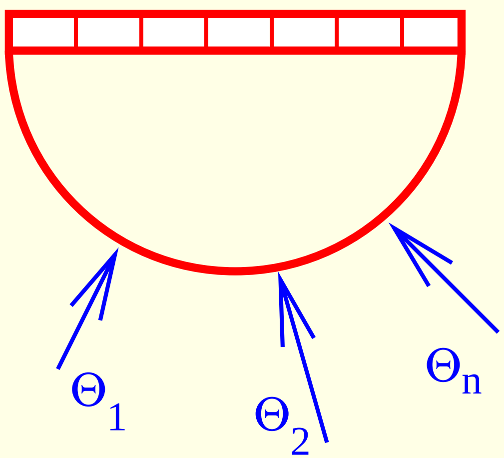
Diagrammatically



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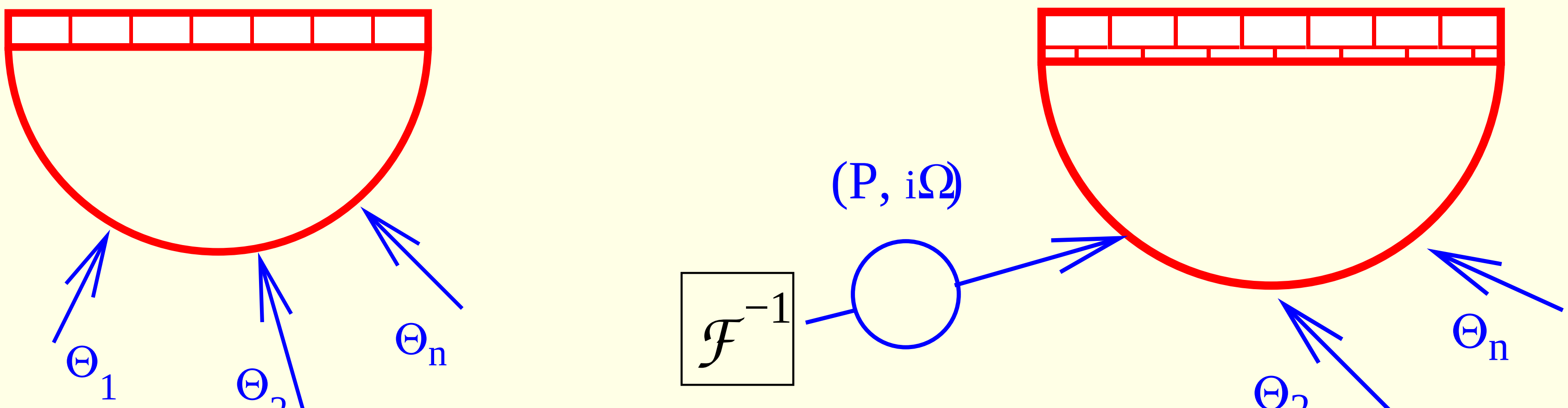

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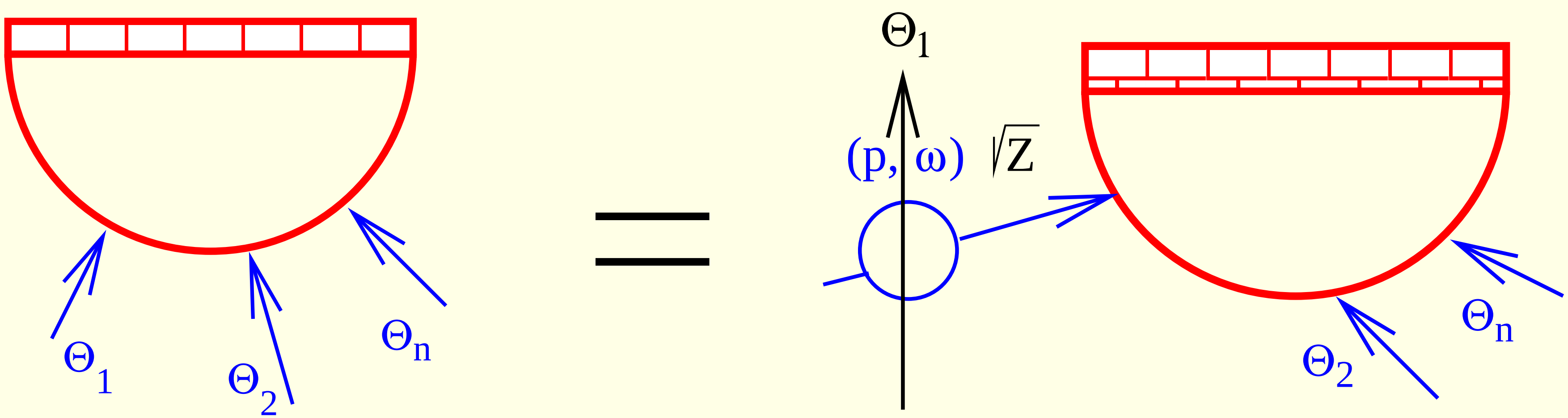

