Gauge/gravity duality, Munich, July 30, 2013

Casimir effect and the quark–anti-quark potential

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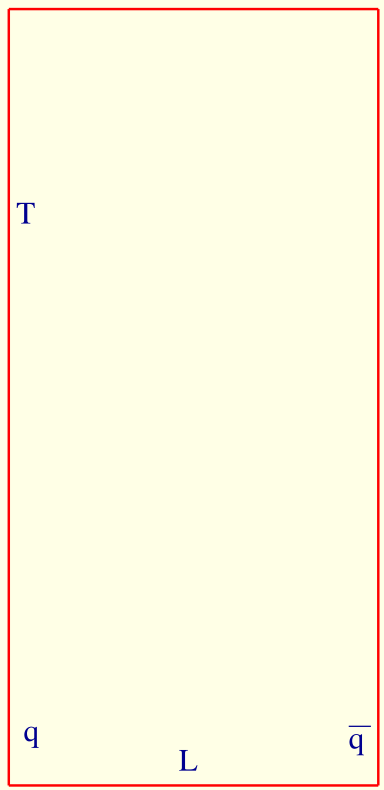
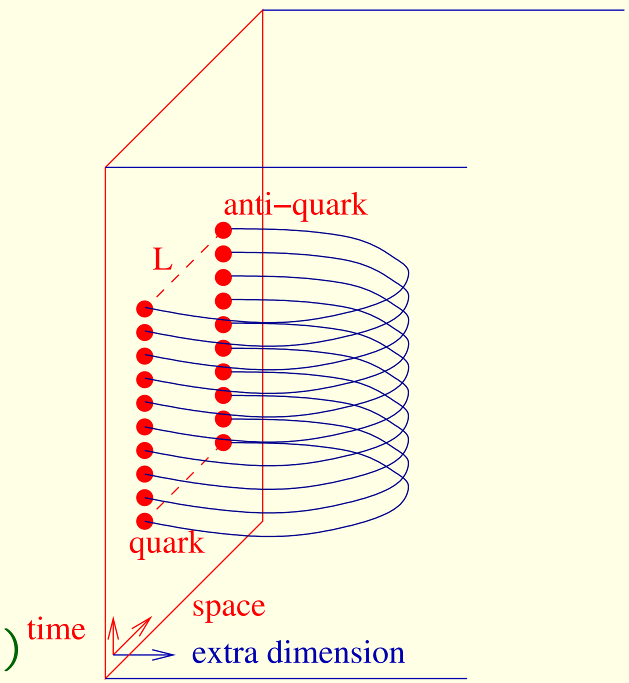
$$\langle W \rangle = e^{-TV_{q\bar{q}}(L,\lambda)}$$



$$= e^{-TE_0(L,\lambda)}$$



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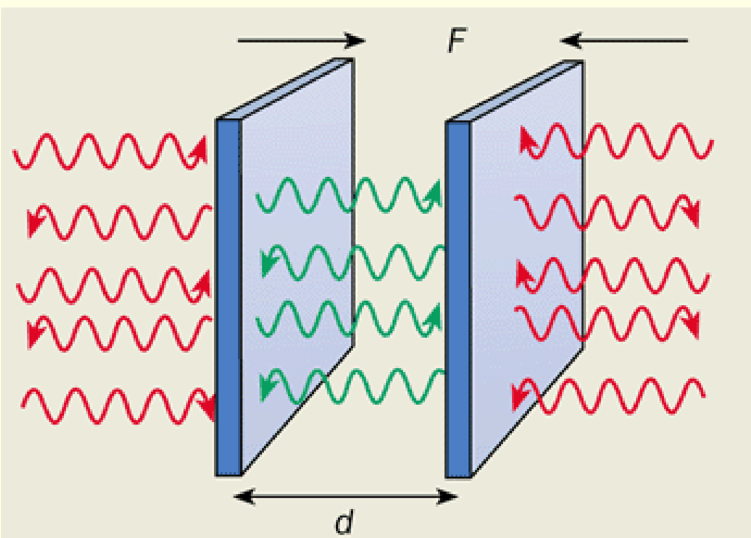
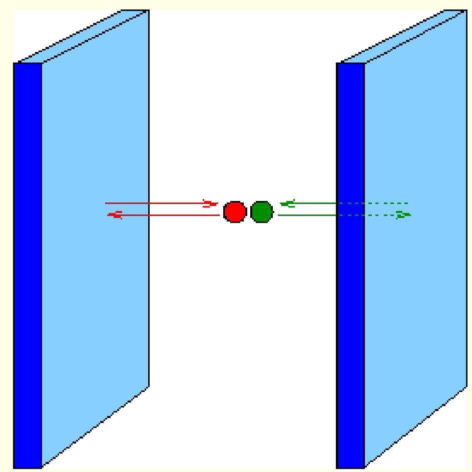
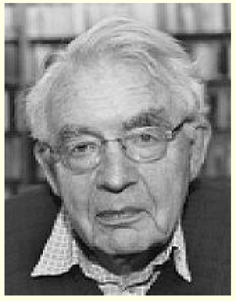


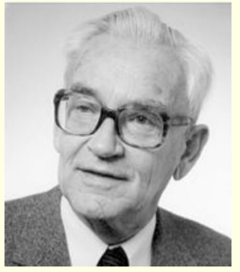


$$F(L) = \frac{dE_0(L)}{dL} = \frac{d\sum_{k(L)}\omega(k(L))}{2dL}$$

$$E_0(L) = -\int \frac{d\tilde{p}}{2\pi}\log(1 + R(-\tilde{p})R(\tilde{p})e^{-2E(\tilde{p})L})$$

Motivation: Casimir-Polder effect





Hendrik Casimir Dirk Polder

colloidal solution: neutral atoms

force not like Van der Waals

$$\frac{F(L)}{A} = -\frac{\hbar c \pi^2}{240 L^4}$$

not a theoretical curiosity!

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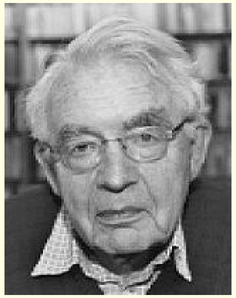

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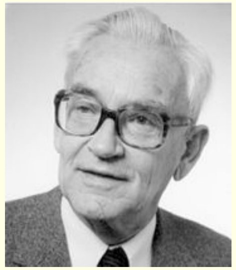

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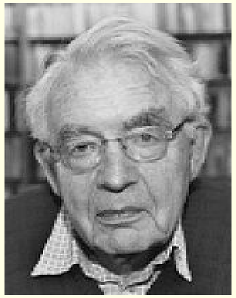

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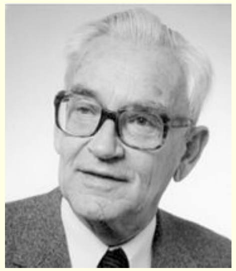

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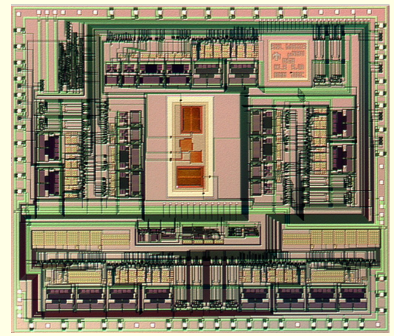

Airbag trigger chip

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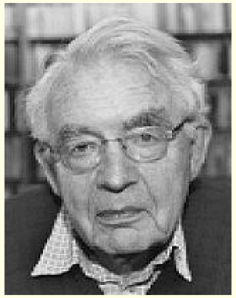

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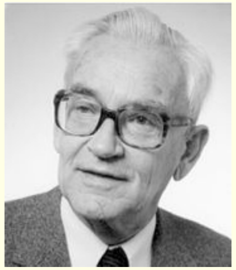

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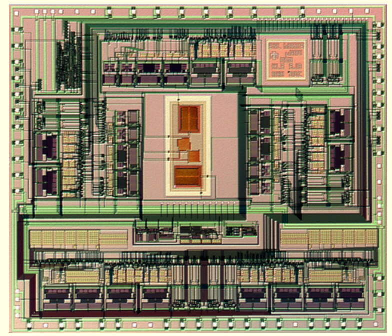

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micromechanical device: pieces stick friction, levitation

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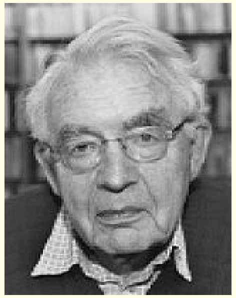

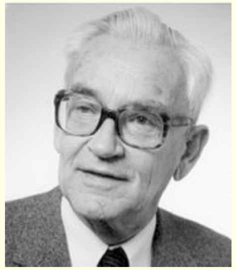

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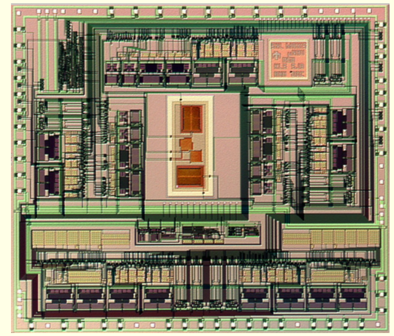

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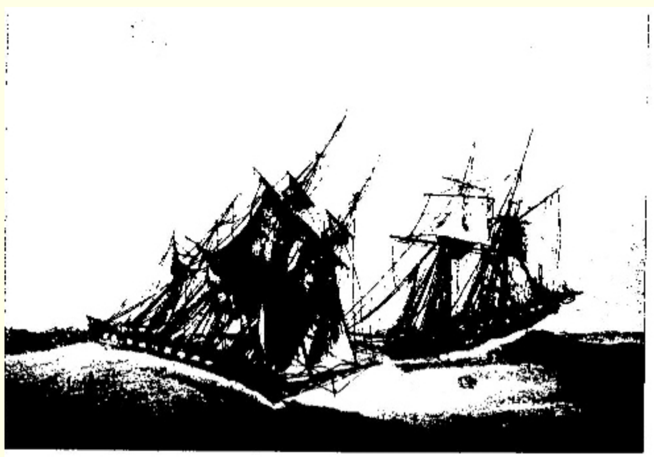
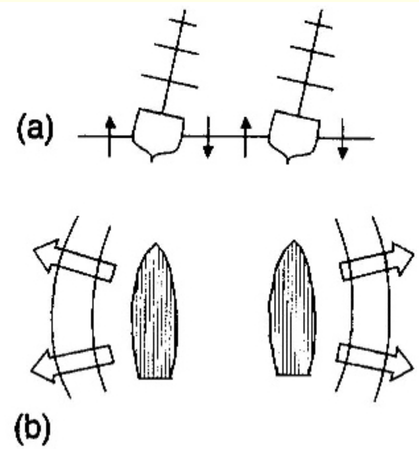
Maritime analogy:





Aim: understand/describe planar Casimir effect

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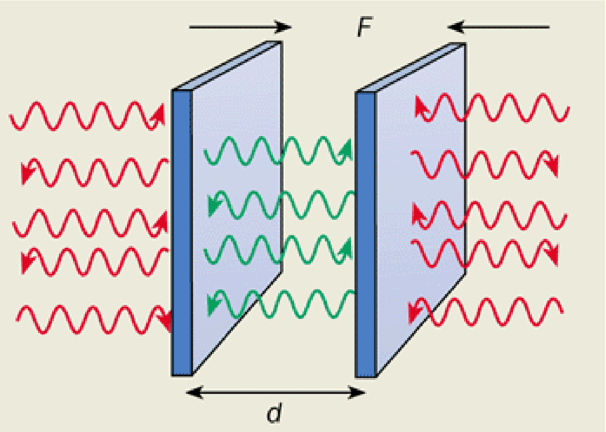

Usual explanation: energy of the vacuum: $E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$

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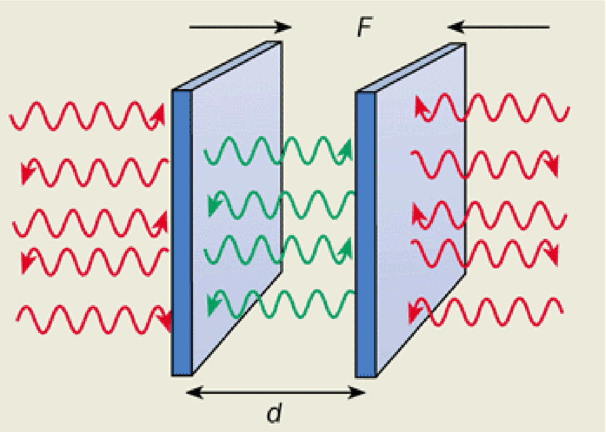

