Integrability in Gauge and String Theory, 29 June - 3 July 2009, Potsdam, Germany

Finite size effects in integrable QFTs

Zoltán Bajnok,

Hungarian Academy of Sciences, Eötvös University, Budapest

AdS $\longleftrightarrow$ integrable model $\longleftrightarrow$ CFT

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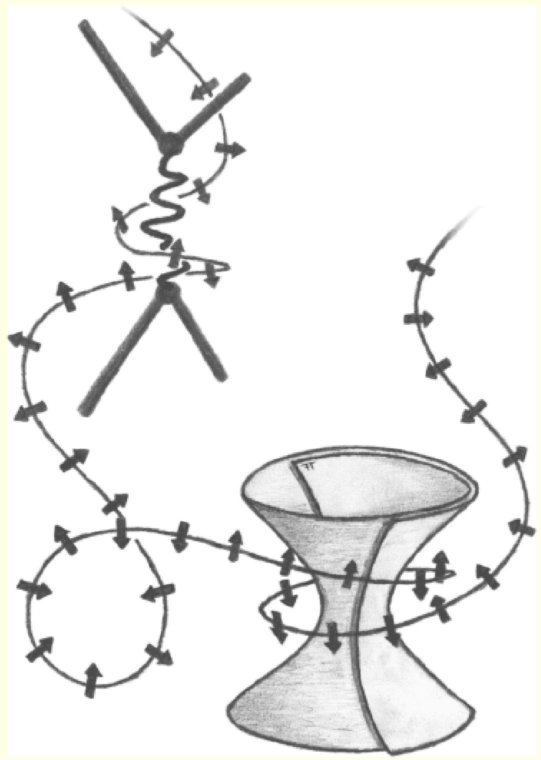

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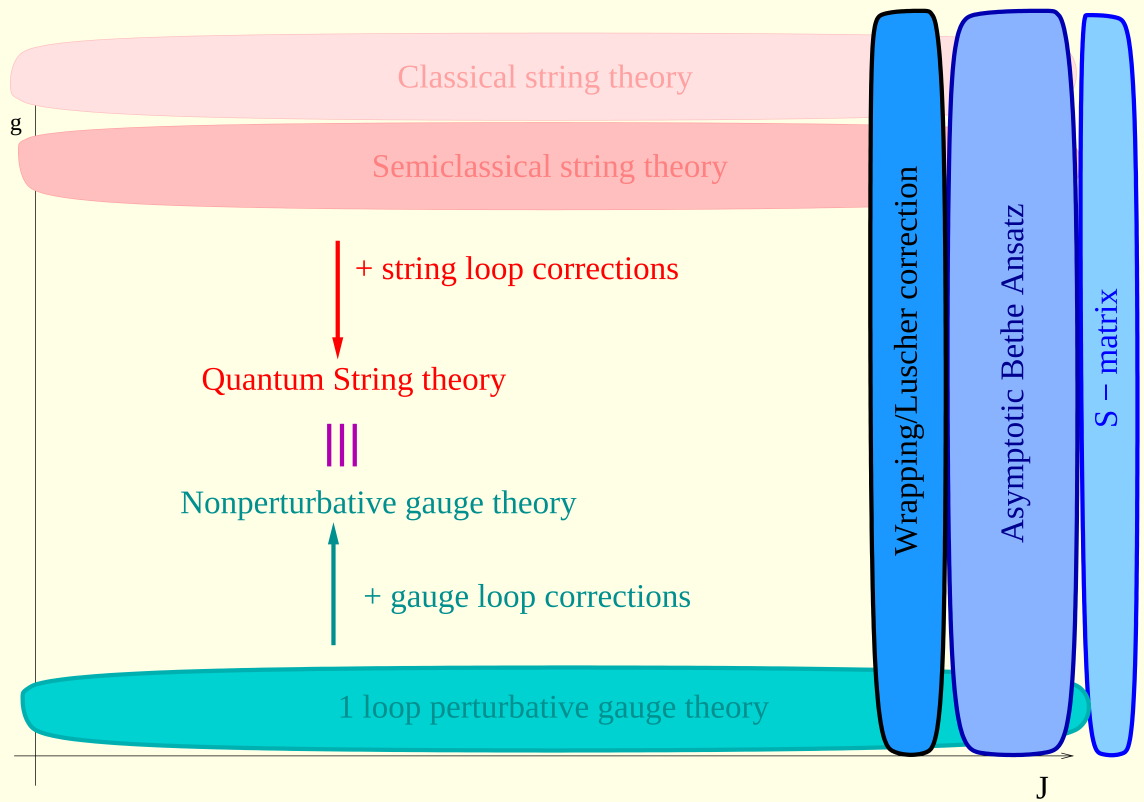

Finite J (volume) integrable models: (Lee-Yang, sinh-Gordon, sine-Gordon) $\longleftrightarrow$ AdS/CFT

Motivation: AdS/CFT

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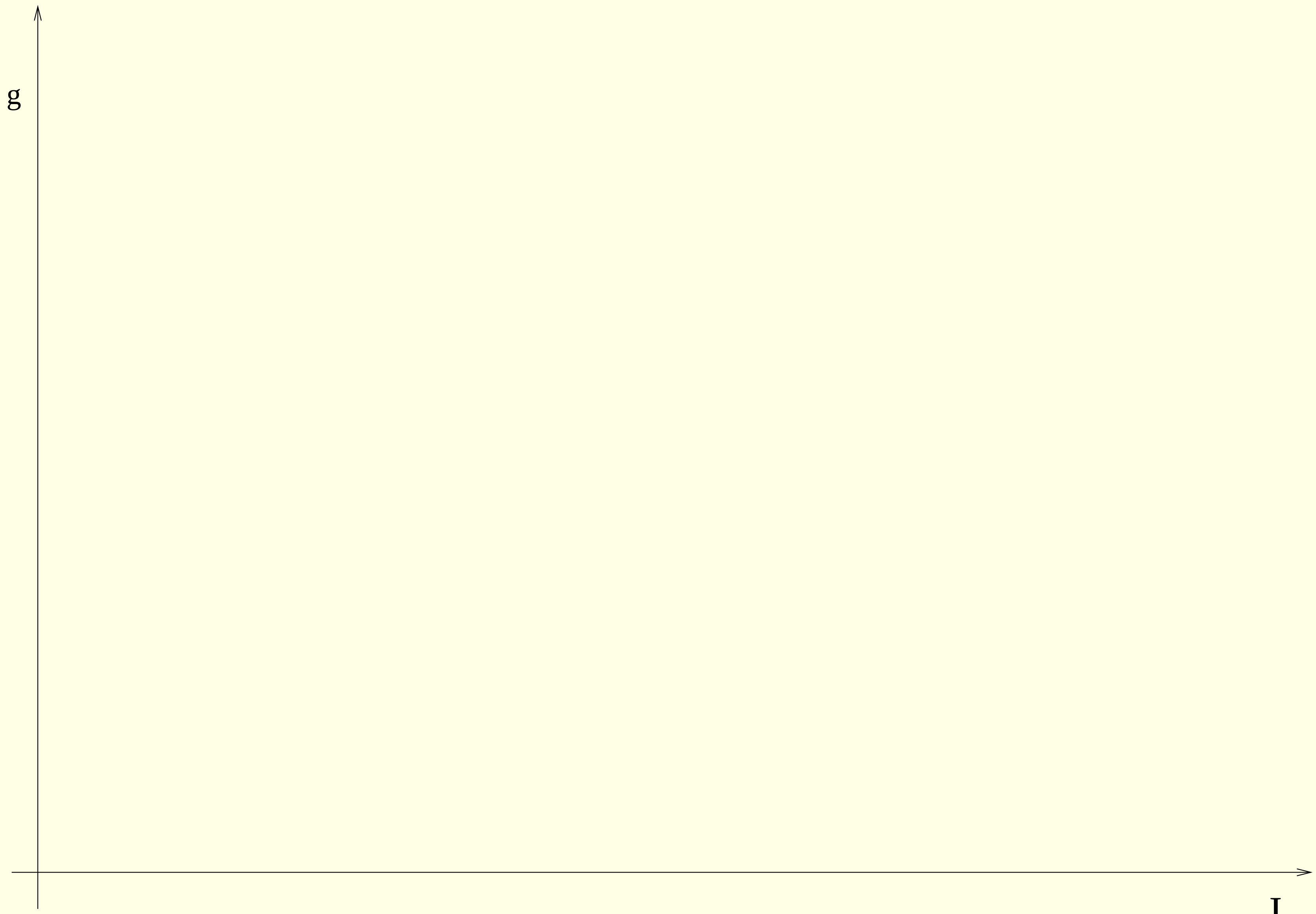

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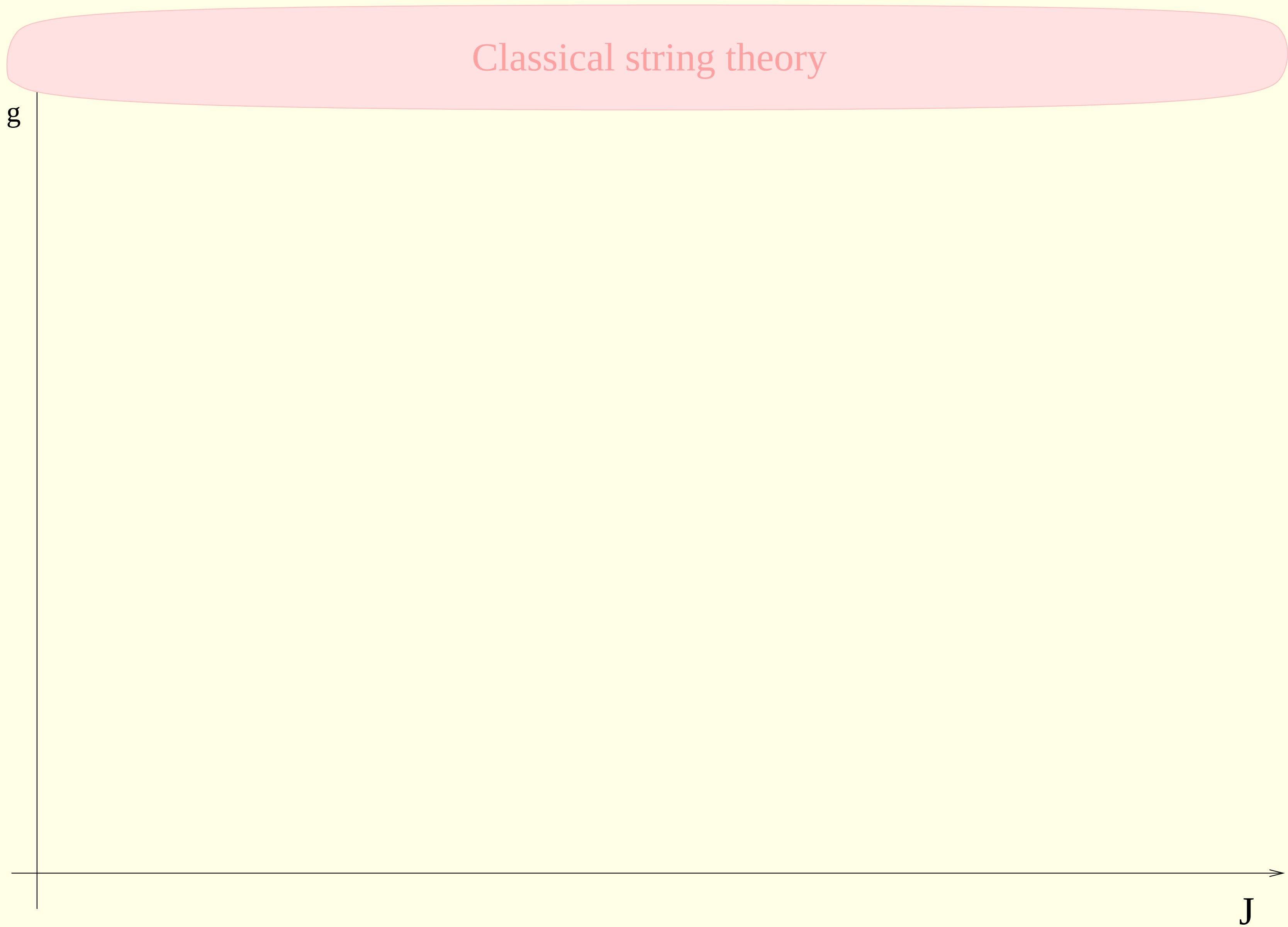

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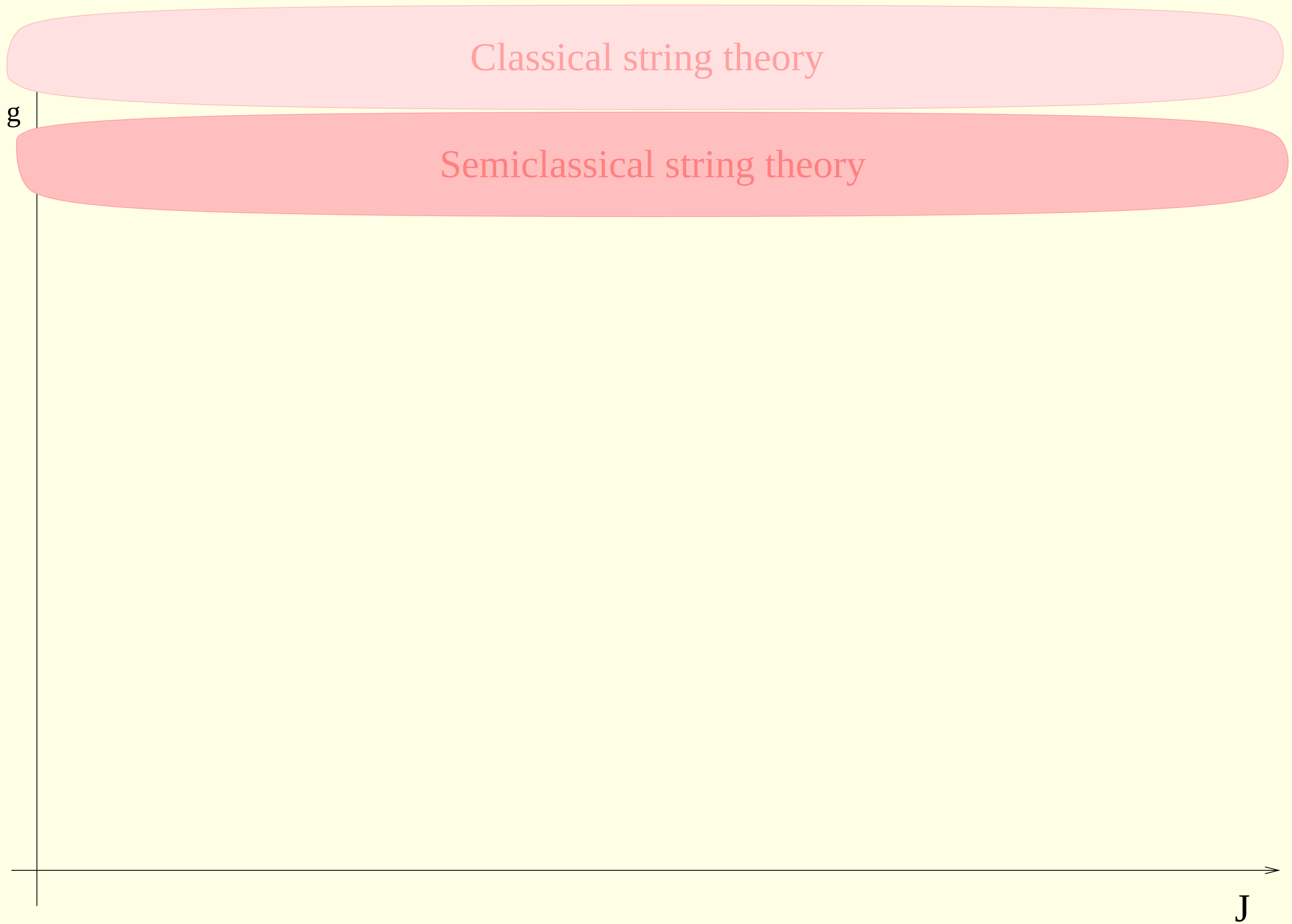

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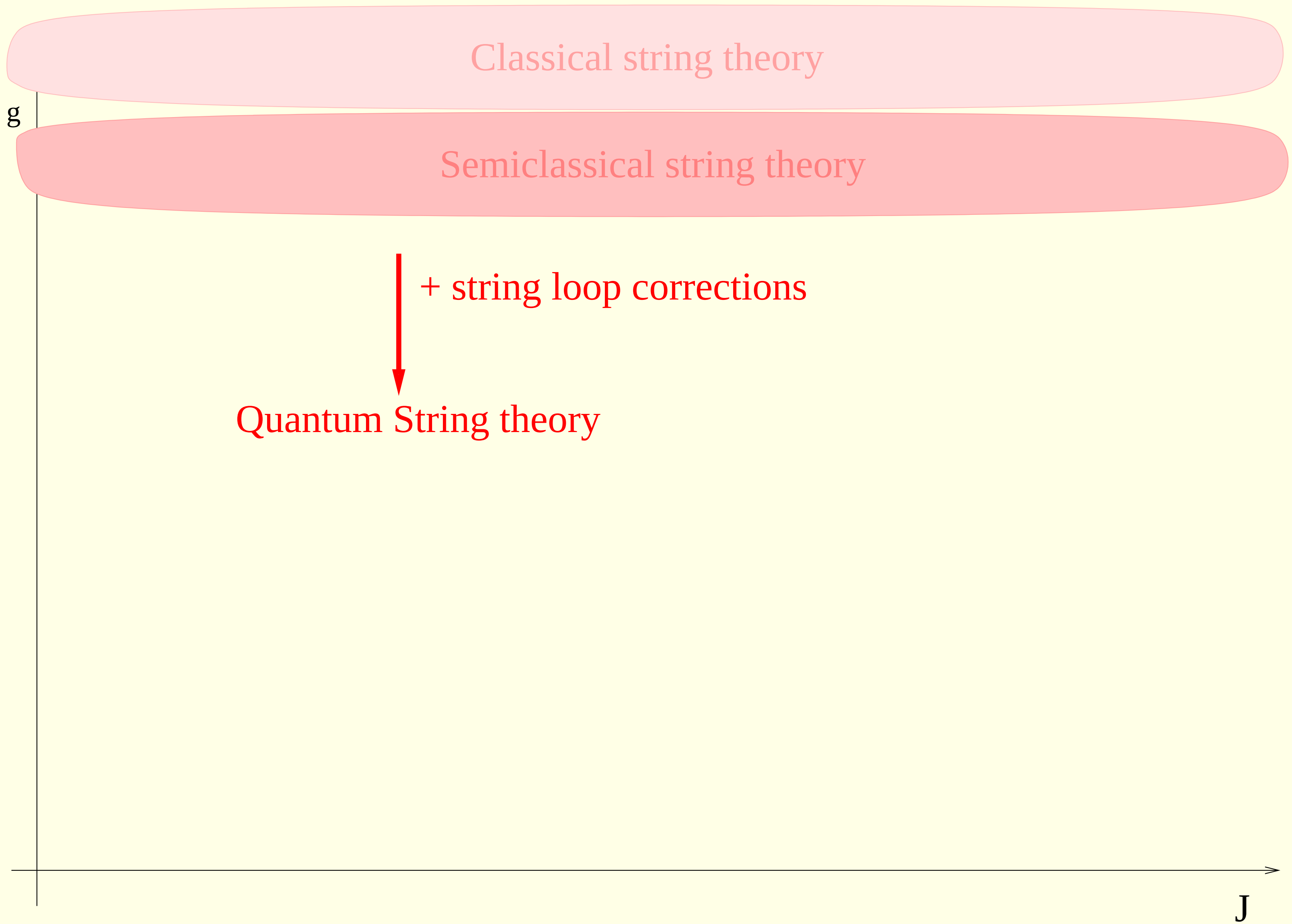

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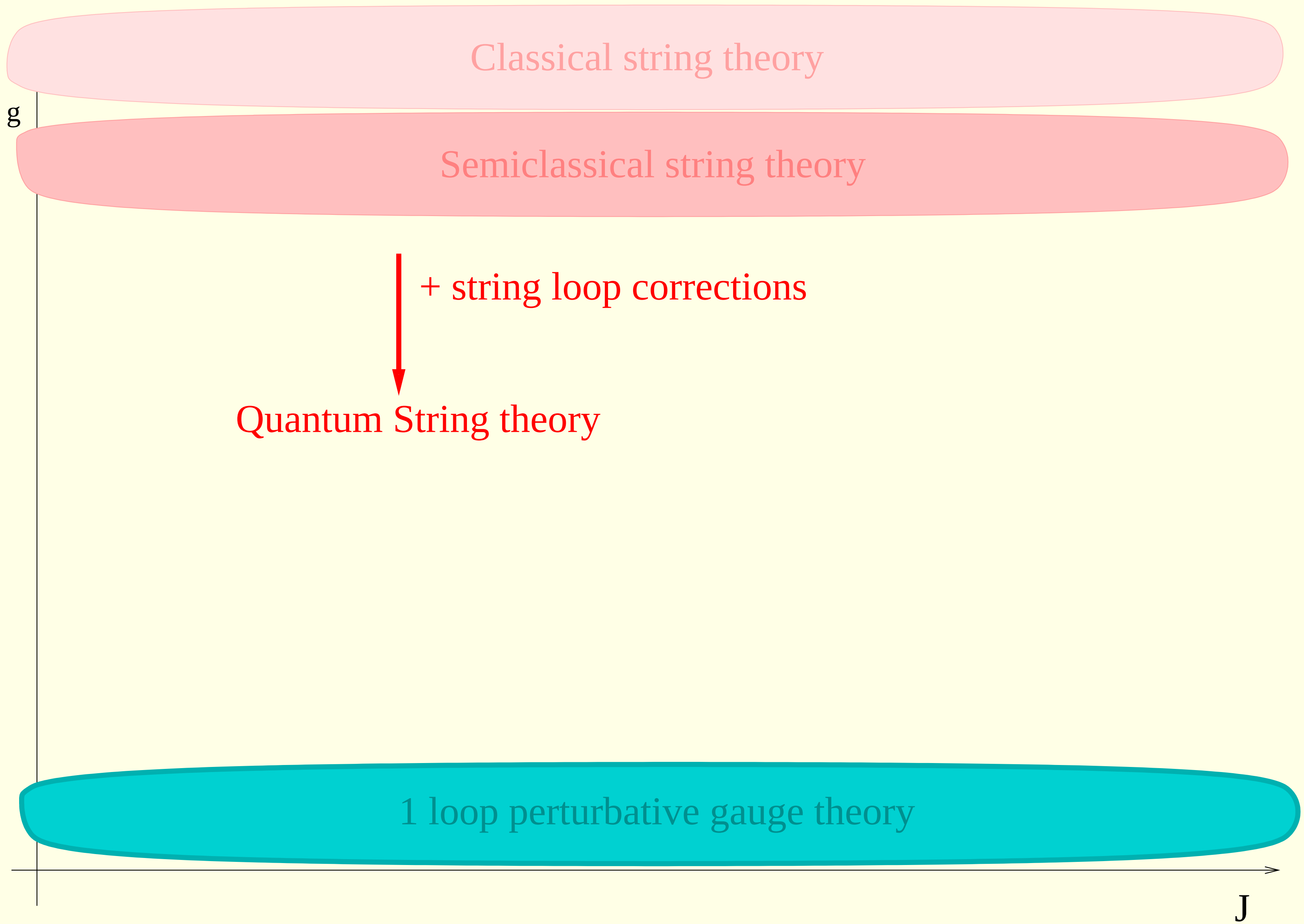

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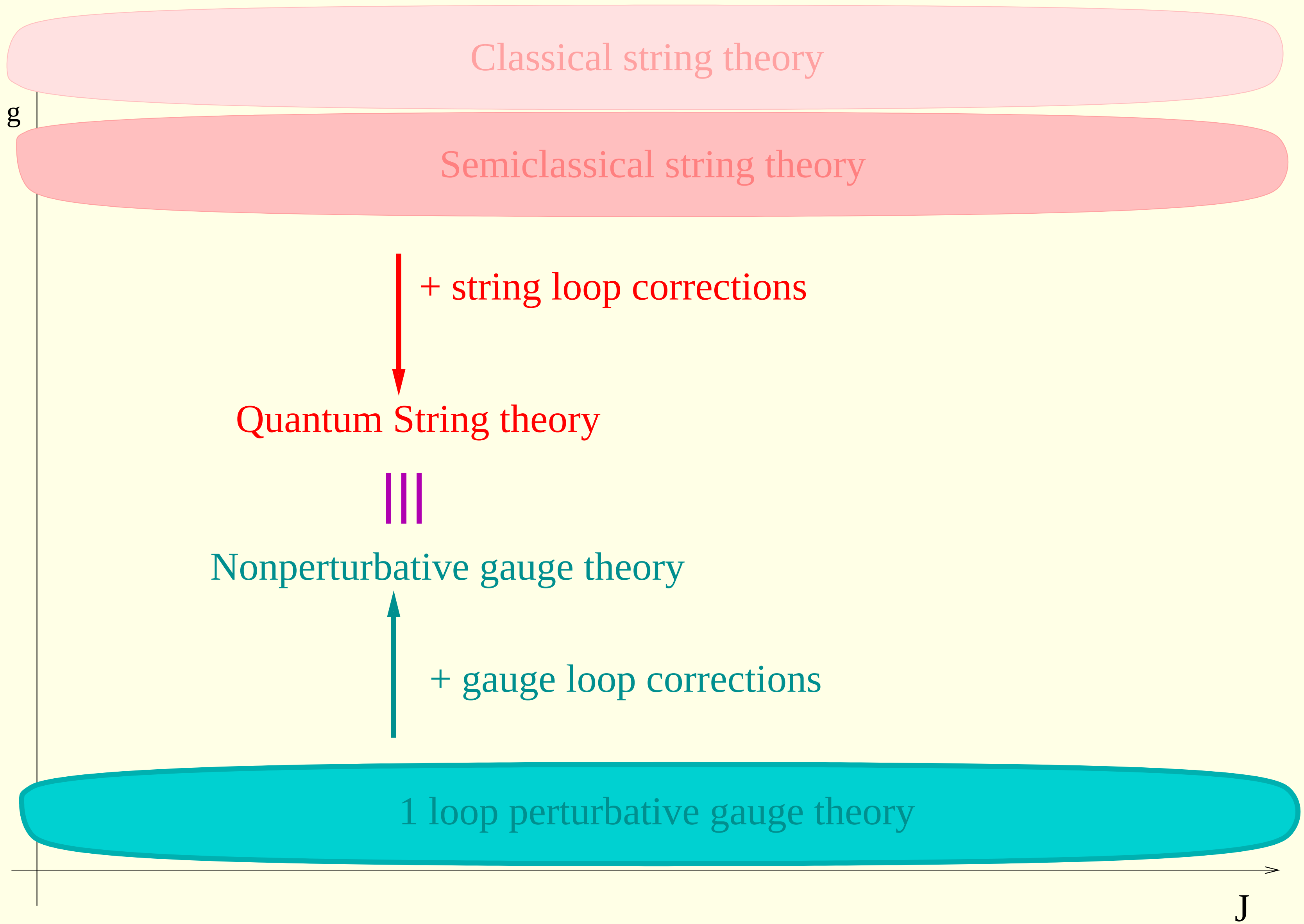

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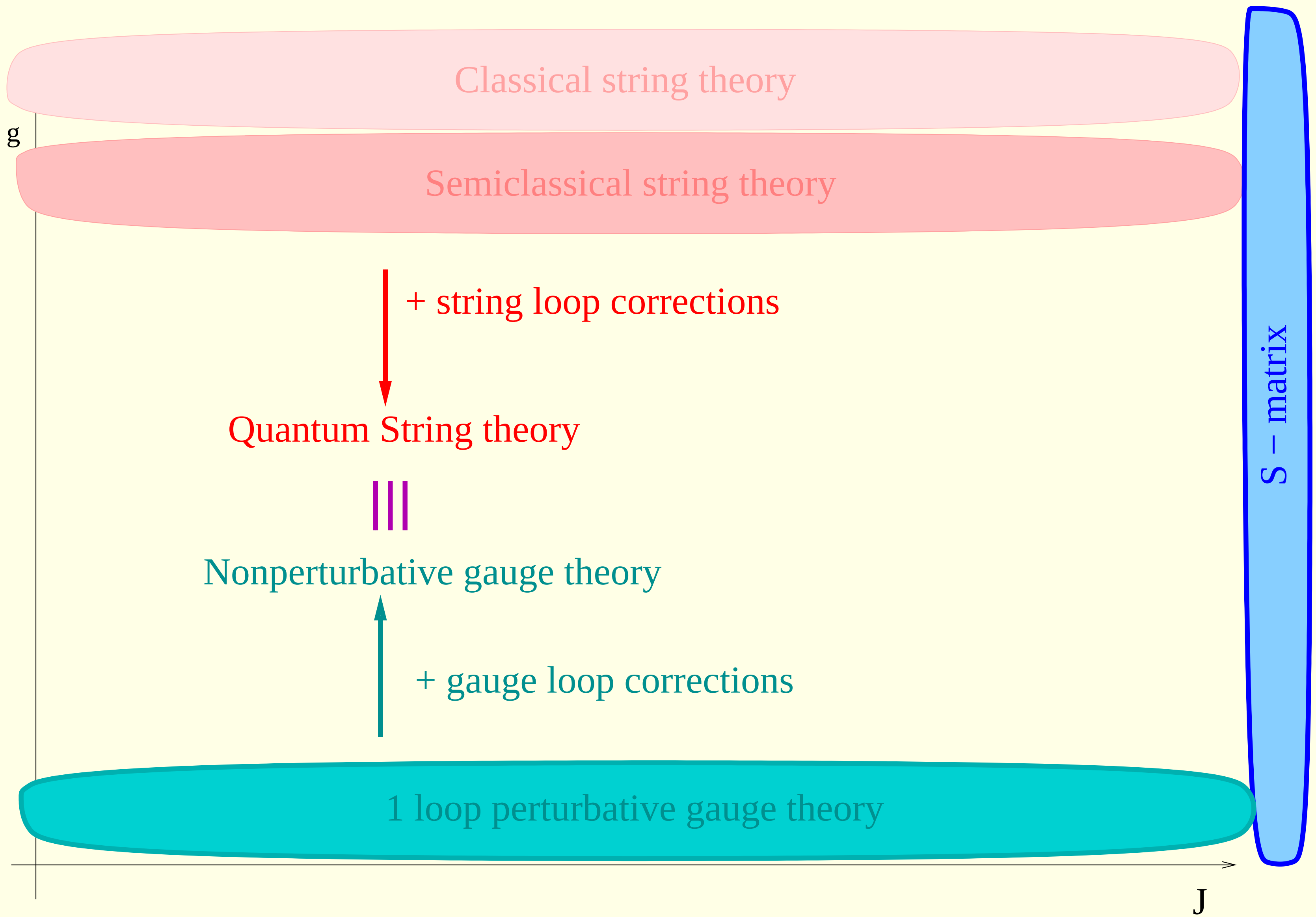

