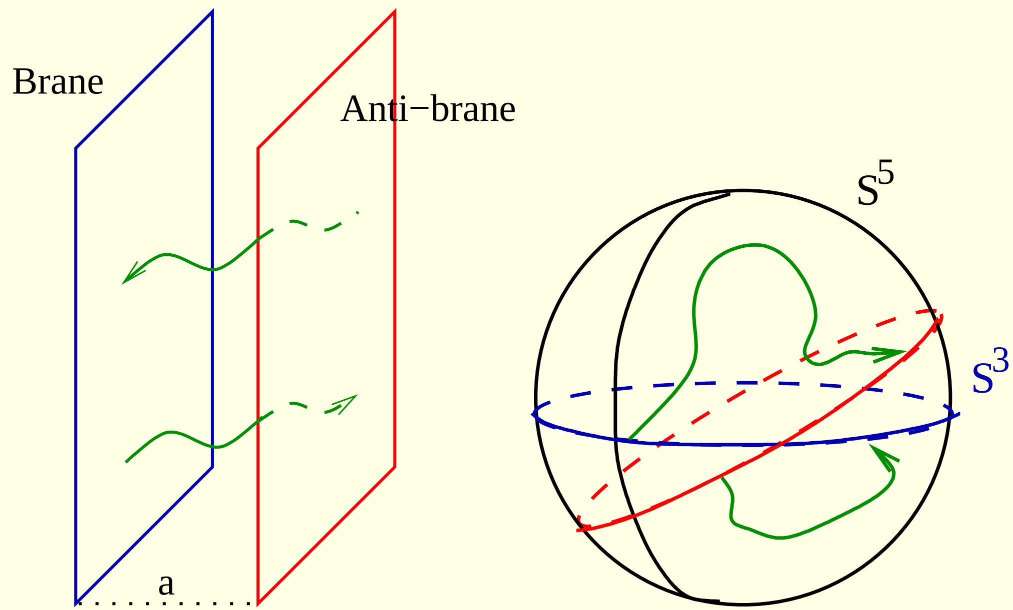
Integrability in Gauge and String Theory Utrecht 19 - 23 August 2013

Brane–anti-brane system from an integrable point of view

Z. Bajnok

Holographic QFT Group, Wigner Research Centre for Physics

Hungarian Academy of Sciences, Budapest



$$\mathcal{O}_{Y\bar{Y}}^{Z^J Z^{J'}} = \epsilon_{i_1\dots i_N}^{j_1\dots j_N} Y_{j_1}^{i_1}\dots Y_{j_{N-1}}^{i_{N-1}} (Z^{J'})_{l_N}^{j_N}\ \epsilon_{k_1\dots k_N}^{l_1\dots l_N} \bar{Y}_{l_1}^{k_1}\dots \bar{Y}_{l_{N-1}}^{k_{N-1}} (Z^J)_{i_N}^{k_N}$$

work done in collaboration with:

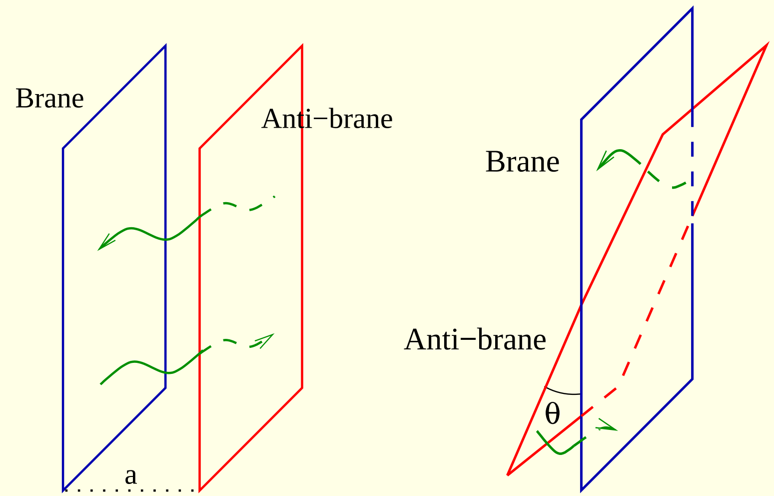
N. Drukker, A. Hegedus, R. Nepomechie, L .Palla, C. Sieg, R. Suzuki

Motivation: Brane–anti-brane system in flat space

Brane–anti-brane system in flat space:

open strings are oriented we need 2

spectrum massive for $a > a_{crit}$



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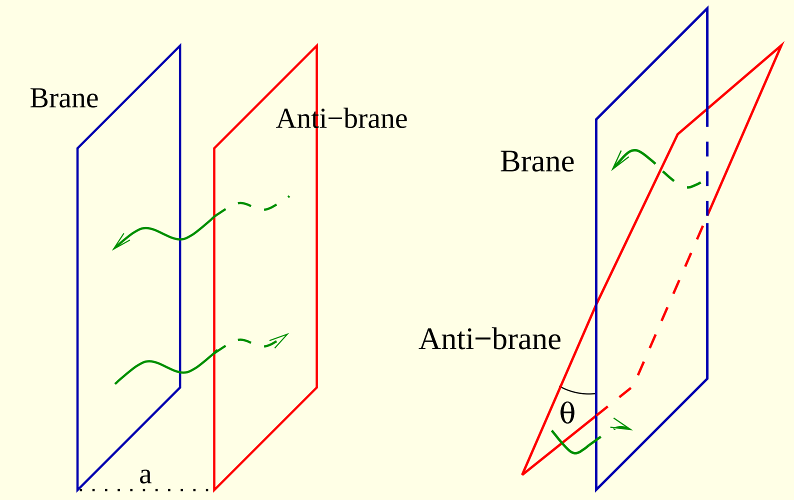

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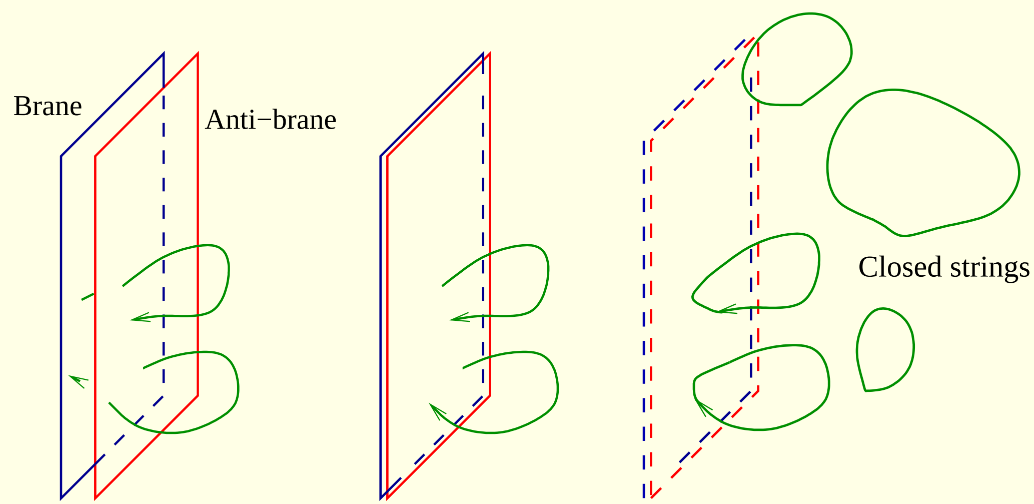
spectrum massive for $a > a_{crit}$

$a = 0$ spectrum contains tachyon

with $m^2 = -\frac{1}{2\alpha'}$

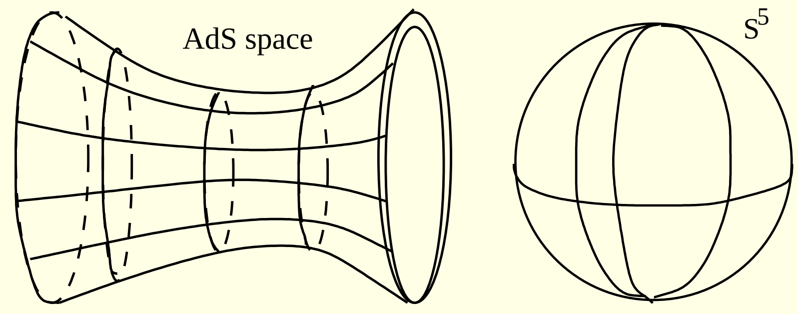
tachyons condensate, branes decay [A. Sen]



Aim: test brane–anti-brane system in curved space using

Motivation: Brane–anti-brane system in curved space

Brane–anti-brane system in AdS dualities:



The most understood AdS duality: $AdS_5 \times S^5$

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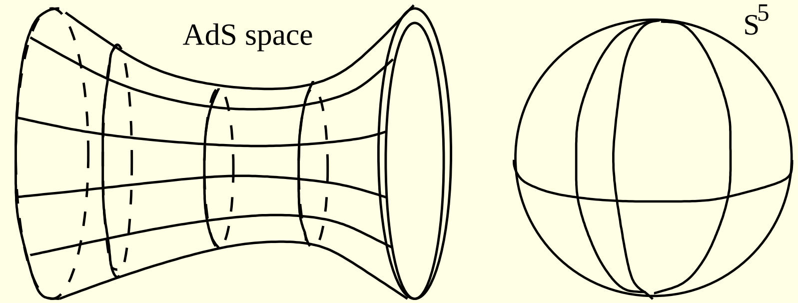

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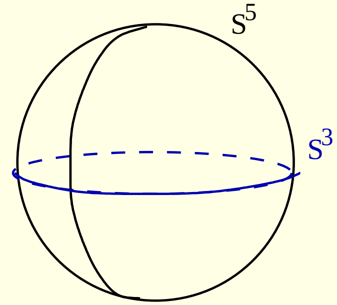
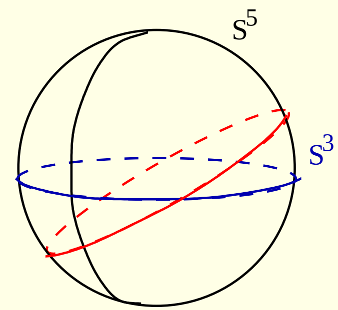
D3 brane wraps $S^3 \subset S^5$