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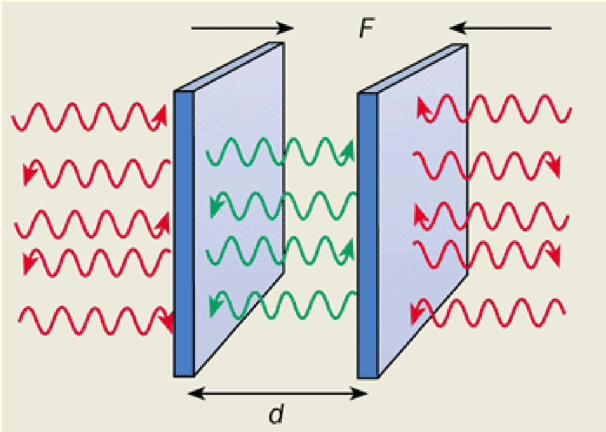

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Lifshitz formula: QED, Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

$$\frac{\Delta E_0(L)}{A} = \int_0^\infty \frac{d^2q}{8\pi^2} d\zeta \log\left[1 - \frac{\epsilon_1\sqrt{\omega^2-q^2} - \sqrt{\epsilon_1\omega^2-q^2}}{\epsilon_1\sqrt{\omega^2-q^2} + \sqrt{\epsilon_1\omega^2-q^2}} \frac{\epsilon_2\sqrt{\omega^2-q^2} - \sqrt{\epsilon_2\omega^2-q^2}}{\epsilon_2\sqrt{\omega^2-q^2} + \sqrt{\epsilon_2\omega^2-q^2}} e^{-2L\sqrt{q^2+\zeta^2}}\right]$$

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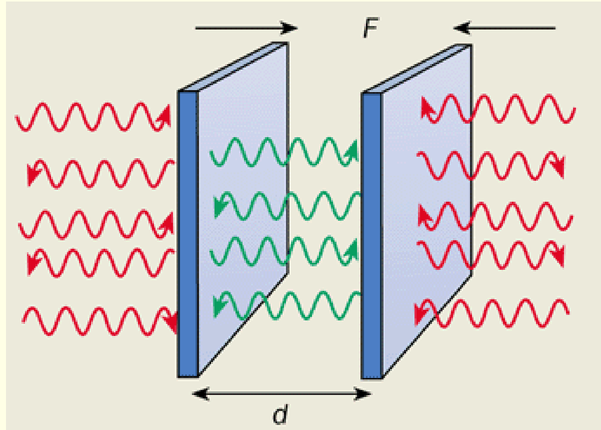

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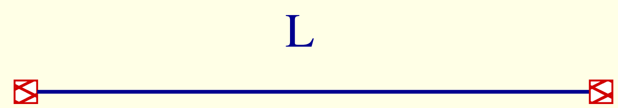
Physics can be understood in 1+1 D QFT

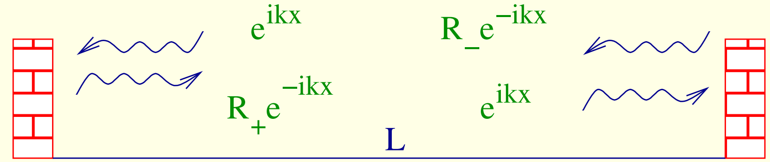


integrability helps to solve the problem even exactly

A simple calculation

Free bulk + interacting boundaries (QED)

$$E_0(L) = \frac{1}{2}\sum_{k(L)} \omega(k(L))$$



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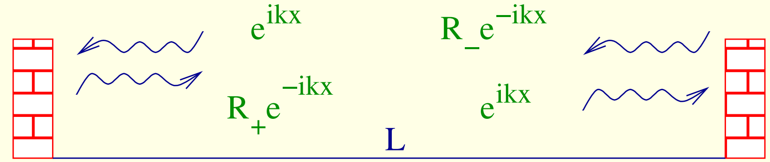

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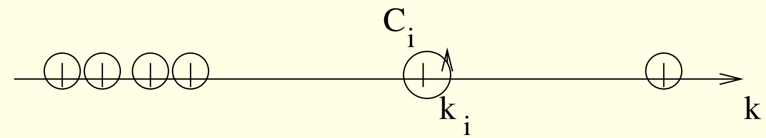

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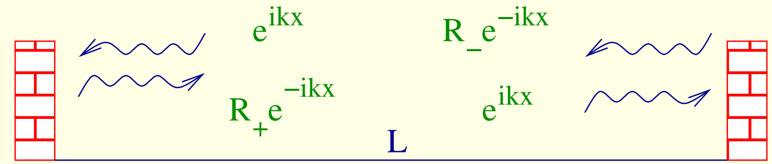

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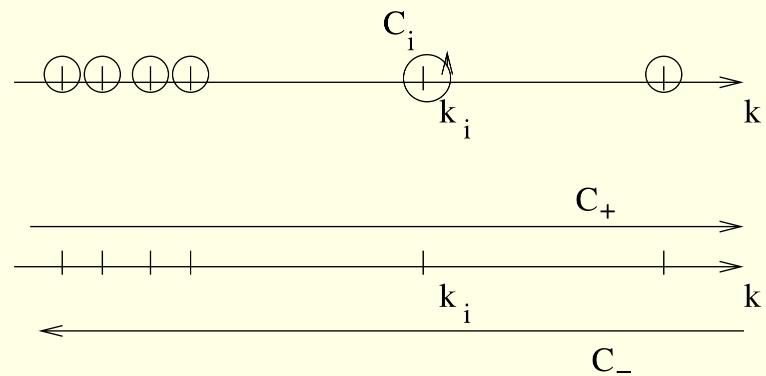

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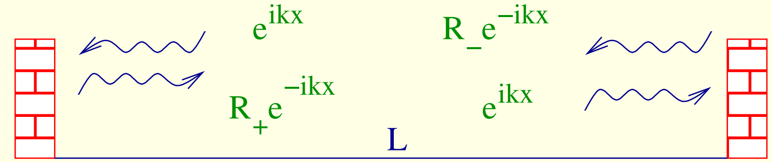

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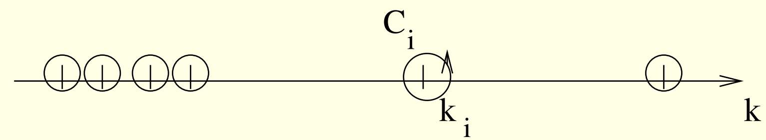

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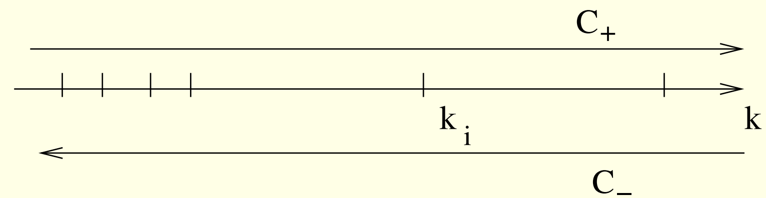

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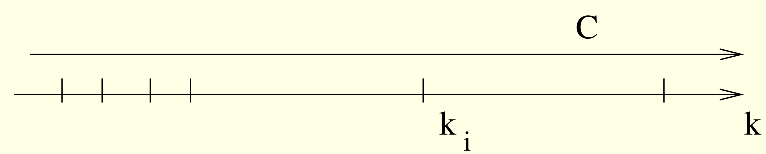

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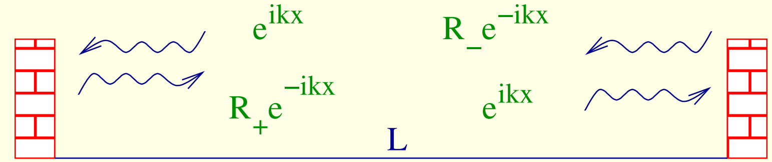

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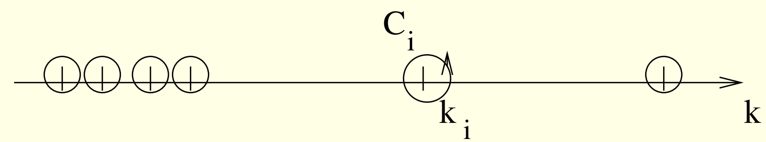

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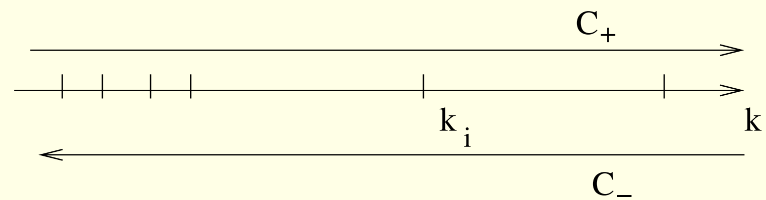

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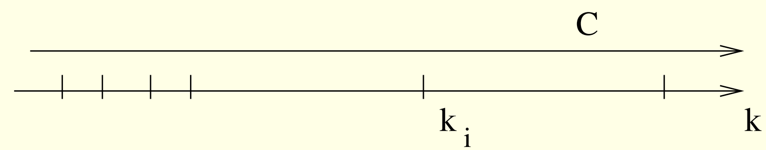

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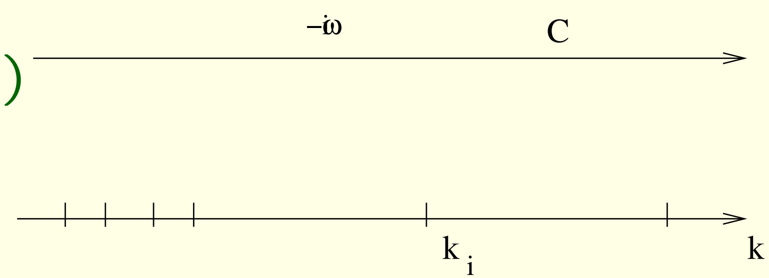

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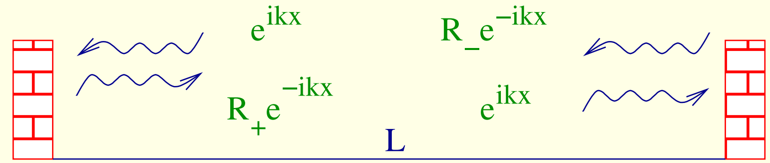

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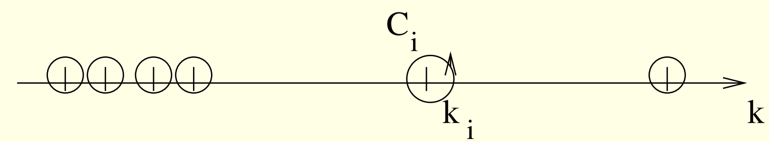

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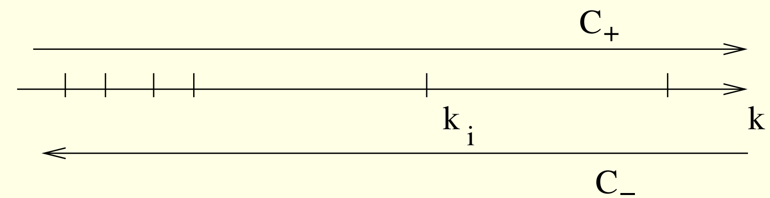

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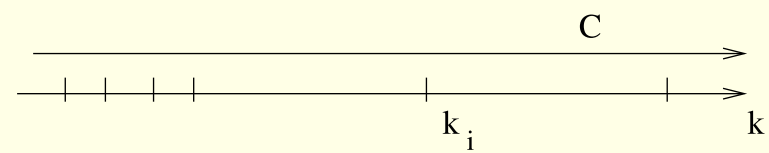

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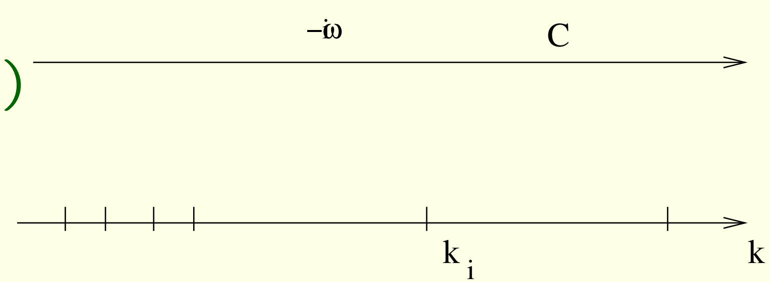
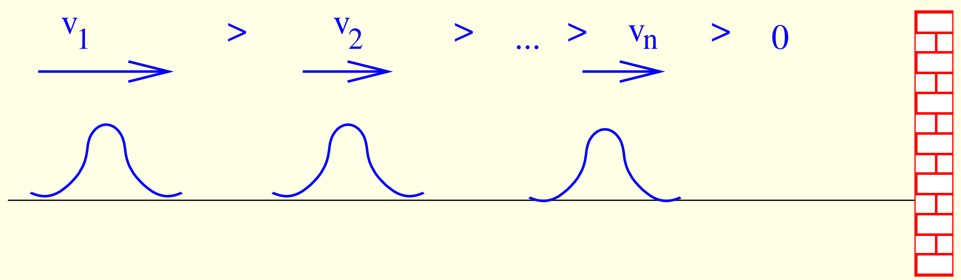


interacting but integrable: similar formula

Integrable boundary field theory: Bootstrap

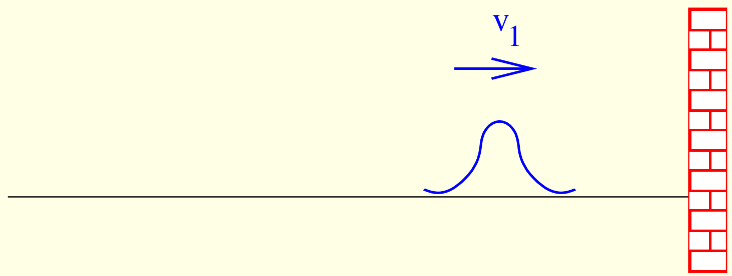
Integrable boundary field theory: Bootstrap