Crossing: Boundary state, reflection

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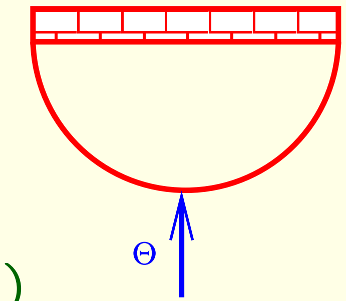
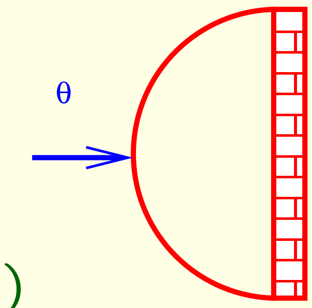
One particle contribution $\leftrightarrow$ One particle emission (reflection)



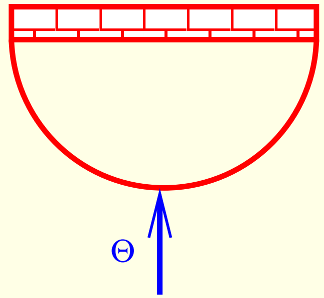
$K^1(\Theta = \frac{i\pi}{2} - \theta)$



$R^1(\theta)$

Crossing: Boundary state, reflection

One particle contribution ↔ One particle emission (reflection)



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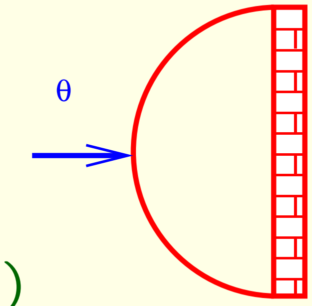
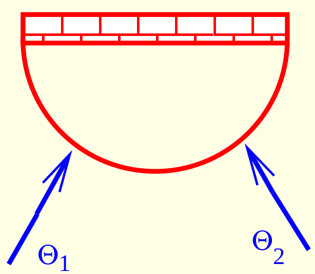
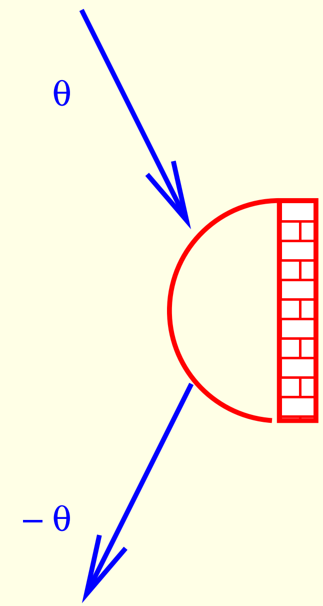