$S^5 = \{|X|^2 + |Y|^2 + |Z|^2 = 1\}$

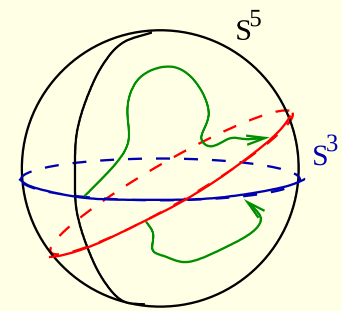
$S^3 = \{Y = 0\}$



2 D3 branes at angles



spectrum of open strings



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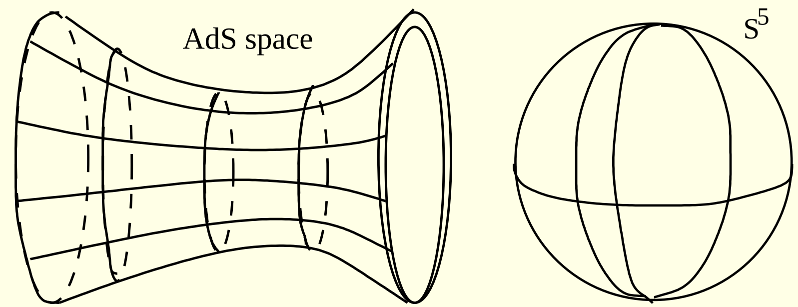

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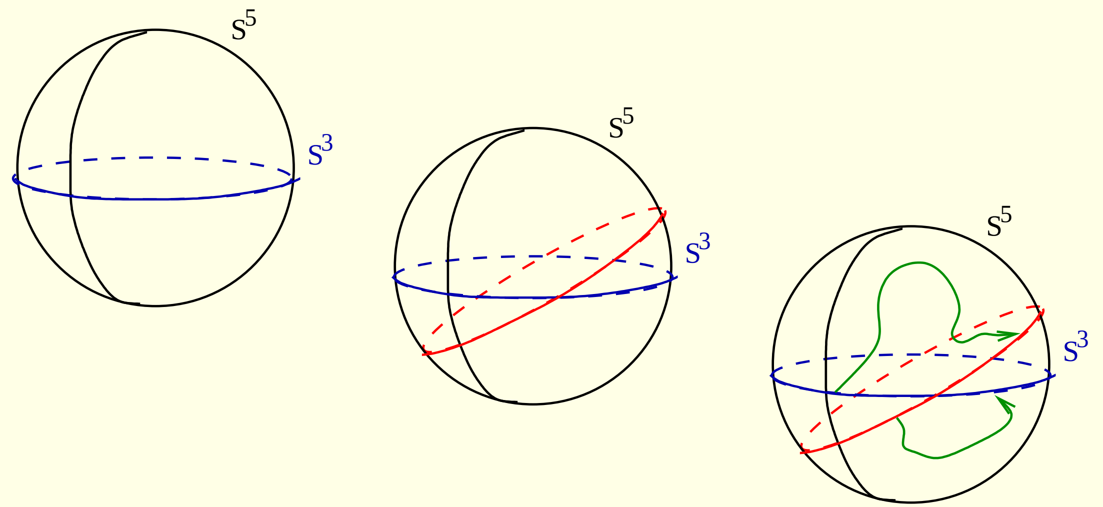

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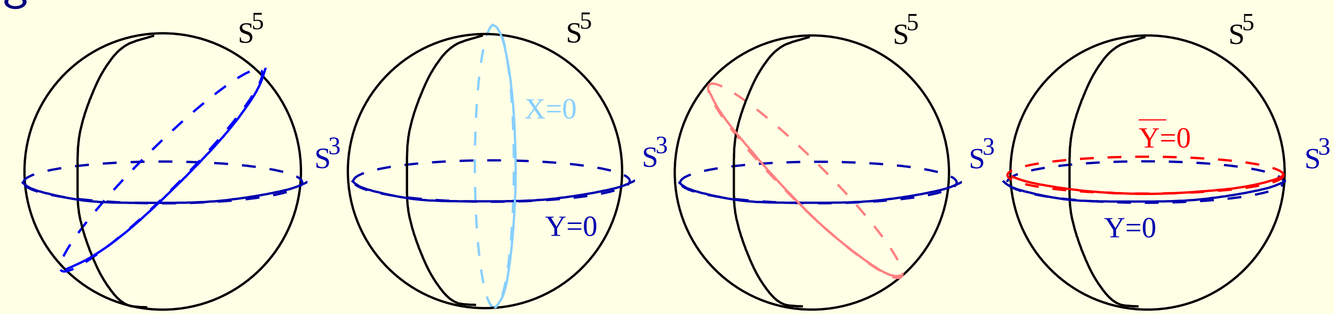
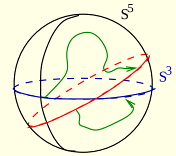
ideal would be to change the angle:

Dbrane-anti-Dbrane: tachyon?



Plan of talk

Open strings connecting brane–anti-brane 

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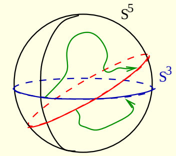
Open strings connecting brane–anti-brane 

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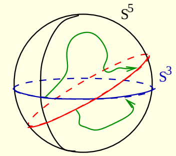
Open strings connecting brane–anti-brane 

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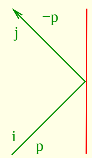
Integrable description: R-matrix 

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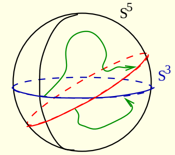
Open strings connecting brane–anti-brane 

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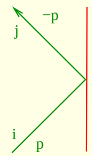
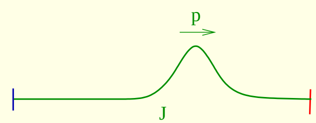
Integrable description: R-matrix 

Finite size spectrum: asymptotic BA: 

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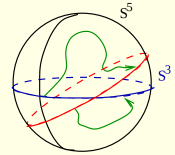
Open strings connecting brane–anti-brane 

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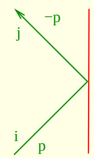
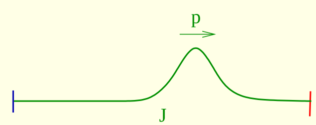
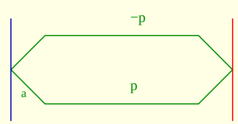
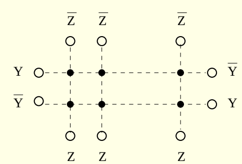
Integrable description: R-matrix 

Finite size spectrum: asymptotic BA: 

Wrapping corrections: 



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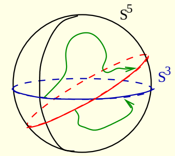
Open strings connecting brane–anti-brane 

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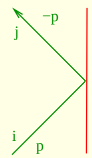
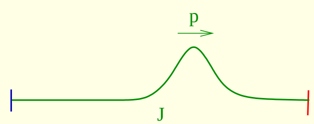
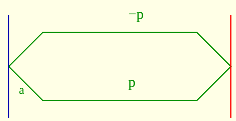
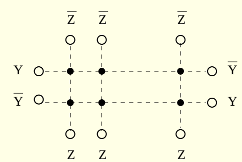
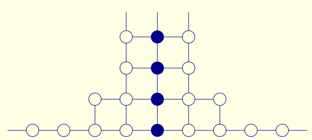
Integrable description: R-matrix 

Finite size spectrum: asymptotic BA: 

Wrapping corrections: 



Y-system $\to$ BTBA 

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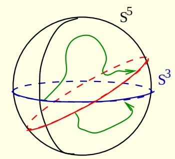
Open strings connecting brane–anti-brane 

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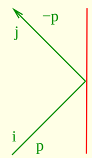
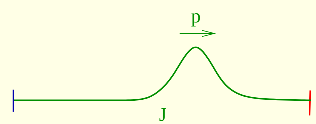
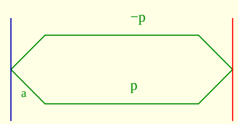
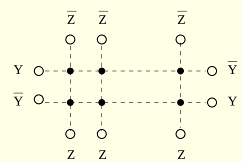
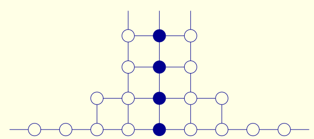
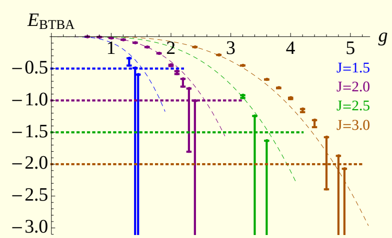
Integrable description: R-matrix 