Boundary multiparticle state: with n particles



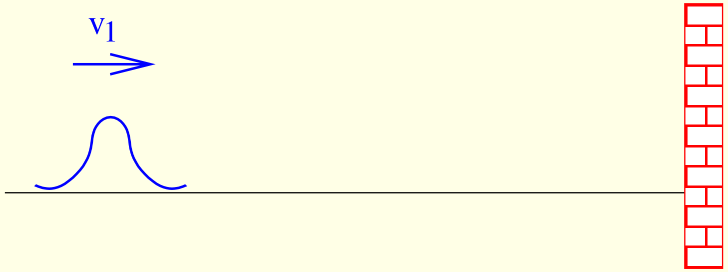
Integrable boundary field theory: Bootstrap

Boundary one particle state:



Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$



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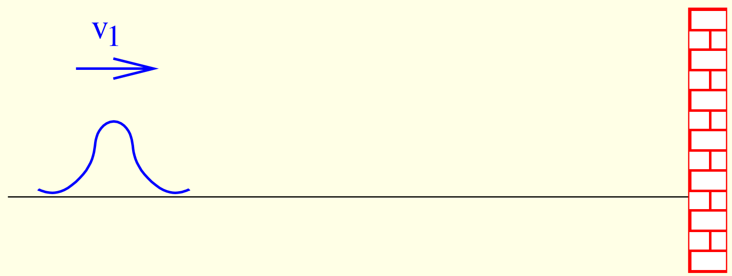
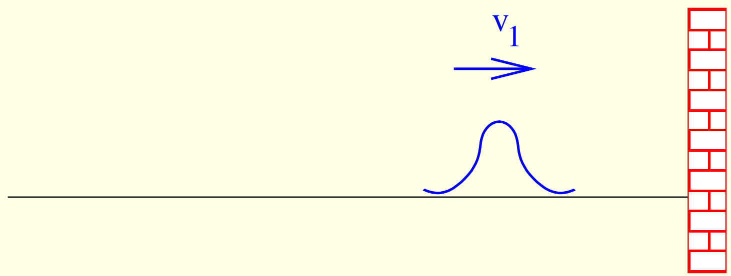
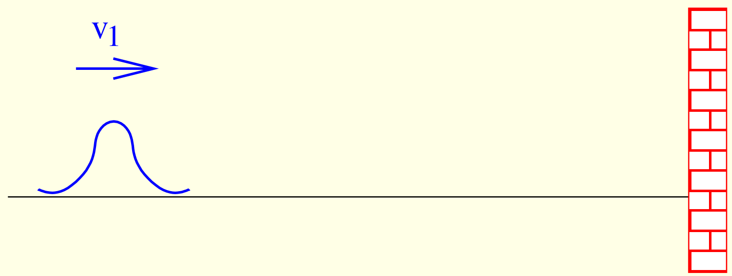


times develop

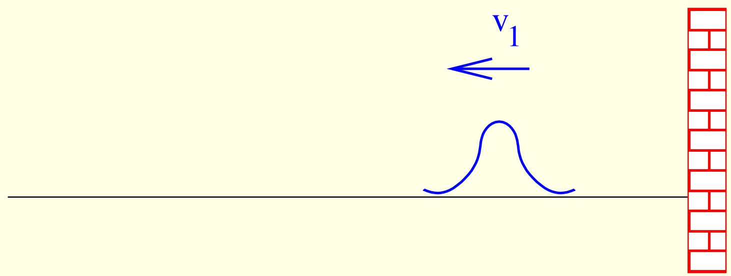


Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$



times develop further



Integrable boundary field theory: Bootstrap

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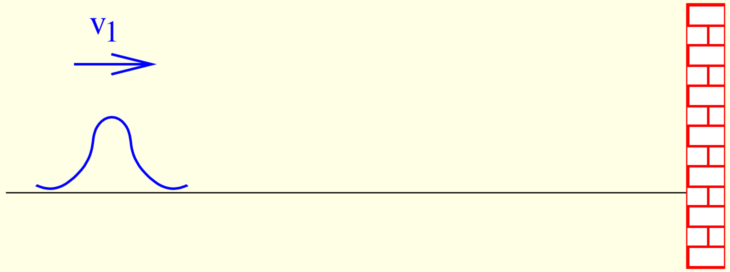
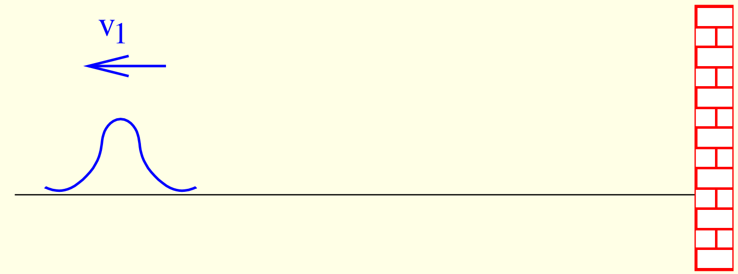


Boundary one pt out state: $t \to \infty$



Integrable boundary field theory: Bootstrap

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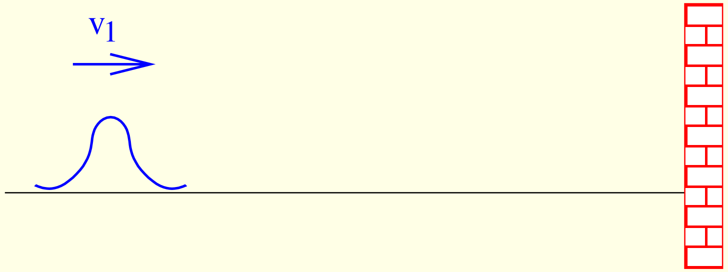

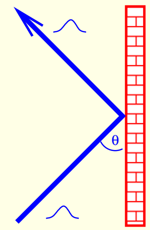

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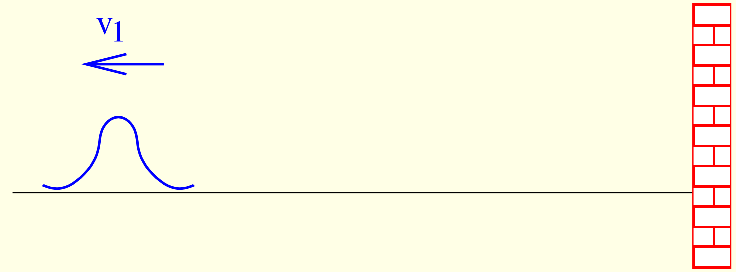

Integrable boundary field theory: Bootstrap

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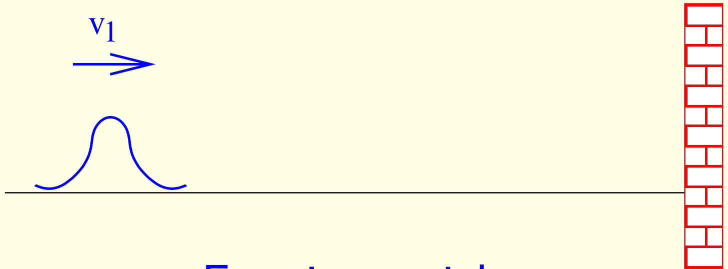
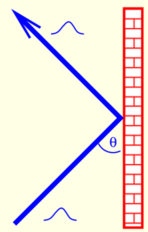
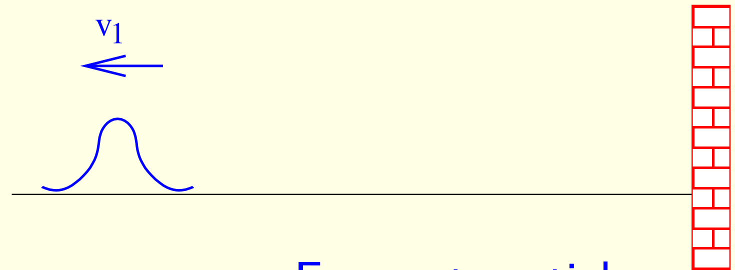


Free in particle



Boundary one pt out state: $t \to \infty$

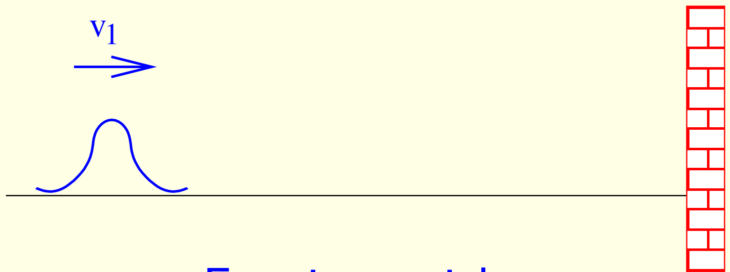


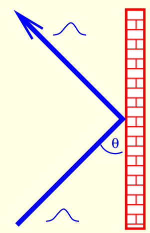
Free out particle

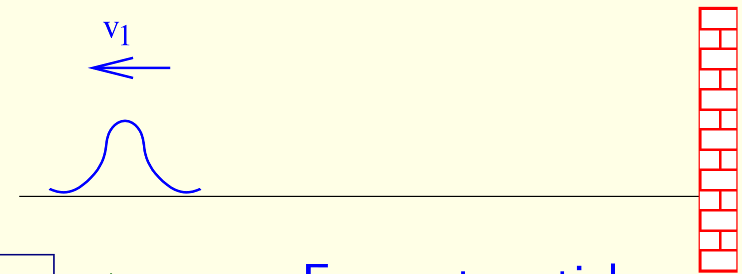
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$

Boundary one pt out state: $t \to \infty$



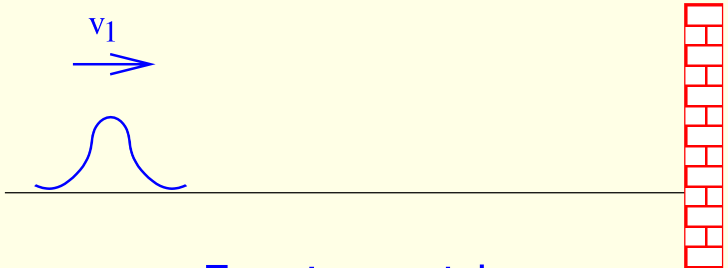


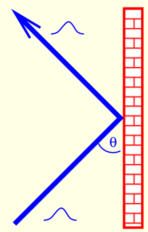


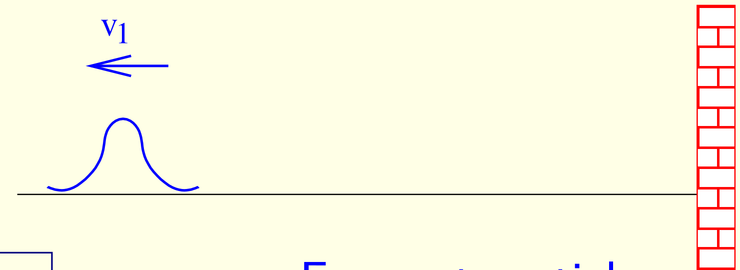
Free in particle $\quad \leftarrow$ $\boxed{\textbf{R-matrix}}$ $\rightarrow \quad$ Free out particle

Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$ Boundary one pt out state: $t \to \infty$





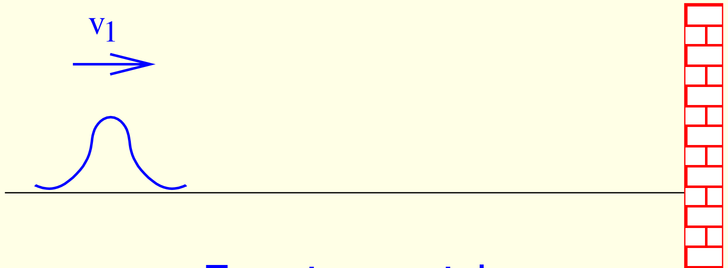


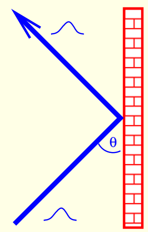
Free in particle $\leftarrow$ $\boxed{\textbf{R-matrix}}$ $\rightarrow$ Free out particle