$R^1(\theta)$

Two particle contribution ↔ One particle reflection



$K^2(\Theta = \frac{i\pi}{2} - \theta)$



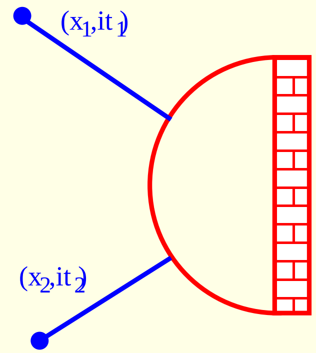
$R_1^1(\theta)$

R^1 from R^1_1: clustering

R^1 from R^1_1: clustering

Reflection factor is related to the twopoint function

$${}_B\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle_B$$



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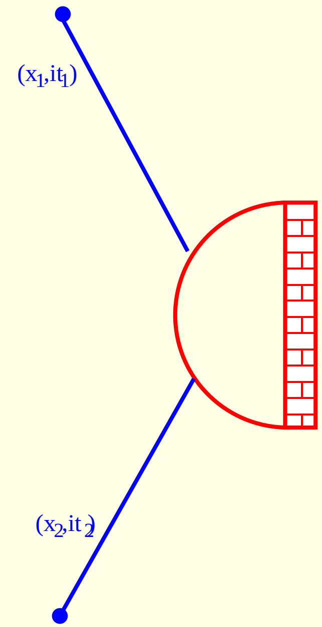
Choosing large separation in $t_1 - t_2$

$$\lim_{(t_1-t_2)\to\infty} {}_B\langle 0|T(\Phi(x_1,it_1)\Phi(x_2,it_2))|0\rangle_B =$$



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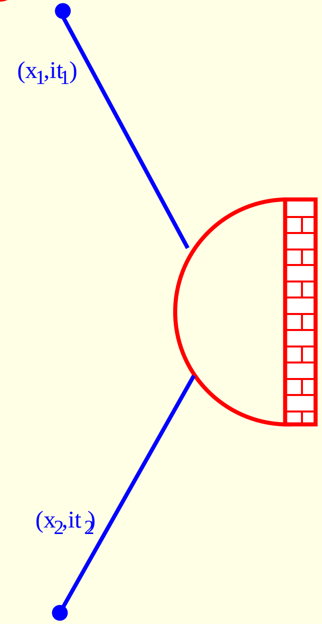

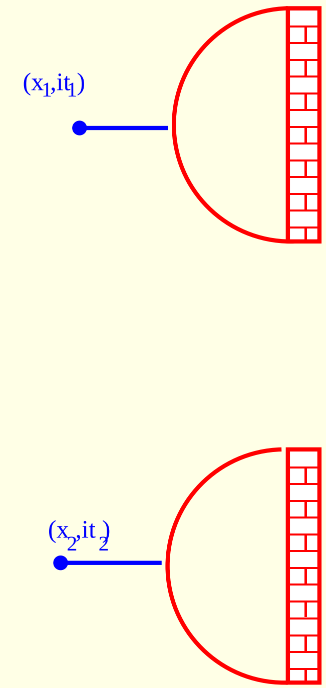

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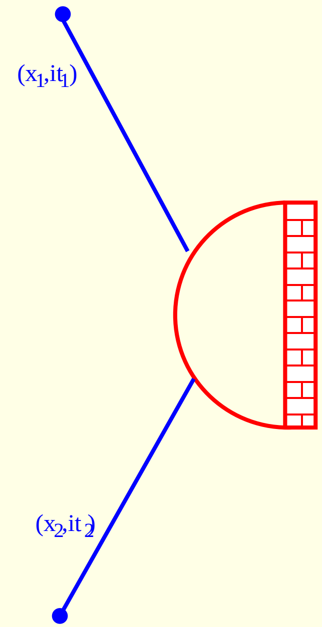
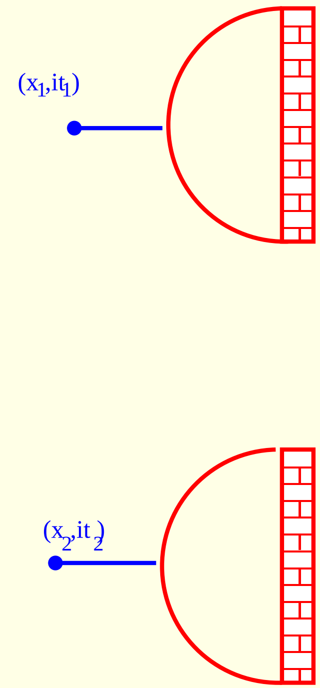