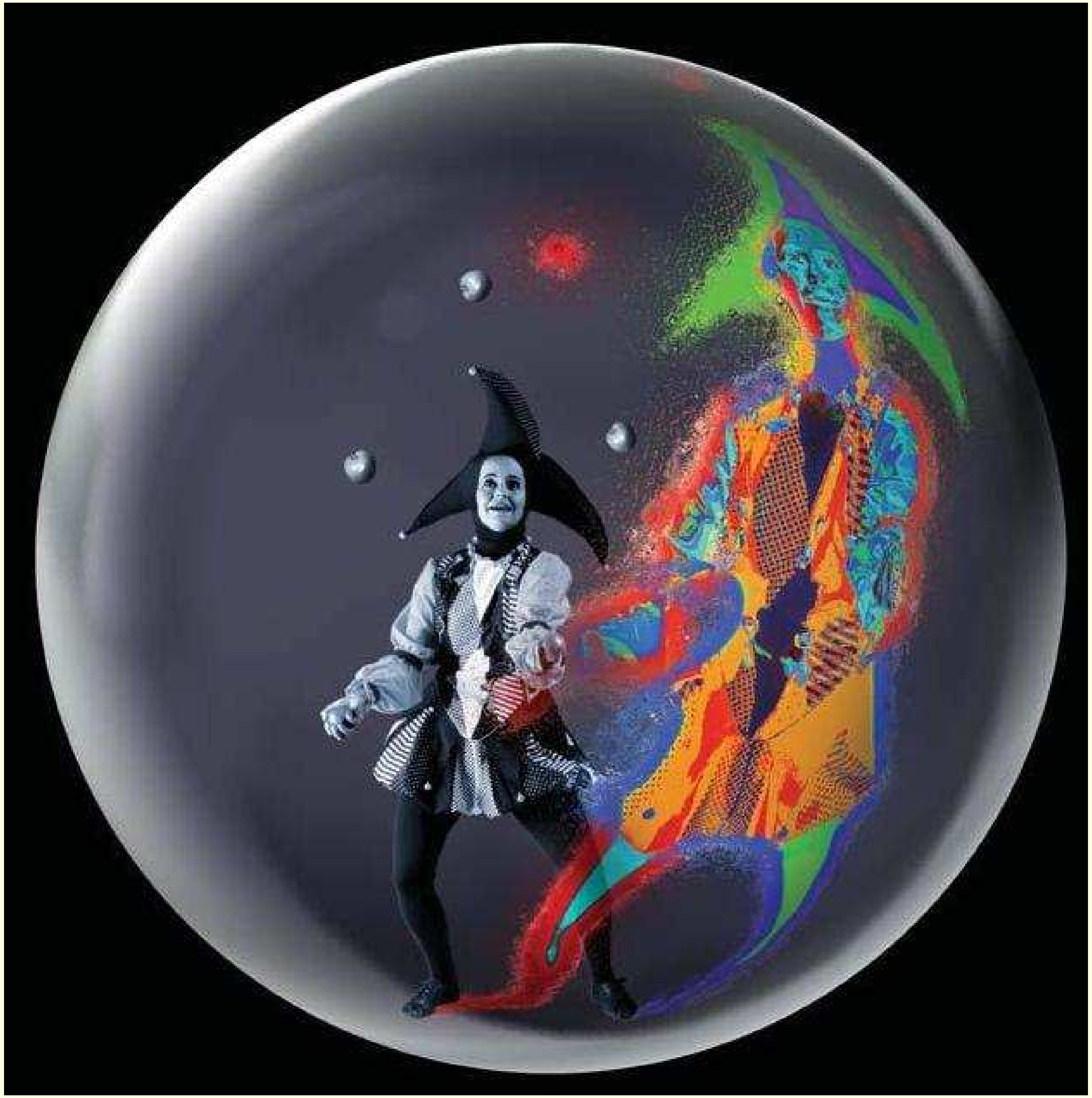
Finite size spectrum: asymptotic BA: 

Wrapping corrections: 



Y-system → BTBA 

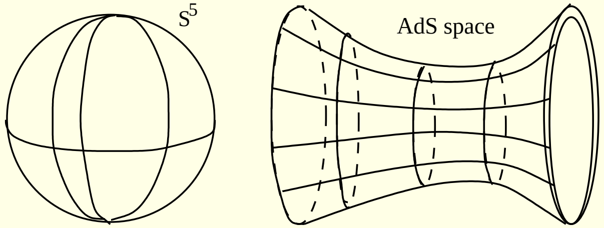
Numerical solution, results: 



The Illusion of Gravity - Juan Maldacena, Scientific American (2005)

AdS/CFT correspondence (Maldacena 1998)

II_B superstring on $AdS_5 \times S^5$



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$\frac{R^2}{\alpha'} \int \frac{d\tau d\sigma}{4\pi} \left(\partial_a Y^M \partial^a Y_M + \partial_a X^M \partial^a X_M \right) + \ldots$

$\equiv$

$\mathcal{N} = 4$ D=4 $SU(N)$ SYM

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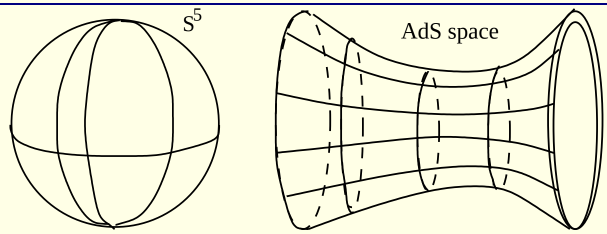

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2D QFT

String energy levels: $E(\lambda)$

$E(\lambda) = E(\infty) + \frac{E_1}{\sqrt{\lambda}} + \frac{E_2}{\lambda} + \ldots$

strong$\leftrightarrow$weak

$\Downarrow$

$\lambda = g_{YM}^2 N$, $N \to \infty$ planar limit

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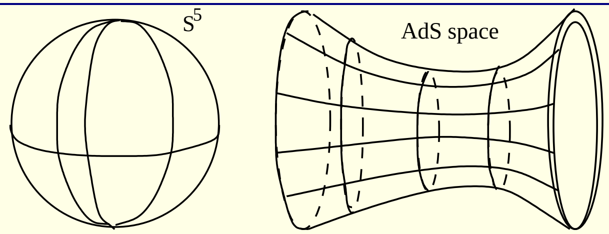

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2D integrable QFT

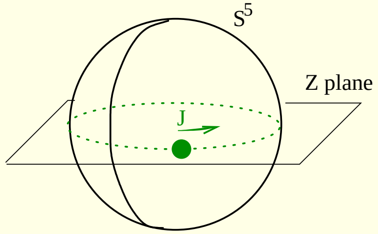
spectrum: $Q = 1, 2, \dots, \infty$, $(\alpha, \dot{\alpha})$ dispersion: $\epsilon_Q(p) = \sqrt{Q^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{p}{2}}$

Exact scattering/reflection matrix: $S_{Q_1 Q_2}(p_1, p_2, \lambda)$, $R_Q(p, \lambda)$

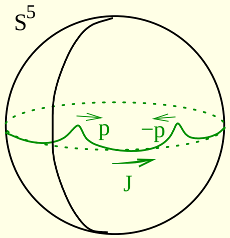
Finite size correction: Lüscher, TBA

AdS/CFT correspondence: closed strings

BPS string configuration:BMN



$$E_{BPS}(\lambda) = J$$



classical energy+loop corrections

$\leftrightarrow$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

supersymmetric BPS operators

$$Z = \Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4$$

$$X = \Phi_1 + i\Phi_2$$

$$\mathcal{O}_{BPS} = \text{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\ldots\uparrow\rangle$$

$$\Delta_{BPS} = J$$

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \text{Tr}(ZYZY + ...) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$$

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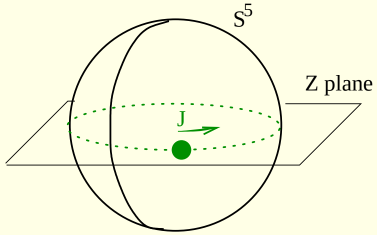
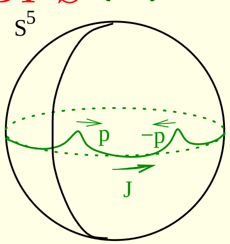


$$E_{BPS}(\lambda) = J$$



classical energy+loop corrections

$\leftrightarrow$

$$V(\Phi,\Psi) = \frac{1}{4}[\Phi,\Phi]^2 + \overline{\Psi}[\Phi,\Psi]$$

supersymmetric BPS operators

$$Z = \Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4$$

$$X = \Phi_1 + i\Phi_2$$

$$\mathcal{O}_{BPS} = \mathrm{Tr}(Z^J) \leftrightarrow |\uparrow\uparrow\dots\uparrow\rangle$$

$$\Delta_{BPS} = J$$

nonsupersymmetric operator: Konishi

$$\mathcal{O}_K = \mathrm{Tr}(ZYZY + ...) \leftrightarrow |\uparrow\downarrow\uparrow\downarrow\rangle + .$$

2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

two particle state

$$E = E_{BPS} + E_{BA} + E_{FSC}$$

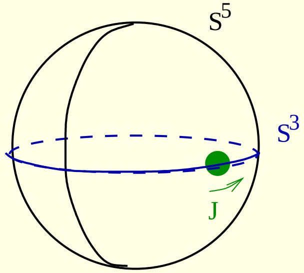
Bethe Ansatz: $e^{ipJ} S(p,-p) = 1$

$$E_{BA} = 2E(p,\lambda) = 2\sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

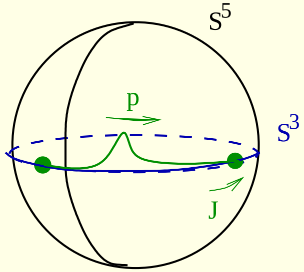
$$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} S_{Q1}(q,p) S_{Q1}(q,-p) e^{-\epsilon_Q L} + \dots = E_{TBA}$$

AdS/CFT correspondence: open strings, single brane

BPS string configuration



$$E_{BPS}(\lambda) = J$$



cl. energy+loop corrections

$\leftrightarrow$

Brane configurations

$$Z = \Phi_5 + i\Phi_6, Y = \Phi_3 + i\Phi_4, X = \Phi_1 + i\Phi_2$$

Y=0 brane

$$\mathcal{O}_Y = \det(Y) = \epsilon^{j_1 \cdots j_N}_{i_1 \ldots i_N} Y^{i_1}_{j_1} \ldots Y^{i_N}_{j_N}$$

BMN string

$$\mathcal{O}_Y^{Z^J} = \epsilon^{j_1 \cdots j_N}_{i_1 \ldots i_N} Y^{i_1}_{j_1} \ldots Y^{i_{N-1}}_{j_{N-1}} (Z^J)^{j_N}_{i_N}$$

$$\Delta_{BPS} = J$$

nonsupersymmetric operator: Konishi type

$$\mathcal{O}_Y^{Z^{J-1}X} = \epsilon^{j_1 \cdots j_N}_{i_1 \ldots i_N} Y^{i_1}_{j_1} \ldots Y^{i_{N-1}}_{j_{N-1}} (Z^{J-1}X)^{j_N}_{i_N}$$

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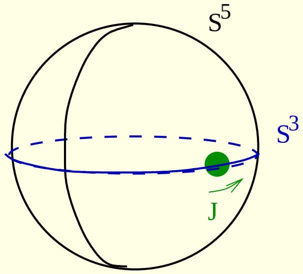

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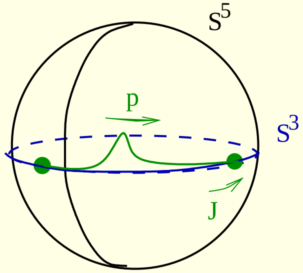

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2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