$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$

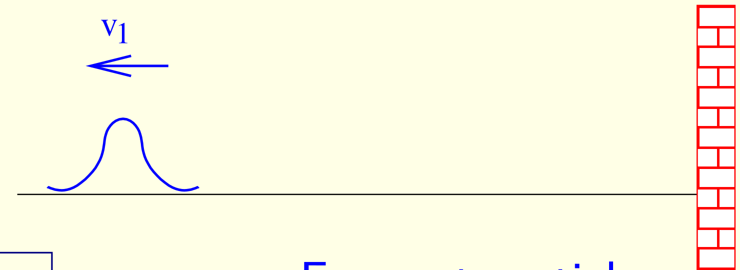
Integrable boundary field theory: Bootstrap

Boundary one particle in state: $t \to -\infty$





Boundary one pt out state: $t \to \infty$

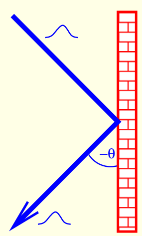


Free in particle $\longleftarrow$ $\boxed{\textbf{R-matrix}}$ $\longrightarrow$ Free out particle

$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$

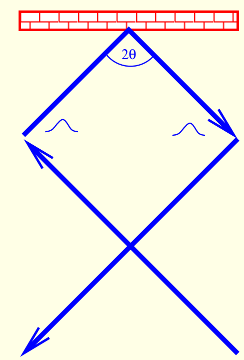
Unitarity

$$R^*(\theta) = R^{-1}(\theta) = R(-\theta)$$



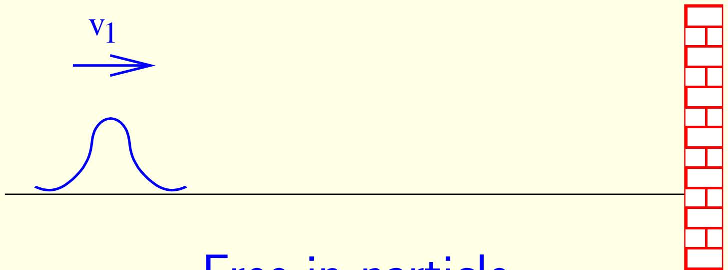
Boundary crossing unitarity

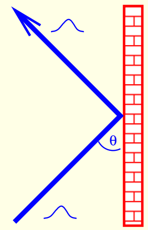
$$R(\tfrac{i\pi}{2} + \theta) = S(2\theta)R(\tfrac{i\pi}{2} - \theta)$$



Integrable boundary field theory: Bootstrap

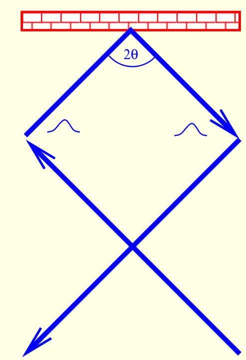
Boundary one particle in state: $t \to -\infty$ Boundary one pt out state: $t \to \infty$





Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

$$|\theta\rangle_B = R(\theta)|-\theta\rangle_B$$



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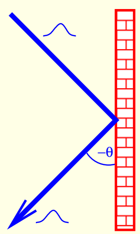

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$$R\left(\frac{i\pi}{2}+\theta\right) = S(2\theta)R\left(\frac{i\pi}{2}-\theta\right)$$

sinh-Gordon $S(\theta) = \frac{\sinh\theta - i\sin\pi p}{\sinh\theta + i\sin\pi p} = [-p] = -(-p)(1+p)\,;\ (p) = \frac{\sinh\left(\frac{\theta}{2}+\frac{i\pi p}{2}\right)}{\sinh\left(\frac{\theta}{2}-\frac{i\pi p}{2}\right)}$

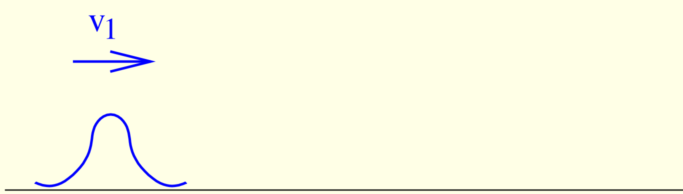
reflection factor $R(\theta) = \left(\frac{1}{2}\right)\left(\frac{1+p}{2}\right)\left(1-\frac{p}{2}\right)\left[\frac{3}{2}-\frac{\eta p}{\pi}\right]\left[\frac{3}{2}-\frac{\Theta p}{\pi}\right]$ Ghoshal-Zamolodchikov '94

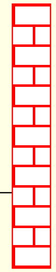
Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu(\cosh b\phi - 1) - \delta\{\mu_+^B e^{\frac{b}{2}\phi} + \mu_-^B e^{-\frac{b}{2}\phi}\}$ $\sqrt{\frac{\mu}{\sin b^2}}\cosh b^2(\eta \pm \Theta) = \mu_\pm^B$

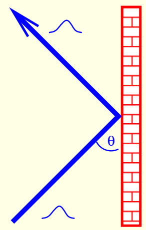
Integrable boundary field theory: Bootstrap

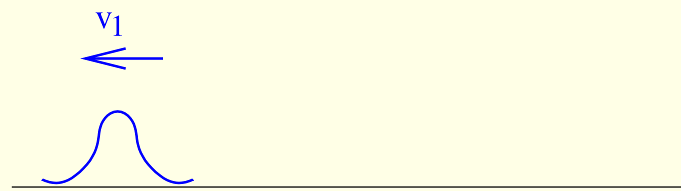
Boundary one particle in state: $t \to -\infty$

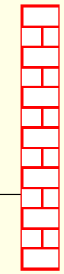
Boundary one pt out state: $t \to \infty$











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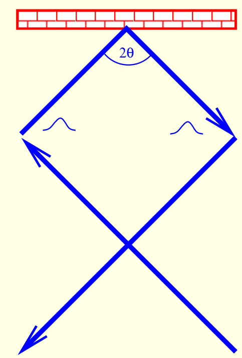
$$|\theta\rangle_B \quad = \quad R(\theta)|-\theta\rangle_B$$



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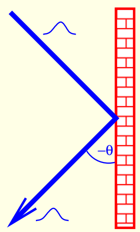

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Integrability $\rightarrow$factorizability: $|\theta_1,\theta_2,\ldots,\theta_n\rangle_B = \prod_{i<j} S(\theta_i-\theta_j)S(\theta_i+\theta_j)\prod_i R(\theta_i)|-\theta_1,-\theta_2,\ldots,-\theta_n\rangle_B$

Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

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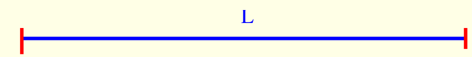
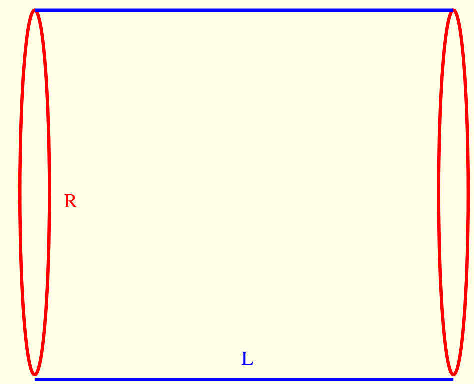


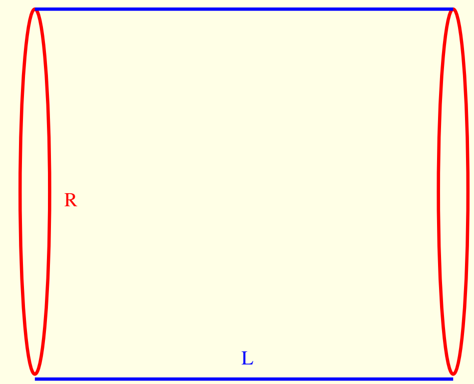
Boundary thermodynamic Bethe Ansatz



Groundstate energy for large L from IR reflection: $E_0(L) =$

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H^B(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log(e^{-E_0(L)R})$$

Boundary thermodynamic Bethe Ansatz



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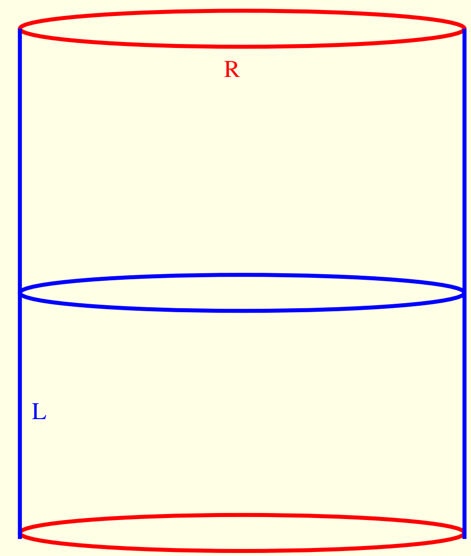

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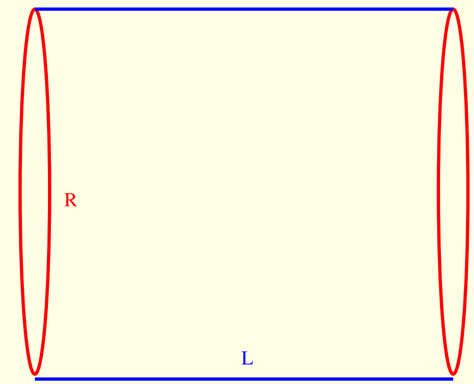

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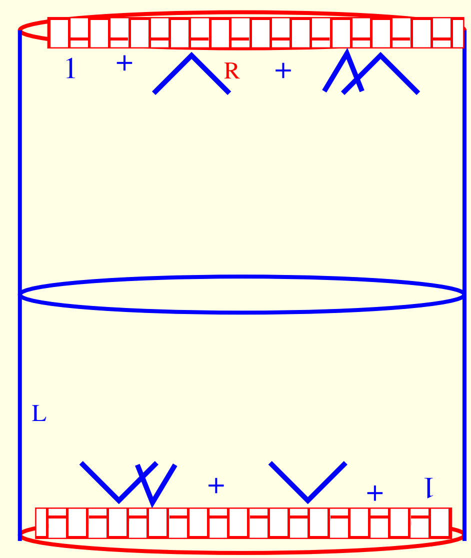
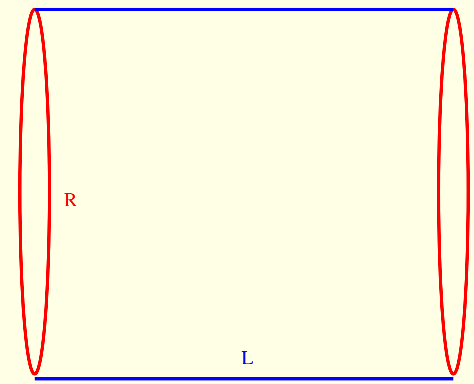
Boundary state $|B\rangle = \exp\left\{\int_{-\infty}^{\infty}\frac{d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)A^+(-\theta)A^+(\theta)\right\}|0\rangle$



Boundary thermodynamic Bethe Ansatz



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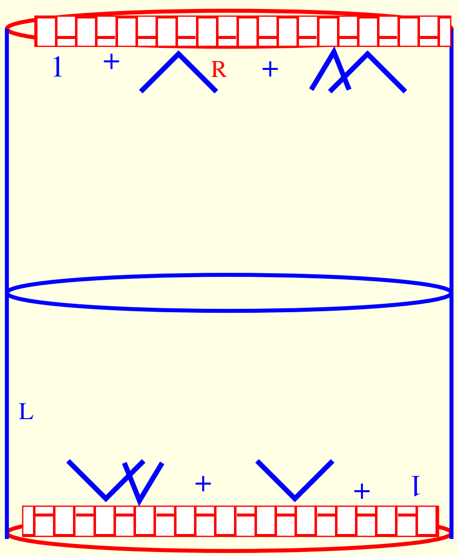
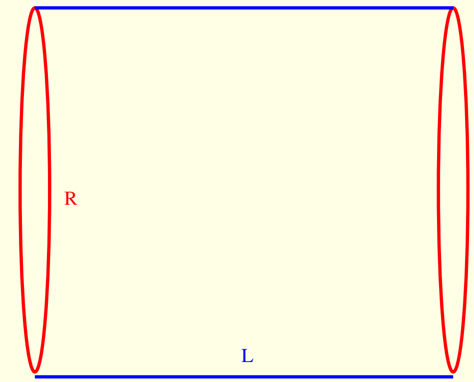


Dominant contribution for large L: two particle term

$$\langle B|e^{-H(R)L}|B\rangle = 1 + \sum_k R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)e^{-2m\cosh\theta_k L} + \dots$$

Boundary thermodynamic Bethe Ansatz



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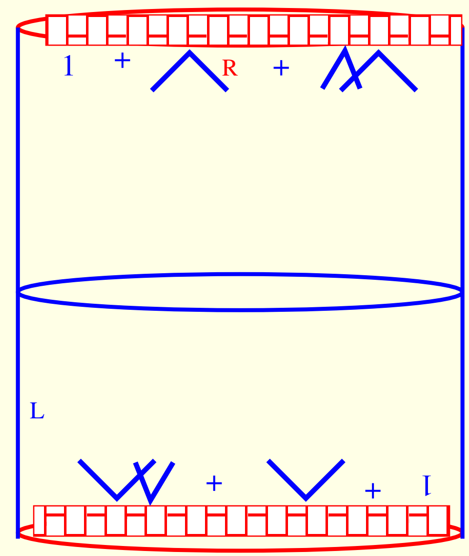