Reflection factor $\boxed{R^1_1(\theta)|_{\theta\to\frac{i\pi}{2}} = \frac{g^2}{2}\frac{1}{i(\theta-\frac{i\pi}{2})} = \frac{g^2}{2}\frac{1}{i\Theta} = K^2(\Theta)}$

'Emission' factor $\boxed{R^1 = \frac{g}{2} = K^1}$

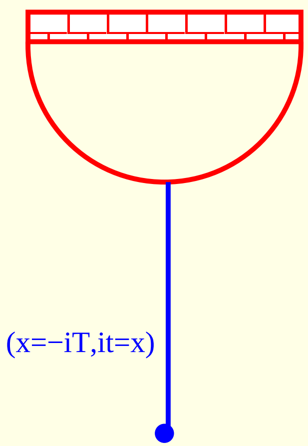
Example $R(\theta) = \frac{\left(\frac{1}{2}\right)\left(1-\frac{p}{2}\right)}{\left(\frac{3}{2}-\frac{p}{2}\right)}\frac{\left(\frac{i\eta p}{\pi}-\frac{1}{2}\right)}{\left(\frac{i\eta p}{\pi}+\frac{1}{2}\right)}\frac{\left(\frac{ip\vartheta}{\pi}-\frac{1}{2}\right)}{\left(\frac{ip\vartheta}{\pi}+\frac{1}{2}\right)}$; $R^1 = \sqrt{\cot\frac{\pi B}{4}\cot\frac{\pi(1-B)}{4}\tan\frac{\eta B}{2}\tanh\frac{\vartheta B}{2}}$

One point function revisited

One point function revisited

One point function:

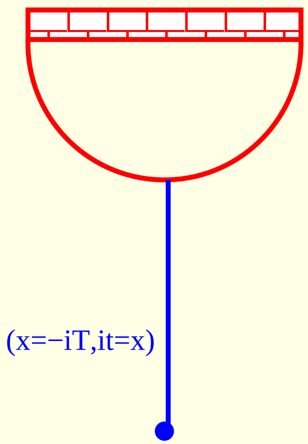
$$\langle B|\Phi(-iT,X)|0\rangle =$$

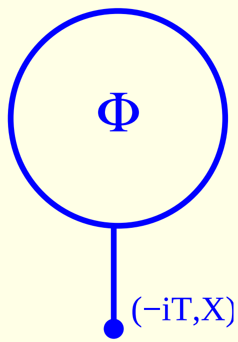


One point function revisited

One point function: Vacuum$+$

$$\langle B|\Phi(-iT,X)|0\rangle = \langle B|0\rangle\times$$
$$\langle 0|\Phi(-iT,X)|0\rangle +$$

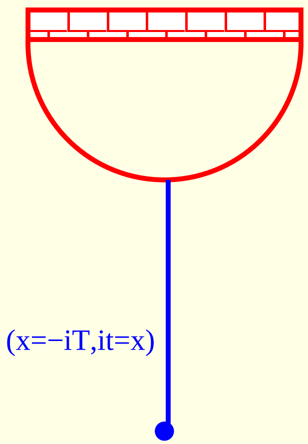


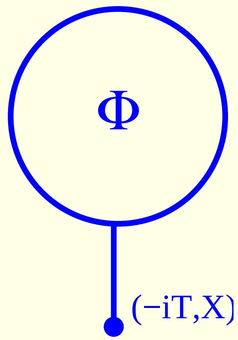


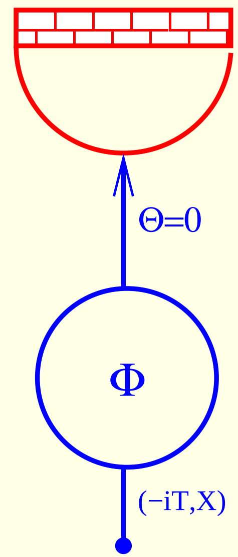
One point function revisited

One point function: Vacuum+ 1pt term+

$$\langle B|\Phi(-iT,X)|0\rangle = \langle B|0\rangle \times \langle 0|\Phi(-iT,X)|0\rangle + \langle B|\Theta\rangle \times \langle\Theta|\Phi(-iT,X)|0\rangle +$$