1 particle state: $E = E_{BPS} + E_{BA} + E_{FSC}$

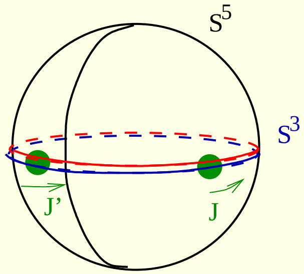
Bethe Ansatz: $e^{2ipJ} R_Y(p) R_Y(p) = 1$

$$E_{BA} = E(p,\lambda) = \sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$

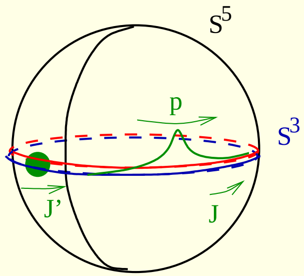
$$E_{FSC} = \sum_Q \int \frac{dq}{2\pi} R_Q(q) R_Q(\bar{q}) e^{-2\epsilon_Q L} + \dots = E_{TBA}$$

AdS/CFT correspondence: open strings, brane–anti-brane

BPS string configuration



$$E_{BPS}(\lambda) = J + J'$$



cl. energy+loop corrections

$\leftrightarrow$

Brane configurations

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$Y - \bar{Y}$ brane

$$\mathcal{O}_{Y\bar{Y}} = \det(Y) = \epsilon^{j_1 \dots j_N}_{i_1 \dots i_N} Y^{i_1}_{j_1} \dots Y^{i_N}_{j_N}$$

2 BMN strings

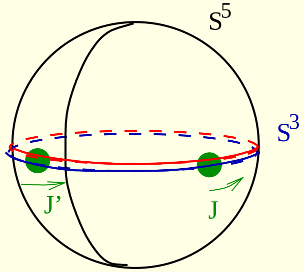
$$\mathcal{O}^{Z^J Z^{J'}}_{Y\bar{Y}} = \epsilon^{j_1 \dots j_N}_{i_1 \dots i_N} Y^{i_1}_{j_1} \dots Y^{i_{N-1}}_{j_{N-1}} (Z^{J'})^{i_N}_{l_N}$$
$$\epsilon^{l_1 \dots l_N}_{k_1 \dots k_N} \bar{Y}^{k_1}_{l_1} \dots \bar{Y}^{k_{N-1}}_{l_{N-1}} (Z^J)^{k_N}_{j_N}$$
$$\Delta_{BPS} = J + J'$$

nonsupersymmetric operator: Konishi type

$$\mathcal{O}^{\mathcal{V}\mathcal{W}}_{Y\bar{Y}} \quad ; \quad \mathcal{W} = Z^{J-1}X \quad ; \quad \mathcal{V} = Z^{J'}$$

AdS/CFT correspondence: open strings, brane–anti-brane

BPS string configuration



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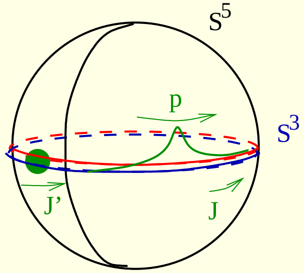

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$$\epsilon^{l_1\ldots l_N}_{k_1\ldots k_N} \bar{Y}^{k_1}_{l_1} \ldots \bar{Y}^{k_{N-1}}_{l_{N-1}} (Z^J)^{k_N}_{j_N}$$

$$\Delta_{BPS} = J + J'$$

nonsupersymmetric operator: Konishi type

$$\mathcal{O}^{\mathcal{V}\mathcal{W}}_{Y\bar{Y}} \quad ; \quad \mathcal{W} = Z^{J-1}X \quad ; \quad \mathcal{V} = Z^{J'}$$

2D integrable QFT

supersymmetric groundstate $E_0(J) = \Delta(\lambda) - J = 0$

$$E = E_{BPS} + E_{BA} + E_{FSC}$$

Bethe Ansatz: $e^{i2pJ} R_Y(p) R_{\bar{Y}}(p) = 1$

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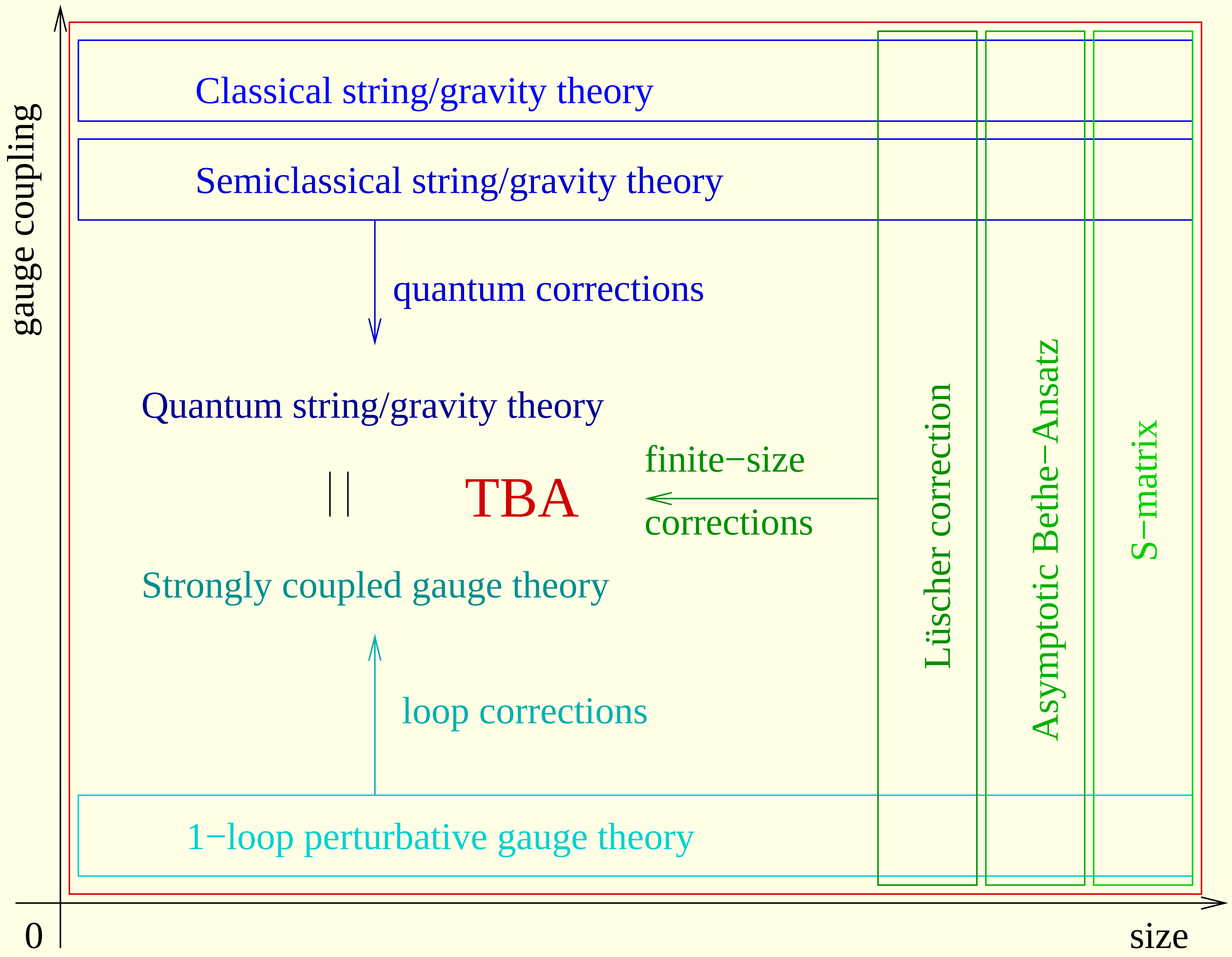
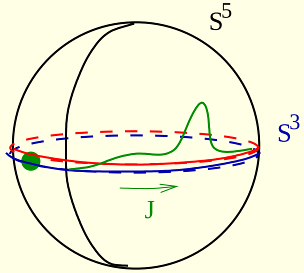
Spectral problem

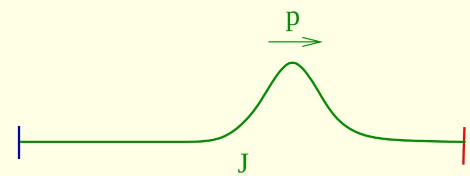


2D $\equiv$ Integrable point of view

2D $\equiv$ Integrable point of view



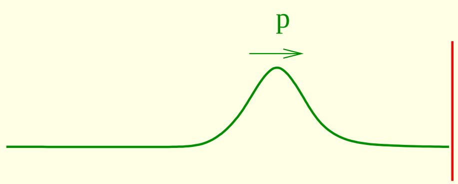
From the string excitation point of view



take $J \to \infty$

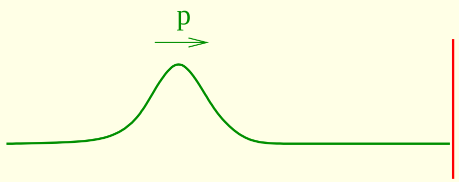
2D $\equiv$ Integrable point of view

Boundary one particle state:



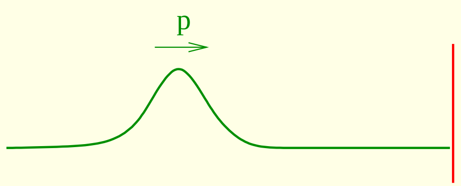
2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$