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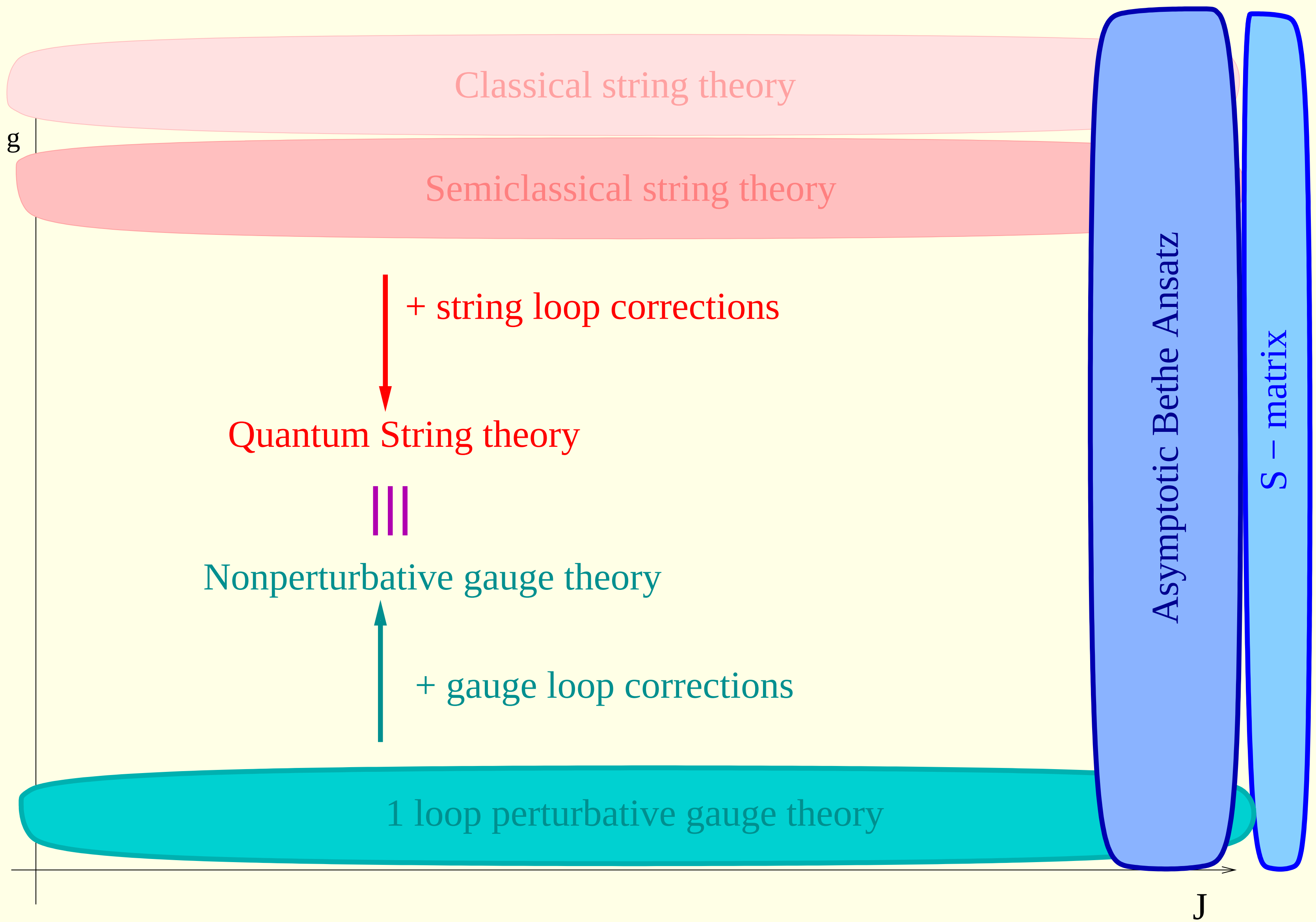

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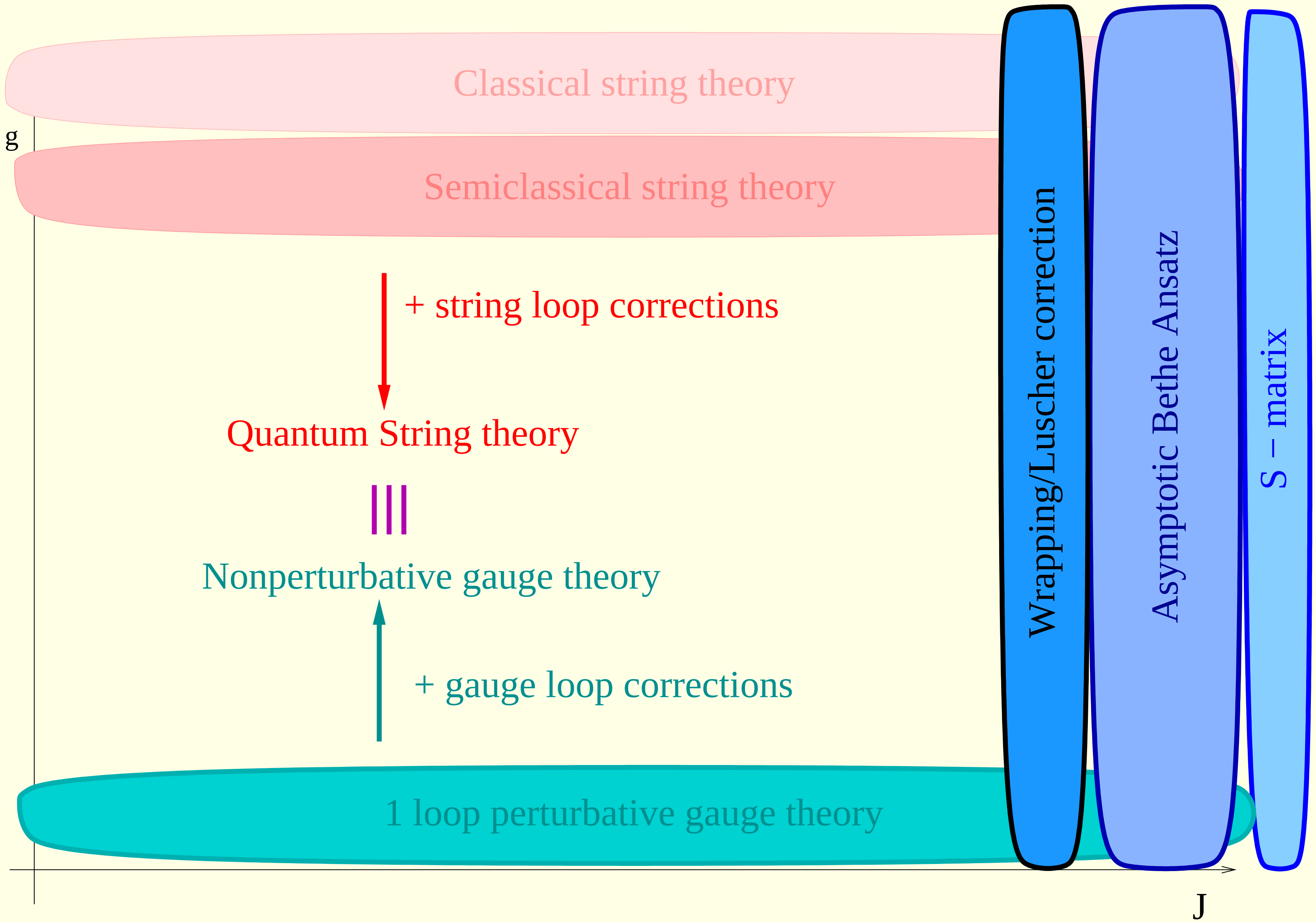
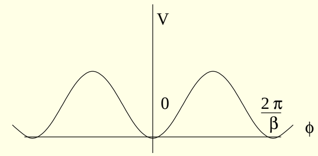


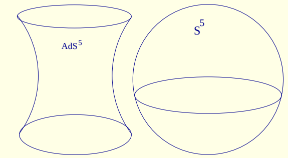
Need finite J (volume) solution of the spectral problem

Plan of talk

Plan of talk

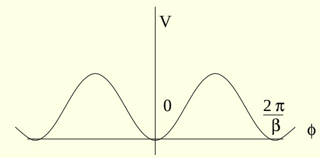
Classical integrable models: sine-Gordon theory

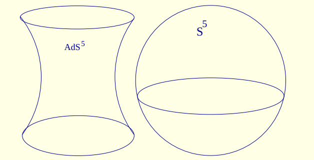




Plan of talk

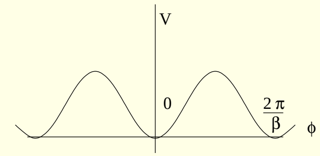
Classical integrable models: sine-Gordon theory

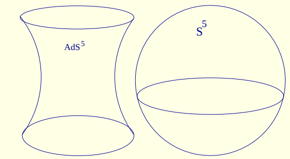




Quantization of integrable models: sine-Gordon model: PCFT

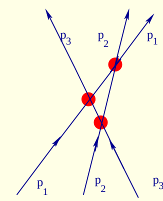
Plan of talk

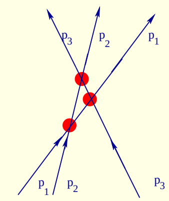




Classical integrable models: sine-Gordon theory

Quantization of integrable models: sine-Gordon model: PCFT

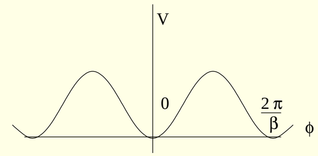


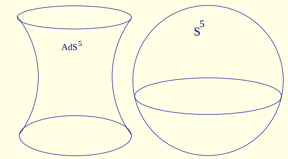


Bootstrap approach to quantum integrable models: S=scalar.matrix

Plan of talk

Classical integrable models: sine-Gordon theory





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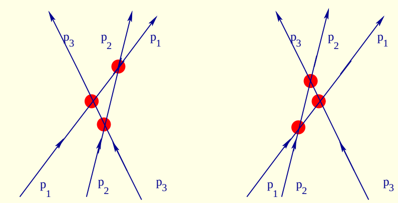

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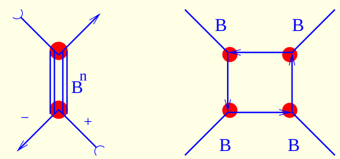
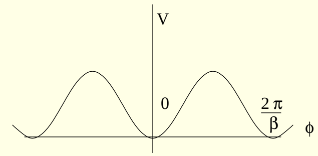
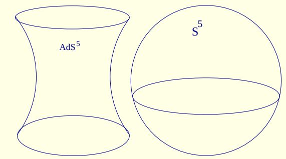


Lee-Yang, sinh-Gordon, sine-Gordon$\leftrightarrow$ AdS σmodel

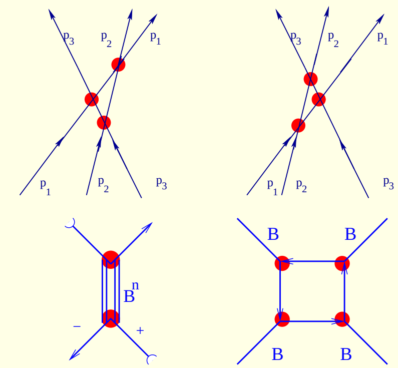
Plan of talk





Classical integrable models: sine-Gordon theory

Quantization of integrable models: sine-Gordon model: PCFT



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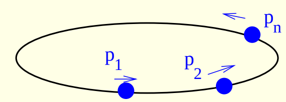
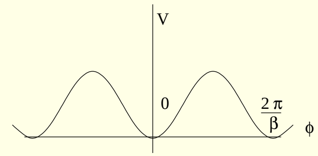
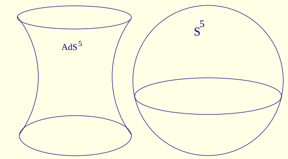


Finite volume: Asymptotic Bethe Ansatz:

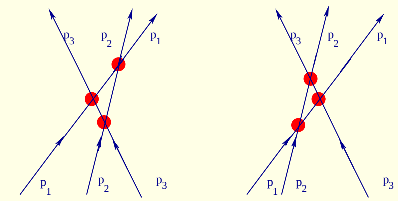
Plan of talk





Classical integrable models: sine-Gordon theory

Quantization of integrable models: sine-Gordon model: PCFT



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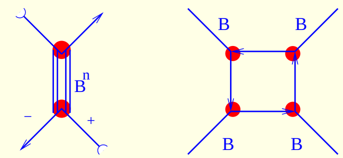

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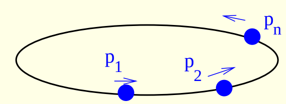

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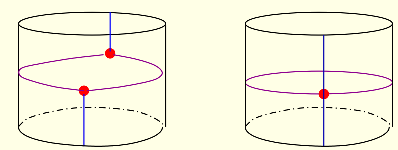
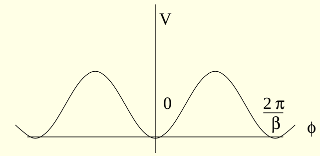
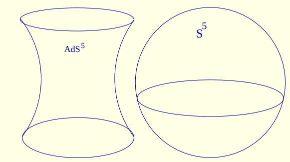


Luscher correction

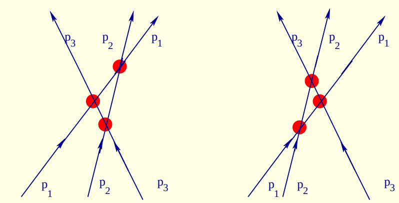
Plan of talk

Classical integrable models: sine-Gordon theory





Quantization of integrable models: sine-Gordon model: PCFT



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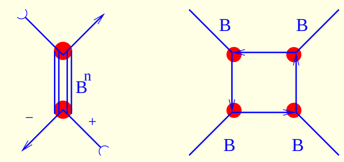

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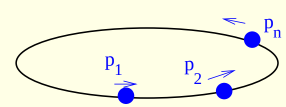
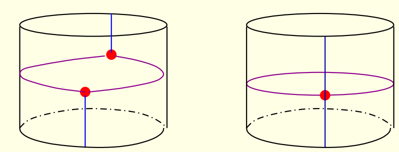
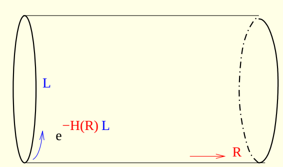
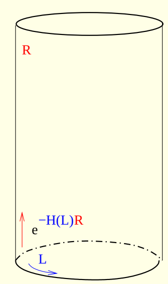




Luscher correction

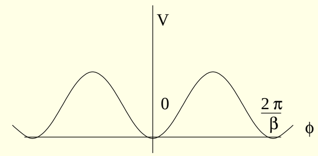
Exact: groundstate TBA,

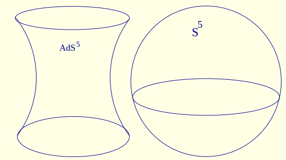




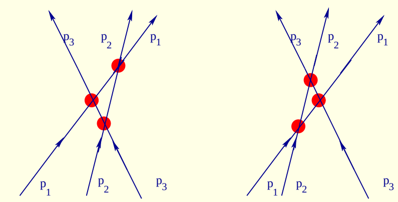
Plan of talk

Classical integrable models: sine-Gordon theory





Quantization of integrable models: sine-Gordon model: PCFT



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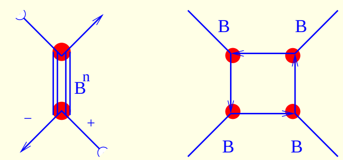
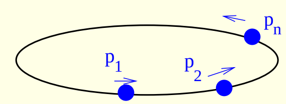
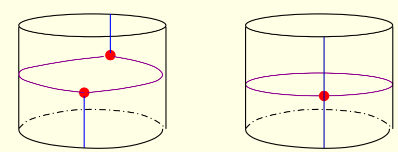


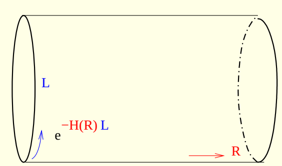
Lee-Yang, sinh-Gordon, sine-Gordon$\leftrightarrow$ AdS σmodel

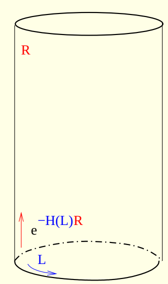


Finite volume: Asymptotic Bethe Ansatz:



Luscher correction





Exact: groundstate TBA,

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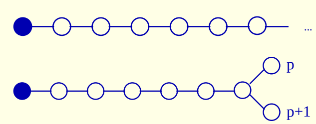
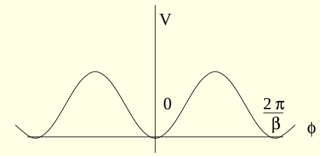
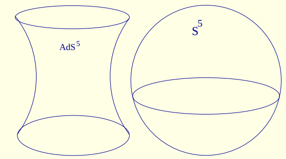


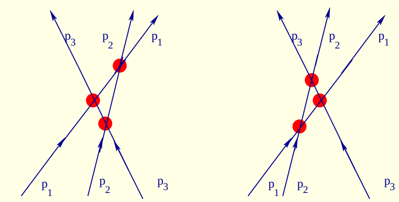
Plan of talk

Classical integrable models: sine-Gordon theory





Quantization of integrable models: sine-Gordon model: PCFT



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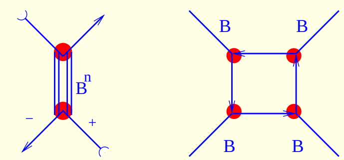

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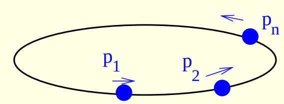
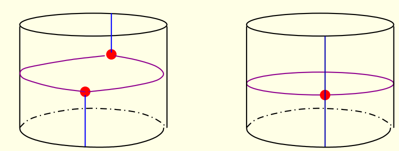
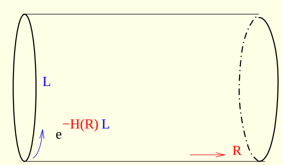
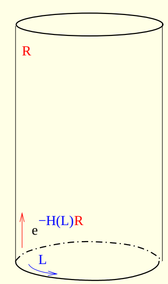




Luscher correction





Exact: groundstate TBA,

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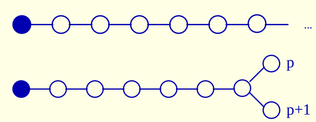
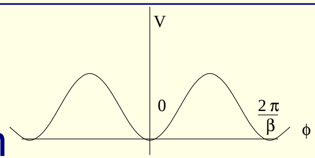
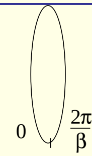
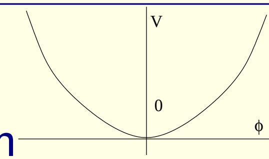
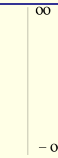
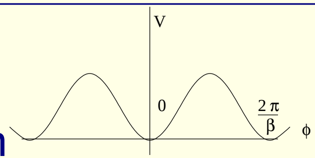
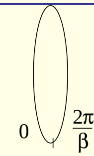
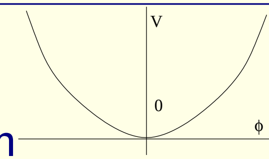
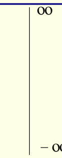


Conclusion

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$		$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{b^2}(\cosh b\phi - 1)$

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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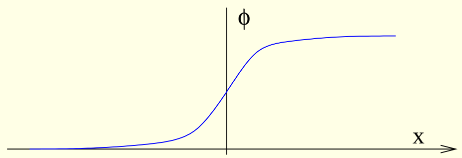
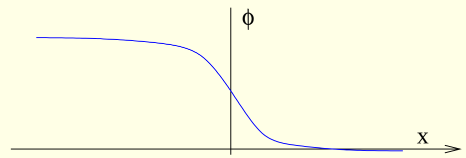
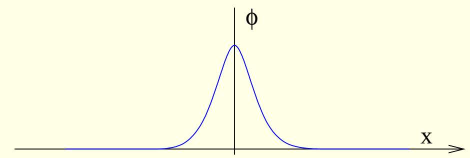
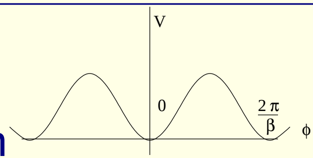
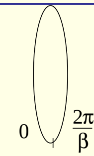
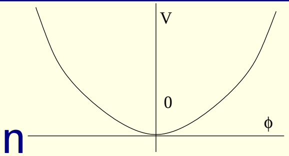
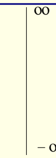
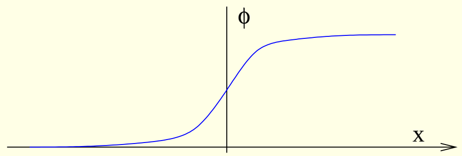
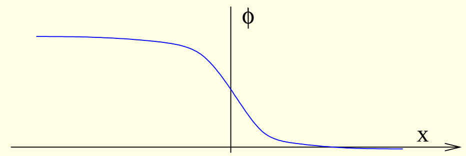
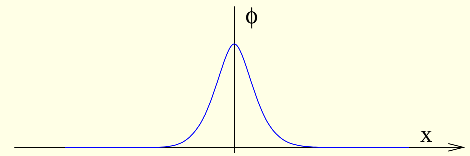
Classical
finite energy
solutions:
sine-Gordon theory

		
soliton	anti-soliton	breather

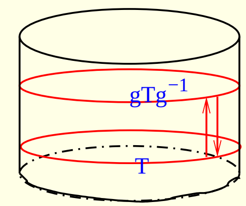
Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$		$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{b^2}(\cosh b\phi - 1)$

Classical finite energy solutions: sine-Gordon theory

		
soliton	anti-soliton	breather

Integrability: $\partial_x A_t - \partial_t A_x + [A_x, A_t] = 0 \leftrightarrow T(\mu) = P \exp \oint A(x)_\nu dx^\nu$



$$A_x(\mu) = \frac{i}{2}\begin{pmatrix} 2\mu & \beta\partial_+\varphi \\ -\beta\partial_+\varphi & -2\mu \end{pmatrix} \qquad A_t(\mu) = \frac{1}{4i\mu}\begin{pmatrix} \cos\beta\varphi & -i\sin\beta\varphi \\ i\sin\beta\varphi & -\cos\beta\varphi \end{pmatrix}$$

Classical integrable models: sin(e/h)-Gordon theory

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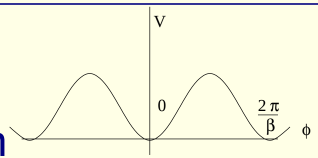
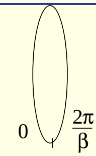
<td>sine-Gordon 

target 
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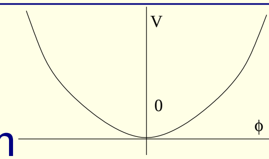
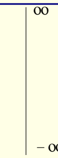
<td>$\beta \leftrightarrow ib$</td>
<td>sinh-Gordon 

target 
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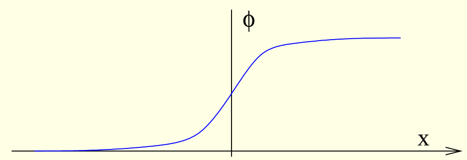
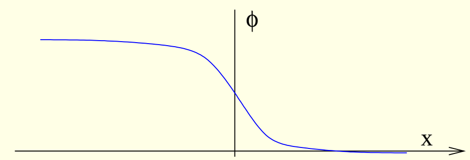
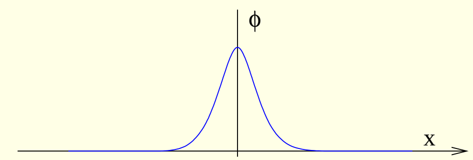
Classical
finite energy
solutions:
sine-Gordon theory







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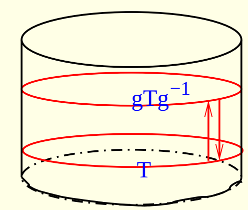

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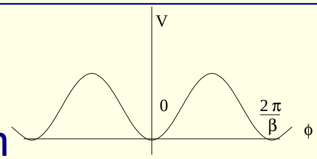
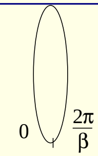
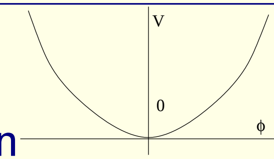
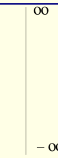
conserved $Q_{\pm 1}[\varphi] = E[\varphi] \pm P[\varphi] = \int \left\{ \frac{1}{2}(\partial_\pm\varphi)^2 + \frac{m^2}{\beta^2}(1 - \cos\beta\varphi) \right\} dx$

charges: $Q_{\pm 3}[\varphi] = \int \left\{ \frac{1}{2}(\partial_\pm^2\varphi)^2 - \frac{1}{8}(\partial_\pm\varphi)^4 + \frac{m^2}{\beta^2}(\partial_\pm\varphi)^2(1 - \cos\beta\varphi) \right\}$

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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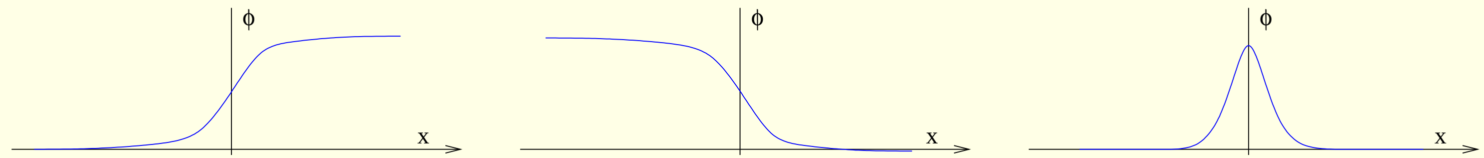
Classical finite energy solutions: sine-Gordon theory



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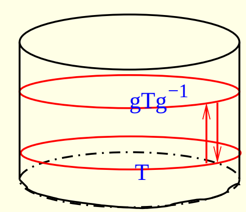

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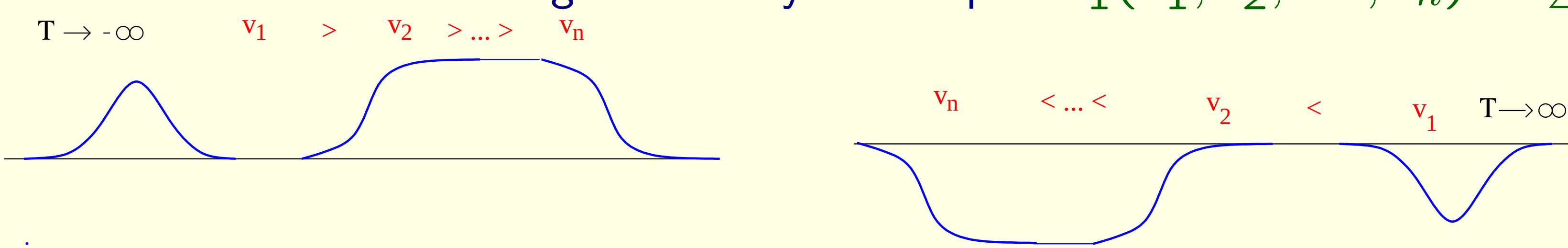
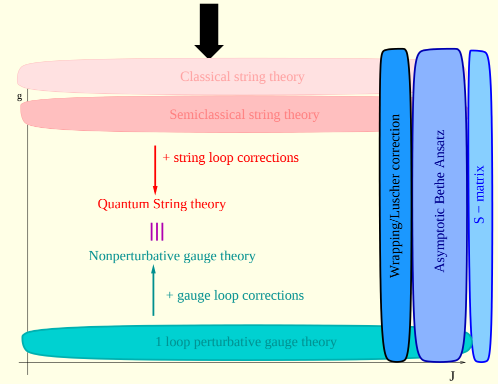
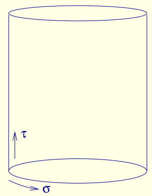
Classical factorized scattering: time delays sums up $\Delta T_1(v_1, v_2, \ldots, v_n) = \sum_i \Delta T_1(v_1, v_i)$