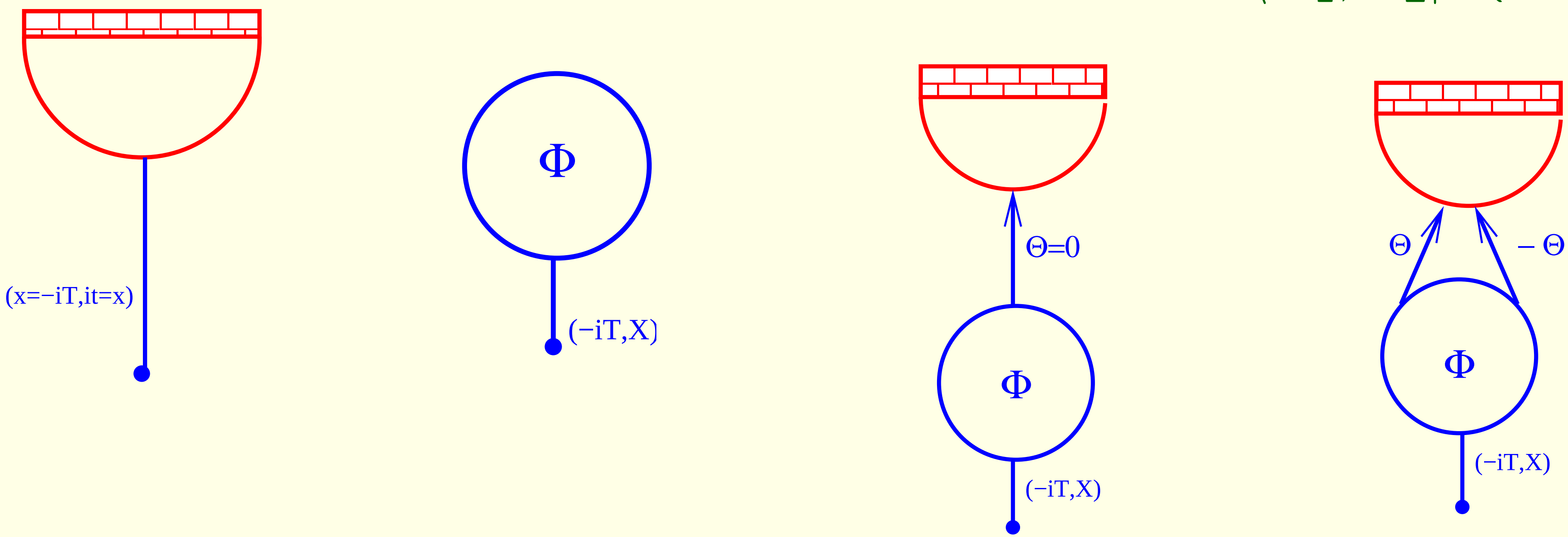




One point function revisited

One point function: Vacuum+ 1pt term+ 2pt term+

$$\langle B|\Phi(-iT,X)|0\rangle = \langle B|0\rangle\times\langle 0|\Phi(-iT,X)|0\rangle + \langle B|\Theta\rangle\times\langle\Theta|\Phi(-iT,X)|0\rangle + \langle B|\Theta_1,\Theta_2\rangle\times\langle\Theta_1,\Theta_2|\Phi(-iT,X)|0\rangle$$