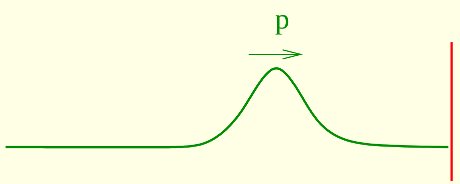


2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$

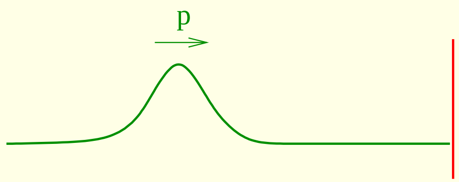


times develop

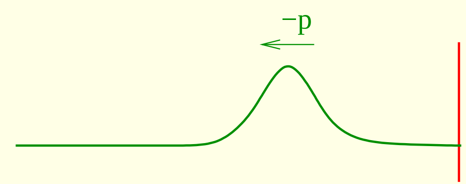


2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$

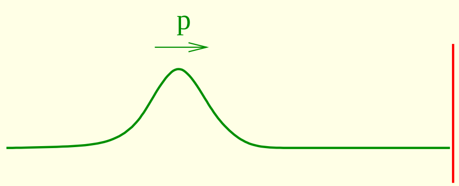


times develop further

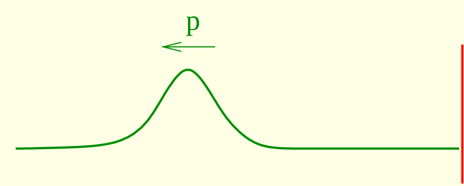


2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$

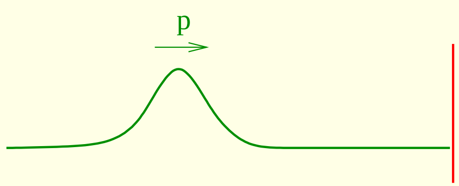


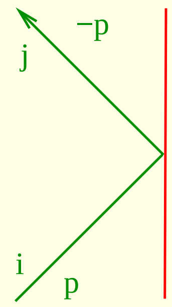
Boundary one pt out state: $t \to \infty$



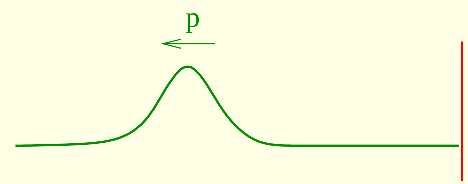
$2D \equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$



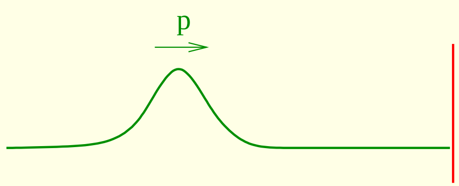


Boundary one pt out state: $t \to \infty$

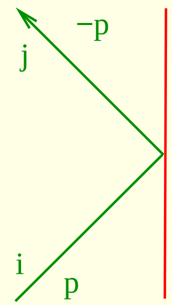


2D $\equiv$ Integrable point of view

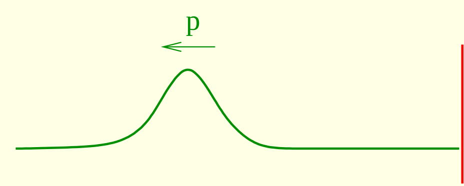
Boundary one particle in state: $t \to -\infty$



Free in particle



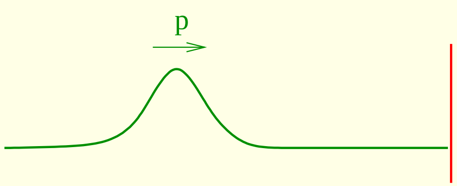
Boundary one pt out state: $t \to \infty$

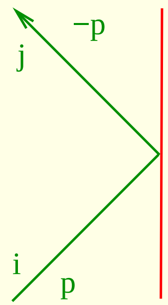


Free out particle

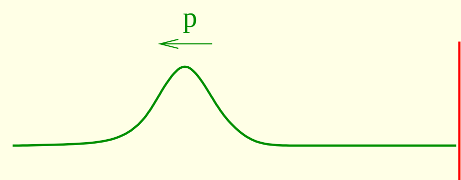
2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$





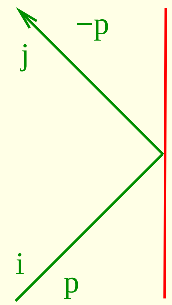
Boundary one pt out state: $t \to \infty$



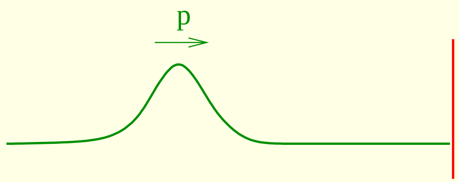
Free in particle $\longleftarrow$ **R-matrix** $\longrightarrow$ Free out particle

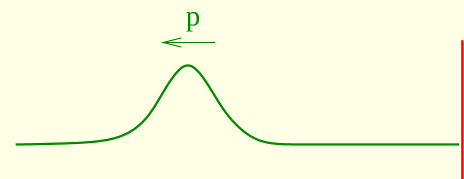
2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$



Boundary one pt out state: $t \to \infty$



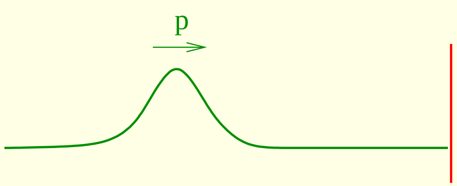


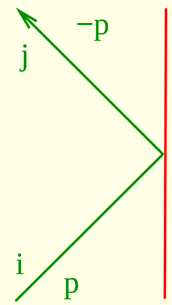
Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

$$|p\rangle_i \quad = \quad R_i^j(p)|-p\rangle_j$$

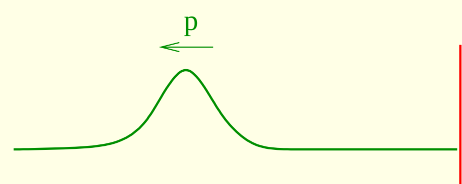
2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$





Boundary one pt out state: $t \to \infty$



Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

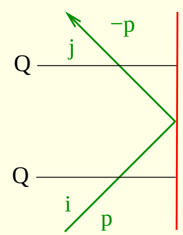
$$|p\rangle_i \quad = \quad R_i^j(p)|-p\rangle_j$$

R-matrix=scalar.matrix

matrix

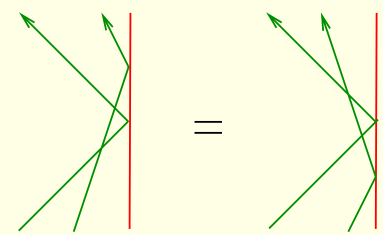
Conserved charges

$$[R(p), Q] = 0$$



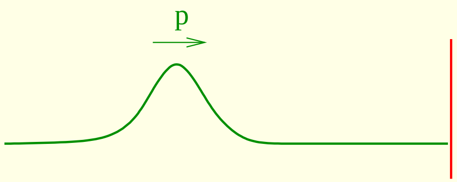
Yang-Baxter equation

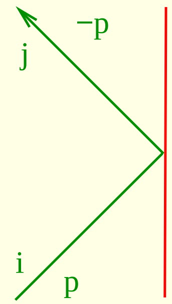
$$S_{12}R_1S_{2\bar{1}}R_2 = R_2S_{1\bar{2}}R_1S_{\bar{2}\bar{1}}$$



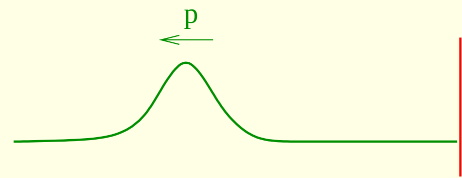
2D $\equiv$ Integrable point of view

Boundary one particle in state: $t \to -\infty$





Boundary one pt out state: $t \to \infty$



Free in particle $\leftarrow$ **R-matrix** $\rightarrow$ Free out particle

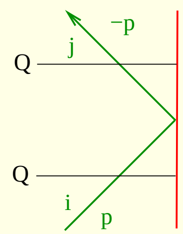
$$|p\rangle_i \quad = \quad R_i^j(p)|-p\rangle_j$$

R-matrix=scalar.matrix

matrix

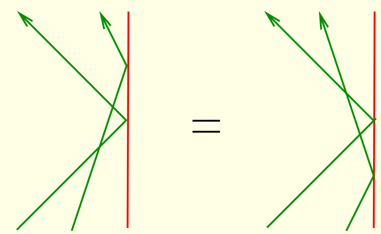
Conserved charges

$$[R(p), Q] = 0$$



Yang-Baxter equation

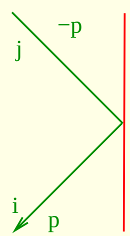
$$S_{12}R_1S_{2\bar{1}}R_2 = R_2S_{1\bar{2}}R_1S_{\bar{2}\bar{1}}$$