Classical integrability: AdS



AdS σ model



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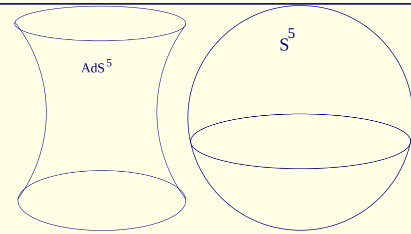
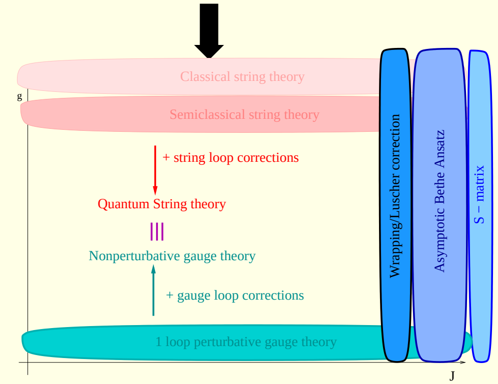


$$\mathcal{L} = \frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermions$$

Classical integrability: AdS



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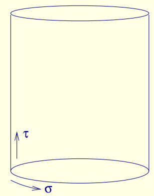

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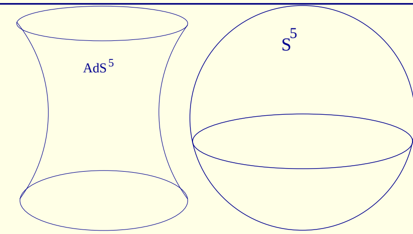

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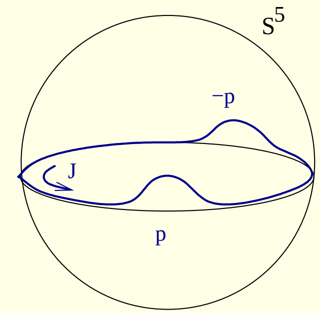
Classical solutions are found, for example magnon:



[Zarembo]

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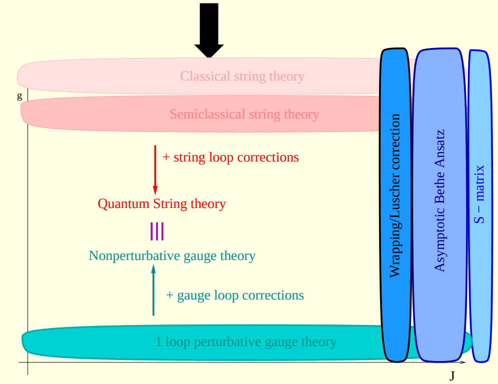
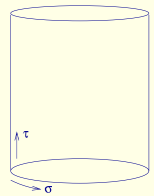
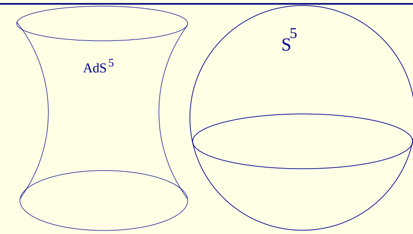


AdS σ model 

target 

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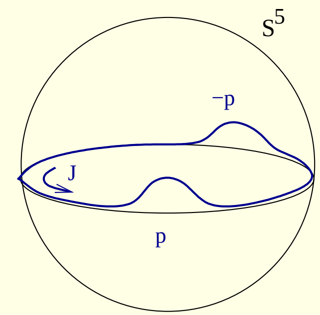


Classical solutions are found, for example magnon:

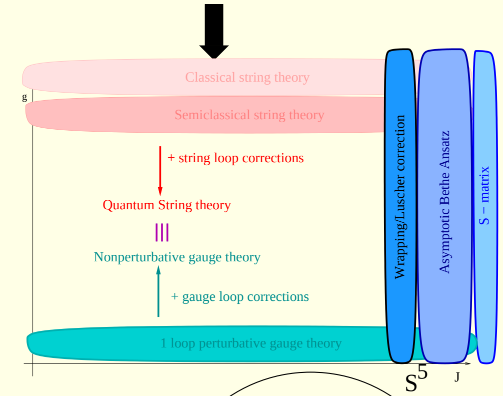
[Zarembo]

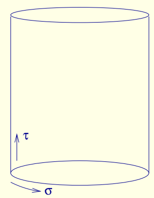
Coset NLσ model: $g \in \frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $\qquad J = g^{-1}dg = J_{||} + J_{\perp}$

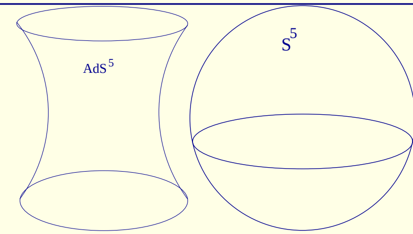
Z_4 graded structure: [Metsaev, Tseytlin 03]

$J_{\perp} \to J_0, J_1, J_2, J_3$ $\qquad \mathcal{L} \propto \mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3)$

Classical integrability: AdS



AdS σ model 

target 

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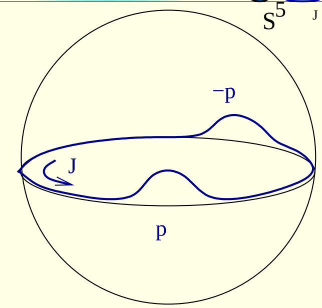

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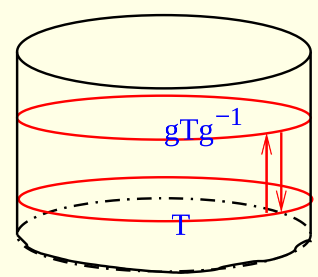
Integrability from flat connection: $dA - A \wedge A = 0$

$$A(\mu) = J_0 + \mu^{-1} J_1 + (\mu^2 + \mu^{-2}) J_2/2 + (\mu^2 + \mu^{-2}) J_2/2 + \mu J_3$$

Conserved charges from the trace of the monodromy matrix

$$T(\mu) = \mathcal{P} \exp \oint A(x)_\mu dx^\mu$$



Quantum integrability: sine-Gordon $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$

Perturbed Conformal Field Theory	Lagrangian perturbation theory
$\mathcal{L}_{CFT} + \lambda\mathcal{L}_{pert} = \frac{1}{2}(\partial\phi)^2 + \lambda(V_\beta + V_{-\beta})$	$\mathcal{L}_0 + V_{pert} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \beta^2 U$
$h_\beta = \beta^2$ definite scaling $V_\beta =: e^{i\beta\phi}:$	semiclassical=free

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Quantum conservation laws [Zamolodchikov]
$\partial_-\Lambda_4 = 0 \to \partial_-\Lambda_4 = \lambda\partial_+\Theta_2$
$[\lambda] = 2 - h_\beta$, $[\Lambda_4] = 4$,
Nonlocal symmetries $U_q(\hat{sl}_2)$

Correlators=$\sum_{loops} Feynman\, diagrams$
Asymptotic states $E(p) = \sqrt{p^2 + m^2}$
S-matrix$\leftrightarrow$correlators LSZ
unitarity, crossing symmetry, analyticity

Bootstrap scheme

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Bootstrap scheme

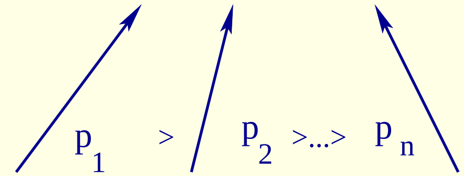
Quantum integrability: AdS no proof !

Perturbative integrability see [Zarembo]s talk and also

[Lipatov, Zarembo, Minahan, Staudacher, Beisert, Kristjansen, Bena, Polchinski, Roiban]

Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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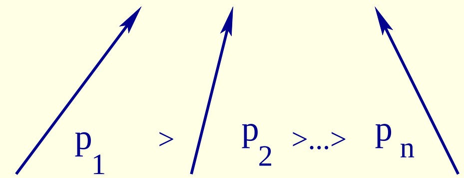

Lorentz: $P = \sum_i p_i \;\; E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$

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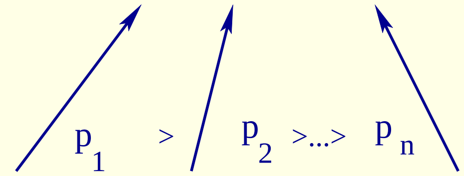

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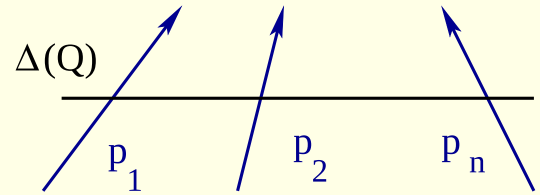

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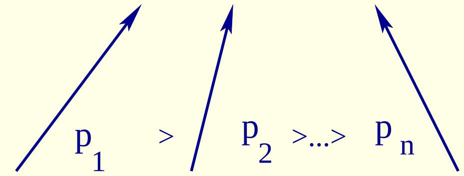

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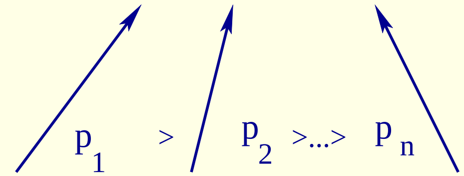
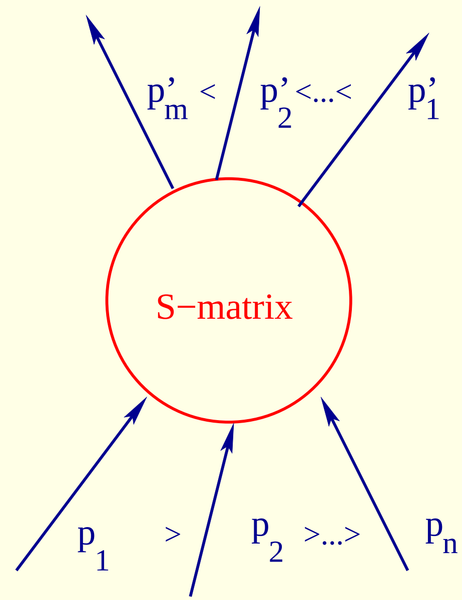
Scattering matrix S: $|out\rangle \to |in\rangle$
commutes with symmetry $[S, \Delta(Q)] = 0$ [Torielli]

Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



Lorentz: $P = \sum_i p_i \;\; E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$



Scattering matrix S: $|out\rangle \to |in\rangle$
commutes with symmetry $[S, \Delta(Q)] = 0$ [Torielli]

Bootstrap program

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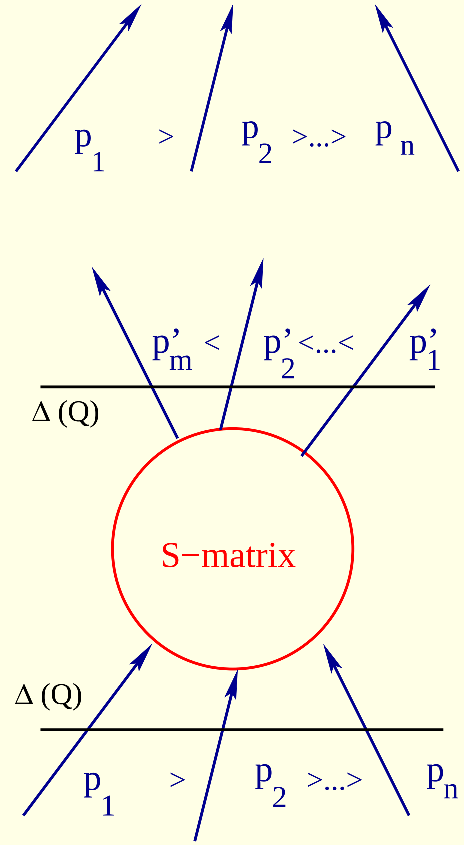

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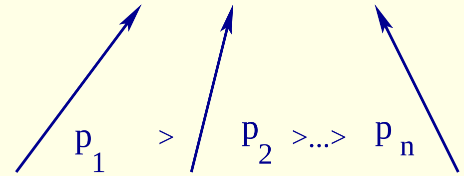

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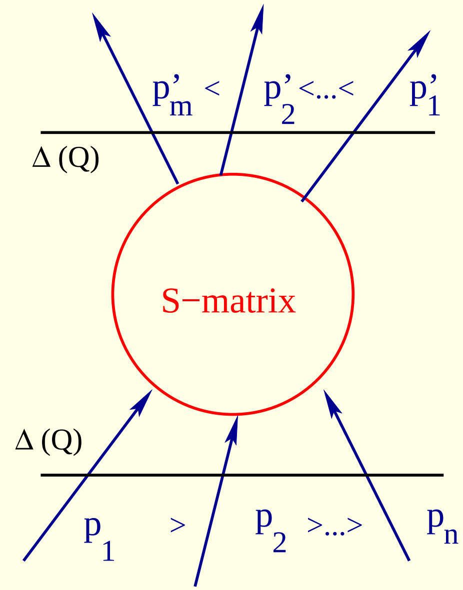

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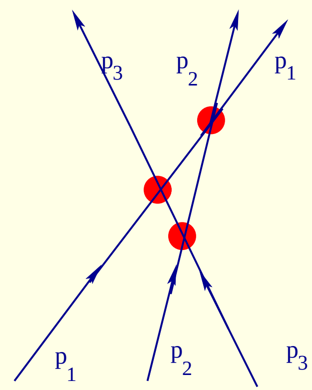
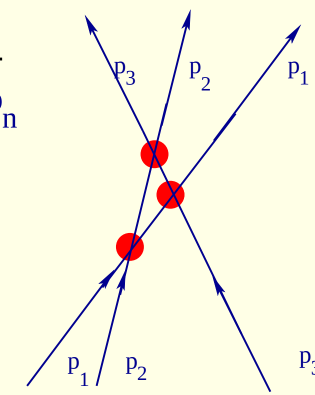
Higher spin concerved charge
factorization + Yang-Baxter equation
$S_{123} = S_{23} S_{13} S_{12} = S_{12} S_{13} S_{23}$





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Bootstrap program

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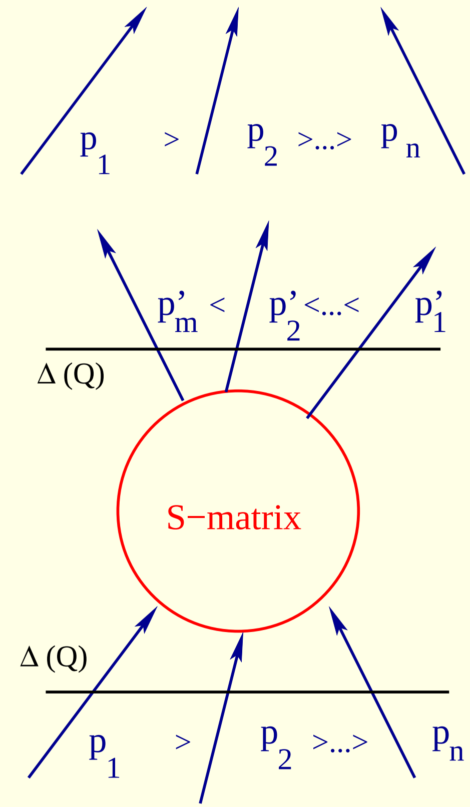

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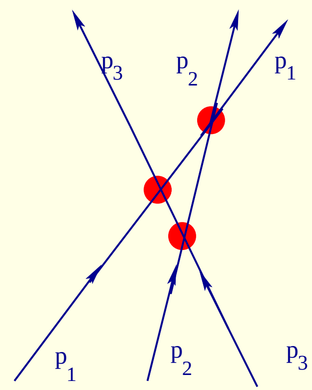

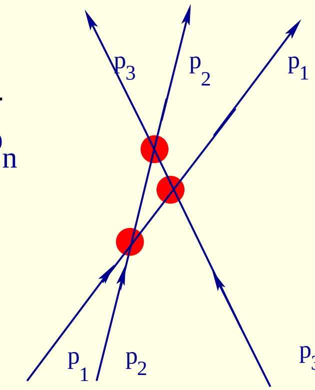

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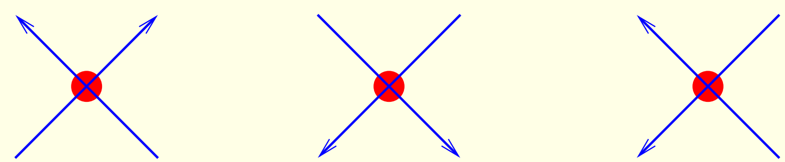
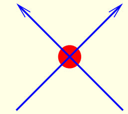


Unitarity $S_{12}S_{21} = Id$
Crossing symmetry $S_{12} = S_{2\bar{1}}$
Maximal analyticity: all poles have physical origin $\longrightarrow$ boundstates, anomalous thresholds

Bootstrap program: diagonal

Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$

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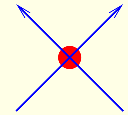
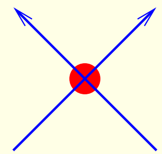
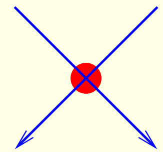
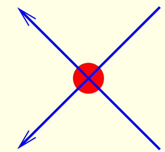
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$







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Maximal analyticity: all poles have physical origin

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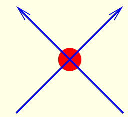
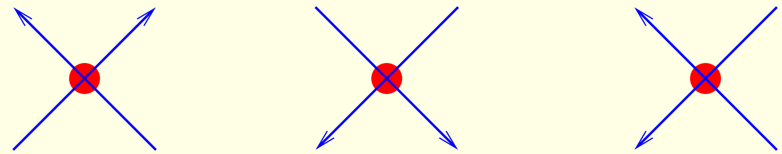
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$



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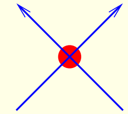
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$

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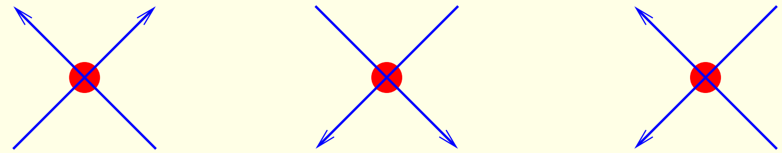

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CDD factor: $S(\theta) = \frac{\sinh\theta - i \sin p\pi}{\sinh\theta + i \sin p\pi}$

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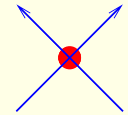
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$

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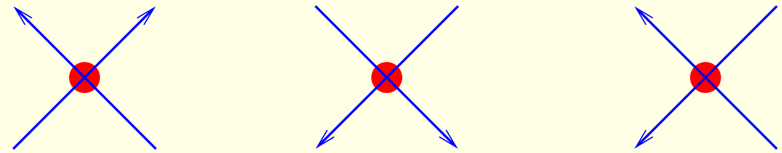

Maximal analyticity: all poles have physical origin

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$V \sim \cosh b\phi \leftrightarrow \frac{b^2}{8\pi + b^2} = p > 0$

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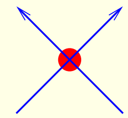
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1,p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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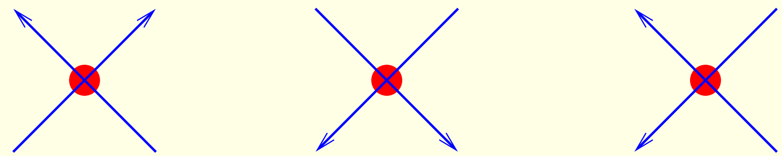

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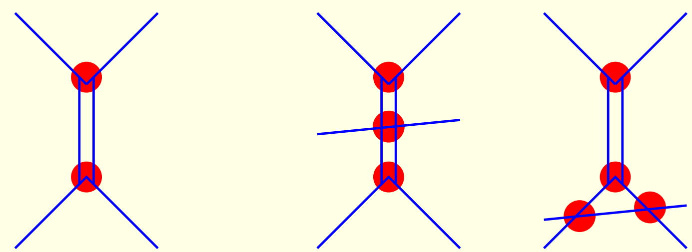
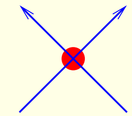
pole at $\theta = ip\pi$ $\rightarrow$boundstate B^2



bootstrap: $S_{12}(\theta) = S_{11}(\theta - \frac{ip\pi}{2})S_{11}(\theta + \frac{ip\pi}{2})$

new particle if $p \neq \frac{2}{3}$ otherwise Lee-Yang

Bootstrap program: diagonal

Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1,p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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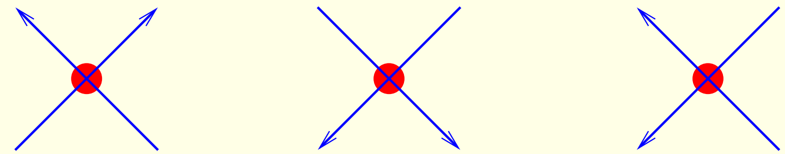

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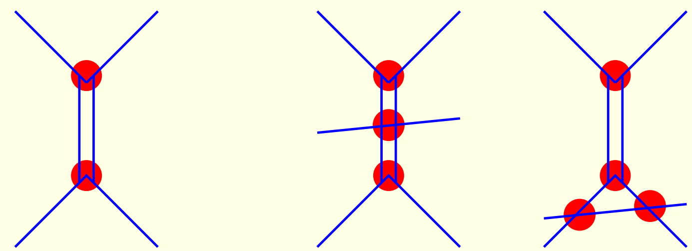

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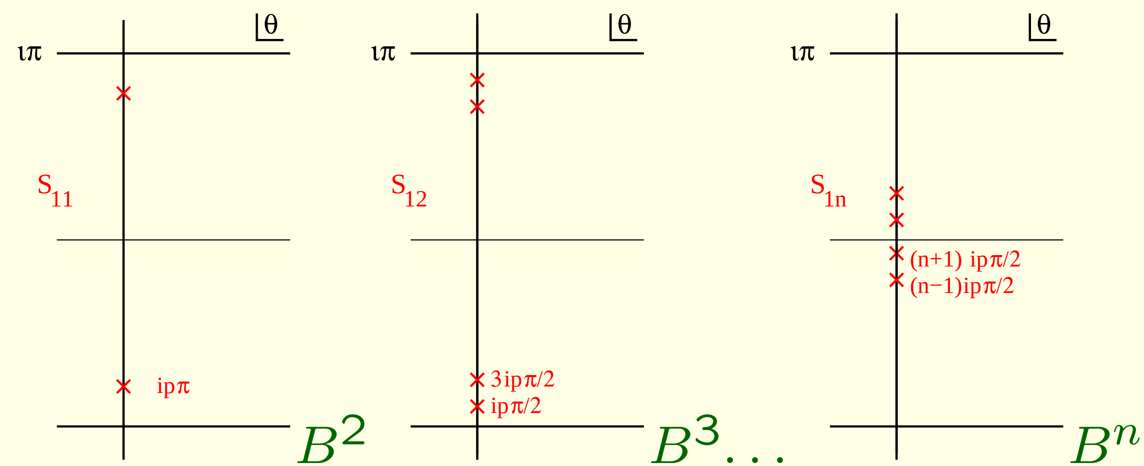
$\rightarrow$sine-Gordon solitons



Bootstrap program: sine-Gordon

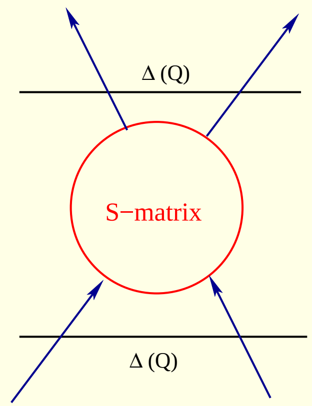
Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar . Matrix}}$ soliton doublet $\begin{pmatrix} s \\ \bar{s} \end{pmatrix}$

Matrix:

global symmetry $U_q(s\hat{l}_2)$

2d evaluation reps

$[S, \Delta(Q)] = 0$



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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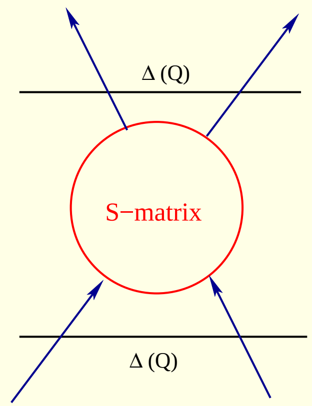

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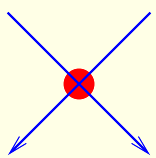
Unitarity

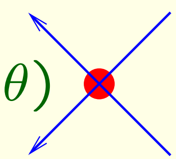
$\mathcal{S}(\theta)\mathcal{S}(-\theta) = 1$



$$\prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma(2l\lambda+1+\frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda+1+\frac{\lambda i\theta}{\pi})}/(\theta\to-\theta)\right]$$

Crossing symmetry

$\mathcal{S}(\theta) = \mathcal{S}^{c_1}(i\pi-\theta)$



Bootstrap program: sine-Gordon

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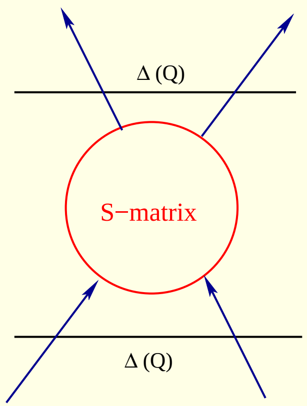

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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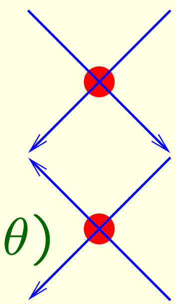

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Maximal analyticity:
all poles have physical origin
either boundstates or
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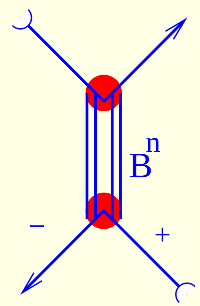
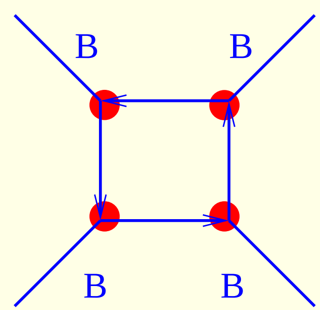
$p = \lambda^{-1}$
[Zamolodchikov^2]





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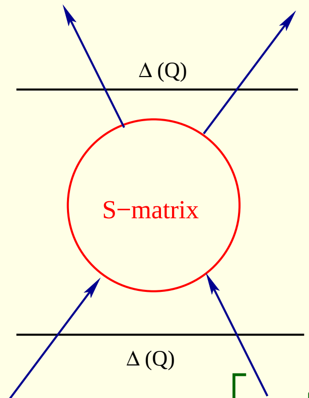

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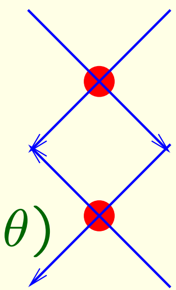

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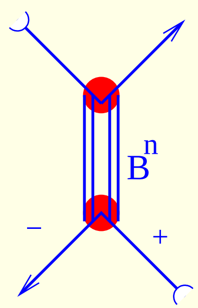
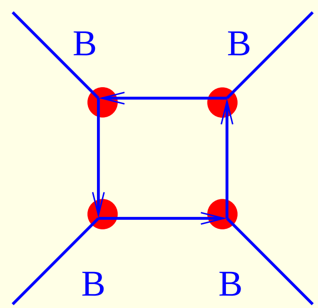
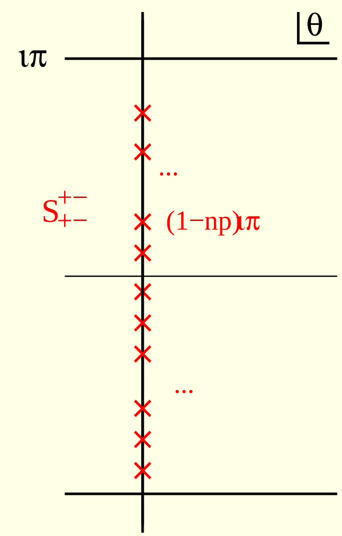
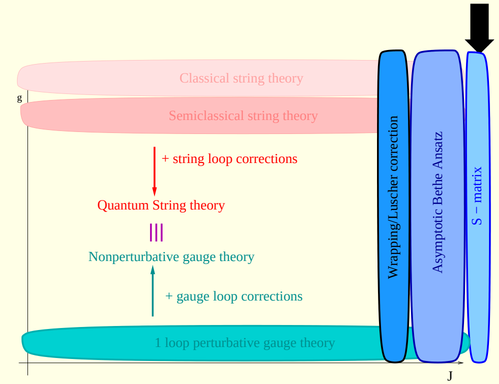