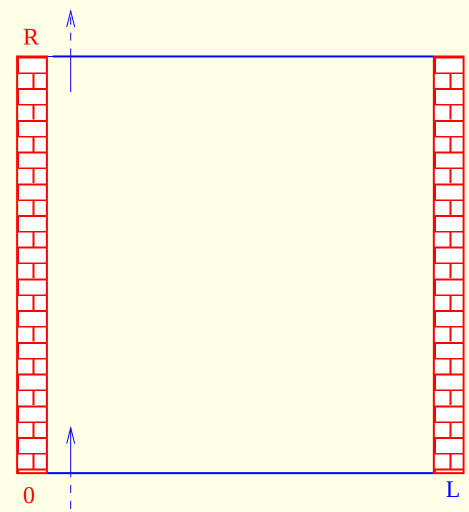
Ground state energy: partition function

Ground state energy: partition function

Ground state energy from the partition function

$$\lim_{R\to\infty} Z(L,R) = \lim_{R\to\infty} Tr(e^{-H_B(L)R})$$

$$= e^{-E_0(L)R} \to E_0(L) = -\lim_{R\to\infty} R^{-1}\log Z(L,R)$$



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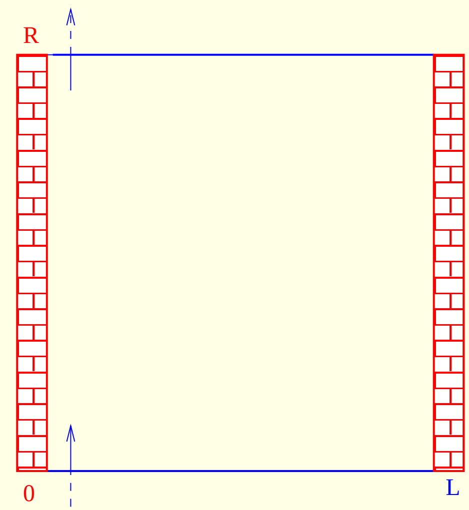
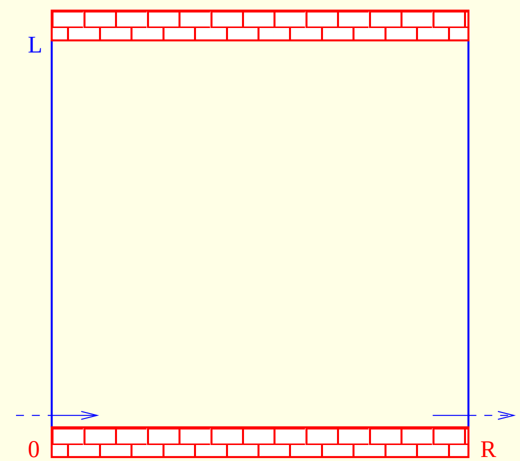


Alternative calculation from the crossed channel

$$\lim_{R\to\infty} Z(L,R) =$$

$$\lim_{R\to\infty} \langle B_0 | e^{-H(R)L} | B_L \rangle$$

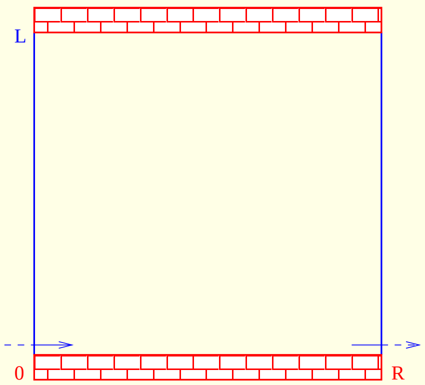


Cluster (large L) expansion

Cluster (large L) expansion

Partition function:

$$Z(L,R) = \langle B_0 | e^{-H(R)L} | B_L \rangle$$



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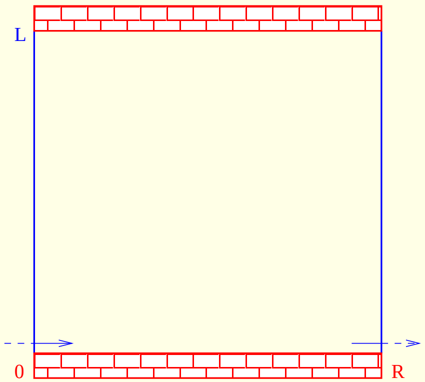
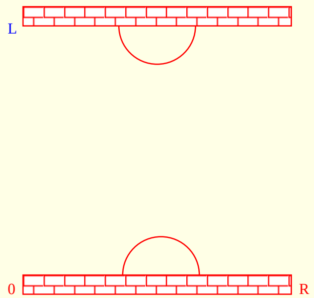