scalar

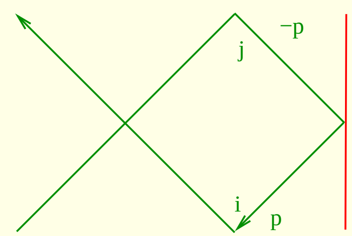
Unitarity

$$R(-p) = R^{-1}(p)$$



crossing

$$R(p) = S(p, -\bar{p})R(-\bar{p})$$

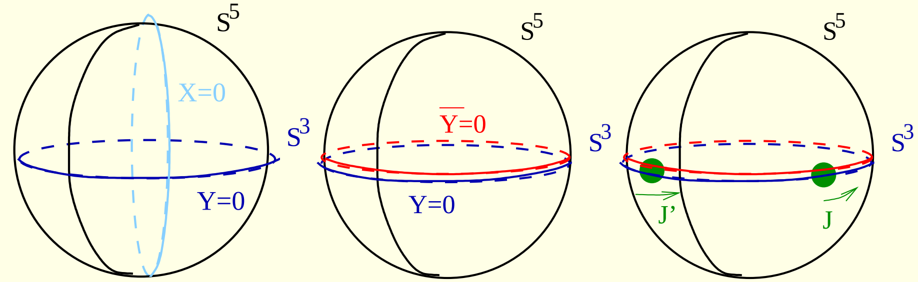


Y=0 brane

parametrization:$X = (1, \dot{1})$,$Y = (1, \dot{2})$, $\bar{Y} = (2, \dot{1})$

BMN Tr(Z^J): $PSU(2,2|4) \longrightarrow PSU(2|2)^2$

Brane $PSU(2|2)^2 \to SU(1|2)^2$



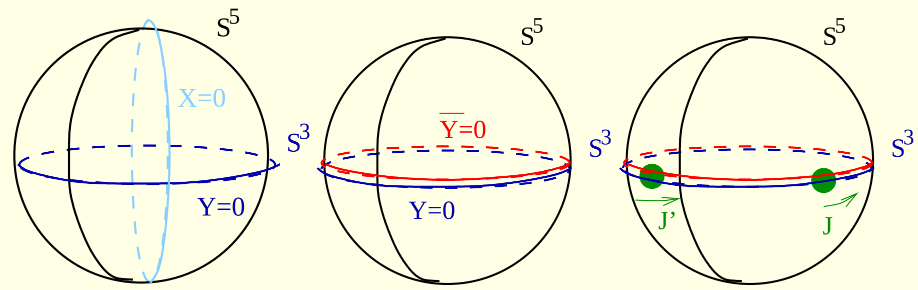
Y=0 brane

parametrization:$X = (1, \dot{1}), Y = (1, \dot{2}), \bar{Y} = (2, \dot{1})$

BMN Tr(Z^J): $PSU(2,2|4) \longrightarrow PSU(2|2)^2$

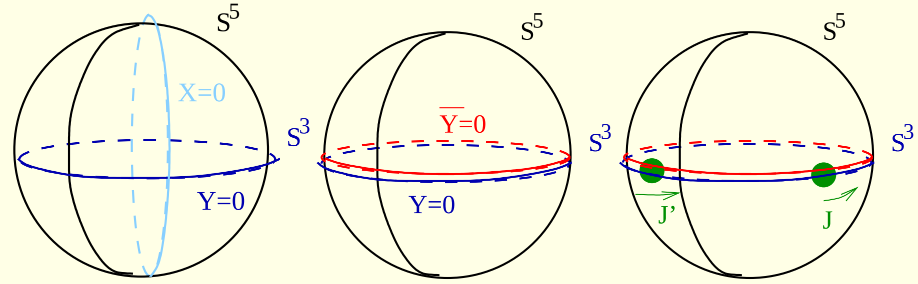
Brane $PSU(2|2)^2 \to SU(1|2)^2$

$(1,2) \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} (1,2)$ is broken



$\theta = \frac{\pi}{2}, \dot{\theta} = 0 \quad \theta = \dot{\theta} = \frac{\pi}{2},$

Y=0 brane



parametrization: $X = (1, \dot{1}), Y = (1, \dot{2}), \bar{Y} = (2, \dot{1})$

BMN Tr(Z^J): $PSU(2,2|4) \longrightarrow PSU(2|2)^2$

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R-matrix=scalar.matrix

matrix $\quad Q = 1$ reps $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \otimes \begin{pmatrix} \dot{1} \\ \dot{2} \\ \dot{3} \\ \dot{4} \end{pmatrix} \quad [R, \Delta(Q)] = 0 \quad R(p) = \begin{pmatrix} -e^{i\frac{p}{2}} & 0 & 0 & 0 \\ 0 & e^{-i\frac{p}{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

scalar $\quad R_0(z) = -e^{-ip}\sigma(p, -p)$ [Correa,Chen] [Hofman,Maldacena]

Y=0 brane

parametrization:$X=(1,\dot 1)$,$Y=(1,\dot 2)$,$\bar Y=(2,\dot 1)$

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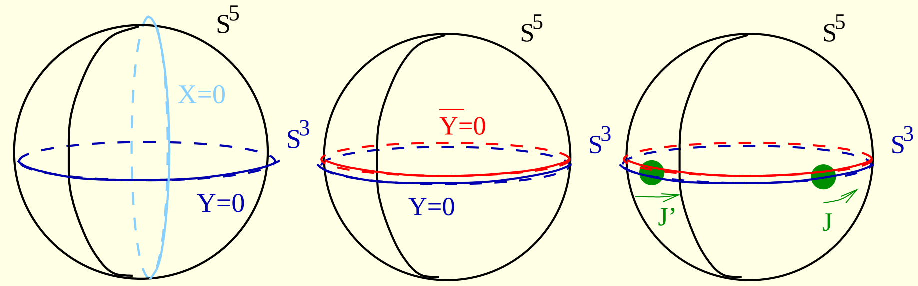

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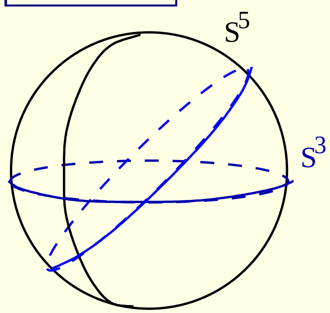
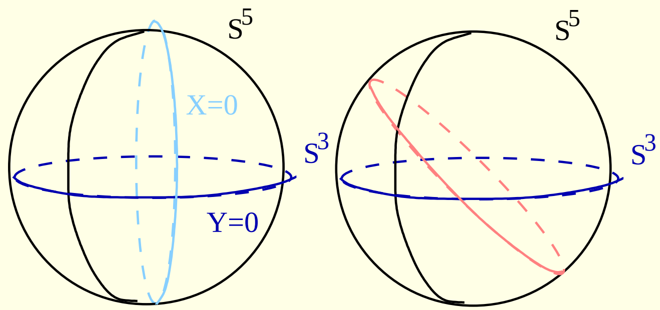
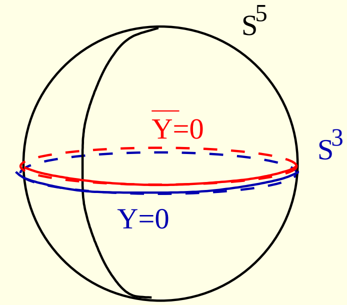
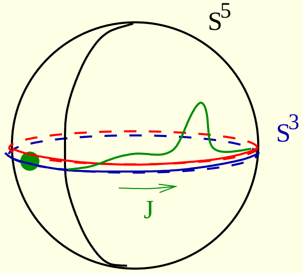




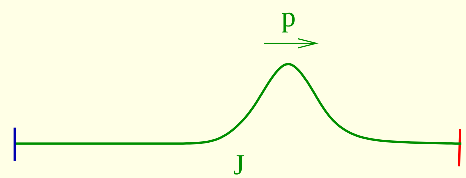


$$\begin{pmatrix}-e^{i\frac{p}{2}} & 0\\ 0 & e^{-i\frac{p}{2}}\end{pmatrix}\rightarrow\begin{pmatrix}\cos^2\theta e^{-i\frac{p}{2}}-\sin^2\theta e^{i\frac{p}{2}} & \sin\theta\cos\theta(e^{-i\frac{p}{2}}+e^{i\frac{p}{2}})\\ \sin\theta\cos\theta(e^{-i\frac{p}{2}}+e^{i\frac{p}{2}}) & \sin^2\theta e^{-i\frac{p}{2}}-\cos^2\theta e^{i\frac{p}{2}}\end{pmatrix}\rightarrow\begin{pmatrix}e^{-i\frac{p}{2}} & 0\\ 0 & -e^{i\frac{p}{2}}\end{pmatrix}$$

$Y - \bar{Y}$ brane system, large J spectrum for 1 particle



From the string excitation point of view



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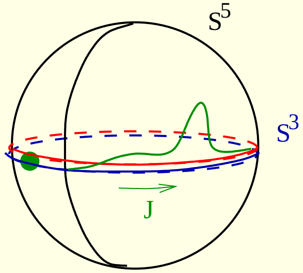

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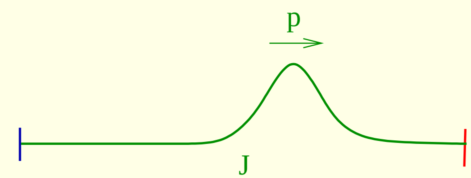
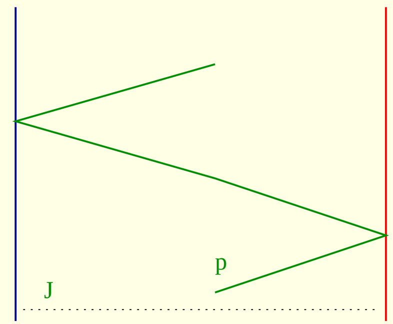


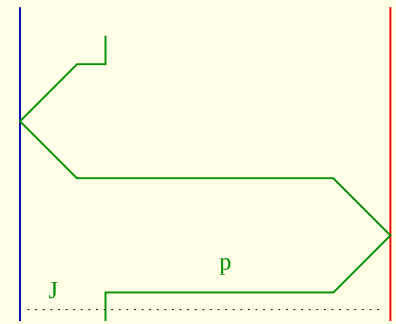
Asymptotic Bethe Ansatz:

periodicity of wavefunction

$$e^{i2pJ} R_Y(p) R_{\bar{Y}}(p) = 1 \rightarrow p$$

$$E_{BA} = E(p, \lambda) = \sqrt{1 + \frac{\lambda}{\pi^2}(\sin\frac{p}{2})^2}$$





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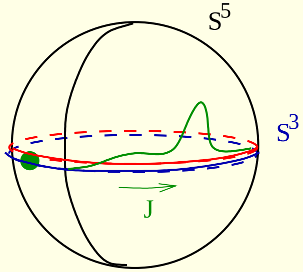

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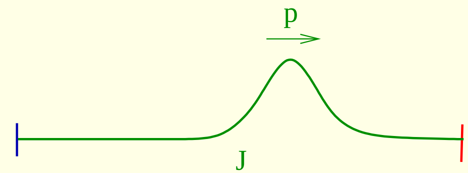

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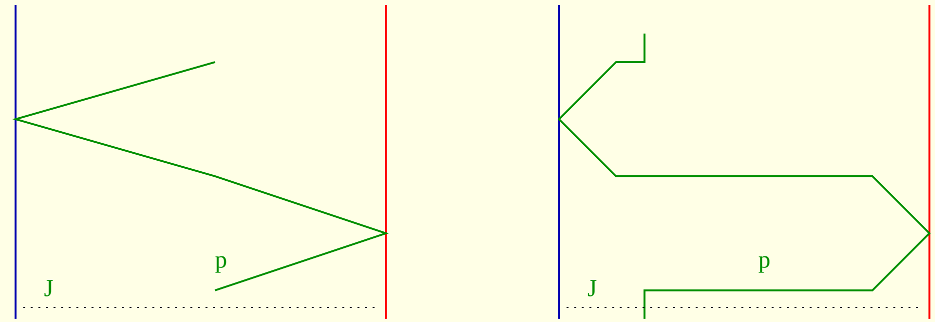