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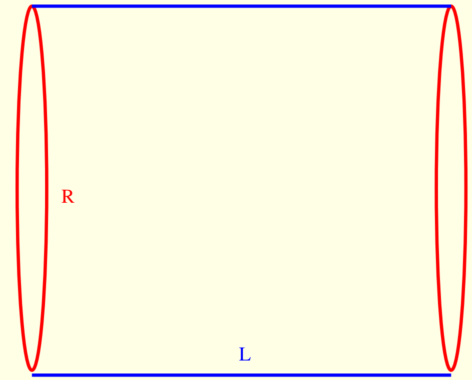
quantization condition: $m\sinh\theta_k = \frac{2\pi}{R}$ $\quad\sum_k \to \frac{Rm}{4\pi}\int d\theta\cosh\theta$

$E_0(L) = -\int\frac{m\cosh\theta d\theta}{4\pi}R(\frac{i\pi}{2}-\theta)R(\frac{i\pi}{2}+\theta)\,e^{-2mL\cosh\theta}$; [Z.B, L. Palla, G. Takacs]

Boundary thermodynamic Bethe Ansatz



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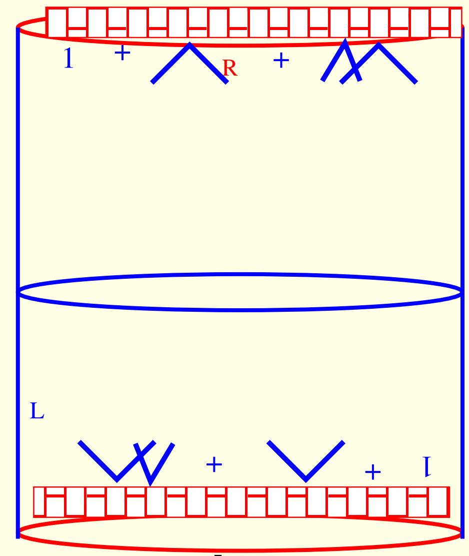

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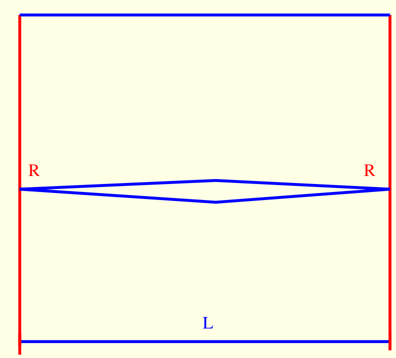
Ground state energy exactly: $\boxed{E_0(L) = -m\int\frac{d\theta}{4\pi}\cosh(\theta)\log(1+e^{-\epsilon(\theta)})}$



$$\epsilon(\theta) = 2mL\cosh\theta - \log(R(\tfrac{i\pi}{2}-\theta)R(\tfrac{i\pi}{2}+\theta))$$
$-\int\frac{d\theta'}{2\pi}\varphi'(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$ [LeClair, Mussardo, Saleur, Skorik]

Casimir effect: Boundary finite size effect

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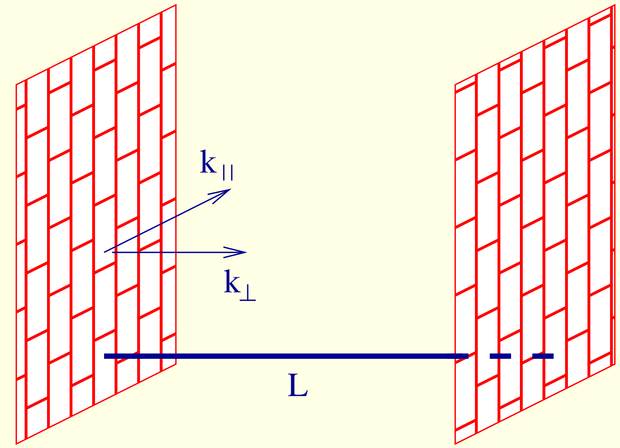
Extension to higher dimensions: $\vec{k}_{\|}$ label

Dispersion $\omega = \sqrt{m^2 + \vec{k}_{\|}^2 + k_\perp^2} = \sqrt{m_{\mathsf{eff}}^2 + k_\perp^2}$

rapidity $\omega = m_{\mathsf{eff}}(k_{\|}) \cosh\theta$, $k_\perp = m_{\mathsf{eff}}(k_{\|}) \sinh\theta$

Reflection $R(\theta, m_{\mathsf{eff}}(k_{\|}))$



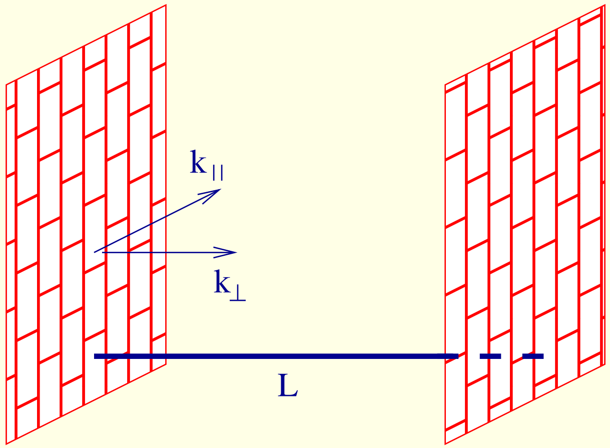
Casimir effect: Boundary finite size effect

Extension to higher dimensions: $\vec{k}_{\|}$ label

Dispersion $\omega = \sqrt{m^2 + \vec{k}_{\|}^2 + k_{\perp}^2} = \sqrt{m_{\text{eff}}^2 + k_{\perp}^2}$

rapidity $\omega = m_{\text{eff}}(k_{\|}) \cosh\theta, \; k_{\perp} = m_{\text{eff}}(k_{\|}) \sinh\theta$

Reflection $R(\theta, m_{\text{eff}}(k_{\|}))$



Bstate: $|B\rangle = \left\{ 1 + \int \frac{d^{D-1}k_{\|}}{(2\pi)^{D-1}} \frac{d\theta}{4\pi} R(\frac{i\pi}{2} - \theta, m_{\text{eff}}(k_{\|})) A^{+}(-\theta, -\vec{k}_{\|}) A^{+}(\theta, \vec{k}_{\|}) + \ldots \right\} |0\rangle$

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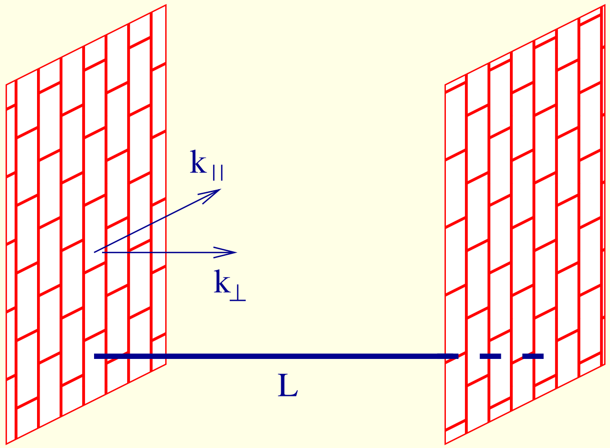

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Ground state energy (for free bulk):

$$\boxed{E_0(L) = \int \frac{d^{D-1}k_{||}}{(2\pi)^{D-1}} \frac{d\theta}{4\pi} \log(1 + R^1(\frac{i\pi}{2} + \theta, m_{\text{eff}}) R^2(\frac{i\pi}{2} - \theta, m_{\text{eff}}) e^{-2m_{\text{eff}} \cosh\theta L})}$$

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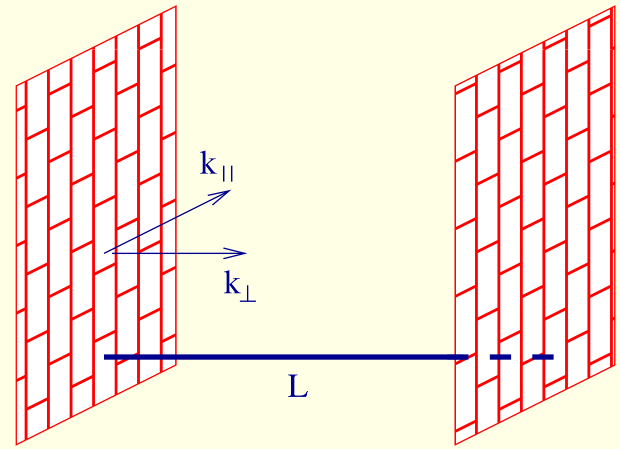

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QED: Parallel dielectric slabs $(\epsilon_1, 1, \epsilon_2)$

reflections $E_{||,\perp}, B_{||,\perp} \longrightarrow R_{||,\perp}$ look it up in Jackson:

$$R_\perp(\omega, k_{||} = q) = \frac{\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}} \quad R_{||}(\omega, k_{||} = q) = \frac{\epsilon\sqrt{\omega^2 - q^2} - \sqrt{\epsilon\omega^2 - q^2}}{\epsilon\sqrt{\omega^2 - q^2} + \sqrt{\epsilon\omega^2 - q^2}}$$

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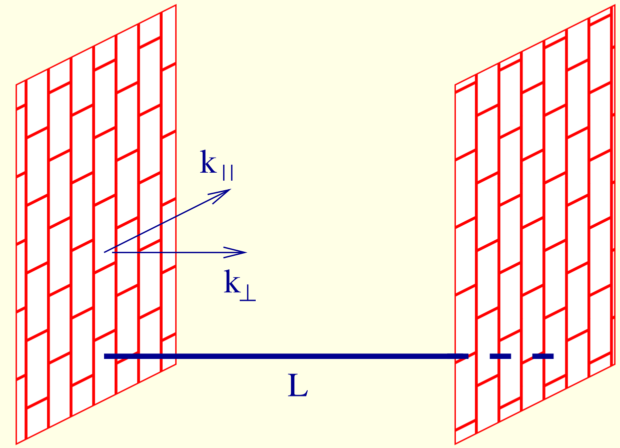

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Lifshitz formula

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Conclusion about Casimir

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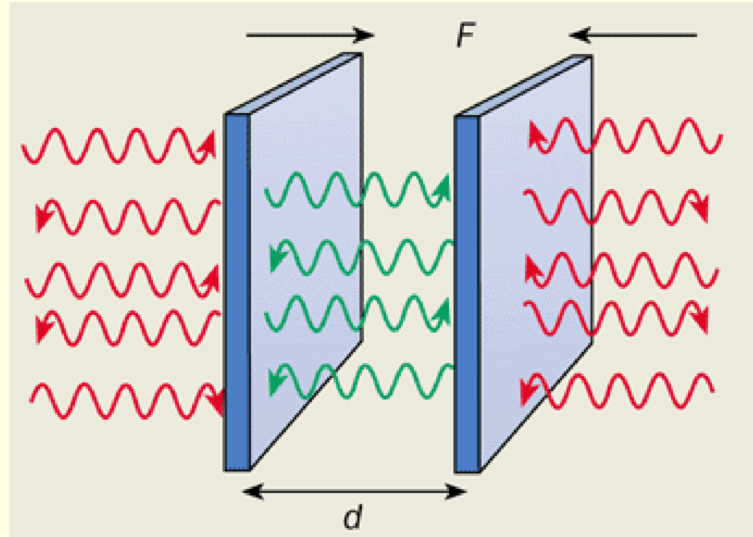


Usual derivation:

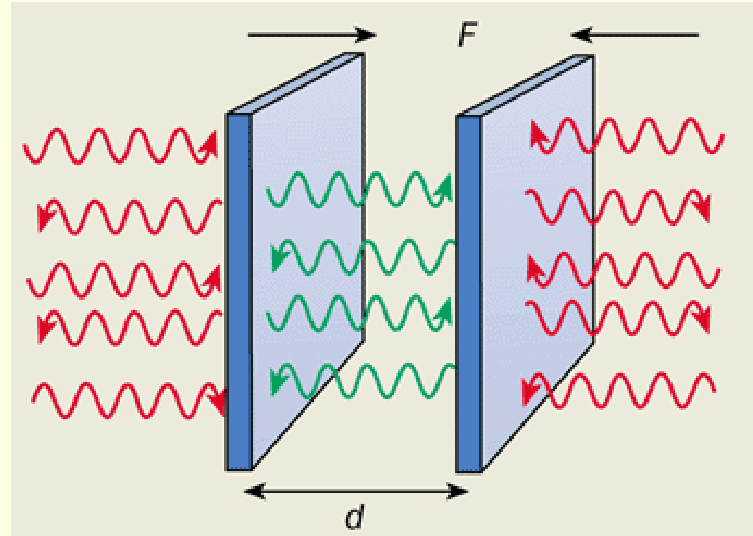
summing up zero freqencies

$E_0(L) = \frac{1}{2} \sum_{k(L)} \omega(k(L)) \propto \infty$

Complicated finite volume problem

$+$ divergencies

Conclusion about Casimir



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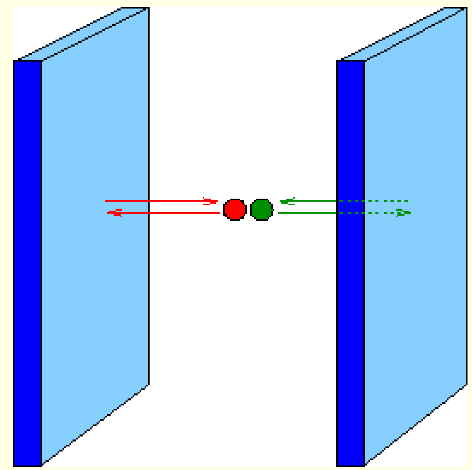
as a boundary finite size effect



$$\boxed{E_0(L) = -\int \frac{d\tilde{p}}{2\pi} \log(1 + R(-\tilde{p})R(\tilde{p})e^{-2\tilde{\epsilon}(\tilde{p})L})}$$

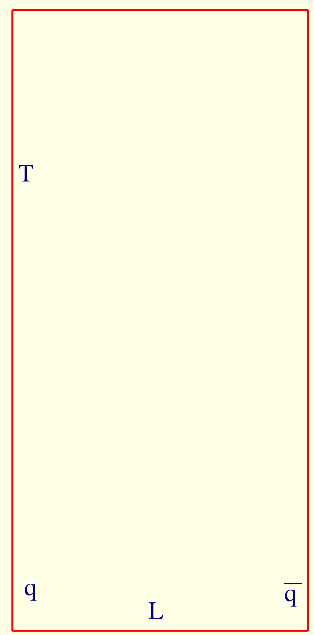
Reflection factor of the IR degrees of freedom:

semi infinite settings,

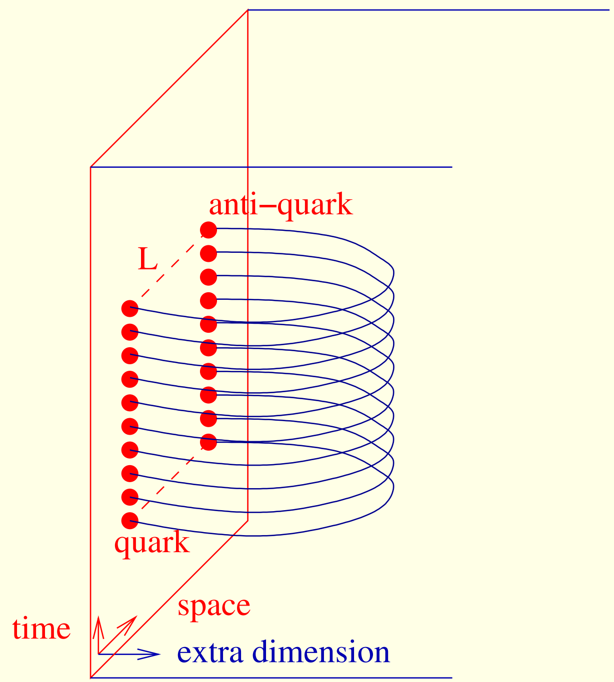
easier to calculate,

no divergences

Main problem: $q-\bar{q}$ potential in $\mathcal{N}=4$ SYM

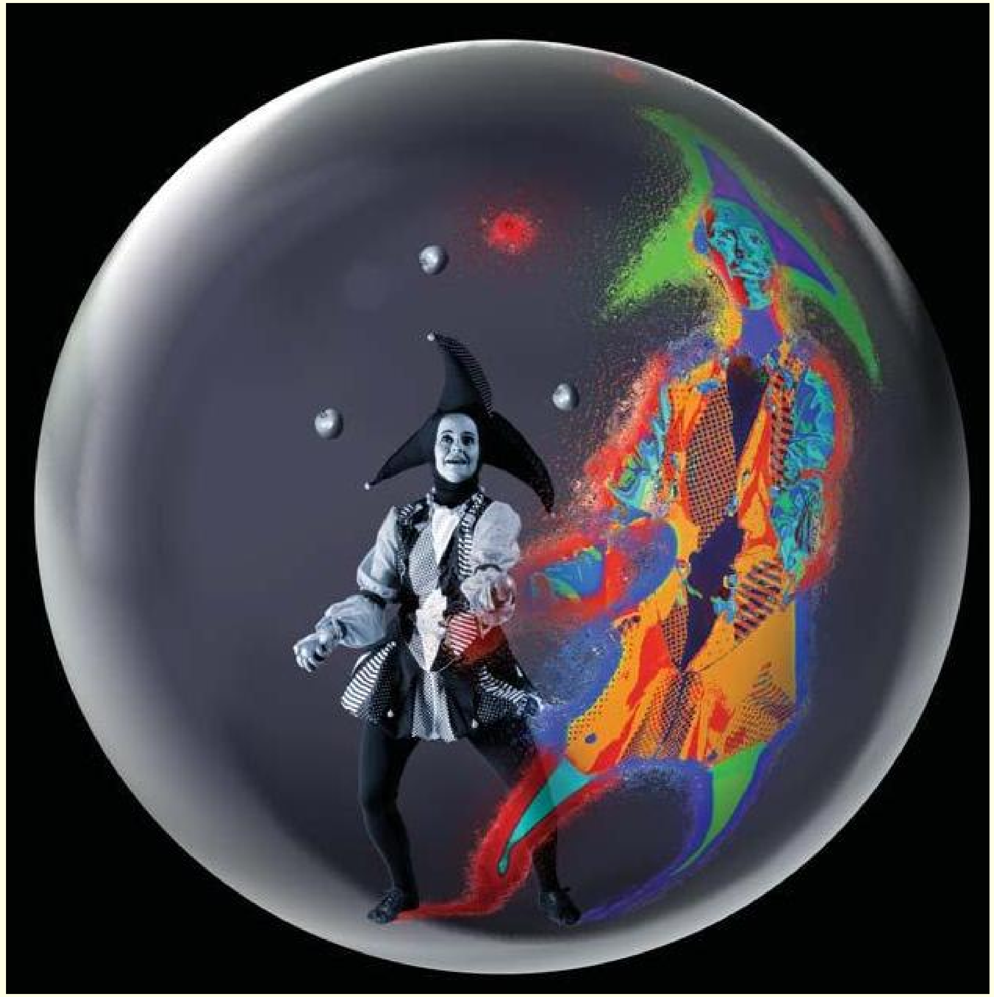


$$\langle W\rangle = e^{-\frac{T}{L}V_{q\bar{q}}(\lambda)}$$



$$= e^{-TE_0(L,\lambda)}$$

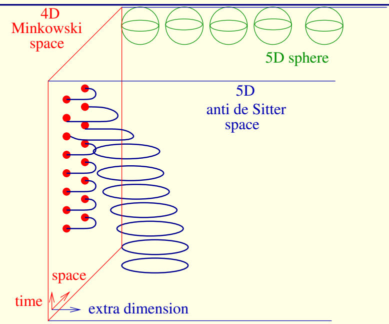
AdS/CFT correspondence (Maldacena 1997)



The Illusion of Gravity - Juan Maldacena, Scientific American (2005)

AdS/CFT correspondence (Maldacena 1997)

$\boxed{\mathrm{II}_B \text{ superstring on } AdS_5 \times S^5}$



$$\sum_1^6 Y_i^2 = R^2 \qquad -++++- = -R^2$$

$$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + \ldots$$

$$\equiv$$

$\boxed{\mathcal{N} = 4 \text{ D=4 } SU(N) \text{ SYM}}$

$$\begin{array}{ccccc} & & \Psi_{1,2,3,4} & & \\ & \nearrow & & \searrow & \\ A_\mu & & & & \Phi_{1,2,3,4,5,6} \\ & \searrow & & \nearrow & \\ & & \overline{\Psi}_{1,2,3,4} & & \end{array}$$

$$\frac{2}{g_{YM}^2} \int d^4x \mathrm{Tr} \left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$$

$$V(\Phi, \Psi) = \frac{1}{4}[\Phi, \Phi]^2 + \overline{\Psi}[\Phi, \Psi]$$

$$\beta = 0 \text{ superconformal } \frac{PSU(2,2|4)}{SO(5)\times SO(1,4)}$$

gaugeinvariants: $\mathcal{O} = \mathrm{Tr}(\Phi^L), \det(\ )$

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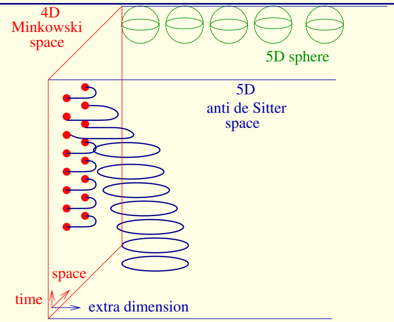

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Dictionary

Couplings: $\sqrt{\lambda} = \frac{R^2}{\alpha'}$, $g_s = \frac{\lambda}{N} \to 0$

2D QFT

String energy levels: $E(\lambda)$

$$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \dots$$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

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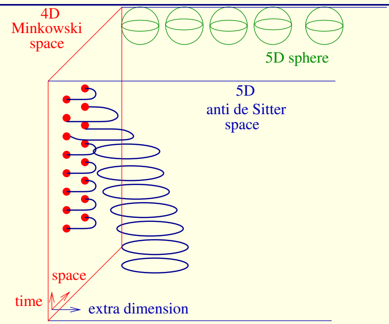

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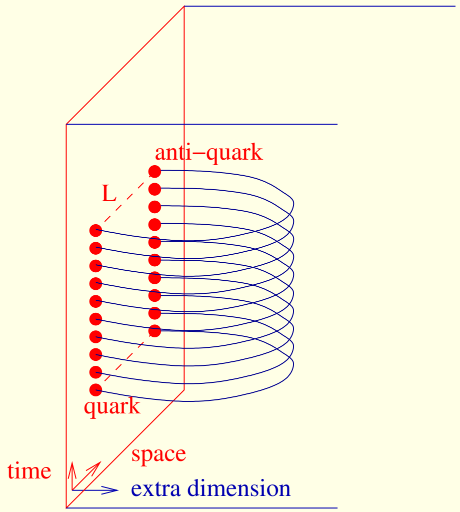
2D integrable QFT

AdS/CFT integrability: $q - \bar{q}$ potential

quark-antiquark potential



$$V(L) = \frac{-\lambda}{4\pi L} + \dots$$

$\equiv$

Wilson loop:

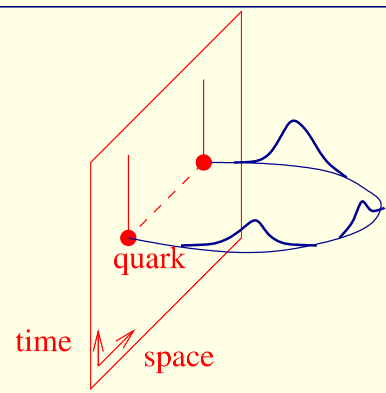
$$\langle \mathcal{P} e^{\oint_C A_\mu dx^\mu + \vec{\Phi}\vec{n}|\dot{x}|ds} \rangle \propto e^{-\frac{T}{L}V_{q\bar{q}}(\lambda,\theta)}$$

strong coupling

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}(1 - \frac{1.3359}{\sqrt{\lambda}} + \dots)$$

minimal surface+fluctuations

Integrable system on the strip



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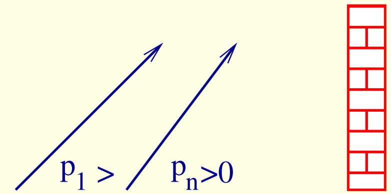
R-matrix bootstrap program

Boundary asymptotic states $|p_1, p_2, \dots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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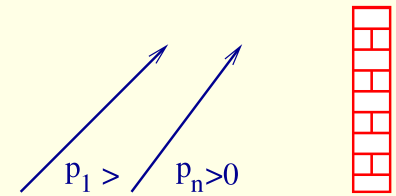

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dispersion relation $E(p) = \sqrt{m^2 + p^2}$

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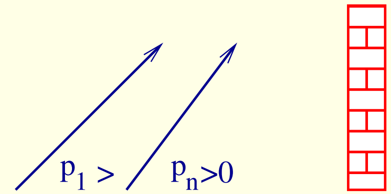

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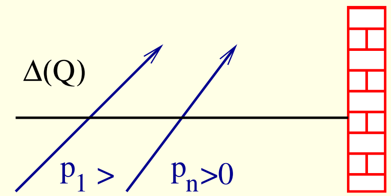