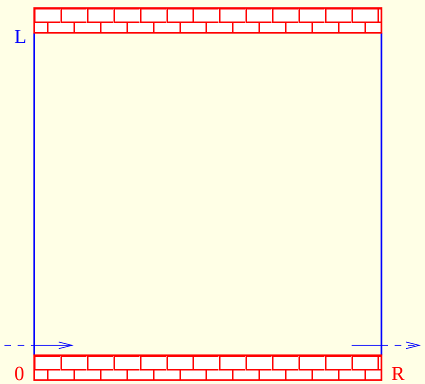
Vacuum+

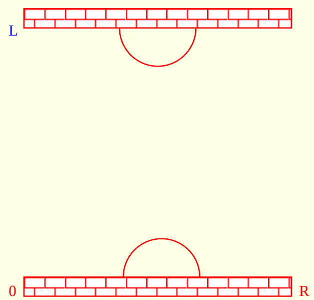
$$\langle B_0 | 0 \rangle \times$$
$$\langle 0 | B_L \rangle +$$

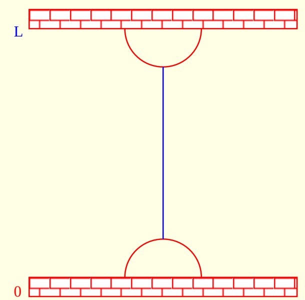


Cluster (large L) expansion

Partition function:	Vacuum+	1pt term+
$Z(L,R) =$	$\langle B_0 \vert 0\rangle \times$	$\langle B_0 \vert \theta\rangle \times$
$\langle B_0 \vert e^{-H(R)L} \vert B_L\rangle$	$\langle 0 \vert B_L\rangle +$	$2m\langle\theta \vert B_L\rangle e^{-mL} +$

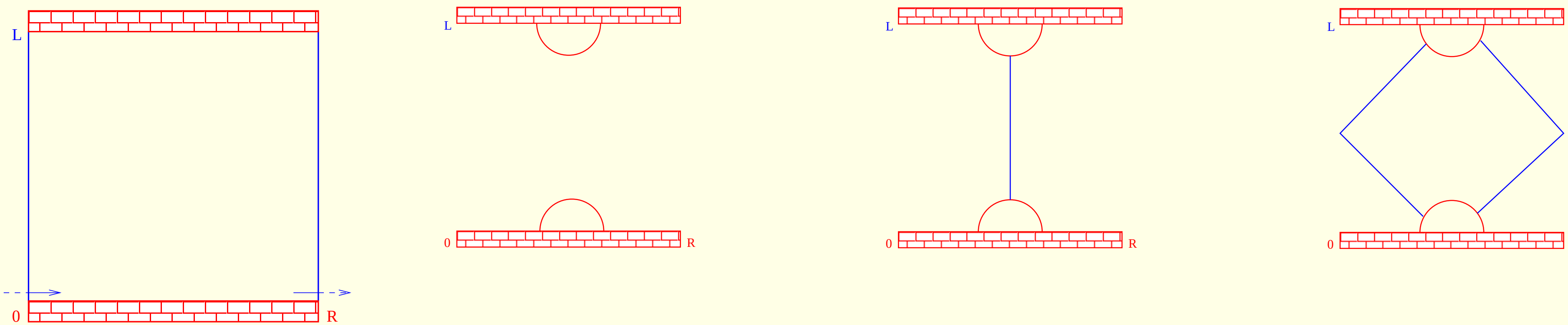






Cluster (large L) expansion

Partition function:	Vacuum+	1pt term+	2pt term+
$Z(L,R) =$	$\langle B_0 \vert 0\rangle \times$	$\langle B_0 \vert \theta\rangle \times$	$\int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta \langle B_0 \vert \theta, -\theta\rangle \times$
$\langle B_0 \vert e^{-H(R)L} \vert B_L\rangle$	$\langle 0 \vert B_L\rangle +$	$2m\langle \theta \vert B_L\rangle e^{-mL} +$	$\langle \theta, -\theta \vert B_L\rangle e^{2m\cosh\theta L} +$