One loop gauge theory check: $\langle \mathcal{O}_{Y\bar{Y}}^{Z^{J-1} X\, Z^{J'}}(0) \mathcal{O}_{\bar{Y}Y}^{\bar{Z}^{J-1} \bar{X}\, \bar{Z}^{J'}}(x) \rangle$

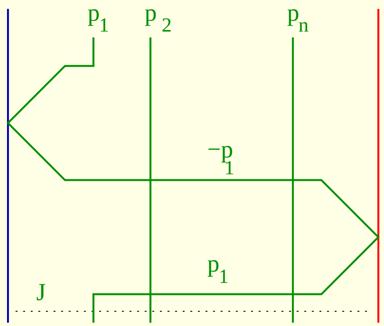
$$\mathcal{O}_{Y\bar{Y}}^{Z^J Z^{J'}} = \epsilon_{i_1 \ldots i_N}^{j_1 \cdots j_N} Y_{j_1}^{i_1} \ldots Y_{j_{N-1}}^{i_{N-1}} (Z^{J'})_{l_N}^{j_N} \epsilon_{k_1 \ldots k_N}^{l_1 \ldots l_N} \bar{Y}_{l_1}^{k_1} \ldots \bar{Y}_{l_{N-1}}^{k_{N-1}} (Z^J)_{i_N}^{k_N}$$

spin chains decouple into two one loop Hamiltonians:

$$H = \frac{\lambda}{8\pi} Q_1^{\bar{Y}} Q_J^Y \left[\sum_{j=1}^{J} (I_{j,j+1} - P_{j,j+1} + \frac{1}{2} K_{j,j+1} + 2 - Q_1^Y - Q_J^{\bar{Y}} \right] Q_J^Y Q_1^{\bar{Y}}$$

agrees with BA!

$Y - \bar{Y}$ brane system, large J spectrum



Asymptotic Bethe Ansatz: periodicity of wavefunction

$e^{i2p_1J}\prod_{i>1} S(p_1,p_i)R_Y(p_1)\prod_{i>1} S(p_i,-p_1)R_{\bar{Y}}(p_1) = 1 \rightarrow p_1$

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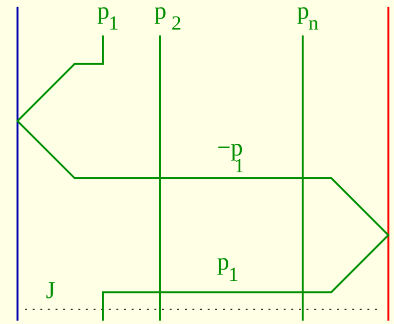

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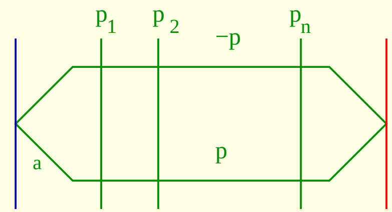

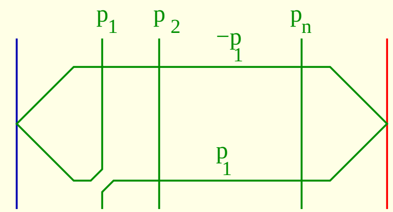

Double row transfer matrix

$T_a(p)=\mathsf{sTr}_a(\prod_i S(p,p_i)R_Y(p)\prod_i S(p_i,-p)R^C_{\bar{Y}}(p))$

Bethe Ansatz: $e^{2ip_1J}T_1(p_1)=-1$ Finite size correction $E_{FS}=-\sum_a\int\frac{dq}{2\pi}T_a(q)e^{-2\tilde{\epsilon}_aJ}$

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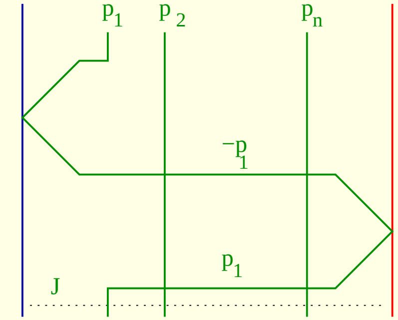

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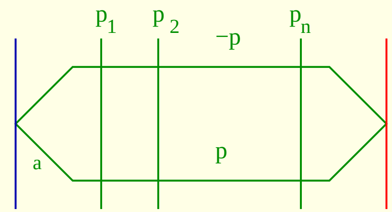

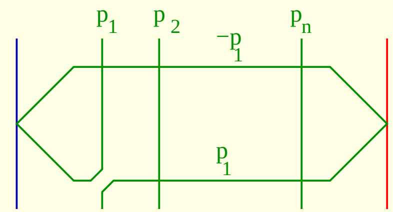

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Generic eigenvalue in terms of $Q_2(u)=\prod_{l=1}^{m_2}(u-\tilde{\mu}_l)(u+\tilde{\mu}_l)\,,B_1R_3=\prod_{j=1}^{m_1}(x(p)-y_j)\,(x(p)+y_j)$

$$T(p)=\left(\frac{x^+(p)}{x^-(p)}\right)^{m_1}\frac{R^{(-)+}}{R^{(+)+}}\rho_1\left[\frac{R^{(+)+}}{R^{(-)+}}\frac{B_1^-R_3^-}{B_1^+R_3^+}+\frac{B_1^-R_3^-}{B_1^+R_3^+}\frac{Q_2^{++}}{Q_2}+\frac{u^+R_1^+B_3^+}{u^-R_1^-B_3^-}\frac{Q_2^{--}}{Q_2}+\frac{u^+B^{(-)-}}{u^-B^{(+)-}}\frac{R_1^+B_3^+}{R_1^-B_3^-}\right]$$

Magnonic BA: $\frac{R^{(+)+}Q_2}{R^{(-)+}Q_2^{++}}|_{x^+(p)=y_j}=-1$; $\frac{\rho_3}{\rho_4}\frac{R_1^-B_1^-R_3^-B_3^-Q_2^{++}}{R_1^+B_1^+R_3^+B_3^+Q_2^{--}}|_{u=\tilde{\mu}_l}=-1$

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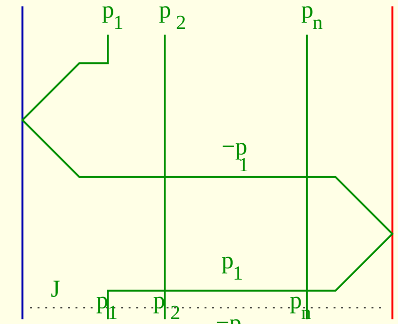

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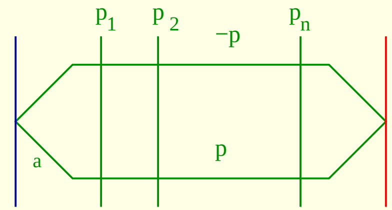

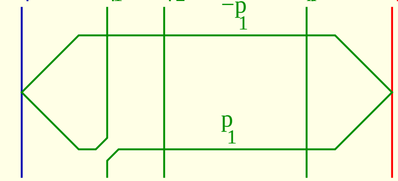

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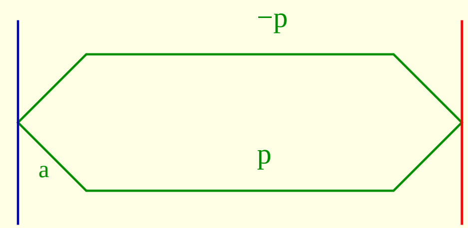
One particle Lüscher correction: $J=1$; $p=\frac{\pi}{2}$: $E_{FS}=8g^8(4\zeta_3-5\zeta_5)$

Ground state energy: large volume

Asymptotic BA trivial, finite size correction from transfer matrices

$$E_{FS} = -\sum_a \int \frac{dq}{2\pi} \mathsf{sTr}_a (R_Y(q) R^C_{\bar{Y}}(\bar{q}) e^{-2\tilde{\epsilon}_a J} = -\frac{4g^{4J}}{4J-1}\binom{4J}{2J}\zeta_{4J-3}$$



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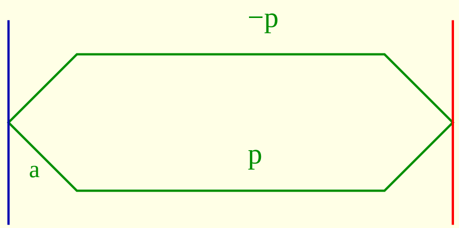

divergent for $L = 1$, tachyon??

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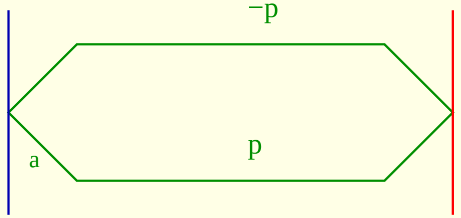

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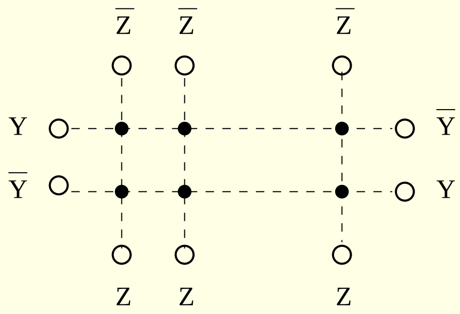
wrapping contribution:



$$\langle \mathsf{Tr}(Y(x)Z^J(0)\bar{Y}(x)Y(0)\bar{Z}^J\bar{Y}(0))\rangle$$
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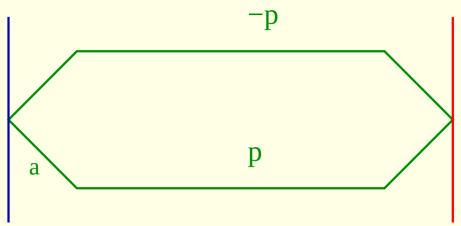

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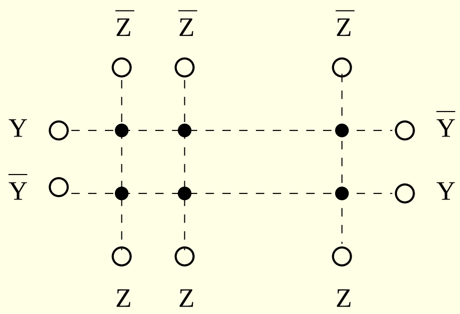

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$I_2 = \frac{\lambda^2}{(4\pi)^4}(\frac{1}{2\epsilon^2} - \frac{1}{2\epsilon} + \mathsf{reg})$ [Schnetz]: $I_{2j} = -4(\frac{\lambda^2}{4\pi})^{2J} \frac{1}{J^2}\binom{4J-2}{2J-1}\zeta_{4J-3}$

for $L=1$ we are missing something! Large N limit?