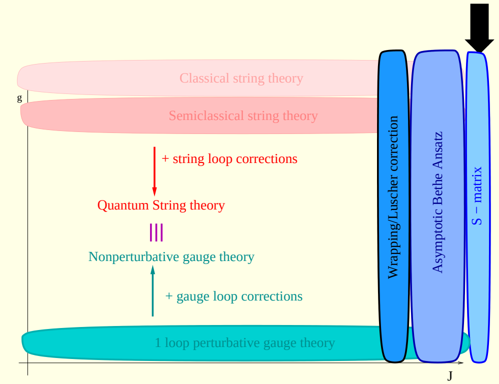




Bootstrap program: AdS

Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar . Matrix}}$

Bootstrap program: AdS



Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar} \; . \; \text{Matrix}}$

Matrix: [Beisert]

global symmetry $PSU(2|2)^2$

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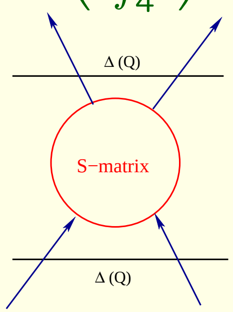
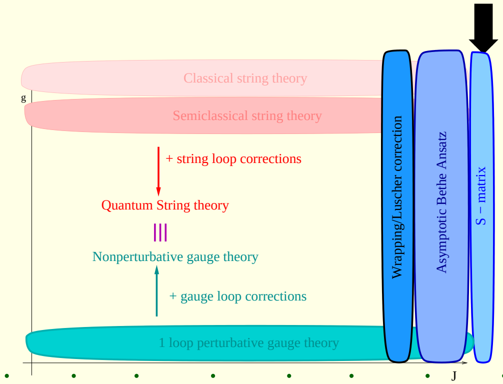
$[S, \Delta(Q)] = 0$





Bootstrap program: AdS

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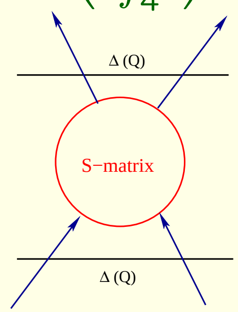

$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
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\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
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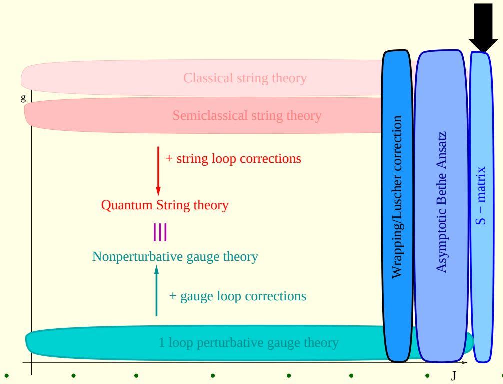
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
\end{pmatrix}$$



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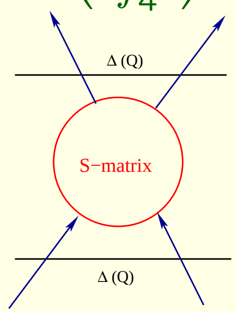

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\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
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\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
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$$\mathcal{S}(z_1, z_2)\mathcal{S}(z_2, z_1) = 1$$

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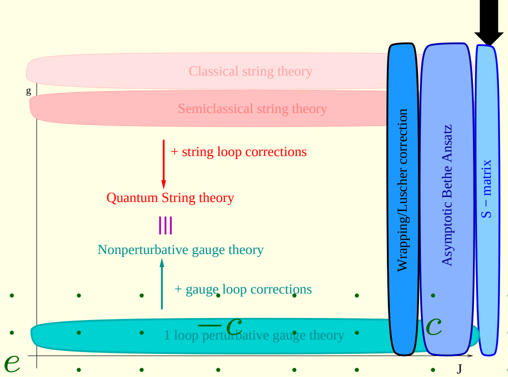
$$S_{11}^{11} = \frac{u_1 - u_2 - i}{u_1 - u_2 + i} e^{i2\theta(z_1, z_2)}$$

$$u = \tfrac{1}{2} \cot \tfrac{p}{2} E(p)$$

[Beisert,Eden,Staudacher]

$$p = 2\, \mathrm{am}(z)$$

Bootstrap program: AdS



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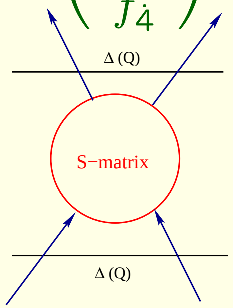

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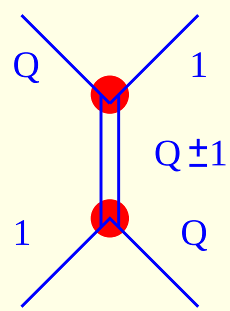
Maximal analyticity:

boundstates atyp symrep: $Q \in \mathbf{N}$

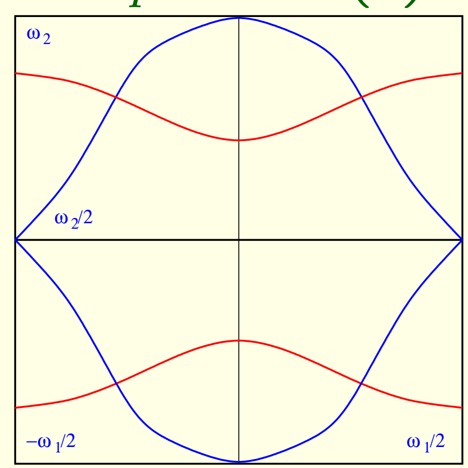
anomalous thresholds

[N.Dorey,Maldacena,Hofman,Okamura]



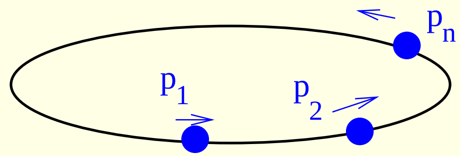
Physical domain?

[Frolov]



QFTs in finite volume

Finite volume spectrum



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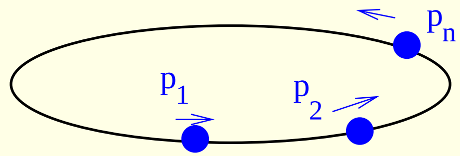

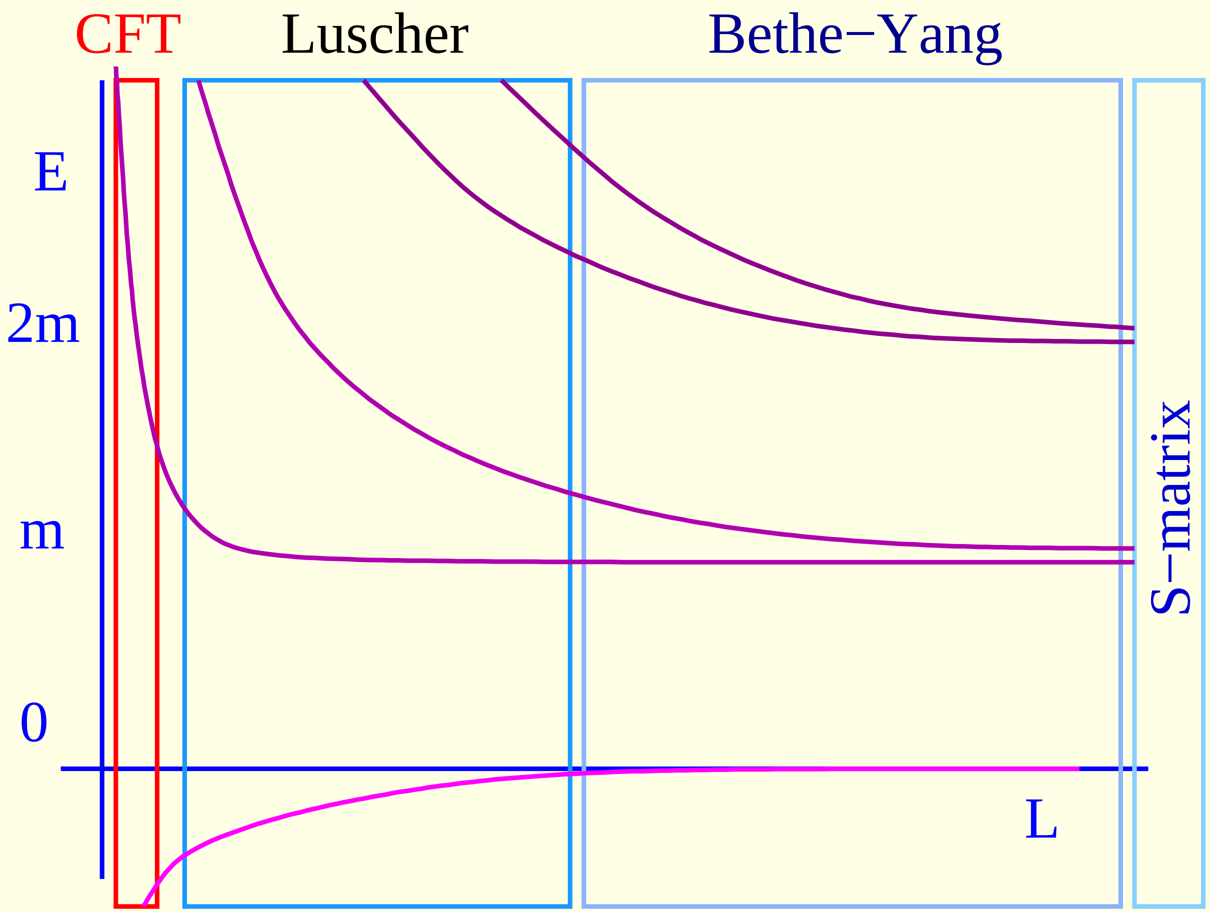

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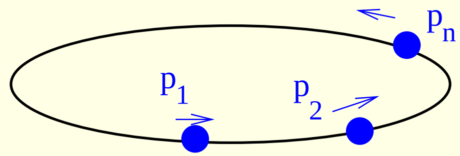

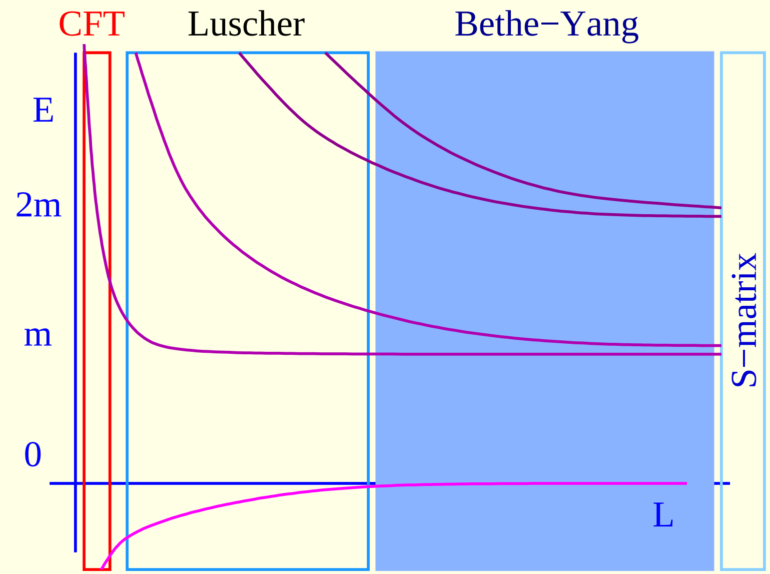

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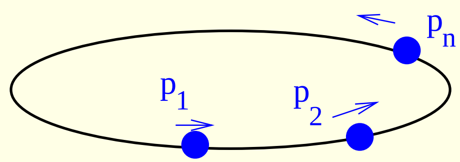
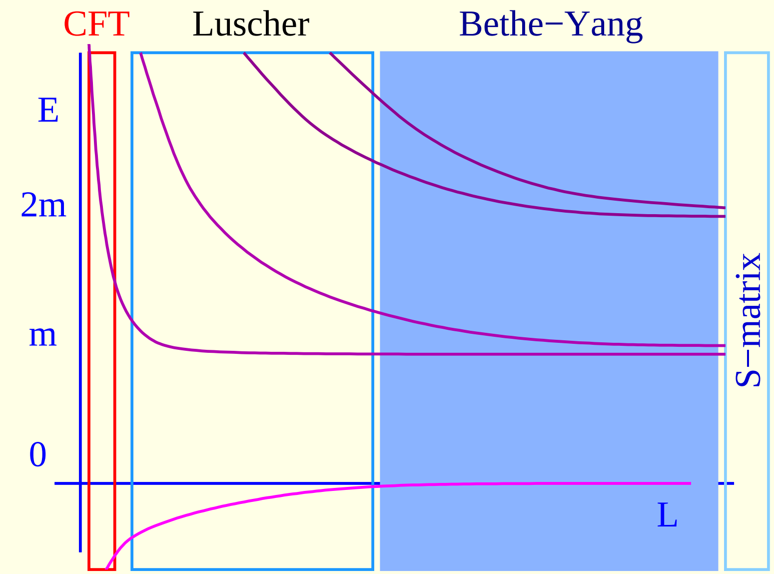
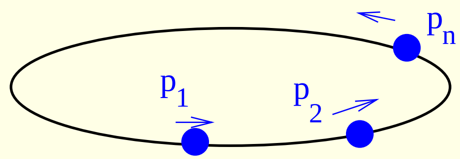


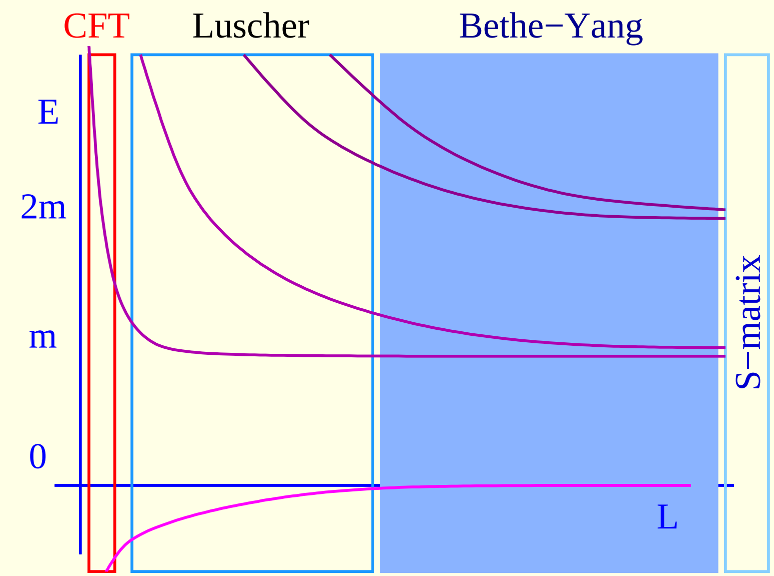


Infinite volume spectrum:

$$E(p_1, \ldots, p_n) = \sum_i E(p_i) \qquad p_i \in R$$

QFTs in finite volume





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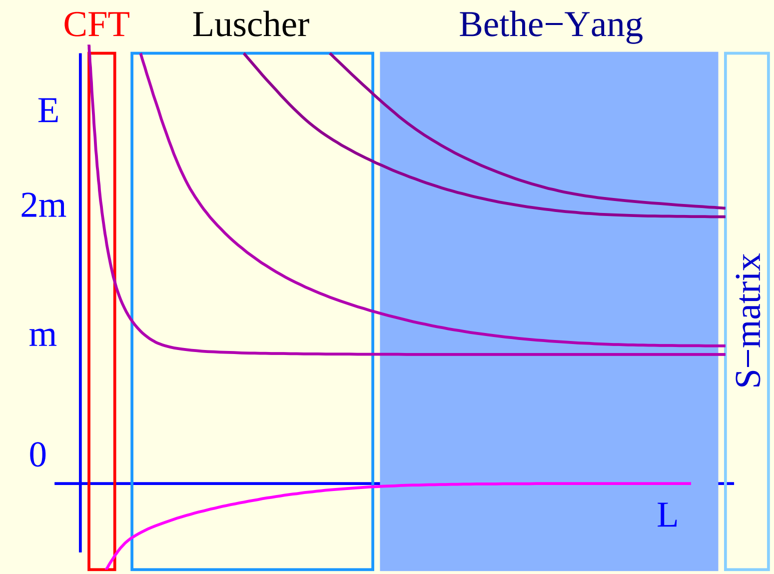

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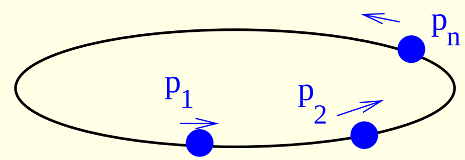

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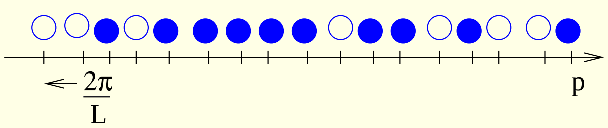

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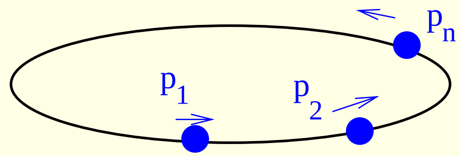

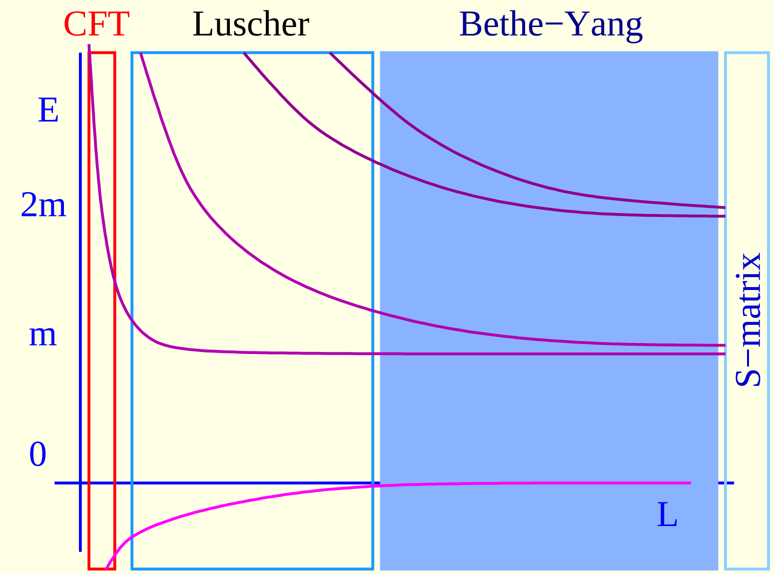

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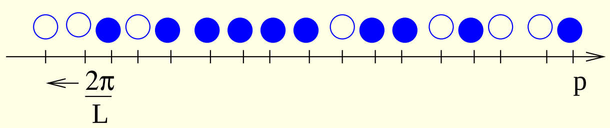

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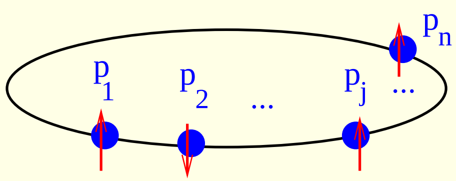

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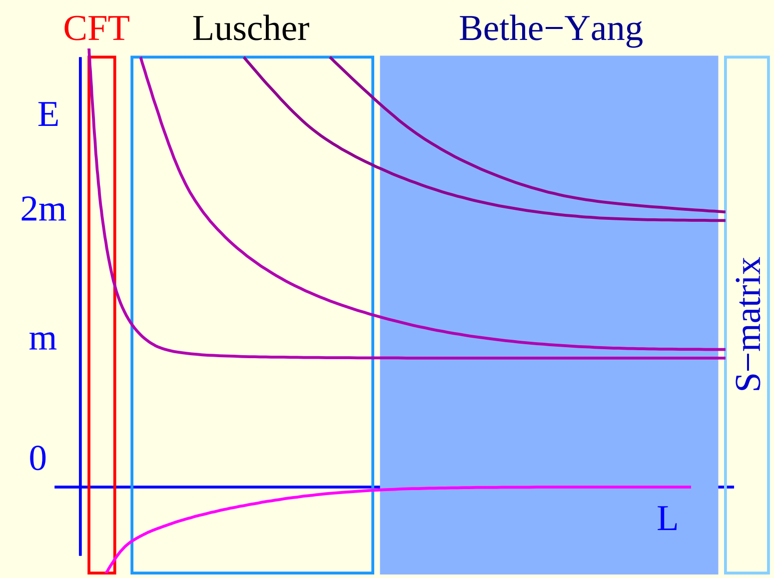

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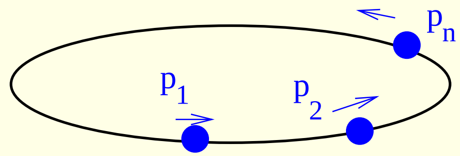

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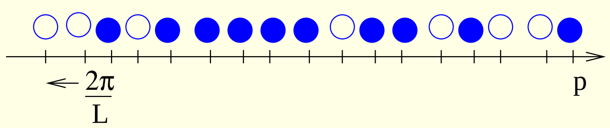

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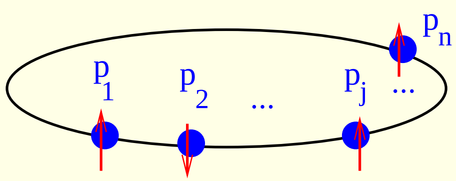
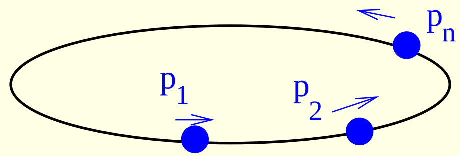
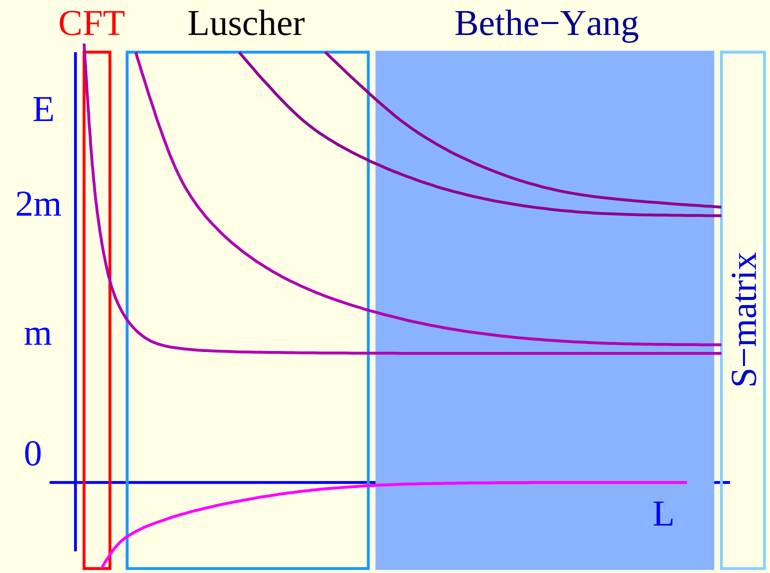


Inhomogenous XXZ spin-chain spectral problem $e^{iL\sinh\theta_j} T(\theta_j) S_0(\theta_j) = -1$

QFTs in finite volume

Finite volume spectrum





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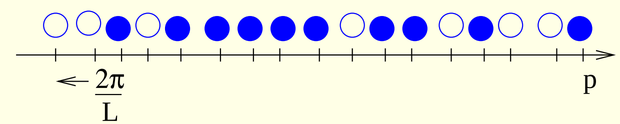

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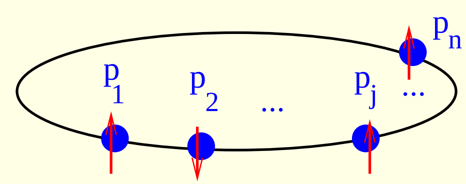

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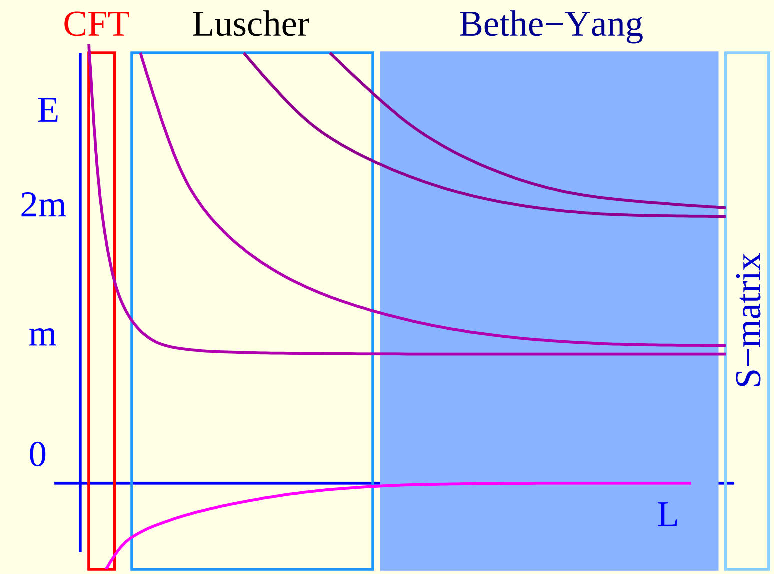
$$T(\theta)Q(\theta)=Q(\theta+i\pi)T_0(\theta-\tfrac{i\pi}{2})+Q(\theta-i\pi)T_0(\theta+\tfrac{i\pi}{2})=Q^{++}T^{-}+Q^{--}T^{+}$$

QFTs in finite volume



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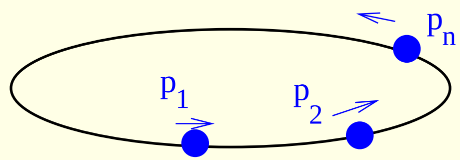

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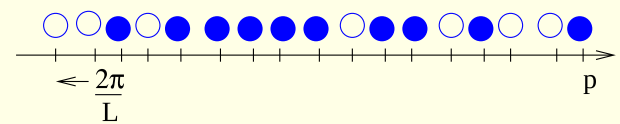
$$E(p_1,\dots,p_n)=\sum_i E(p_i) \qquad p_i\in R$$

Polynomial volume corrections:

Bethe-Yang; p_i quantized. Diagonal

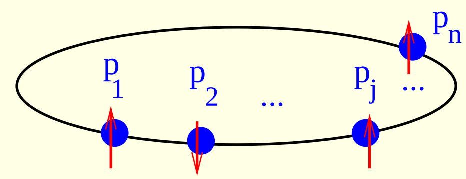
$$e^{ip_jL}S(p_j,p_1)\dots S(p_j,p_n)=-1 \quad ; \quad S(0)=-1$$



$$p_jL+\sum_k \frac{1}{i}\log S(p_j,p_k)=(2n+1)i\pi$$

Non-diagonal, sine-Gordon

$$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-P$$



Inhomogenous XXZ spin-chain spectral problem $e^{iL\sinh\theta_j}T(\theta_j)S_0(\theta_j)=-1$

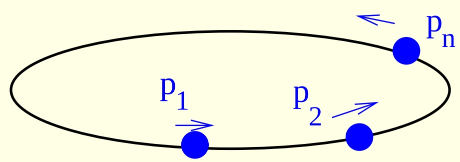
$$T(\theta)Q(\theta)=Q(\theta+i\pi)T_0(\theta-\tfrac{i\pi}{2})+Q(\theta-i\pi)T_0(\theta+\tfrac{i\pi}{2})=Q^{++}T^-+Q^{--}T^+$$

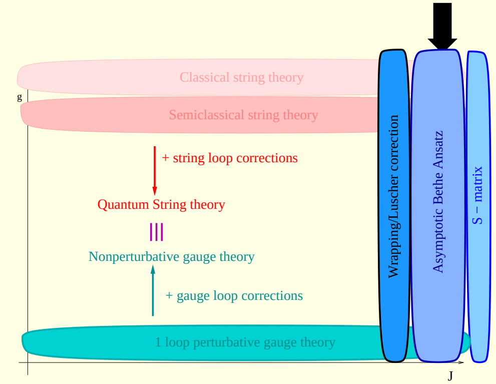
$Q(\theta)=\prod_\beta \sinh(\lambda(\theta-w_\beta))$ Bethe Ansatz: $\dfrac{T_0(w_\alpha-\frac{i\pi}{2})Q(w_\alpha+i\pi)}{T_0(w_\alpha+\frac{i\pi}{2})Q(w_\alpha-i\pi)}=\dfrac{T_0^-Q^{++}}{T_0^+Q^{--}}|_\alpha=-1$

$T_0(\theta)=\prod_j \sinh(\lambda(\theta-\theta_j))$

Bethe-Yang=Asymptotic Bethe Ansatz

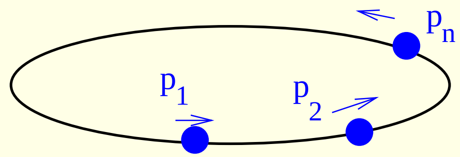
Finite volume spectrum

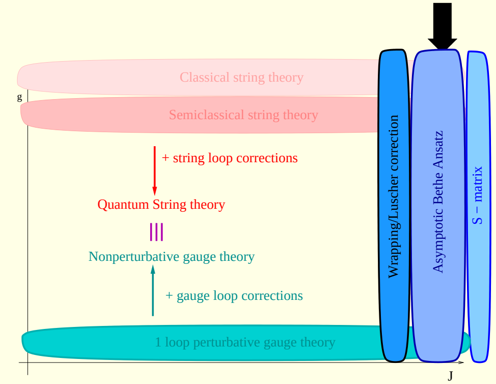




Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum



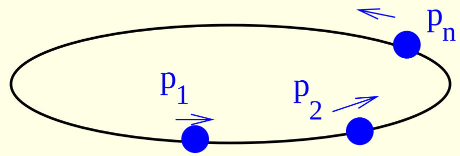


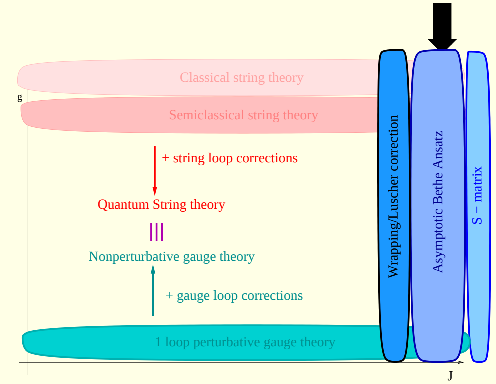
Infinite volume spectrum:

$E(p_1,\dots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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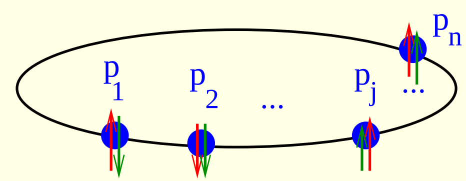
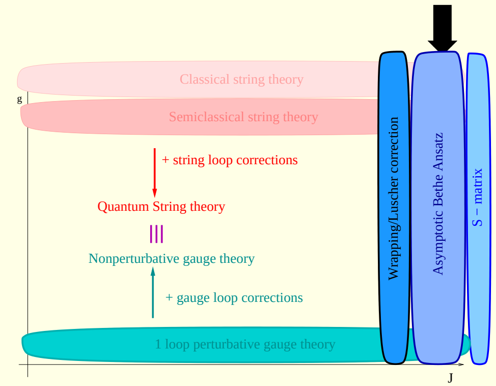
Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

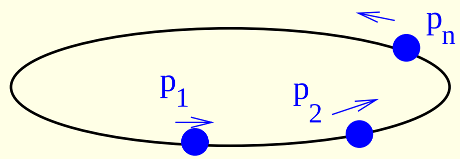
$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$



Bethe-Yang=Asymptotic Bethe Ansatz



Finite volume spectrum



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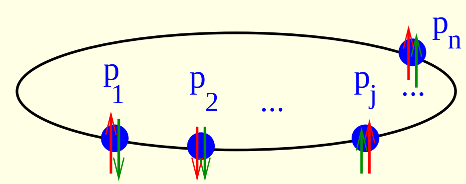
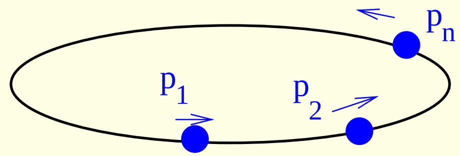
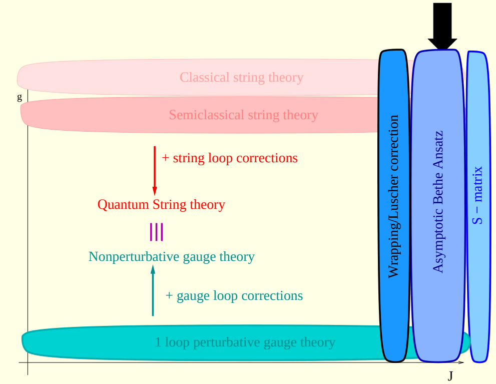


Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\dfrac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





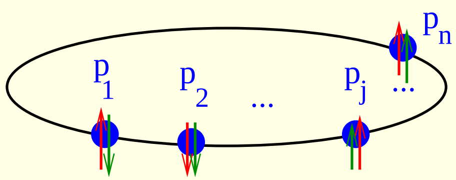
Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

$\boxed{\text{Polynomial volume corrections:}}$

Asymptotic Bethe Ansatz; p_i quantized, .