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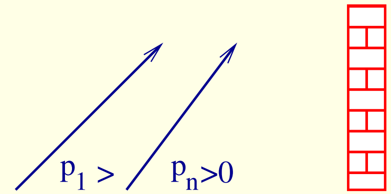

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Reflection matrix R: $|out\rangle \rightarrow |in\rangle$
commutes with sym. $[R, \Delta(Q)] = 0$

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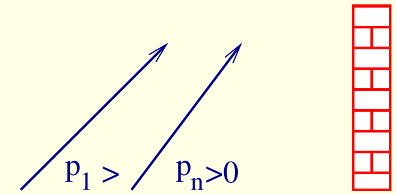

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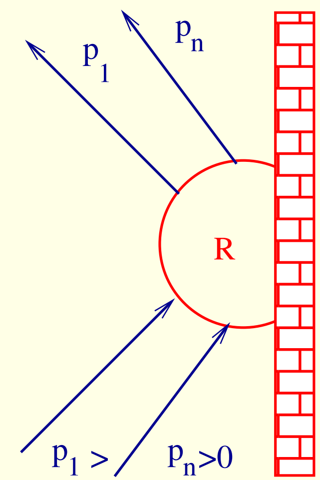

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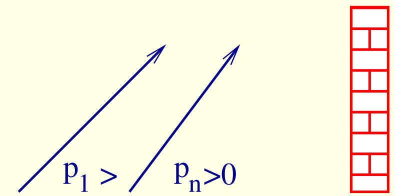

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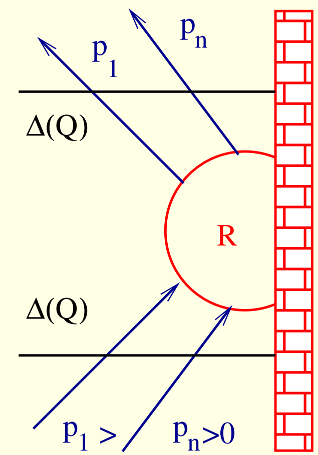

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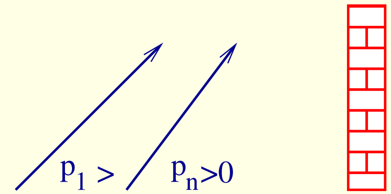

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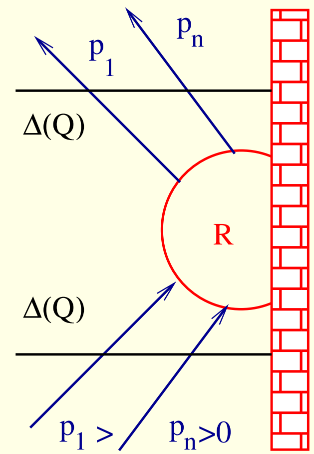
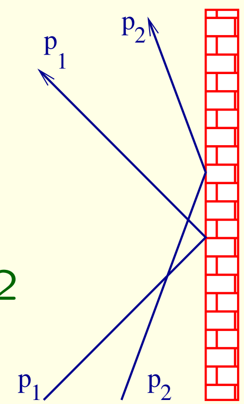
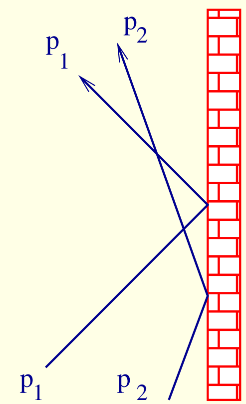


Reflection matrix R: $|out\rangle \rightarrow |in\rangle$
commutes with sym. $[R, \Delta(Q)] = 0$
Higher spin concerved charge
factorization + Bdry Yang-Baxter equation

$$R_{12} = S_{12} R_1 \bar{S}_{21} R_2 = R_2 \bar{S}_{21} R_1 S_{12}$$





$\boxed{\text{R-matrix} = \text{scalar . Matrix}}$

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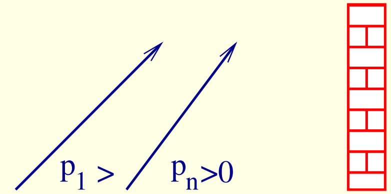

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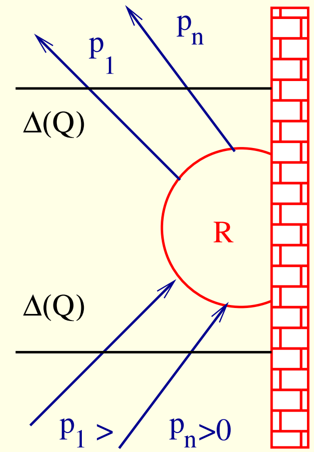

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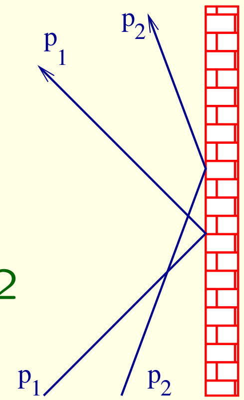
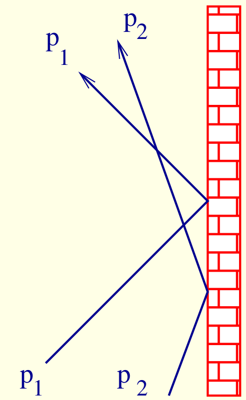
$R_{12} = S_{12} R_1 \bar{S}_{21} R_2 = R_2 \bar{S}_{21} R_1 S_{12}$

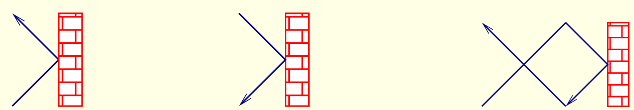




$\boxed{\text{R-matrix} = \text{scalar . Matrix}}$

Unitarity $R(p)R(-p) = Id$

Boundary crossing symmetry $R(p) = S(p, -p)R(\bar{p})$



Maximal analyticity: all poles have physical origin $\to$ boundstates, anomalous thresholds

R-matrix bootstrap program: AdS

Nondiagonal scattering: $\boxed{\text{R-matrix} = \text{scalar} \; . \; \text{Matrix}}$

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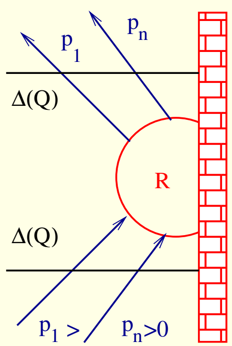
R-matrix: [Correa-Maldacena-Sever,Drukker]

global symmetry $PSU(2|2)_{diag}$

$$Q = 1 \text{ reps} \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_{\dot{1}} \\ b_{\dot{2}} \\ f_{\dot{3}} \\ f_{\dot{4}} \end{pmatrix}$$

$$[R, \Delta(Q)] = 0$$



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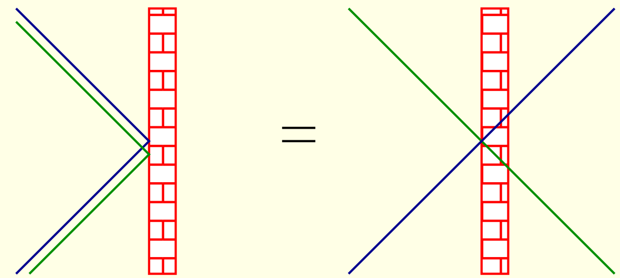

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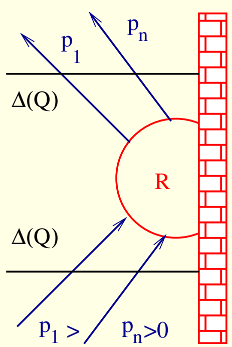

$$R^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}(p) = S^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}(p, -p) R_0(p)$$

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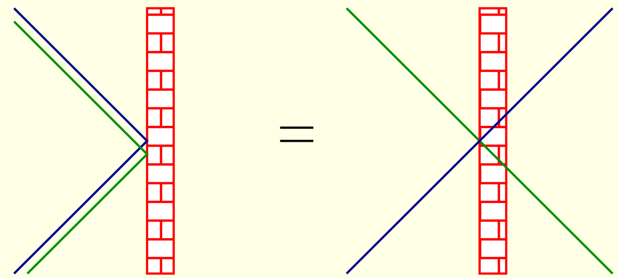

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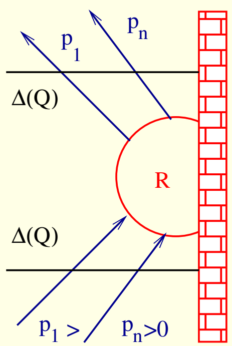

$$R^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}(p) = S^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}(p, -p) R_0(p)$$

Unitarity

$$R(z)R(-z) = 1$$

Crossing symmetry

$$R(z) = S(z, -z)R(\omega_2 - z)$$

$$\boxed{\begin{gathered} R_0(p) = \frac{\sigma_B(p)}{\sigma(p,-p)} \\ \sigma_B = e^{i\chi(x^+)-i\chi(x^-)} \end{gathered}}$$

boundary dressing phase

$$\chi(x) = \oint \frac{dz}{2\pi} \frac{1}{x-z} \frac{\sinh(2\pi g(z+z^{-1}))}{2\pi g(z+z^{-1})}$$

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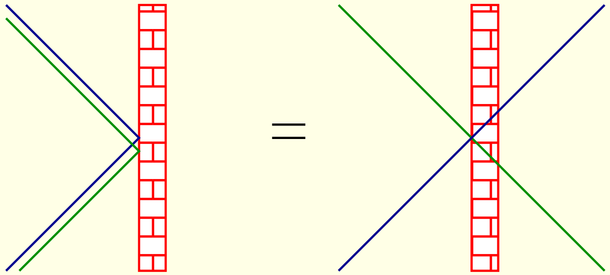

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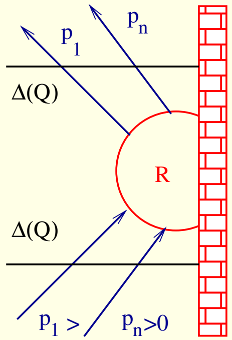

$$R_{\alpha\dot\alpha}^{\beta\dot\beta}(p) = S_{\alpha\dot\alpha}^{\beta\dot\beta}(p,-p)R_0(p)$$

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Maximal analyticity: no boundstates

Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

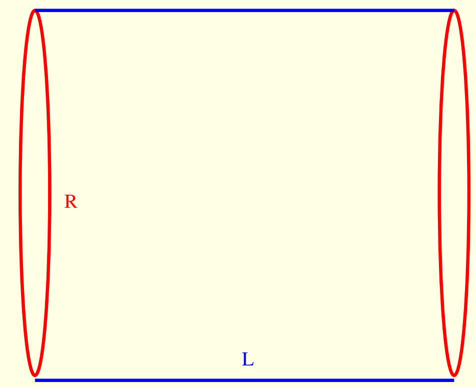
Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

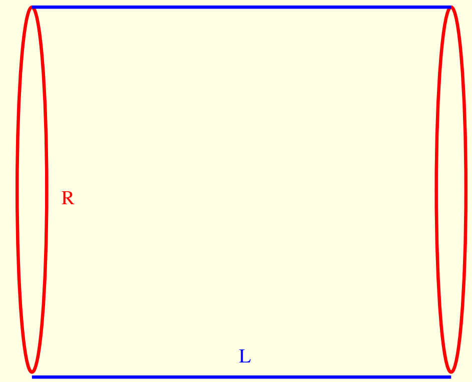
Boundary thermodynamic Bethe Ansatz

Groundstate energy for large L from IR reflection: $E_0(L) =$

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H^B(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log(e^{-E_0(L)R})$$

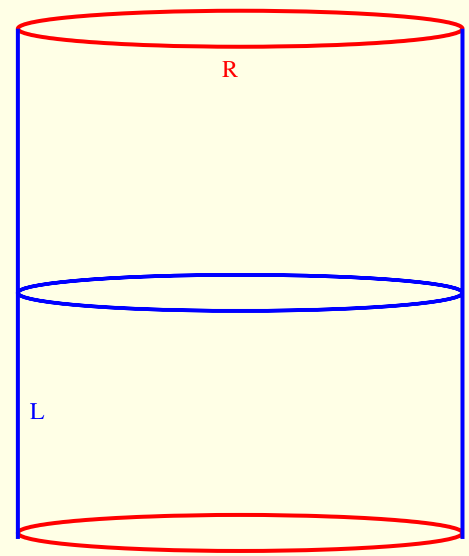


Boundary thermodynamic Bethe Ansatz



Groundstate energy for large L from IR reflection: $E_0(L) =$

$$-\lim_{R\to\infty}\frac{1}{R}\log(\mathrm{Tr}(e^{-H^B(L)R})) = -\lim_{R\to\infty}\frac{1}{R}\log\langle B|e^{-H(R)L}|B\rangle$$