Ground state energy: BTBA

We need strong coupling $\longrightarrow$ BTBA: Claim there is no BTBA for non-diagonal bulk scatterings

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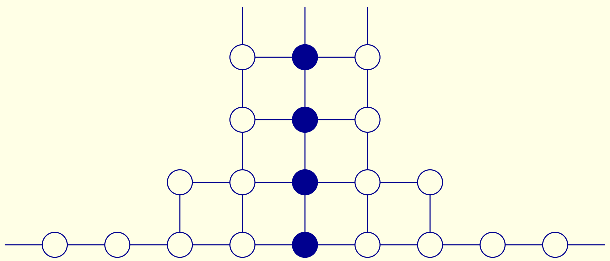
T-system: $T^{+}_{a,s}T^{-}_{a,s}=T_{a-1,s}T_{a+1,s}+T_{a,s-1}T_{a,s+1}$

Y-system: $Y_{a,s}=\frac{T_{a,s+1}T_{a,s-1}}{T_{a-1,s}T_{a+1,s}}$; $Y_{a,0}=\frac{u^{[-a]}}{u^{[a]}}T^{2}_{a,1}\left(\frac{z^{-}}{z^{+}}\right)^{2L}$



Ground state energy: BTBA

We need strong coupling $\rightarrow$ BTBA: Claim there is no BTBA for non-diagonal bulk scatterings

Exceptions: $q-\bar{q}$ potential [Correa,Maldacena,Sever] [Drukker] (but it is not really boundary)

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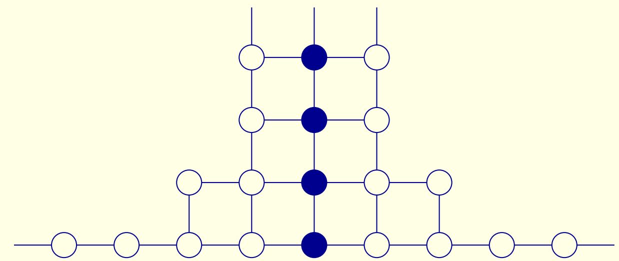

generating functional for generic angle: $\mathcal{W}_{su(2)}=\sum_{s=0}^{\infty}\mathcal{D}^{s}T_{1,s}\mathcal{D}^{s}$

$\mathcal{W}_{su(2)}=f(\mathcal{D}\frac{u^{+}}{u^{-}}\mathcal{D})^{-1}(1-\mathcal{D}\frac{u^{+}}{u^{-}}\mathcal{D})(1-\mathcal{D}^{2})f(\mathcal{D}^{2})^{-1}$ with $f(z)=\sqrt{1-2\cos(2\theta)z+z^{2}}$

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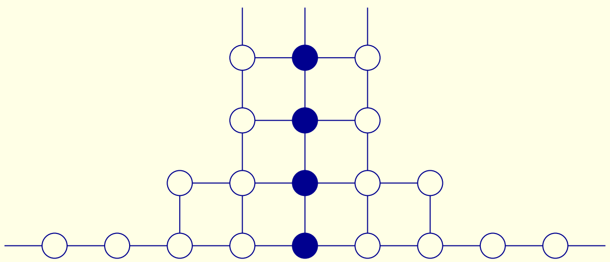

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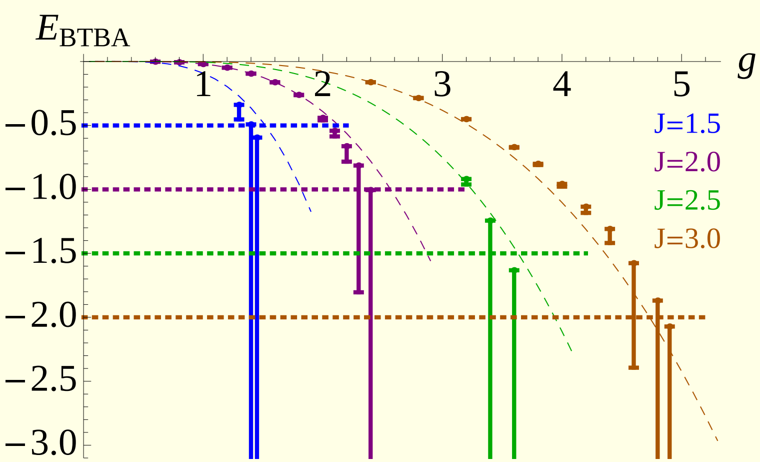
Y-system+discontinuity = BTBA equations:

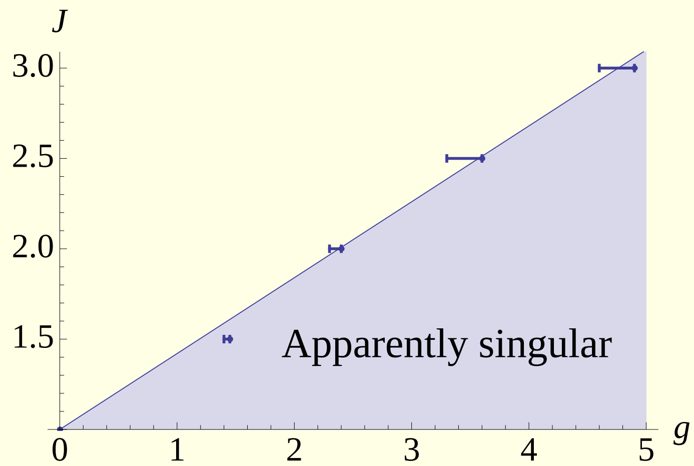
$$\epsilon^j(q) = \nu^j(q) + \delta^j_Q\tilde{E}_Q(q)L - \int K^j_i(q,q')\log(1+e^{-\epsilon^i(q)})dq'$$

Energy: $E(J,g) = -\sum_Q\int\frac{dq}{2\pi}\log(1+Y_Q(q))$

Ground state energy: Solving the BTBA equations

Numerical solution: BTBA breaks down at $g_{crit}(J)$:





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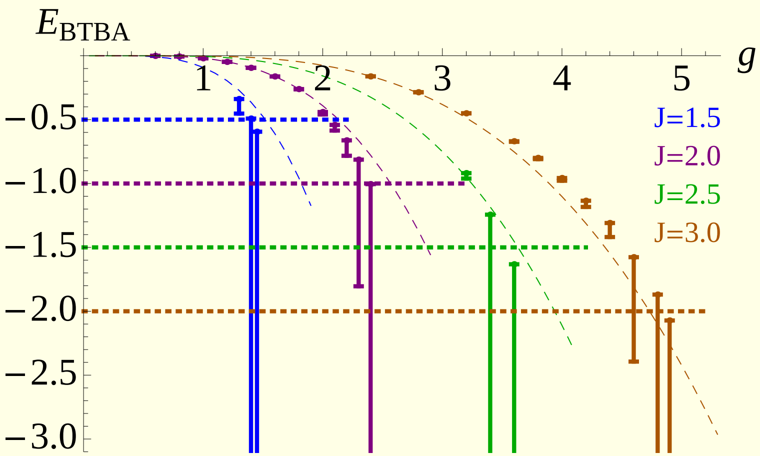
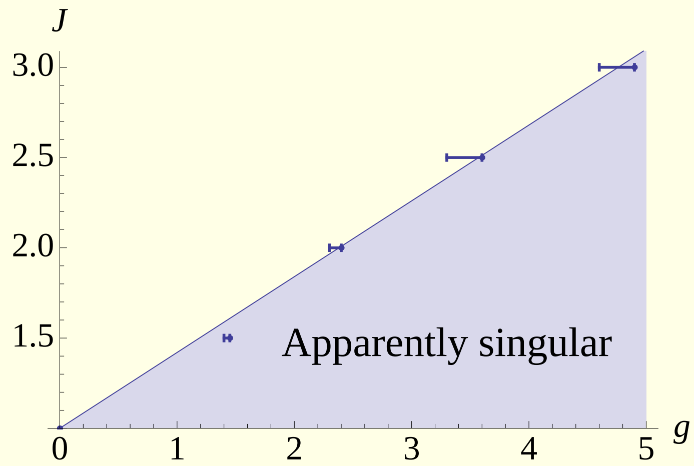




TBA cannot describe state with $E < E_{crit}$

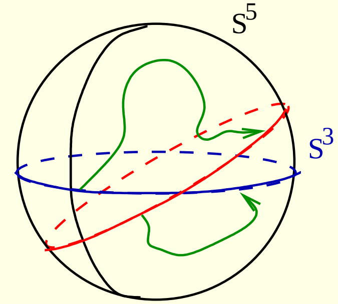
Analytically: $E(J,g) = -\sum_Q \int \frac{dq}{2\pi} \log(1+Y_Q(q))$ should makes sense

For $q \to \infty$: $Y_Q \sim q^{-4(J+E)}$ integral exists if $E > -J + \frac{1}{4}$

For $Q \to \infty$: $Y_Q \sim Q^{3-4(J+E)}$ sum exists if $E > -J + 1$

compatible with Luscher

Conclusion



We analyzed the spectrum of open strings on a brane-anti-brane system

formulated the gauge theory dual and derived a one-loop integrable open spin chain

described the asymptotic large volume spectrum

compared the vacuum Luscher correction to gauge theory calculations

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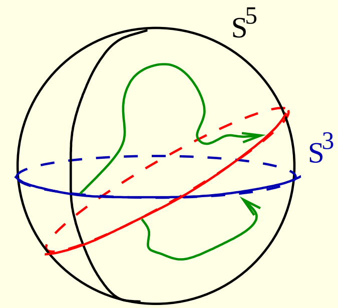

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Open problems

derive asymptotic BA for generic angle

understan the $L = 1$ discrepancy from the gauge theory point of view

derive BTBA for generic angle

go beyond the critical coupling by FiNLIE or $P\mu$-system