$e^{ip_jL}\mathcal{S}(p_j,p_1)\ldots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$



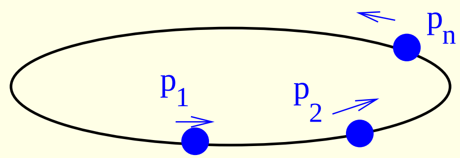
Inhomogenous Hubbard[2] spin-chain: $e^{iLp_j}S_0^2(u_j)\frac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

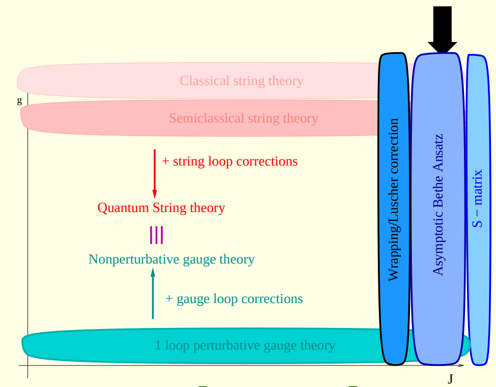
$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$ [Beisert,Staudacher]

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$ $\qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$ $\qquad$ [Kazakov,Gromov]

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

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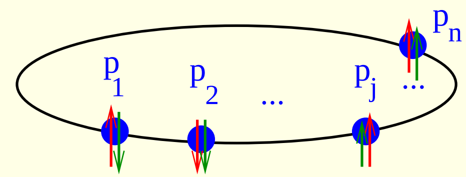

Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\frac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

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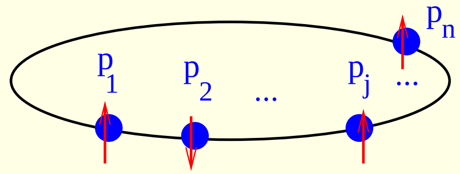
Bethe Ansatz: $\frac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1 \quad \frac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1 \quad \frac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$ [Frolov]

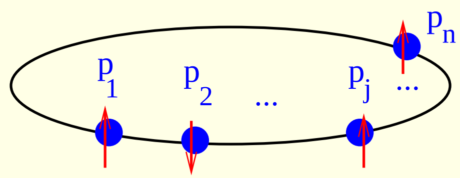
Lüscher correction of multiparticle states

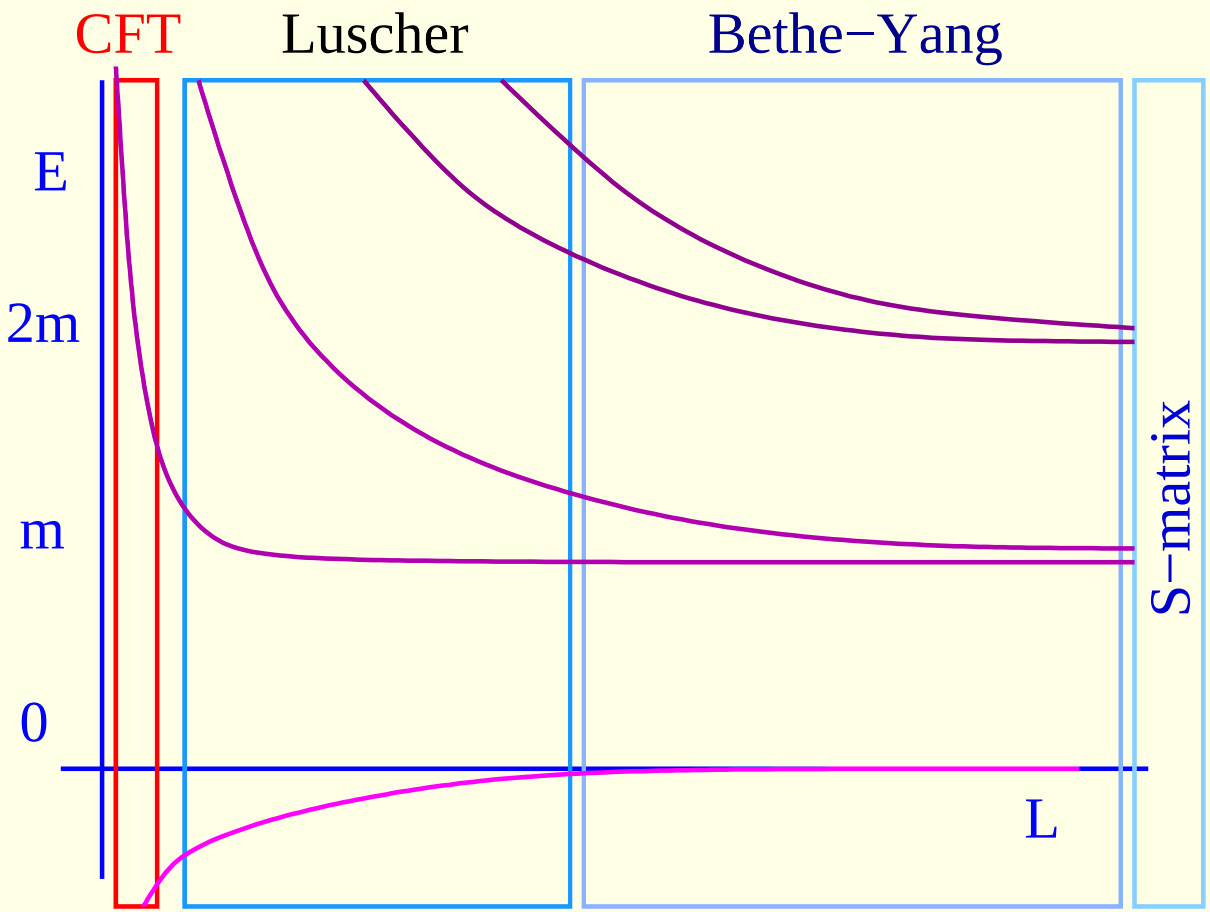
Finite volume spectrum



Lüscher correction of multiparticle states

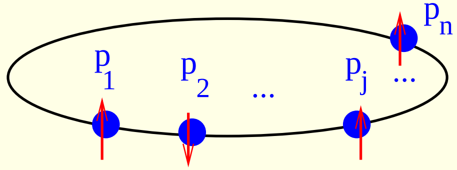
Finite volume spectrum

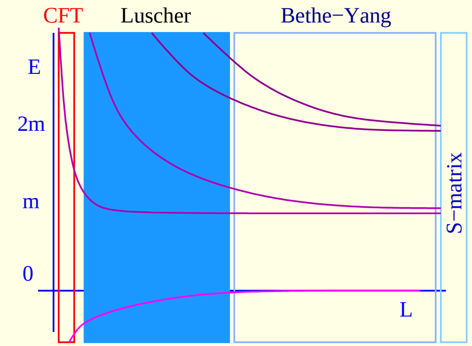




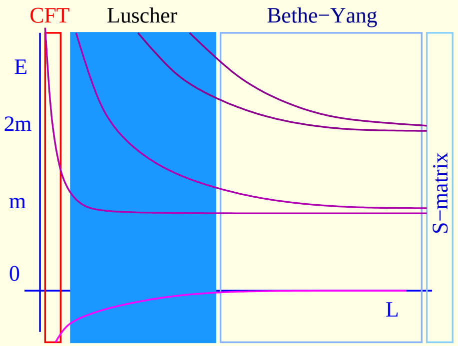
Lüscher correction of multiparticle states

Finite volume spectrum

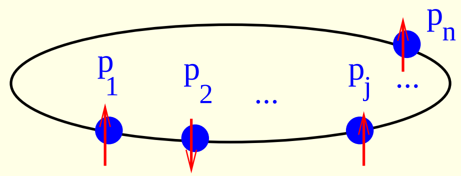




Lüscher correction of multiparticle states



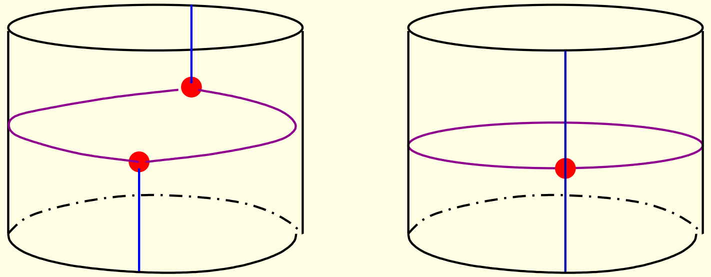
Finite volume spectrum



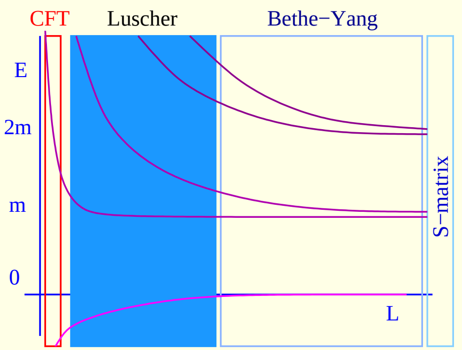
Lüscher originally: $O(e^{-mL})$ mass correction

$$m(L) = \frac{\sqrt{3}}{2} m \left(-i \mathrm{Res}_{\theta=\frac{2i\pi}{3}} S\right) e^{-\frac{\sqrt{3}}{2} mL}$$

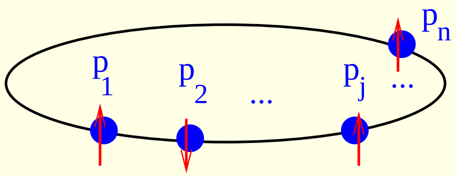
$$-\int \frac{d\theta}{2\pi} \cosh\theta \left(S\left(\theta + \frac{i\pi}{2}\right) - 1\right) e^{-mL\cosh\theta}$$



Lüscher correction of multiparticle states



Finite volume spectrum



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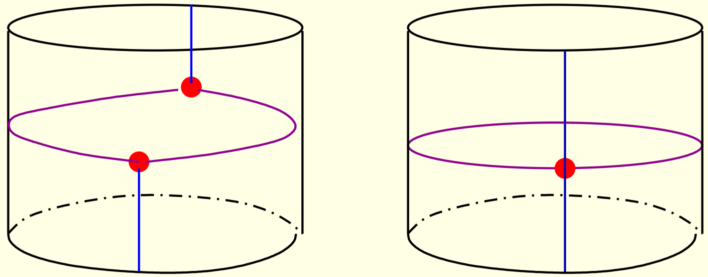
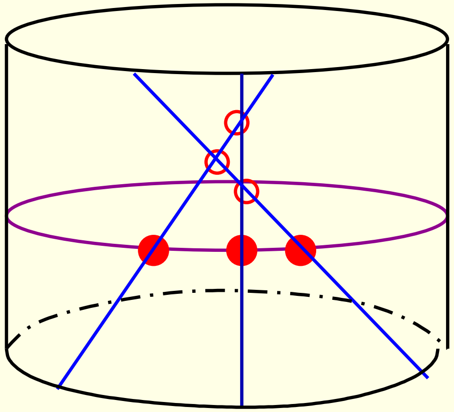


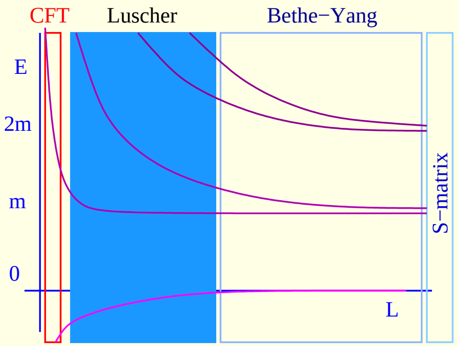
$\boxed{\text{Multiparticle Lüscher correction}}$ [ZB,Janik]



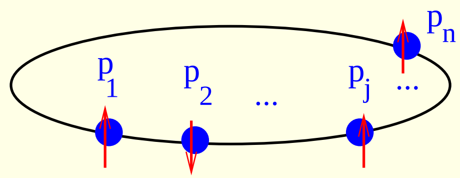
BY: $\mathcal{S}(p_j, p_1) \ldots \mathcal{S}(p_j, p_n)\Psi = -e^{-ip_jL}\Psi$

$T(p, p_1, \ldots, p_n)\Psi = t(p, p_1, \ldots, p_n, \Psi)\Psi$

Lüscher correction of multiparticle states



Finite volume spectrum



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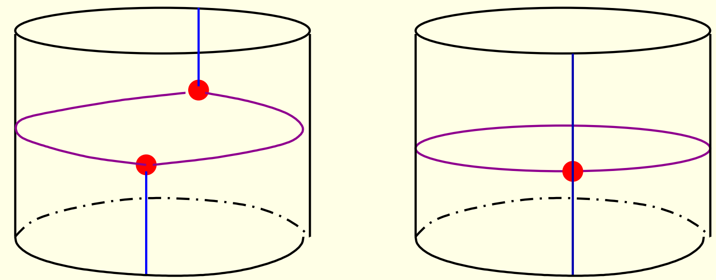

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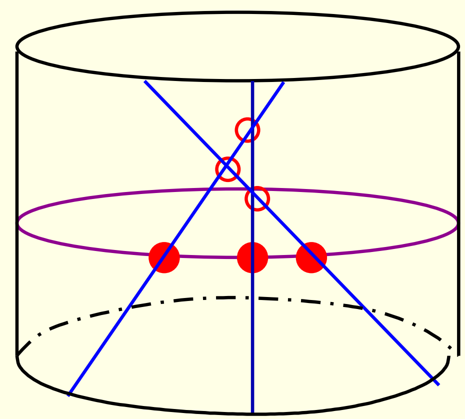

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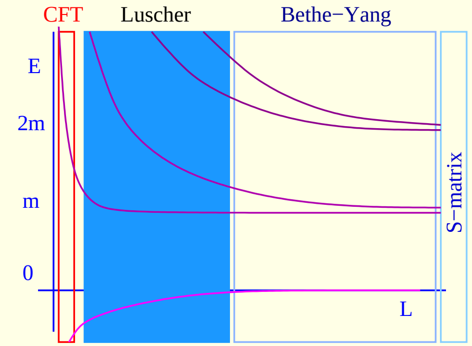
$T(p, p_1, \dots, p_n)\Psi = t(p, p_1, \dots, p_n, \Psi)\Psi$

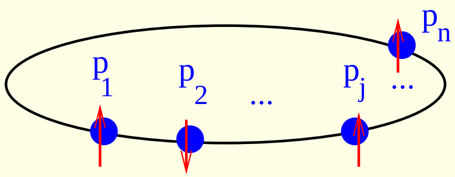
Modified momenta:

$$p_j L - i \log t(p_j, p_1, \dots, p_n, \Psi) = (2n+1)\pi + \Phi_j$$
$$\Phi_j = \int \frac{dq}{2\pi} \frac{d}{dp_j} t(q, p_1, \dots, p_n, \Psi) e^{-LE(q)}$$

Lüscher correction of multiparticle states



Finite volume spectrum



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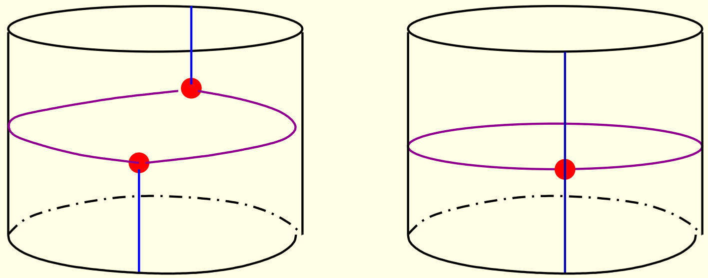

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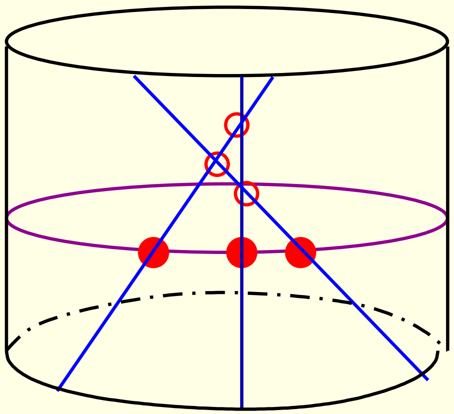

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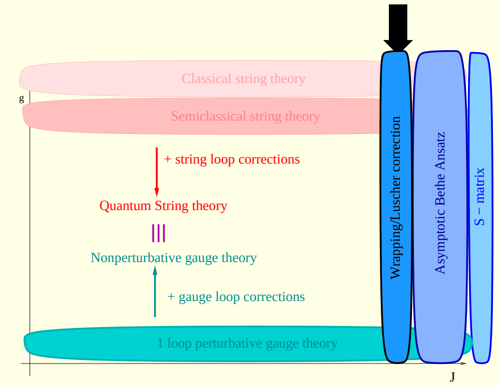
$$\Phi_j = \int \frac{dq}{2\pi} \frac{d}{dp_j} t(q, p_1, \dots, p_n, \Psi) e^{-LE(q)}$$

Modified energy:

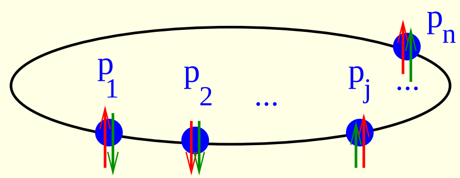
$$E(p_1, \dots, p_n) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi} t(q, p_1, \dots, p_n, \Psi) e^{-LE(q)}$$

Lüscher/wrapping correction in AdS

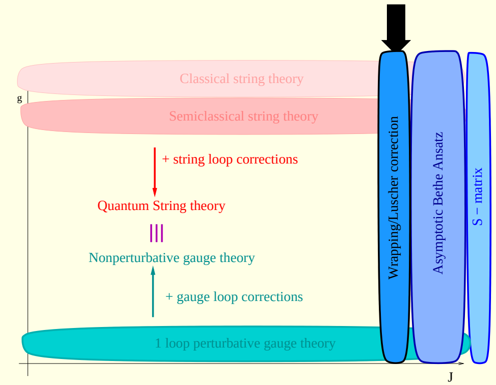


Finite volume spectrum

[Ambjorn,Janik,Kristjansen]

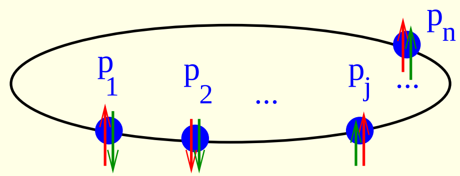


Lüscher/wrapping correction in AdS



Finite volume spectrum

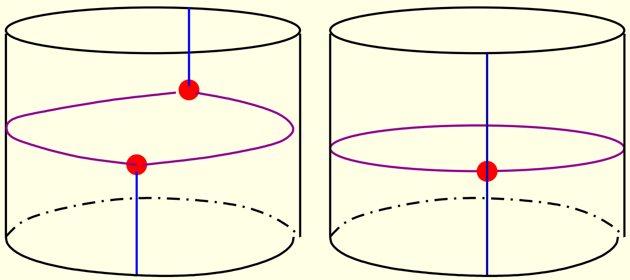
[Ambjorn,Janik,Kristjansen]



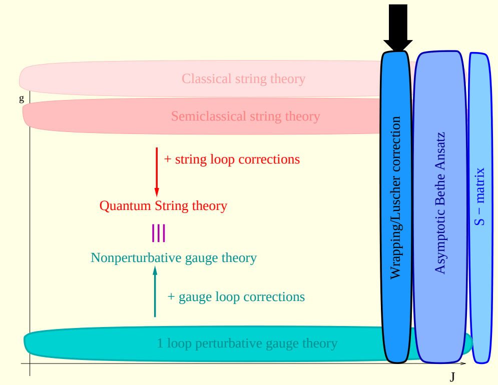
One particle correction: [Janik, Lukowski]

$$\Delta(L) = \left(1 - E'(p)\tilde{E}'(\tilde{p}_0)\right)\left(-i\mathrm{Res}_{\tilde{p}=\tilde{p}_0}\sum_b S\right)e^{-\tilde{E}(\tilde{p})L}$$

$$-\sum_b \int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}\left(1 - E'(p)\tilde{E}'(\tilde{p})\right)\left(S_{ba}^{ba}(\tilde{p},p) - 1\right)e^{-\tilde{E}(\tilde{p})L}$$

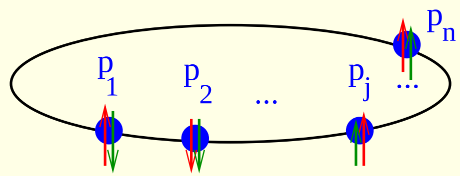


Lüscher/wrapping correction in AdS



Finite volume spectrum

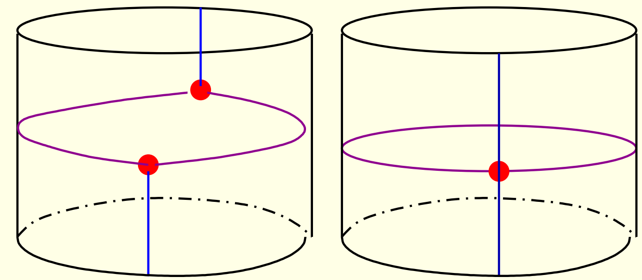
[Ambjorn,Janik,Kristjansen]



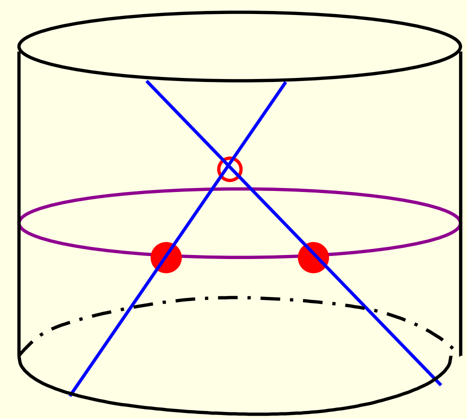
One particle correction: [Janik, Lukowski]

$$\Delta(L) = \left(1 - E'(p)\tilde{E}'(\tilde{p}_0)\right)\left(-i\mathrm{Res}_{\tilde{p}=\tilde{p}_0}\sum_b S\right)e^{-\tilde{E}(\tilde{p})L}$$

$$-\sum_b \int_{-\infty}^{\infty} \frac{d\tilde{p}}{2\pi}\left(1 - E'(p)\tilde{E}'(\tilde{p})\right)\left(S_{ba}^{ba}(\tilde{p},p) - 1\right)e^{-\tilde{E}(\tilde{p})L}$$



Two particle Lüscher correction (Konishi) [ZB,Janik]



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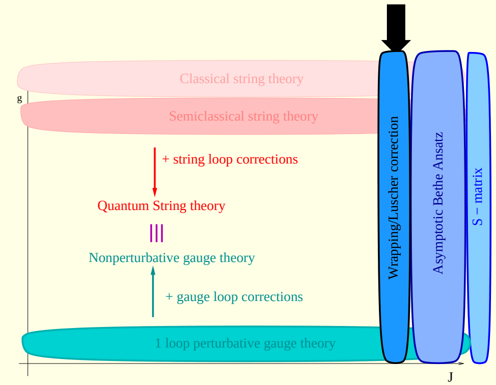
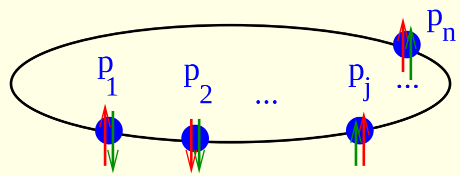
Lüscher/wrapping correction in AdS



Finite volume spectrum

[Ambjorn,Janik,Kristjansen]



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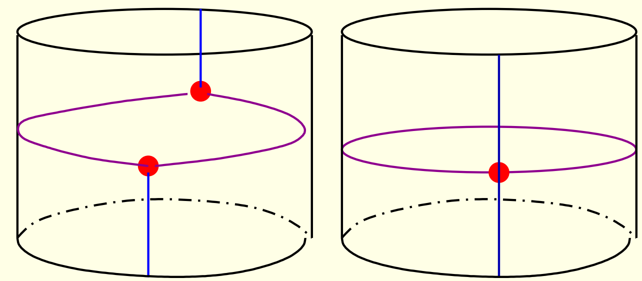

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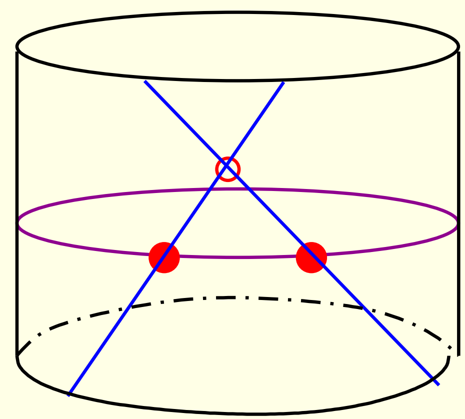

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Modified momenta: [Janik]

$$p_jL - i\log t(p_j,p_1,p_2,\Psi) = (2n+1)\pi + \Phi_j$$

$$\int \frac{d\tilde{p}}{2\pi}\frac{d}{dp_1}t(\tilde{p},p_1,p_2,\Psi)e^{-L\tilde{E}(\tilde{p})} \neq \Phi_1 = -\int\frac{d\tilde{p}}{2\pi}\left(\frac{d}{d\tilde{p}}S(\tilde{p},p_1)\right)S(\tilde{p},p_2)e^{-L\tilde{E}(\tilde{p})}$$

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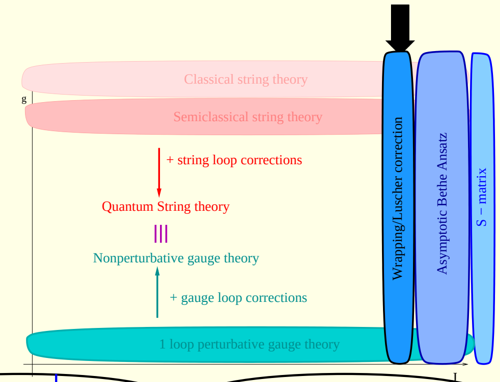

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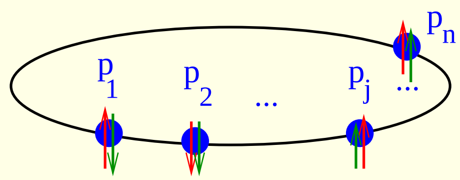