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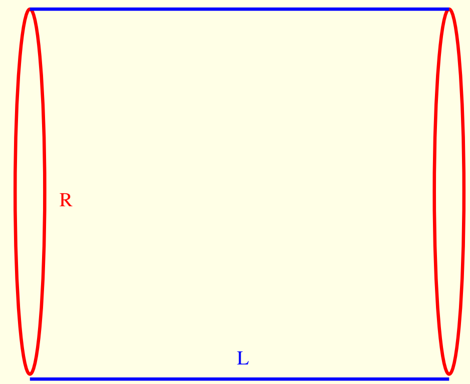

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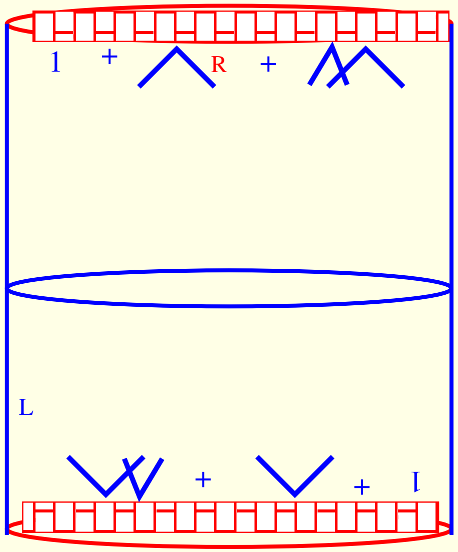
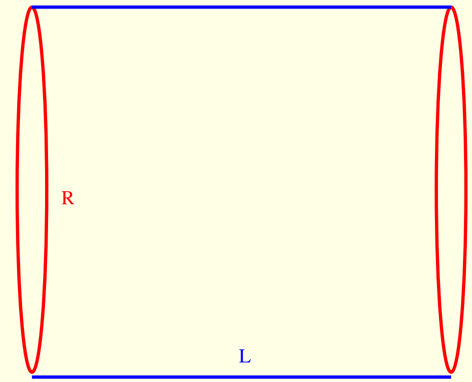


Boundary state $|B\rangle = \exp\left\{\int_{-\infty}^{\infty}\frac{dq}{4\pi}R_a^b(\bar{q})C^{ad}A_b^+(-q)A_d^+(q)\right\}|0\rangle$

Boundary thermodynamic Bethe Ansatz



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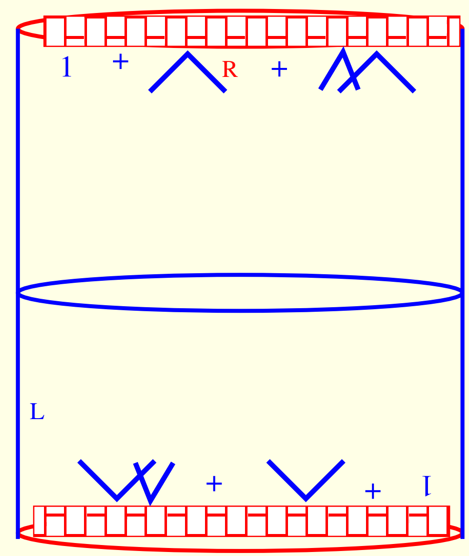
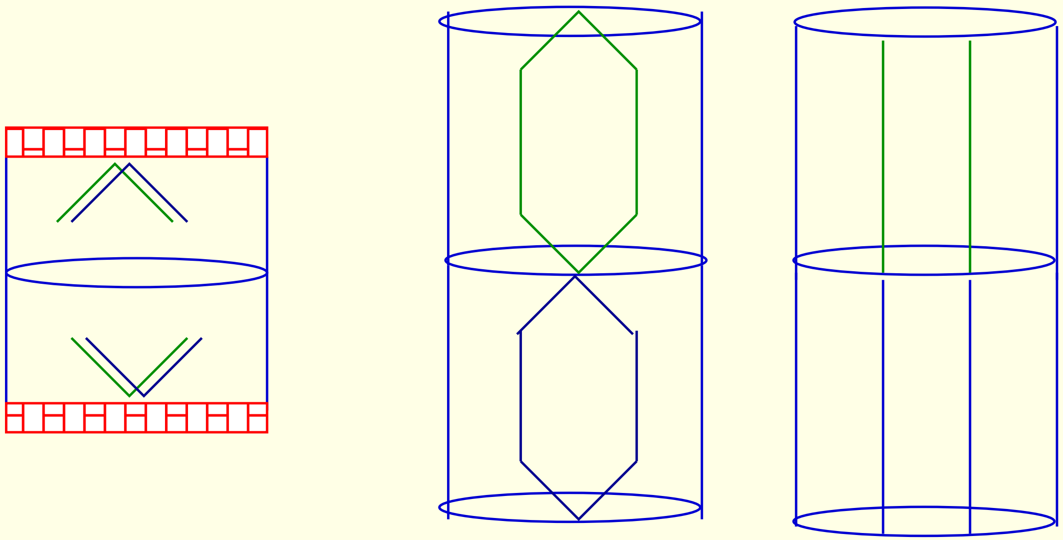
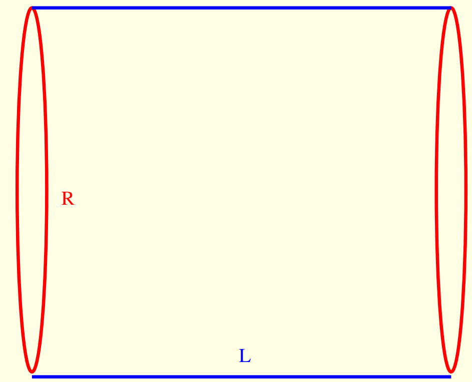




Folding trick:

Boundary thermodynamic Bethe Ansatz



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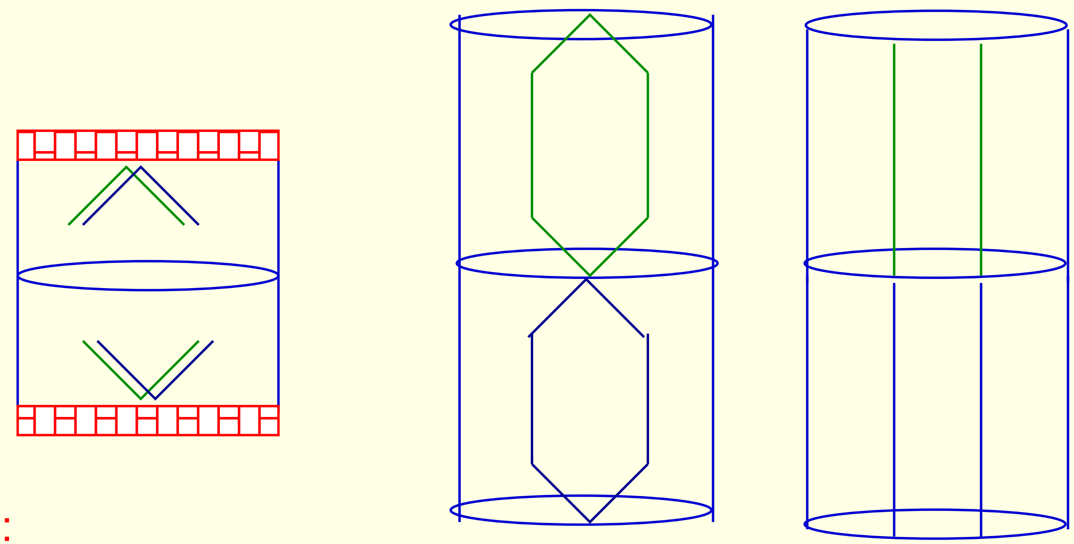
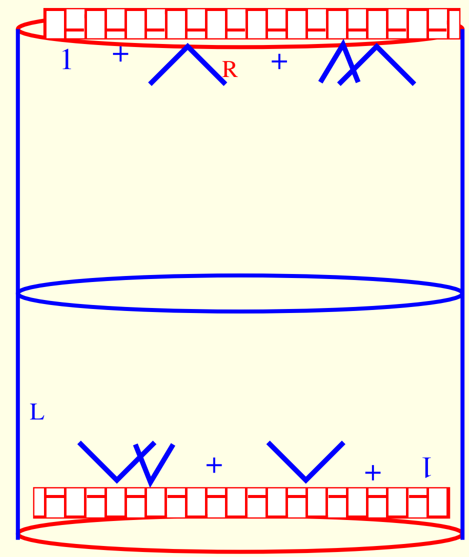
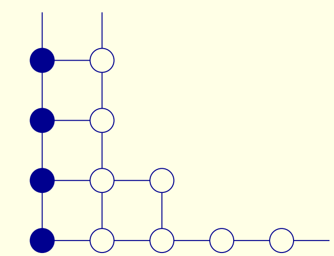




Folding trick:

[Correa,Maldacena,Sever '12][Drukker '12]

Ground state energy exactly: $\boxed{E_0(L) = -\sum_Q \int \frac{d\tilde{p}}{4\pi}\log(1+e^{-\epsilon_Q(\tilde{p})})}$



$$\epsilon^j(\tilde{p}) = \delta_Q^j(\sigma_Q(\tilde{p}) + 2\tilde{E}_Q(\tilde{p})L) - \int K_i^j(\tilde{p},\tilde{p}')\log(1+e^{-\epsilon^i(\tilde{p}')})d\tilde{p}'$$

Regularized $q-\bar{q}$ BTBA equations

Singular boundary fugacity: $\sigma_Q(0)=\infty$, no-obvious weak coupling expansion

shifting countours $\to$ regularization (extra source terms, $\sim$ excited state TBA)

$$
\begin{aligned}
\log Y_Q &= -2(f+\Psi)Q - R\tilde{\epsilon}_Q + \log\sigma_Q + D^{Q'Q}(iu_{Q'}) + \log(1+Y_{Q'})\star_\eta K^{Q'Q} \\
&\quad + \Big[2\log\big(1+Y_{v|1}\big)\star s\,\hat{\star}K_{yQ} + 2\log(1+Y_{v|Q-1})\star s - 2\log\frac{1-Y_-}{1-Y_+}\hat{\star}s\star K^{1Q}_{vx} \\
&\qquad + \log\frac{1-\frac{1}{Y_-}}{1-\frac{1}{Y_+}}\hat{\star}K_Q + \log(1-\frac{1}{Y_-})(1-\frac{1}{Y_+})\hat{\star}K_{yQ}\Big] \\
\log Y_-Y_+ &= 2D_{xvs}(iu_Q) - D_Q(iu_Q) - \log(1+Y_Q)\star_\eta K_Q + 2\log(1+Y_Q)\star K^{Q1}_{xv}\star s + 2\log\frac{1+Y_{v|1}}{1+Y_{w|1}} \\
\log\frac{Y_+}{Y_-} &= D_{Qy}(iu_Q) + \log(1+Y_Q)\star_\eta K_{Qy} \\
\log Y_{v|M} &= -D_s(iu_{M+1}) - \log(1+Y_{M+1})\star_\eta s + I_{MN}\log(1+Y_{v|N})\star s + \delta_{M1}\log\frac{1-Y_-}{1-Y_+}\hat{\star}s \\
\log Y_{w|M} &= I_{MN}\log(1+Y_{w|N})\star s + \delta_{M1}\log\frac{1-\frac{1}{Y_-}}{1-\frac{1}{Y_+}}\hat{\star}s
\end{aligned}
$$

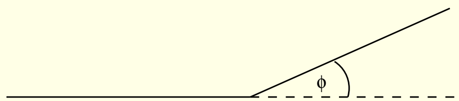
$$f=i(\pi-\phi)\qquad;\qquad \Psi=-i(\pi-\phi)\qquad;\qquad R=2L$$

[ZB, Balog, Hegedus, Toth '13]

$q - \bar{q}$ potential: weak coupling expansion

quark-antiquark potential

$$\langle \mathcal{P} e^{\oint_C A_\mu dx^\mu + \vec{\Phi}\vec{n}_0} Z^J \mathcal{P} e^{\oint_C A_\mu dx^\mu + \vec{\Phi}\vec{n}_\theta} \rangle$$



$$V_{q\bar{q}}(g, \phi, \theta) = \sum \Gamma_k g^{2k}$$

$$\Gamma = \sum_{n=1}^{k} \left(\frac{\cos\phi - \cosh\theta}{\sin\phi}\right)^n \gamma_k^{(n)}$$

$$\gamma_1^{(1)} = \frac{\phi}{2} \quad ; \gamma_2^{(1)} = \frac{\phi}{12}(\phi^2 - \pi^2)$$

$$\gamma_2^{(2)}(0) = \gamma_2^{(2)\prime}(0) = 0$$

$$\gamma_2^{(2)\prime\prime}(\phi) = \frac{\phi}{2} \cot\phi$$

$\equiv$

QMinimal surface

Wilson loop

strong coupling

$$V(r) = -\frac{4\pi^2\sqrt{2\lambda}}{\Gamma(\frac{1}{4})^4}\frac{1}{L}\left(1 - \frac{1.3359}{\sqrt{\lambda}} + \ldots\right)$$

minimal surface+fluctuations