Ground state: Casimir energy

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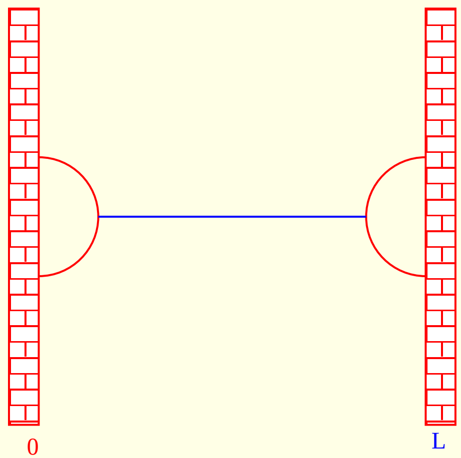
Nonvanishing g:

$$E_0(L) = -2mK_0^1 K_1^{1*} e^{-mL} +$$



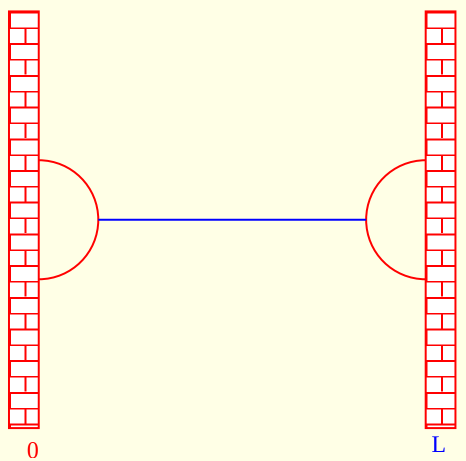
Ground state: Casimir energy

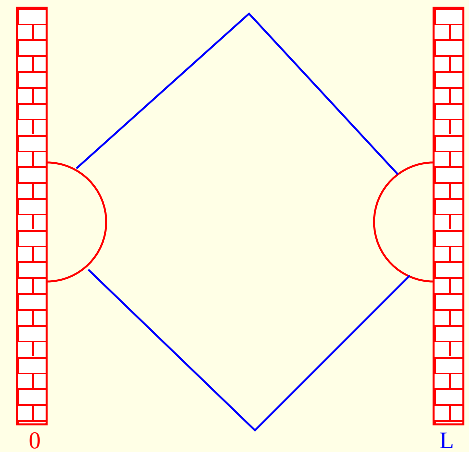
Nonvanishing g:

vanishing g

$$E_0(L) = -2mK_0^1 K_1^{1*} e^{-mL} +$$

$$\int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta K_0^2(\theta) K_L^{2*}(\theta) e^{-2m\cosh\theta L} +$$



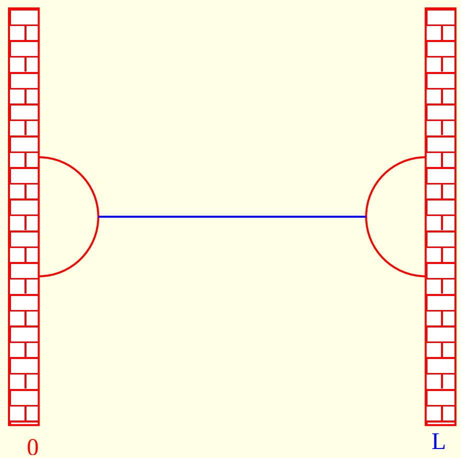


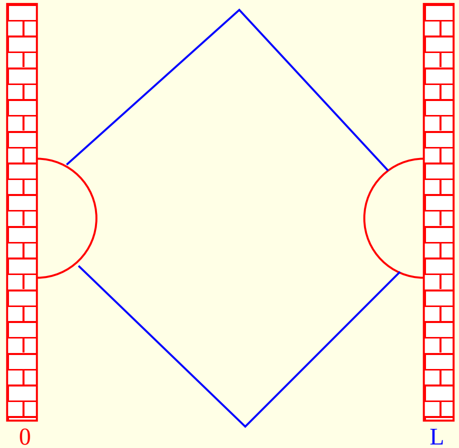
Ground state: Casimir energy

Nonvanishing g:

vanishing g

$$E_0(L) = -2mK_0^1 K_1^{1*} e^{-mL} + \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta K_0^2(\theta) K_L^{2*}(\theta) e^{-2m\cosh\theta L} +$$





free theory

$$E_{0L}^0(L) = \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} m \cosh\theta \log(1 + K_0^2(\theta) K_L^{2*}(\theta) e^{-2m\cosh\theta L})$$