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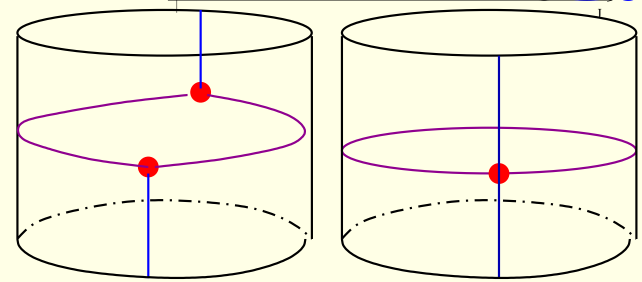

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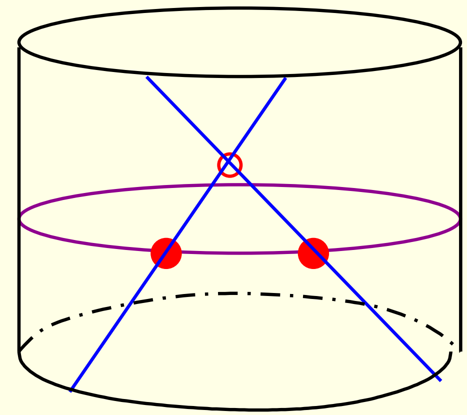

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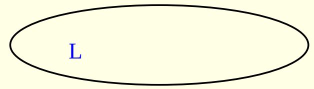
Modified energy:

$$E(p_1,p_2) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi}t(q,p_1,p_2,\Psi)e^{-LE(q)}$$

Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly



[Zamolodchikov]

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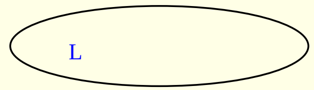

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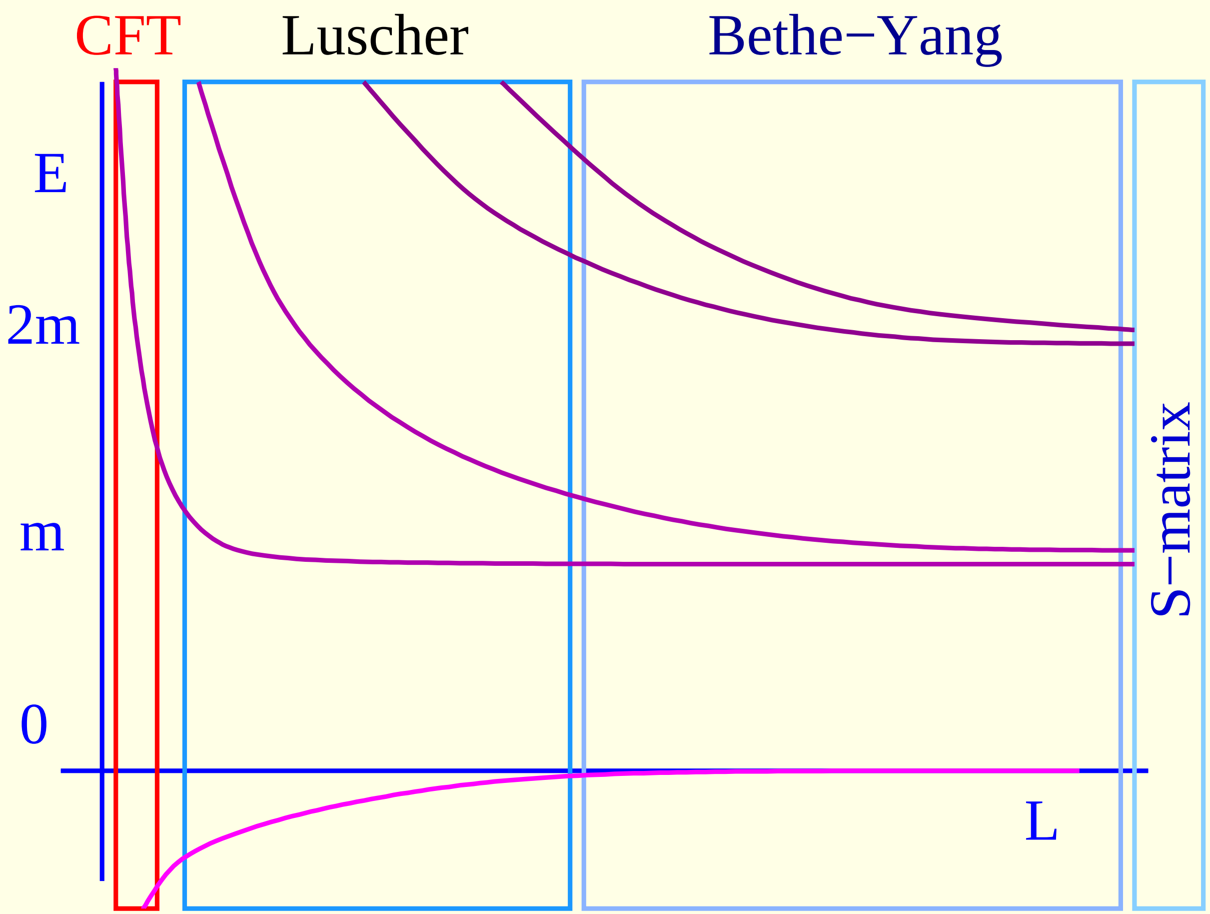

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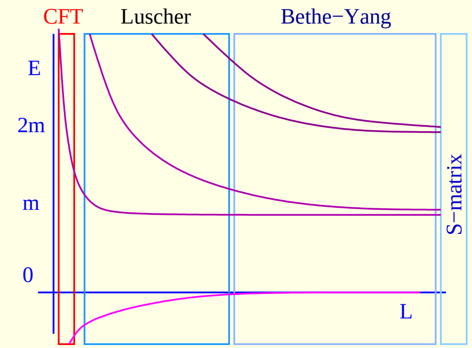

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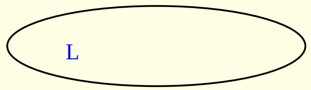

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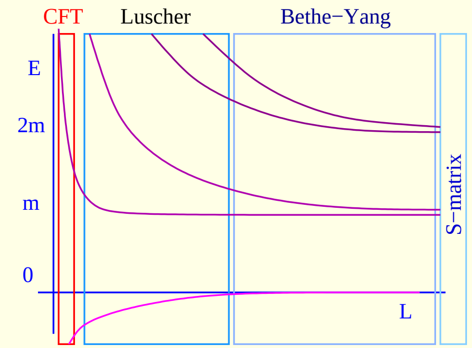

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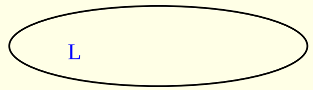
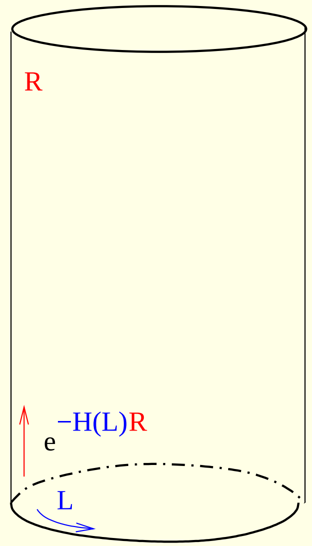


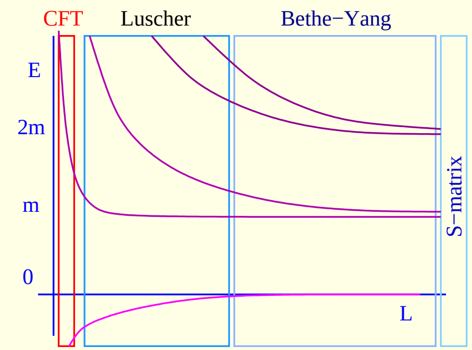
[Zamolodchikov]

Euclidien partition function:



$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(L)R})$$
$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$

Thermodynamic Bethe Ansatz: diagonal



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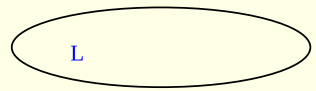

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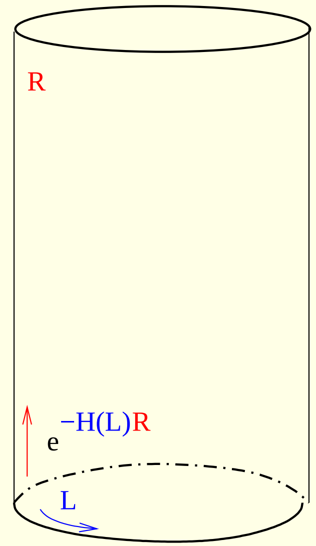

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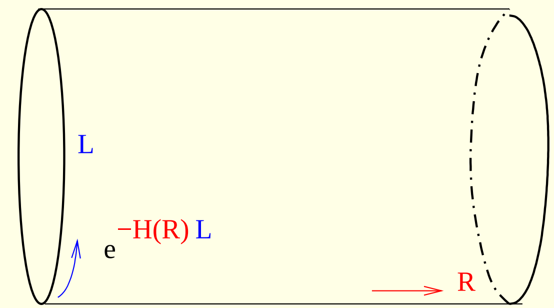
Exchange space and Euclidien time



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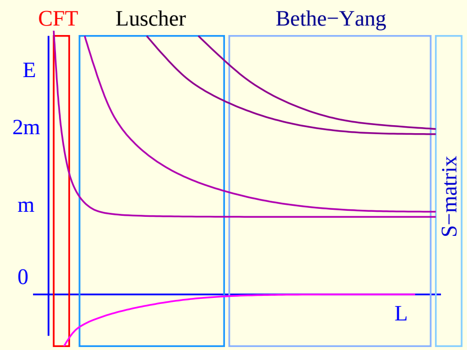

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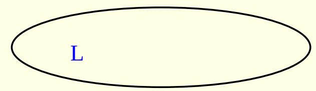

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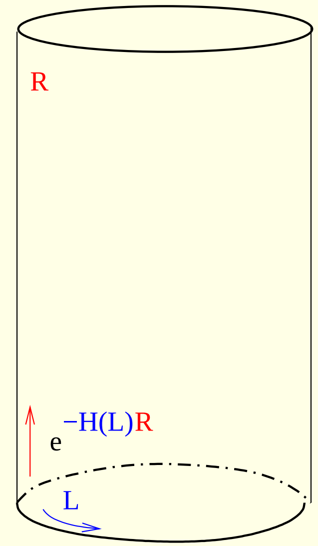

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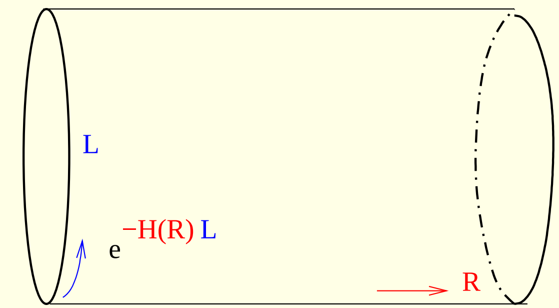

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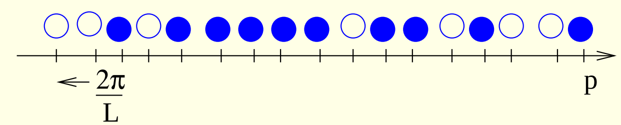
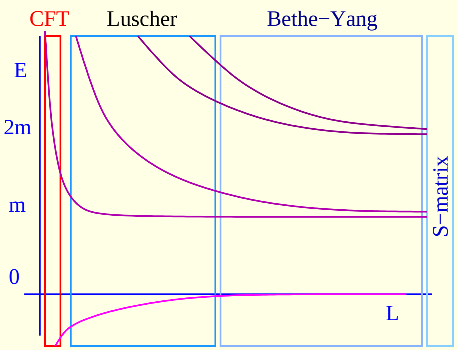


Dominant contribution: finite particle/hole density ρ, ρ_h:

Thermodynamic Bethe Ansatz: diagonal



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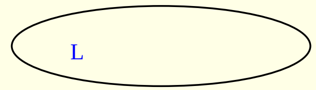

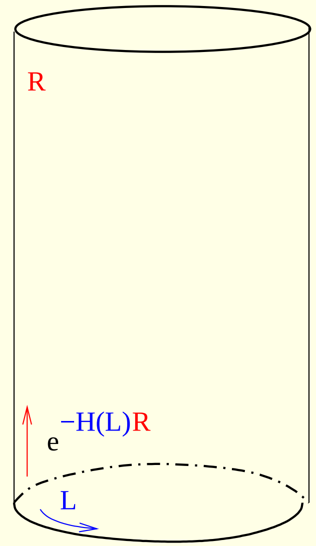

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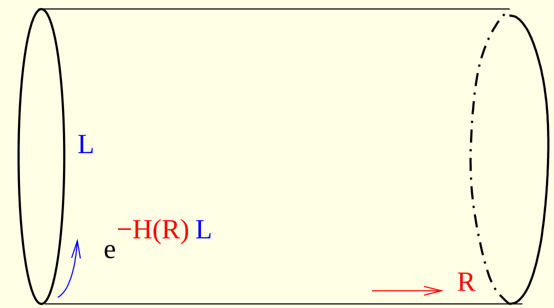

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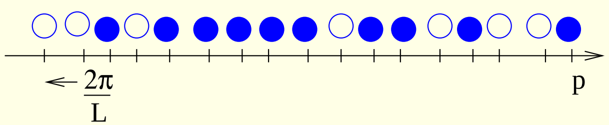

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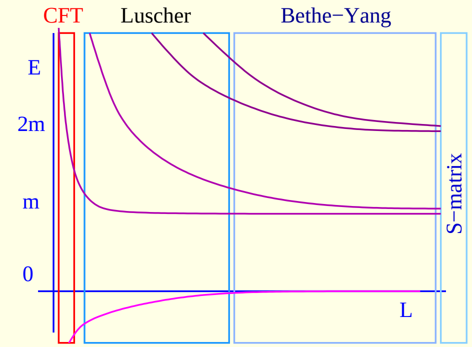

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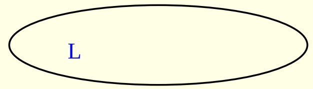

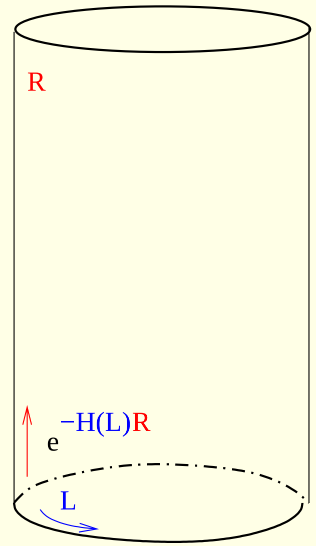

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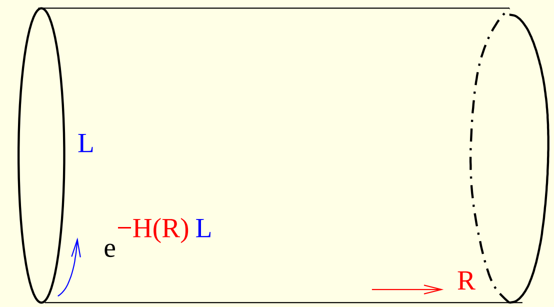

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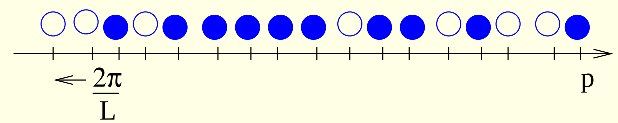

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$$Z(L,R) = \int d[\rho,\rho_h]e^{-LE(R)-\int((\rho+\rho_h)\ln(\rho+\rho_h)-\rho\ln\rho-\rho_h\ln\rho_h)dp}$$

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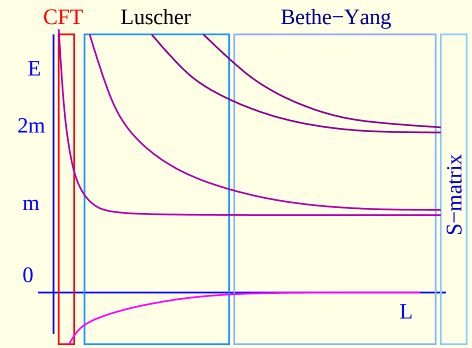
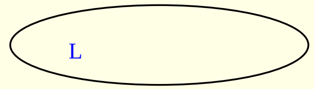
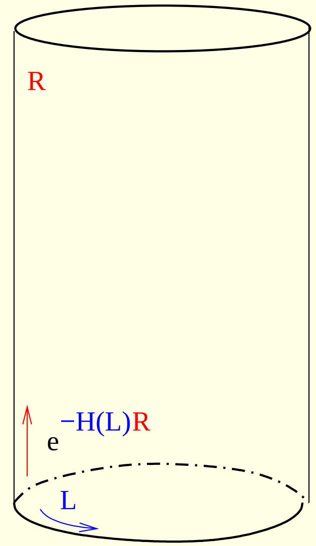


Ground-state energy exactly





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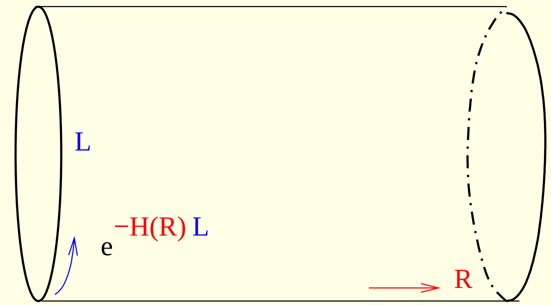

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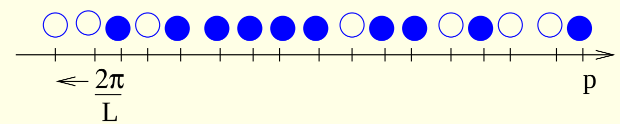

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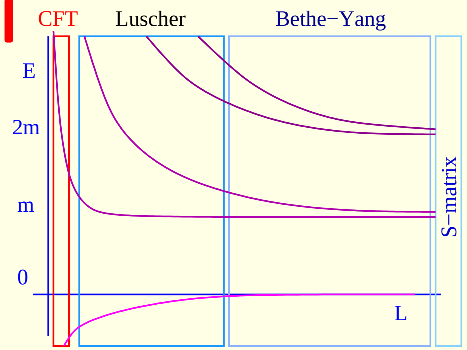
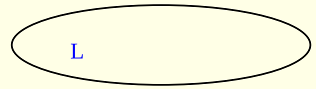
Saddle point : $\epsilon(p) = \ln\frac{\rho_h(p)}{\rho(p)}$ $\boxed{\epsilon(p) = E(p)L + \int\frac{dp}{2\pi}id_p\log S(p',p)\log(1+e^{-\epsilon(p')})}$

Ground state energy exactly: $\boxed{E_0(L) = -\int\frac{dp}{2\pi}\log(1+e^{-\epsilon(p)})}$ Lee-Yang, sinh-Gordon

Thermodynamic Bethe Ansatz: non-diagonal

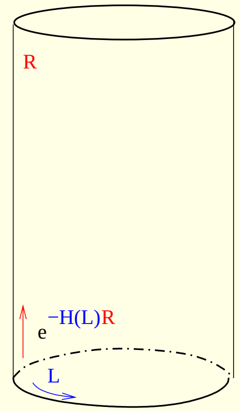


Ground-state energy exactly



[Tateo]

Euclidien partition function:



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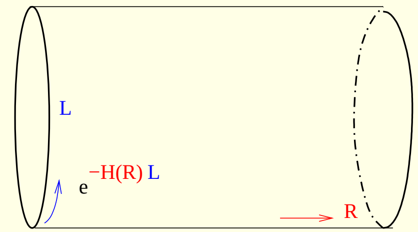

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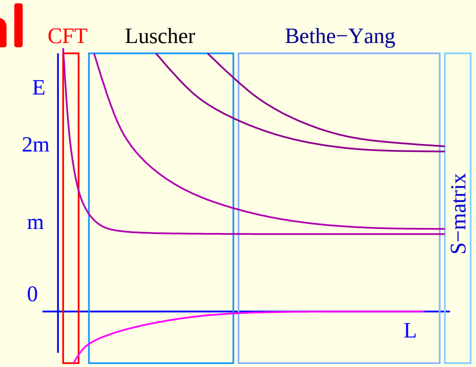
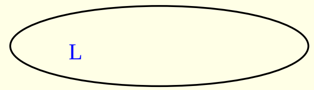
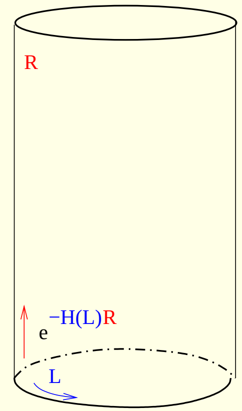


Ground-state energy exactly





[Tateo]

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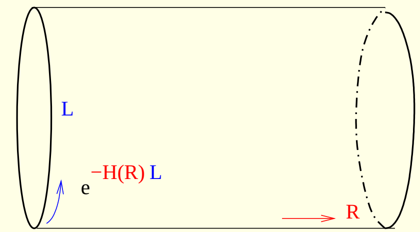

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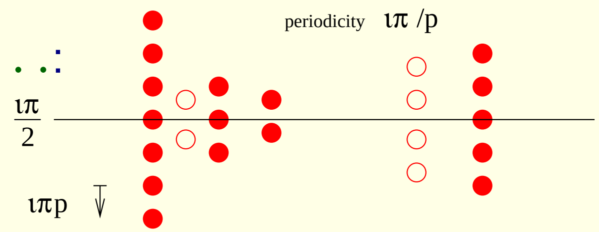
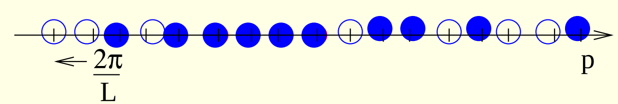
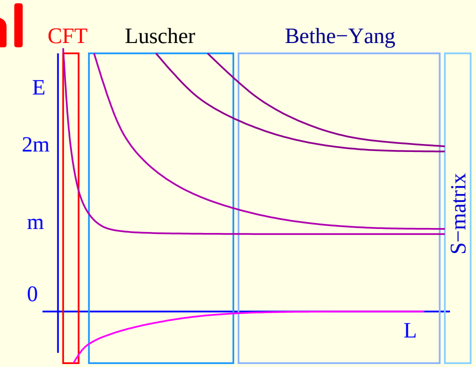




Thermodynamic Bethe Ansatz: non-diagonal



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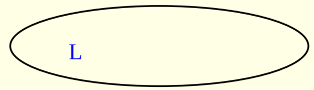

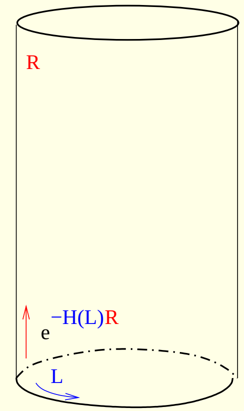

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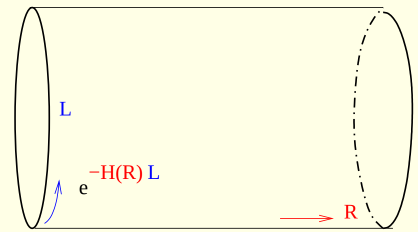

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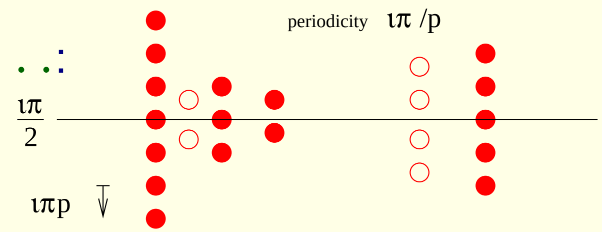

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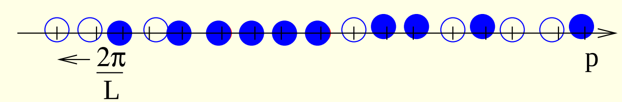
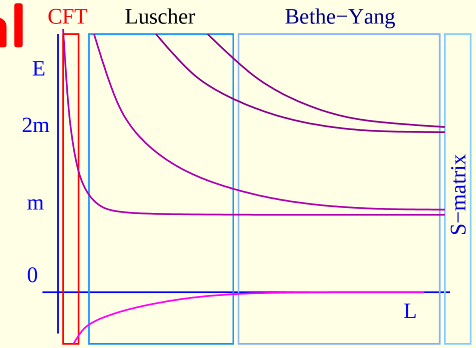


$$R\delta_0^m + \int K_n^m(p,p')\rho^n(p')dp' = 2\pi(\rho^m + \rho_h^m)$$

Thermodynamic Bethe Ansatz: non-diagonal



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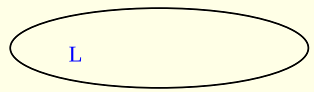

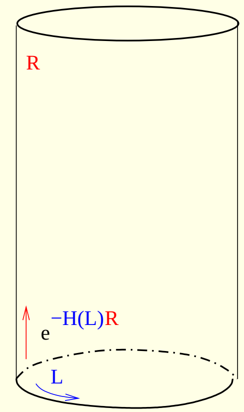

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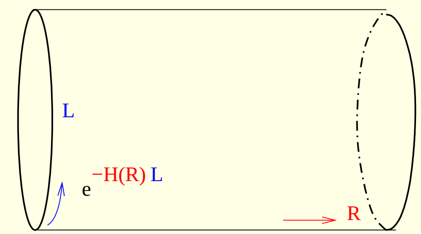

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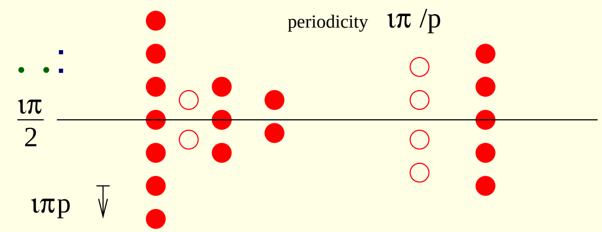

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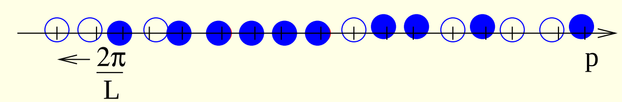

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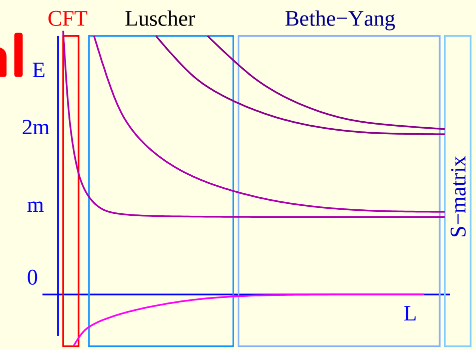
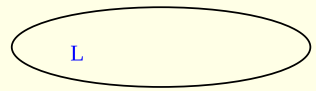
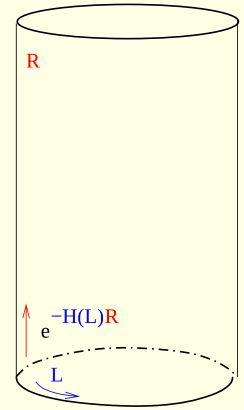
Thermodynamic Bethe Ansatz: non-diagonal



Ground-state energy exactly





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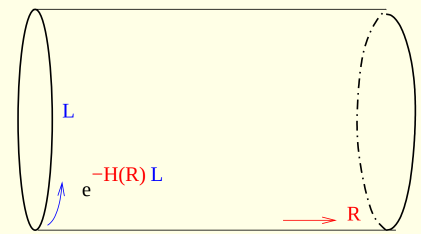

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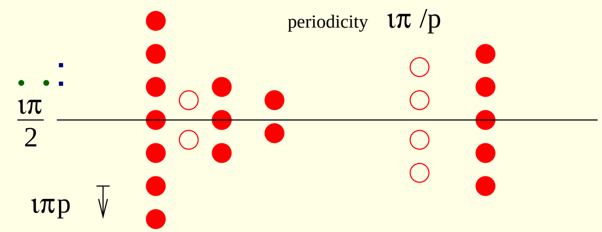

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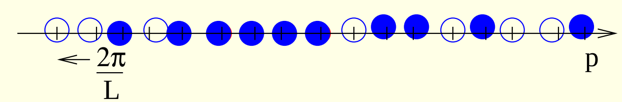

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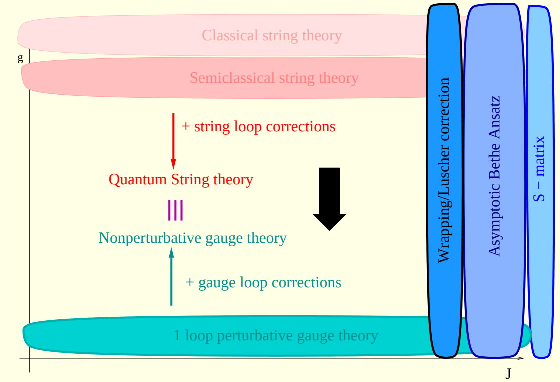
Saddle point : $\epsilon^i(\theta) = -\ln\frac{\rho^i(p)}{\rho^i_h(p)}$ $\boxed{\epsilon^j(\theta) = \delta_0^j E(p)L - \int K_i^j(p',p)\log(1+e^{-\epsilon^i(p')})dp'}$

Ground state energy exactly: $\boxed{E_0(L) = -\int \frac{dp}{2\pi}\log(1+e^{-\epsilon_0(\theta)})d\theta}$

Thermodynamic Bethe Ansatz: AdS



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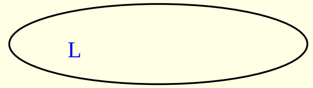

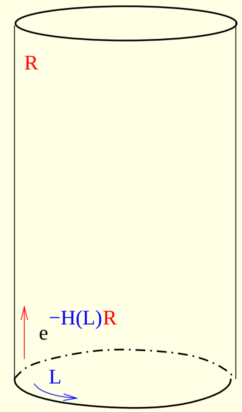

[Bombardelli,Tateo,Fioravanti,Frolov,Arutyunov, Gromov,Kazakov,Viera,Kozak]

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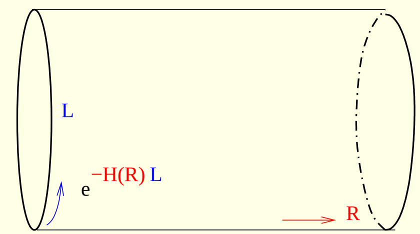

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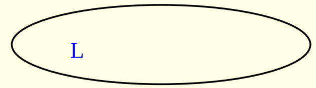

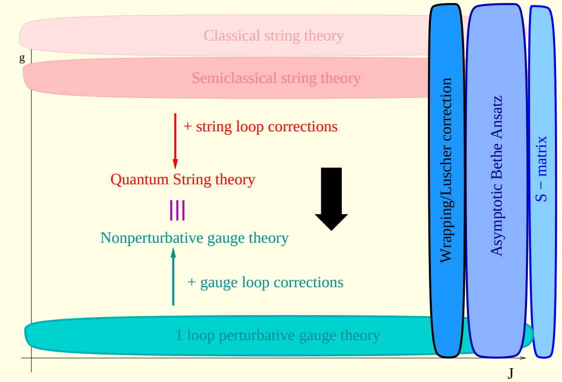

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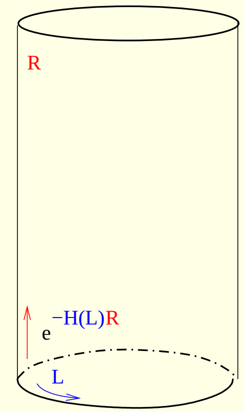

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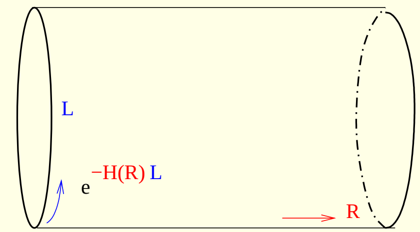

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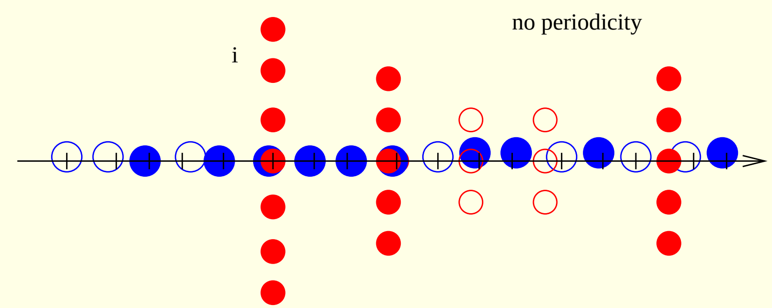

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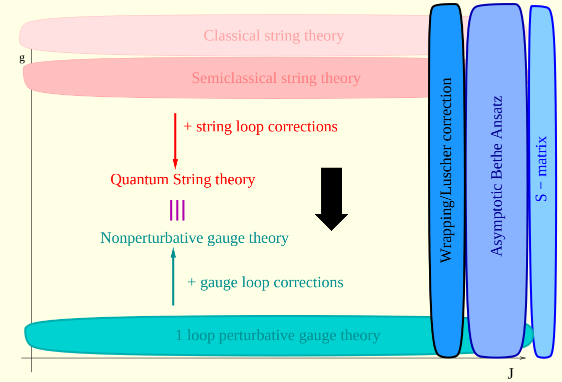

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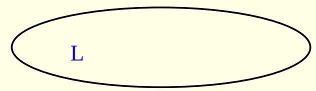

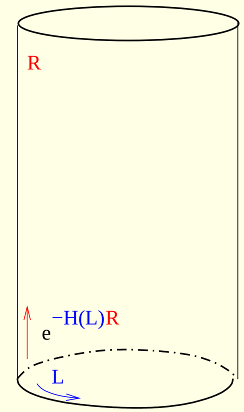

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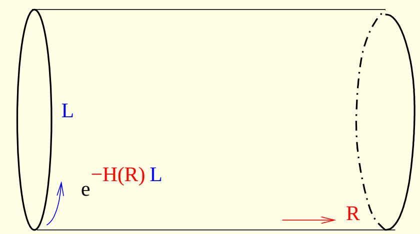

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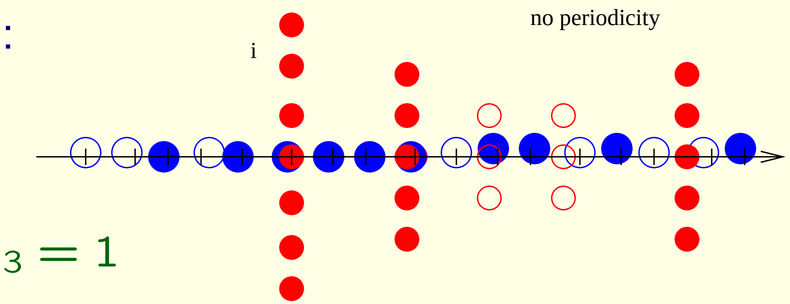

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[Frolov,Kazakov,Gromov]

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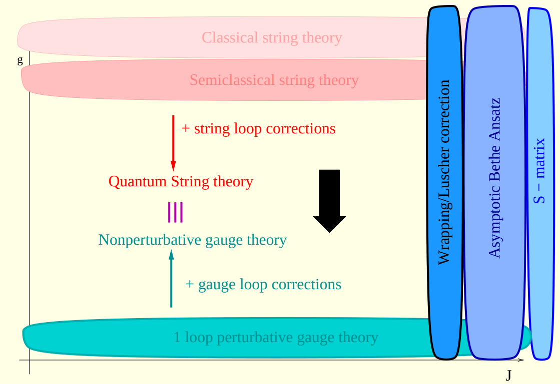
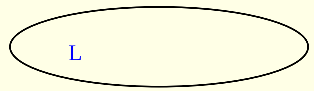
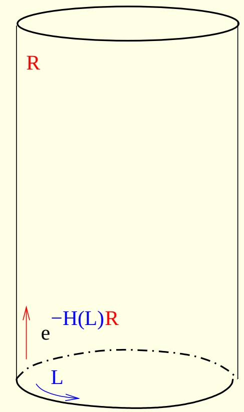


Ground-state energy exactly





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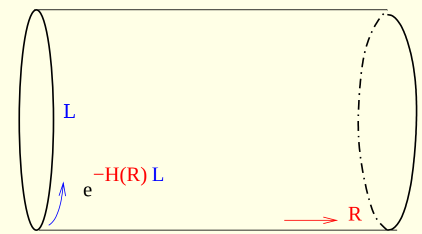

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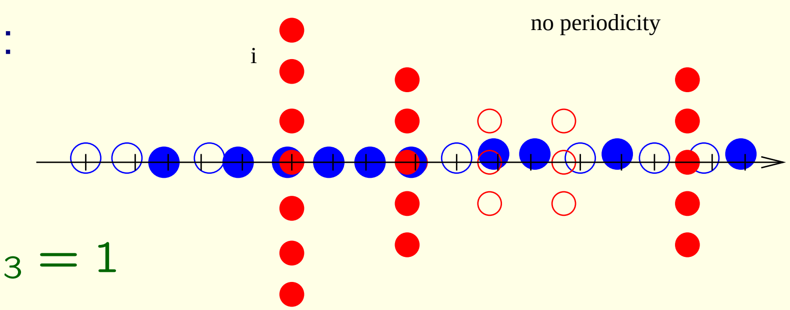

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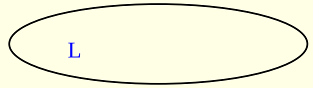

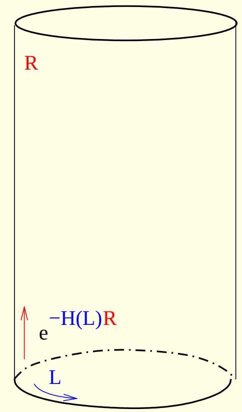

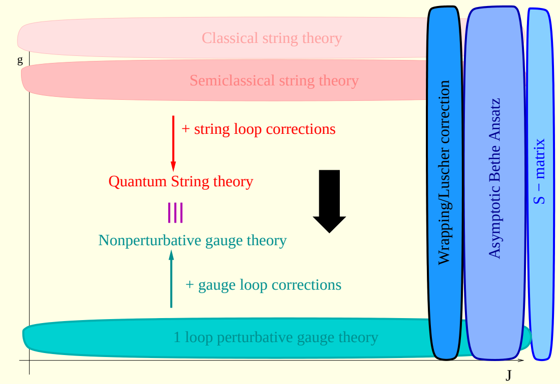

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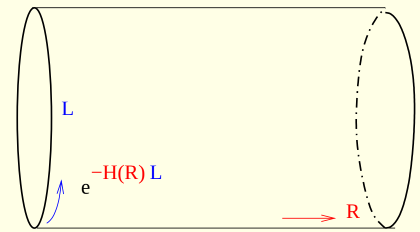

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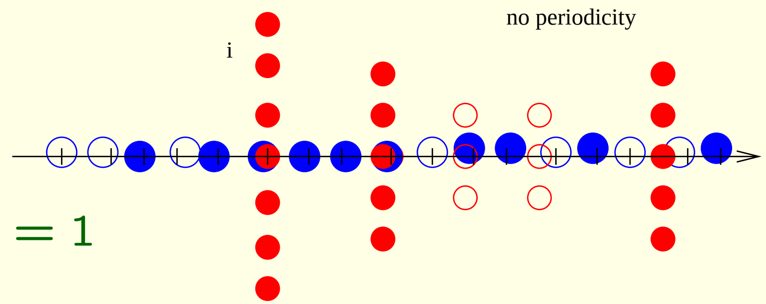

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$$\int K_n^m(\tilde{p},\tilde{p}')\rho^n(\tilde{p}')d\tilde{p}'=2\pi(\rho^m+\rho_h^m)$$ [Frolov, Kazakov, Gromov]

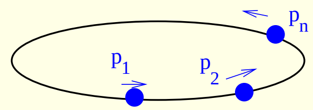
$$Z(L,R)=\int d[\rho^i,\rho_h^i]e^{-LE(R)-\sum_i\int((\rho^i+\rho_h^i)\ln(\rho^i+\rho_h^i)-\rho^i\ln\rho^i-\rho_h^i\ln\rho_h^i)dp}$$

Saddle point : $\epsilon^i(\tilde{p})=-\ln\frac{\rho^i(\tilde{p})}{\rho_h^i(\tilde{p})}$ $\boxed{\epsilon^j(\theta)=\delta_Q^j\tilde{E}_Q(\tilde{p})L-\int K_i^j(\tilde{p},\tilde{p}')\log(1+e^{-\epsilon^i(\tilde{p}')})d\tilde{p}'}$

Ground state energy exactly: $\boxed{E_0(L)=-\sum_Q\int\frac{d\tilde{p}}{2\pi}\log(1+e^{-\epsilon_Q(\tilde{p})})d\tilde{p}}$

Excited states TBA, Y-system: diagonal

Excited states exactly



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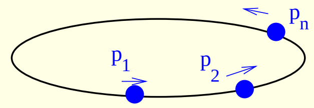

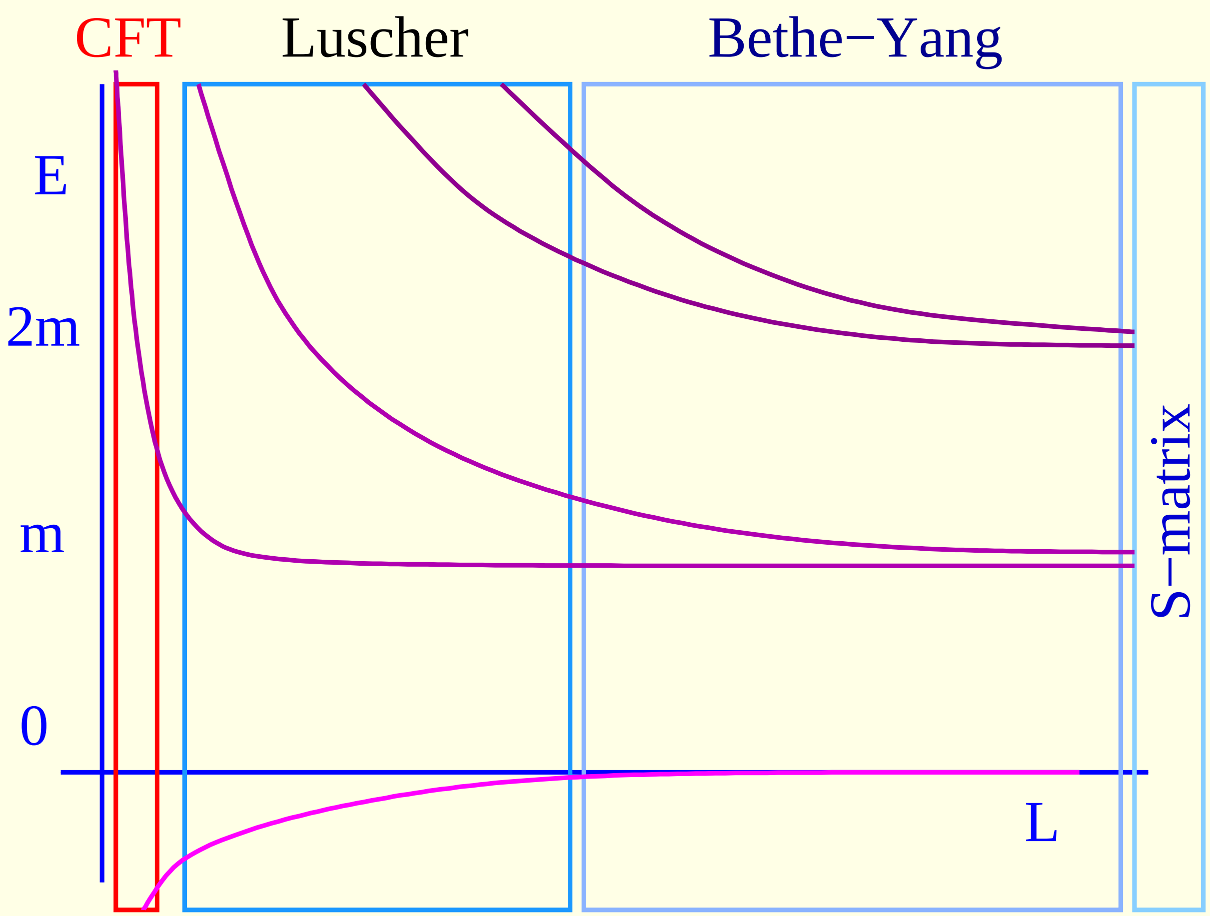

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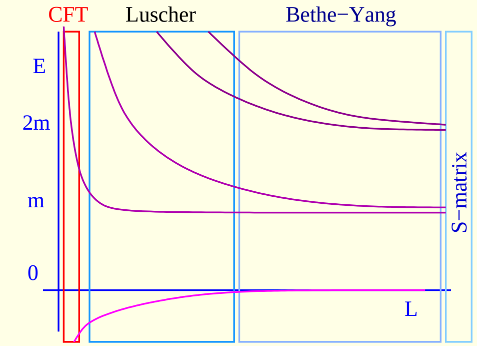

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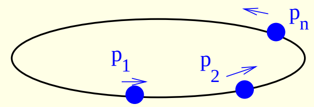

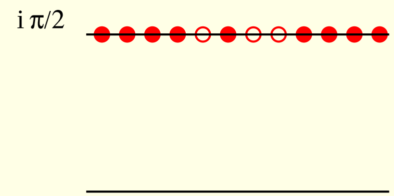

By lattice regularization: sinh-Gordon [Teschner]

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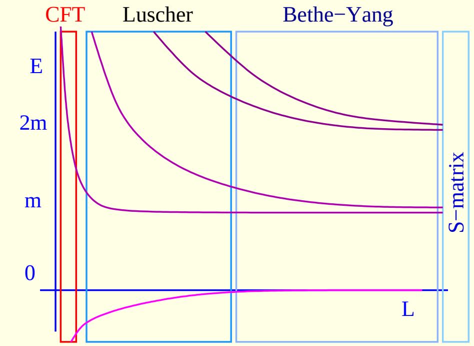

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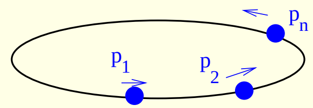

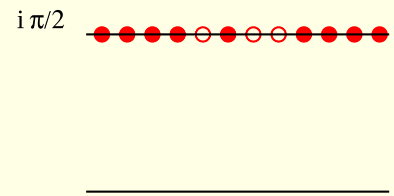

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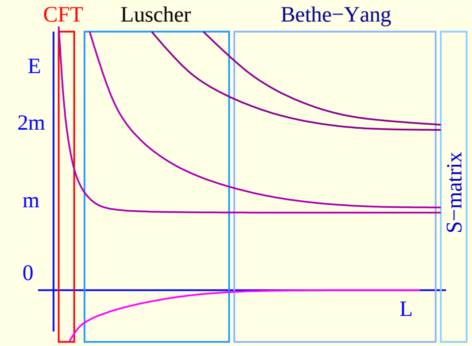

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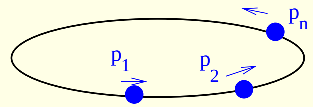

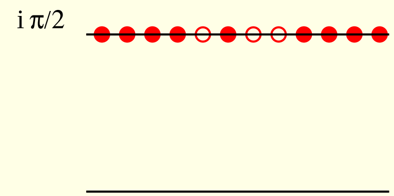

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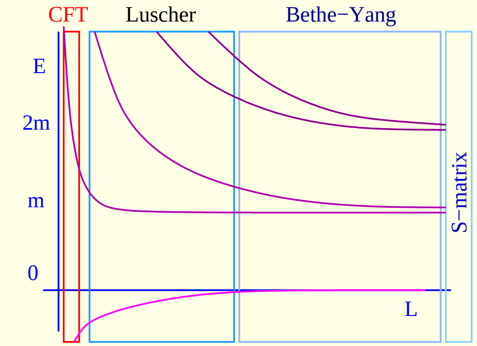

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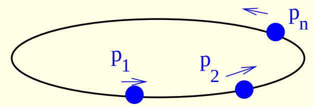

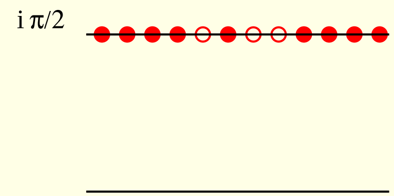

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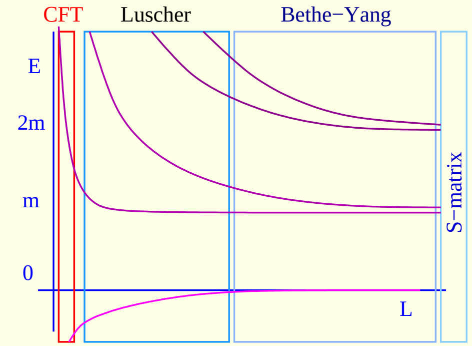

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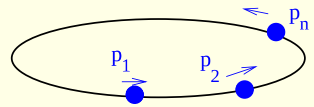

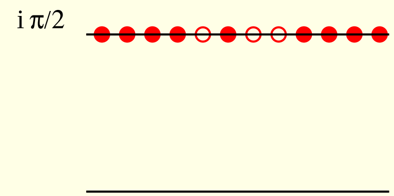

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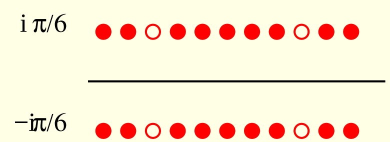

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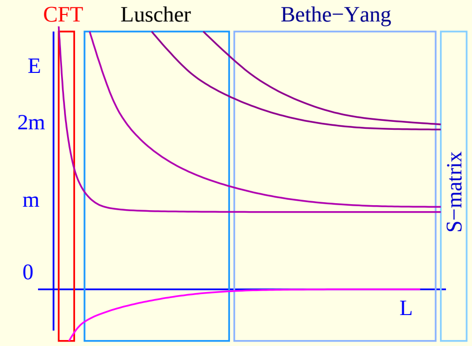
Lüscher corrections: differ by μ term

Excited states TBA, Y-system: diagonal



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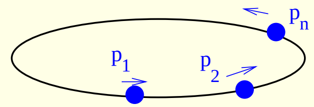

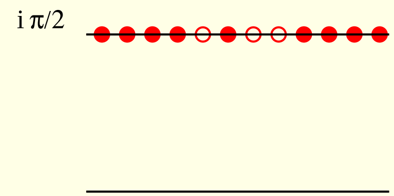

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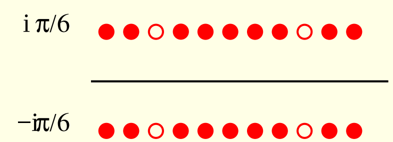

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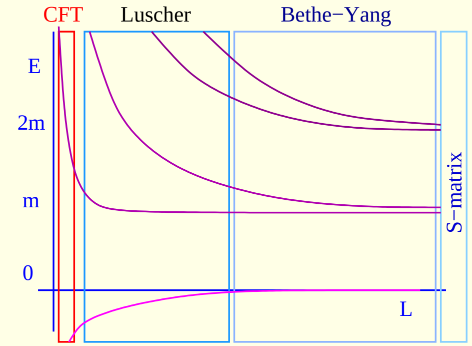

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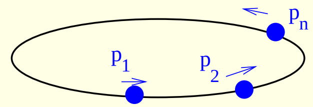

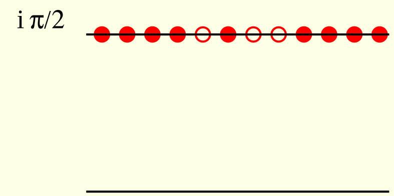

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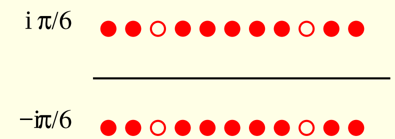

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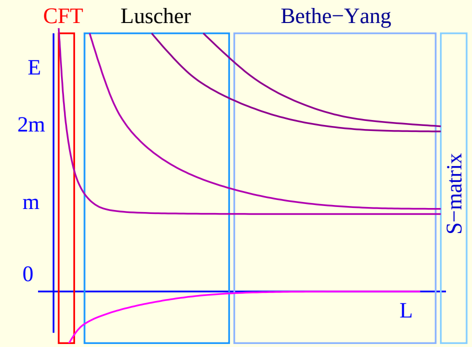
Lüscher corrections: differ by μ term

Excited states TBA, Y-system: diagonal



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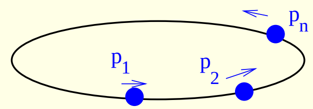

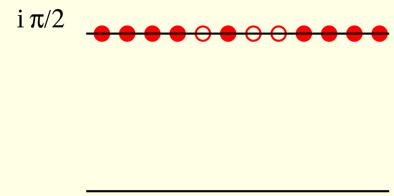

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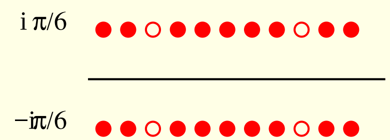

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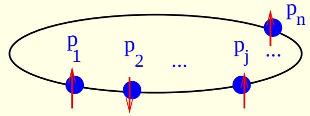
$$S(\theta-\tfrac{i\pi}{3})S(\theta+\tfrac{i\pi}{3})=S(\theta)\to Y(\theta+\tfrac{i\pi}{3})Y(\theta-\tfrac{i\pi}{3})=1+Y(\theta)$$

Y-system+analyticity=TBA $\leftrightarrow$ scalar . Matrix [Bazhanov,Lukyanov,Zamolodchikov]

Excited states TBA, Y-system: Non-diagonal

Excited states exactly



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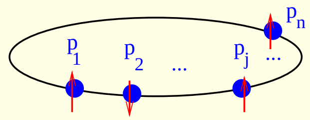
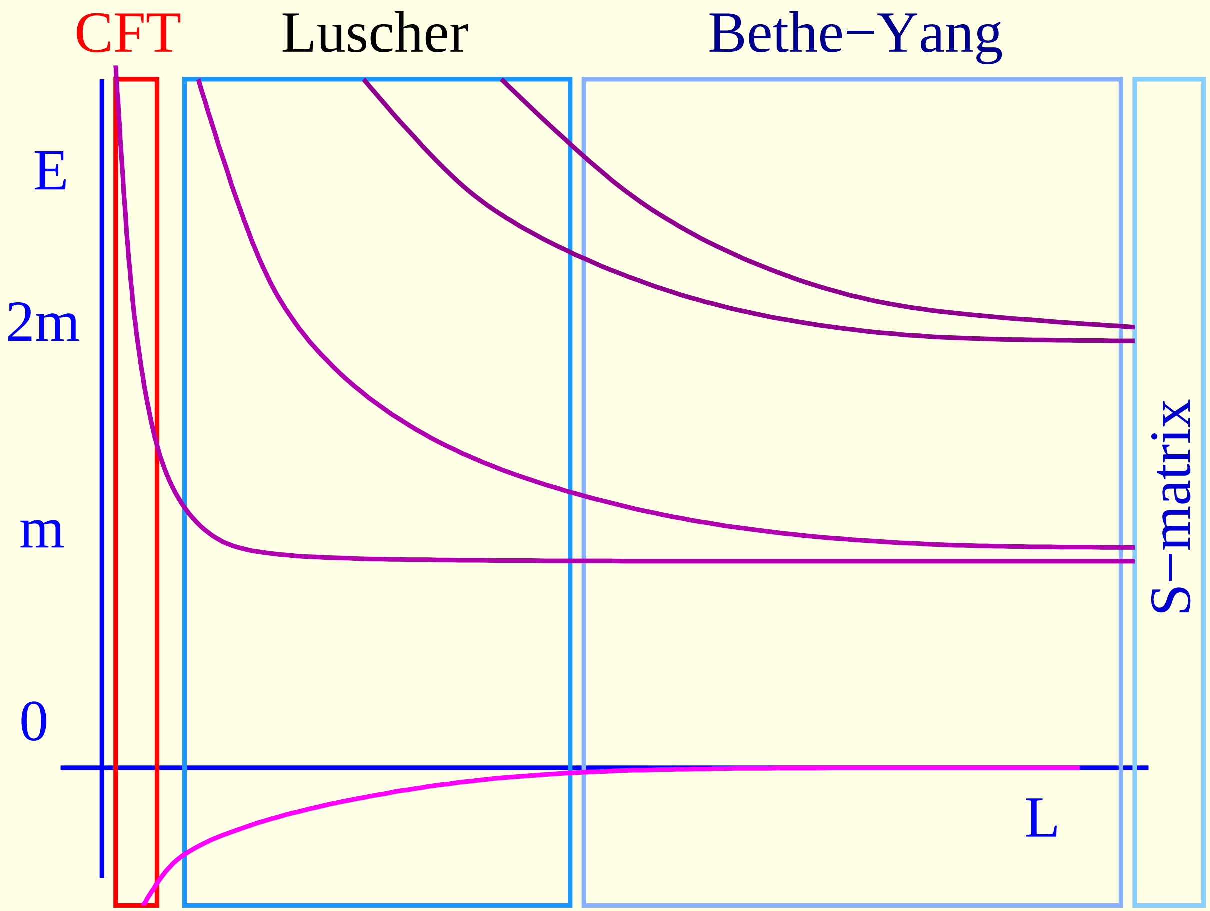
Excited states exactly 



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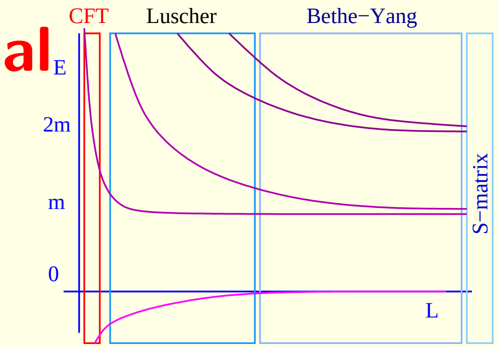

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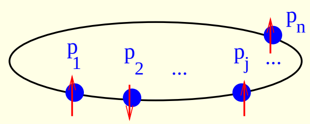

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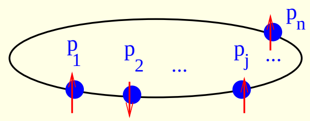

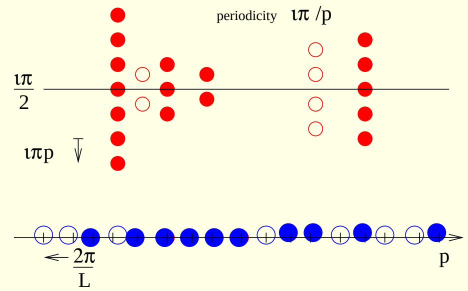

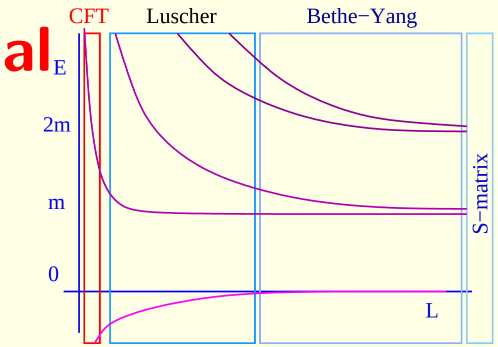

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$$Y_s(\theta+\frac{i\pi p}{2})Y_s(\theta-\frac{i\pi p}{2})=(1+Y_{s-1})(1+Y_{s+1})$$

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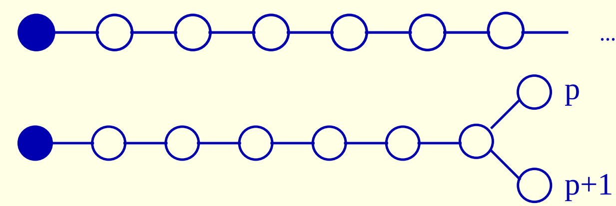

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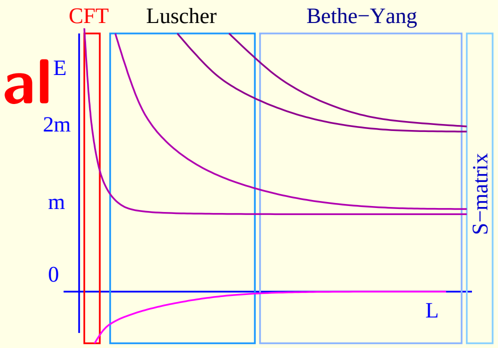

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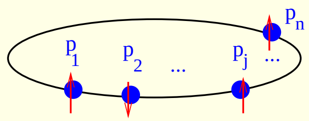

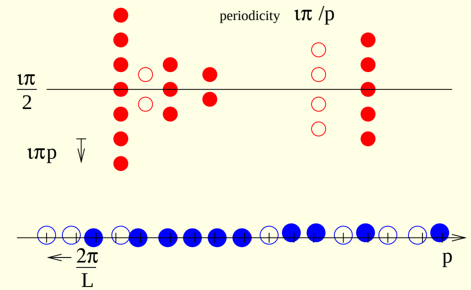

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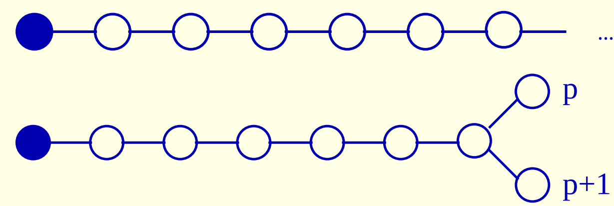

Excited states: analyticity from Lüscher [Balog, Hegedus]

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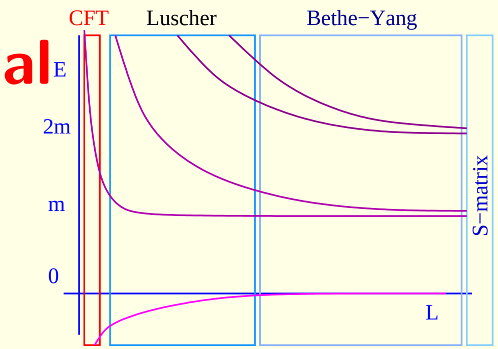

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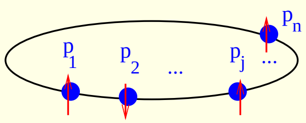

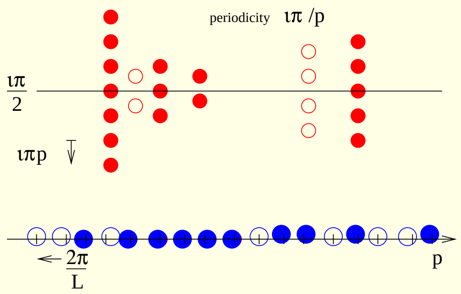

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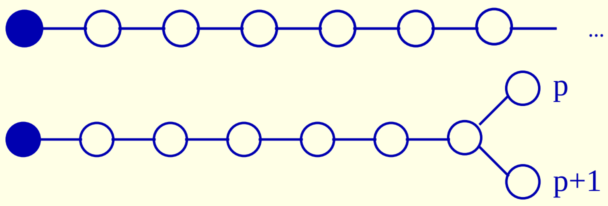

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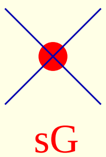
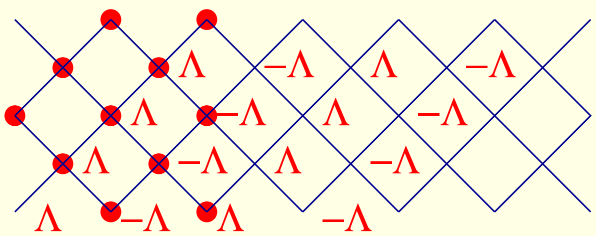
Lattice regularization:

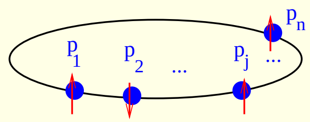
[Destri,de Vega,Ravanini,Fioranvanti,...]

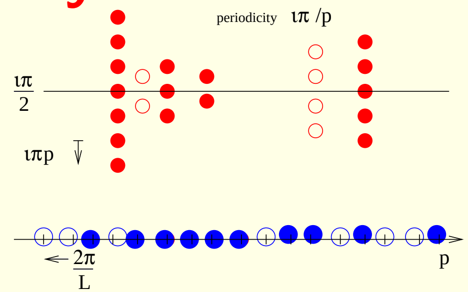


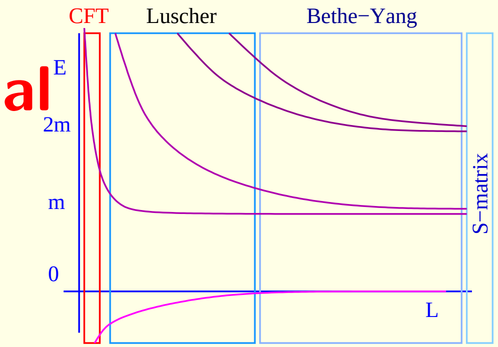


Excited states TBA, Y-system: Non-diagonal

Excited states exactly



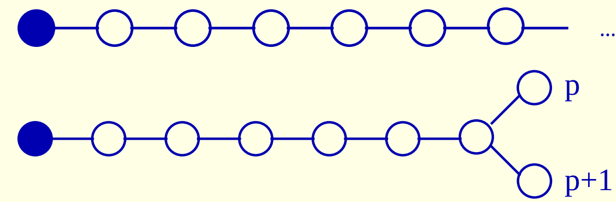




Y-system: sine-Gordon

$$Y_s(\theta+\frac{i\pi p}{2})Y_s(\theta-\frac{i\pi p}{2})=(1+Y_{s-1})(1+Y_{s+1})$$

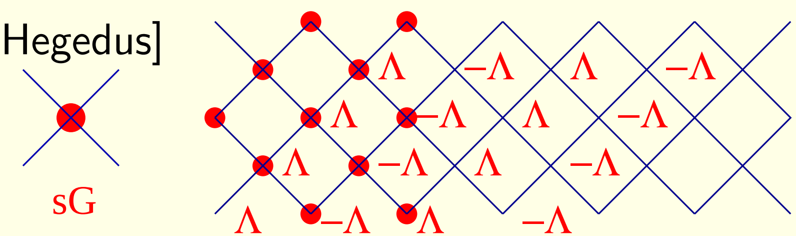
$$(e^{\frac{i\pi p}{2}\partial}+e^{-\frac{i\pi p}{2}\partial})\log Y_s=\sum_r I_{sr}\log(1+Y_r)$$



Excited states: analyticity from Lüscher [Balog, Hegedus]

Lattice regularization:

[Destri,de Vega,Ravanini,Fioranvanti,...]



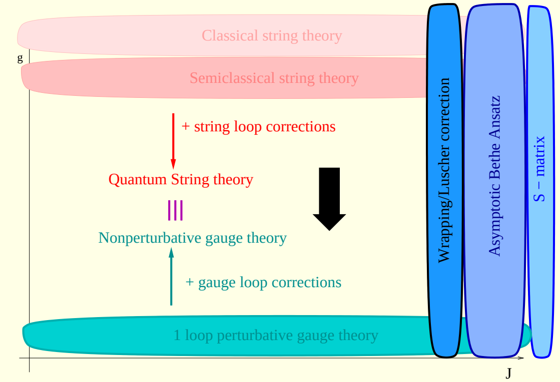
$$Z(\theta)=ML\sinh\theta+\text{source}(\theta|\{\theta_k\})+2\Im m\int dx G(\theta-x-i\epsilon)\log\left[1-(-1)^{\delta}e^{iZ(x+i\epsilon)}\right]$$

$$\text{source}(\theta|\{\theta_k\})=\sum_k \text{sgn}_k(-i)\log S_{++}^{++}(\theta-\theta_k)$$ kernel: $G(\theta)=-i\partial_\theta\log S_{++}^{++}(\theta)$

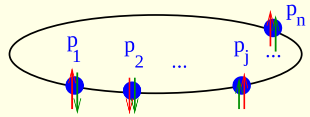
Energy: $E=M\sum_k \text{sgn}_k\cosh\theta_k-2M\Im m\int dx G(\theta+i\epsilon)\log\left[1-(-1)^{\delta}e^{iZ(x+i\epsilon)}\right]$

Bethe-Yang $e^{iZ(\theta_k)}=-1$

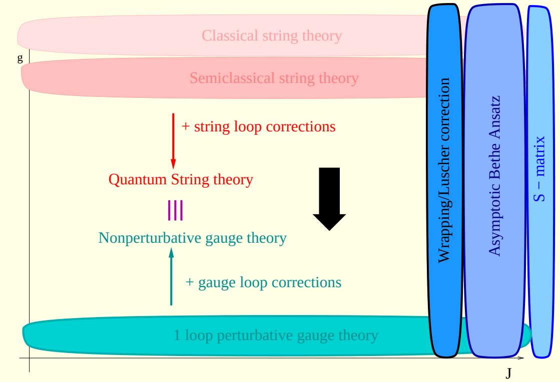
Excited states TBA, Y-system: AdS



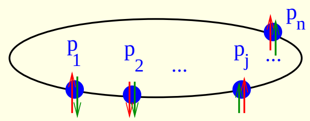
Excited states exactly

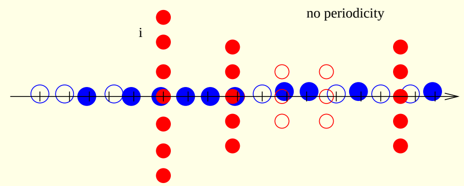


Excited states TBA, Y-system: AdS



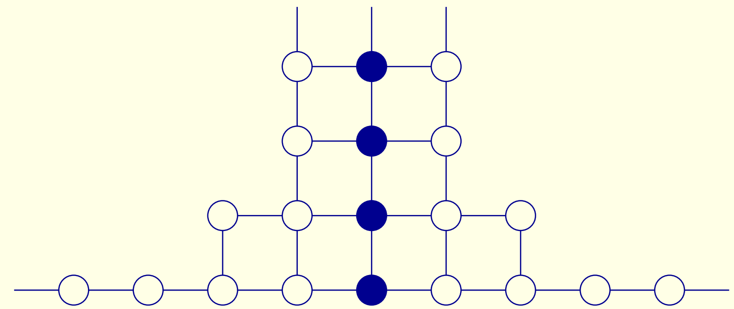
Excited states exactly



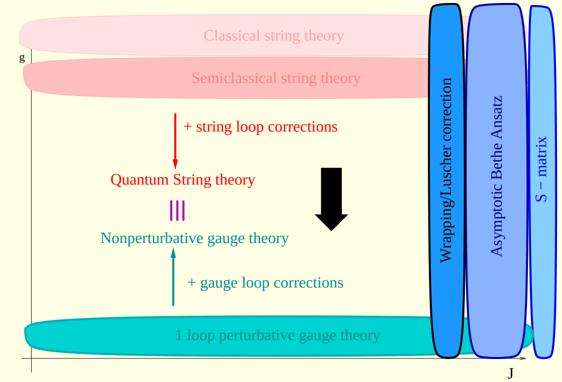


Y-system: AdS [Gromov,Kazakov,Viera]

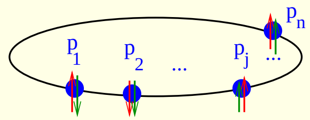
$$\frac{Y_{a,s}(\theta+\frac{i}{2})Y_{a,s}(\theta-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

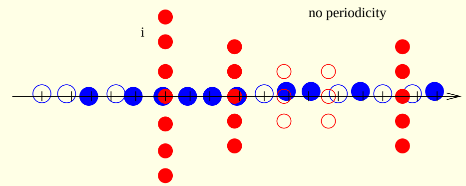


Excited states TBA, Y-system: AdS



Excited states exactly





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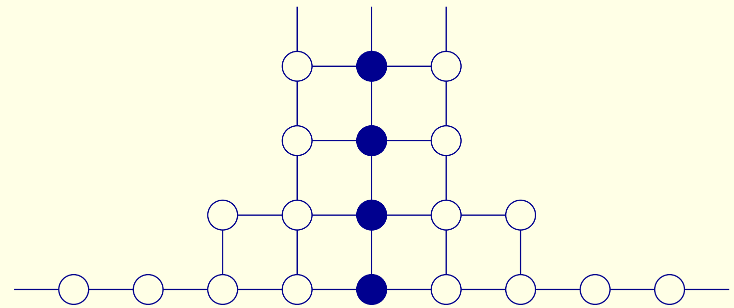


Excited states: analyticity from Lüscher [Gromov,Kazakov]

Assumption on analytical structure $\longrightarrow$excited state TBA [Gromov,Kazakov,Kozak,Viera]

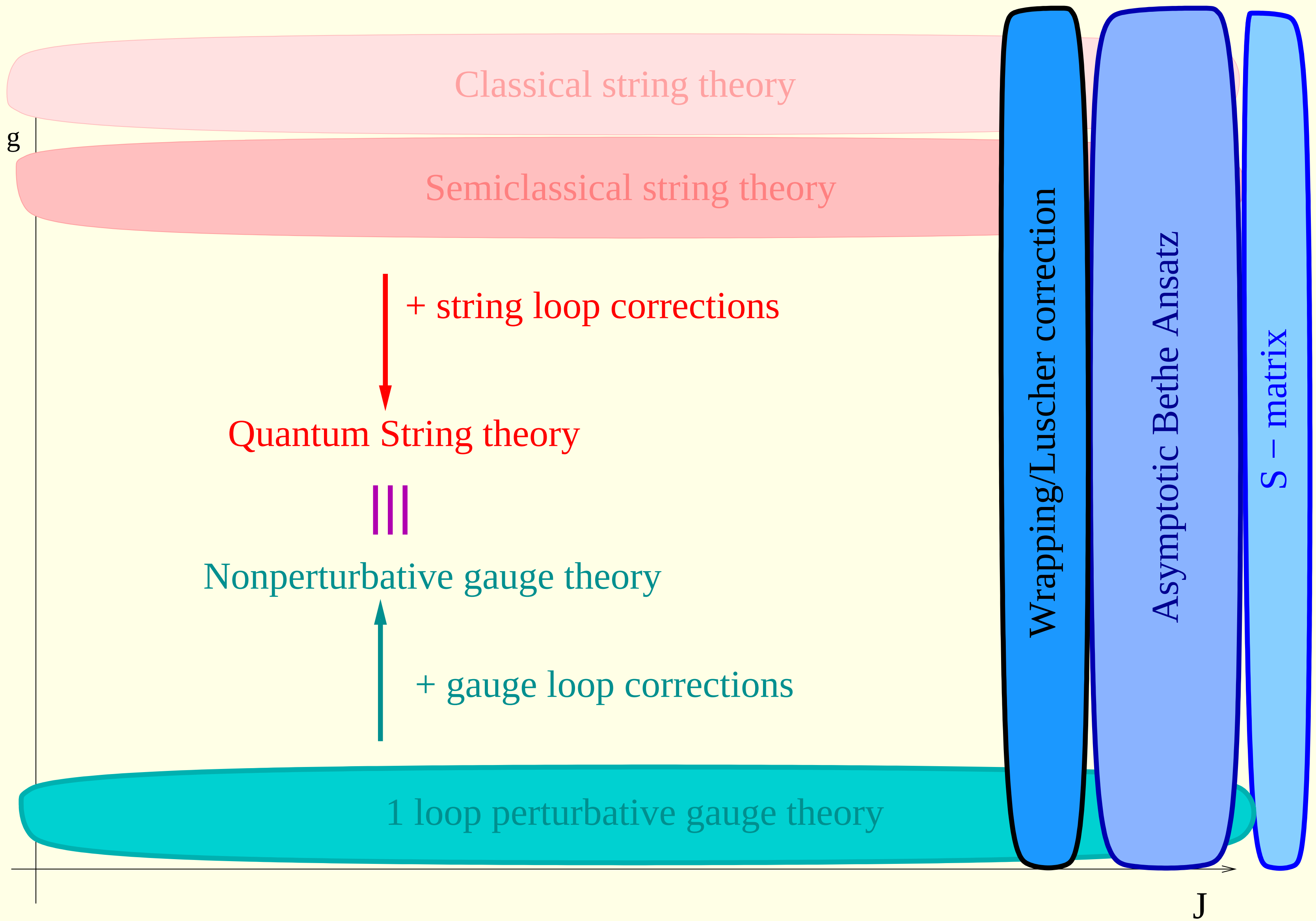
NO μ terms!

NEEDs ANALYTICAL CHECKS: 5 loop Konishi

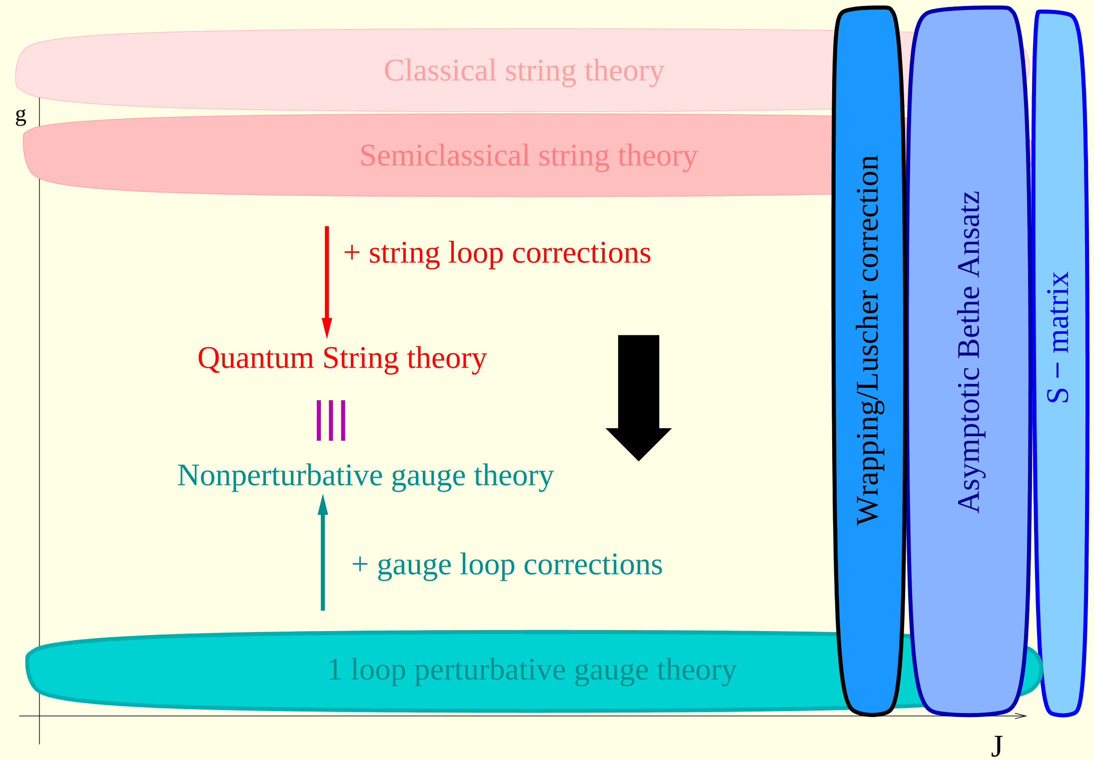
Cannot be the final answer $\longrightarrow$ Lattice regularization: ?

Conclusion

Conclusion


Classical string theory
g
Semiclassical string theory
+ string loop corrections
Quantum String theory
|||
Nonperturbative gauge theory
+ gauge loop corrections
1 loop perturbative gauge theory
J
Wrapping/Luscher correction
Asymptotic Bethe Ansatz
S – matrix

Conclusion



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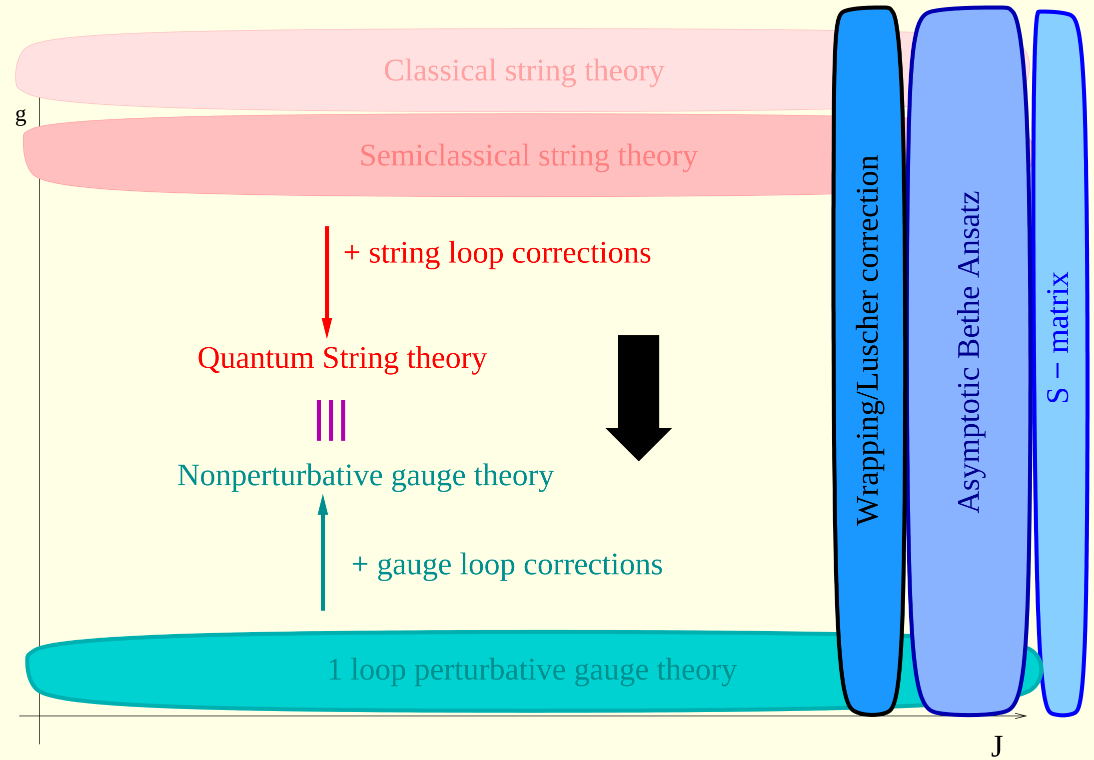
Conclusion

$\boxed{\text{S-matrix} = \text{scalar . Matrix}}$

physical sheet, explanation of all the poles



Conclusion

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physical sheet, explanation of all the poles

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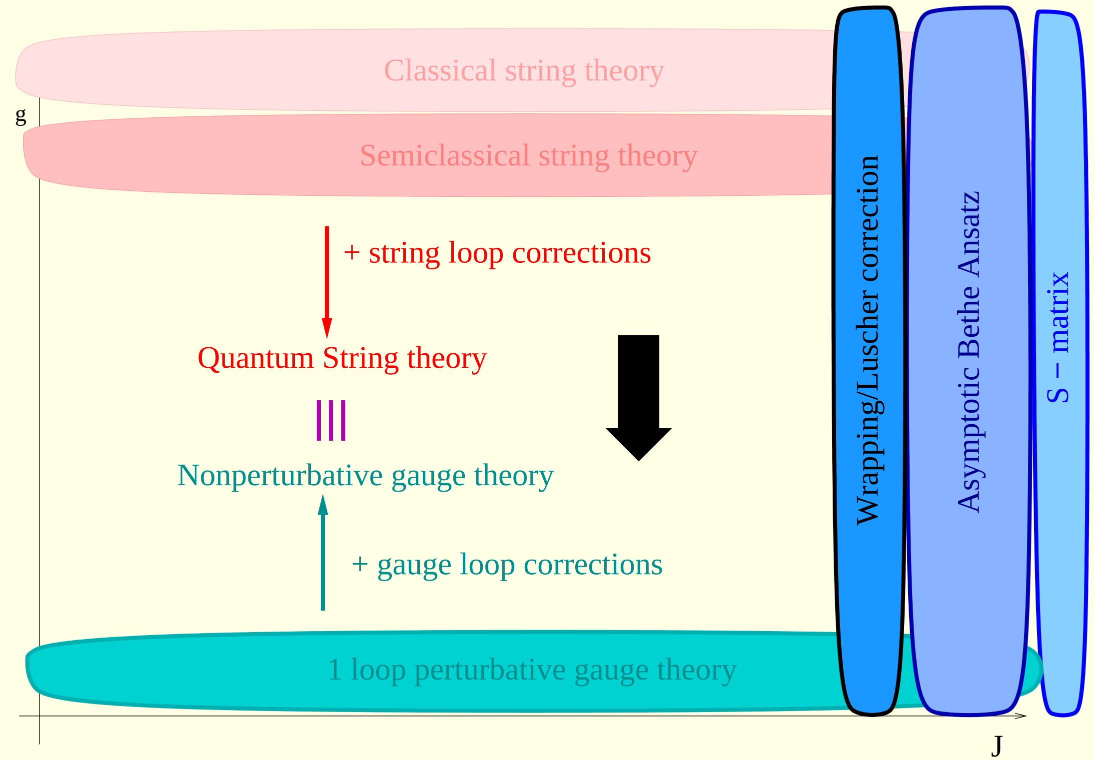
Analytical structure of all excited states



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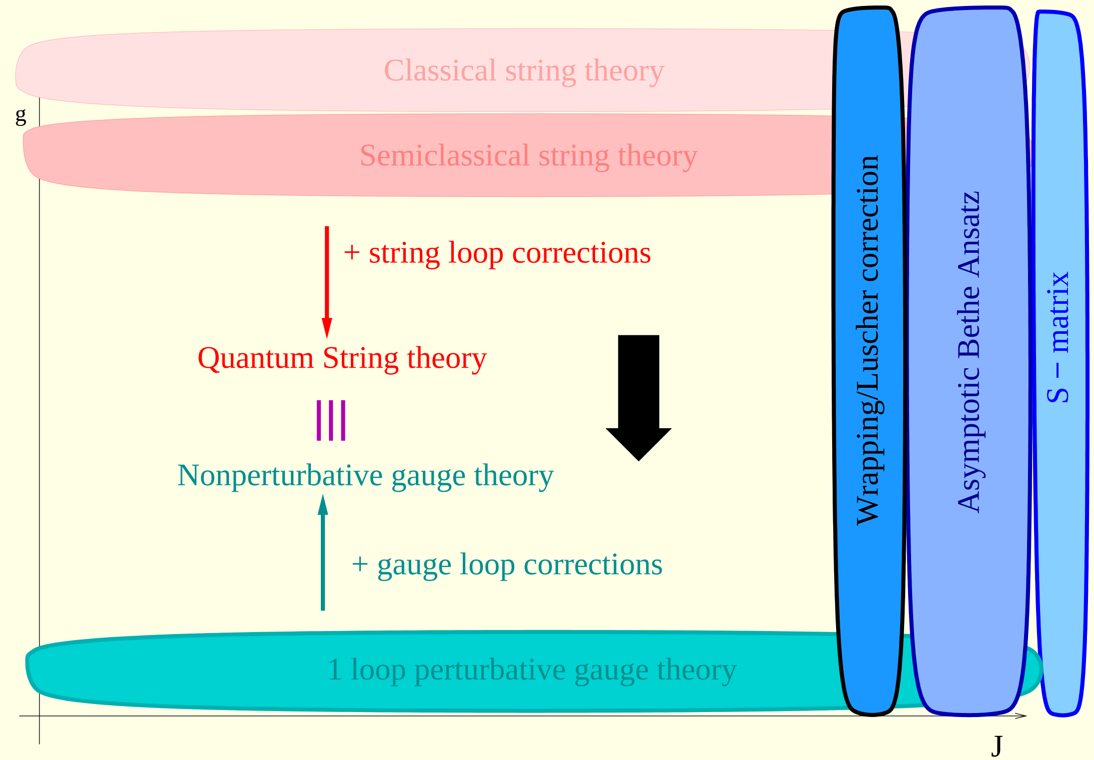
Analytical structure of all excited states

lattice?



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