Statisztikus tanulás az idegrendszerben

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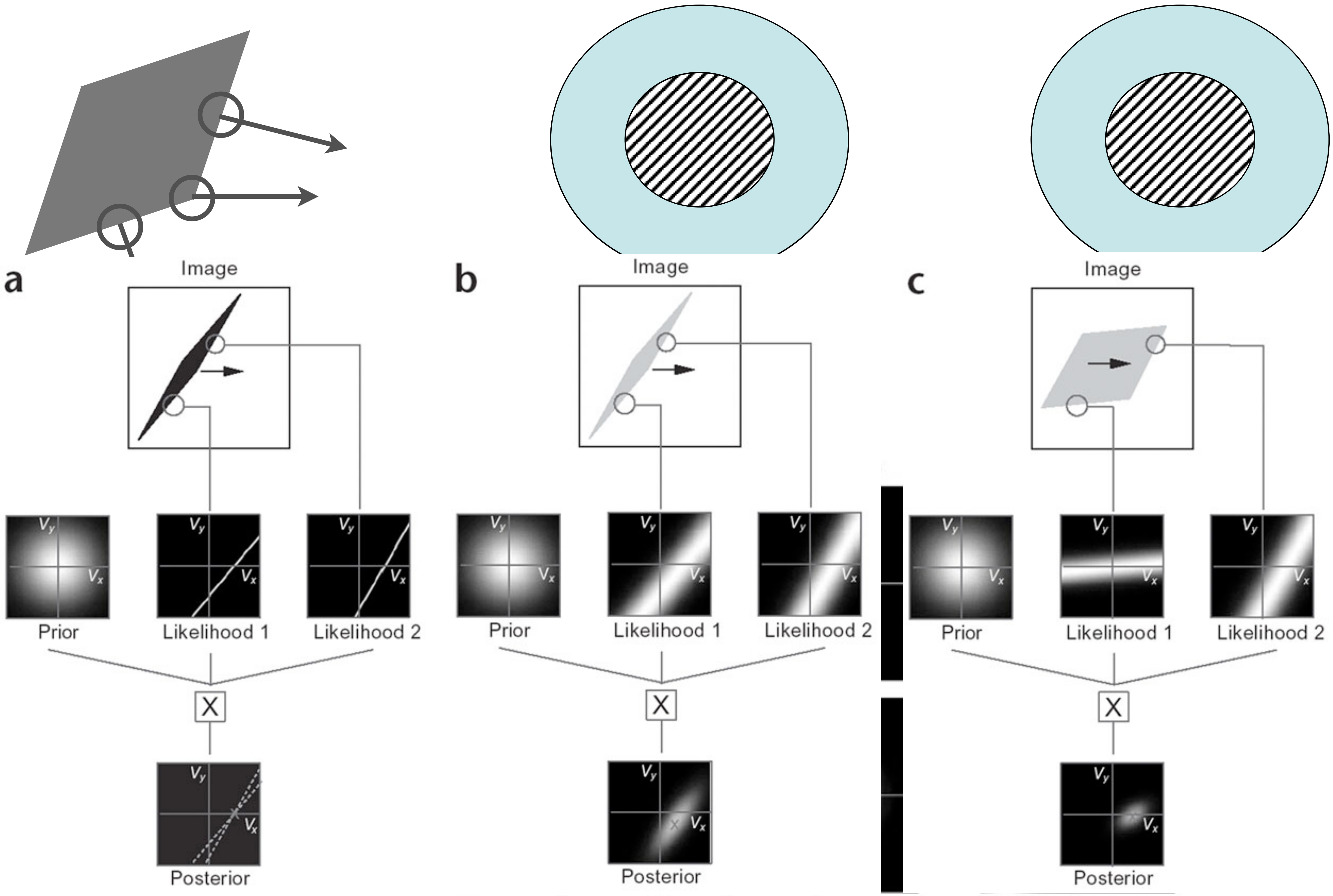
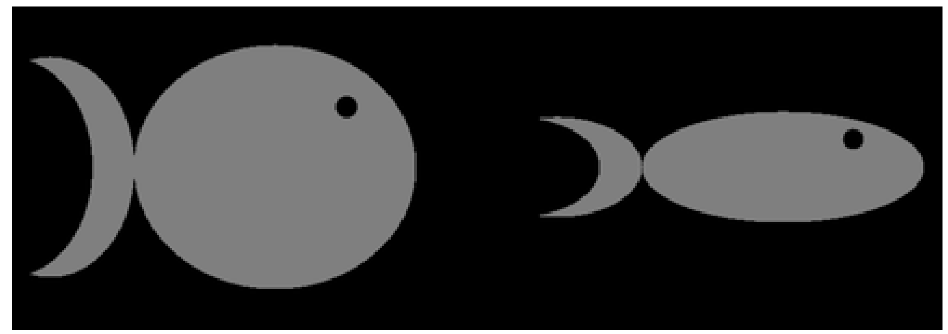
RECAP: Representing priors

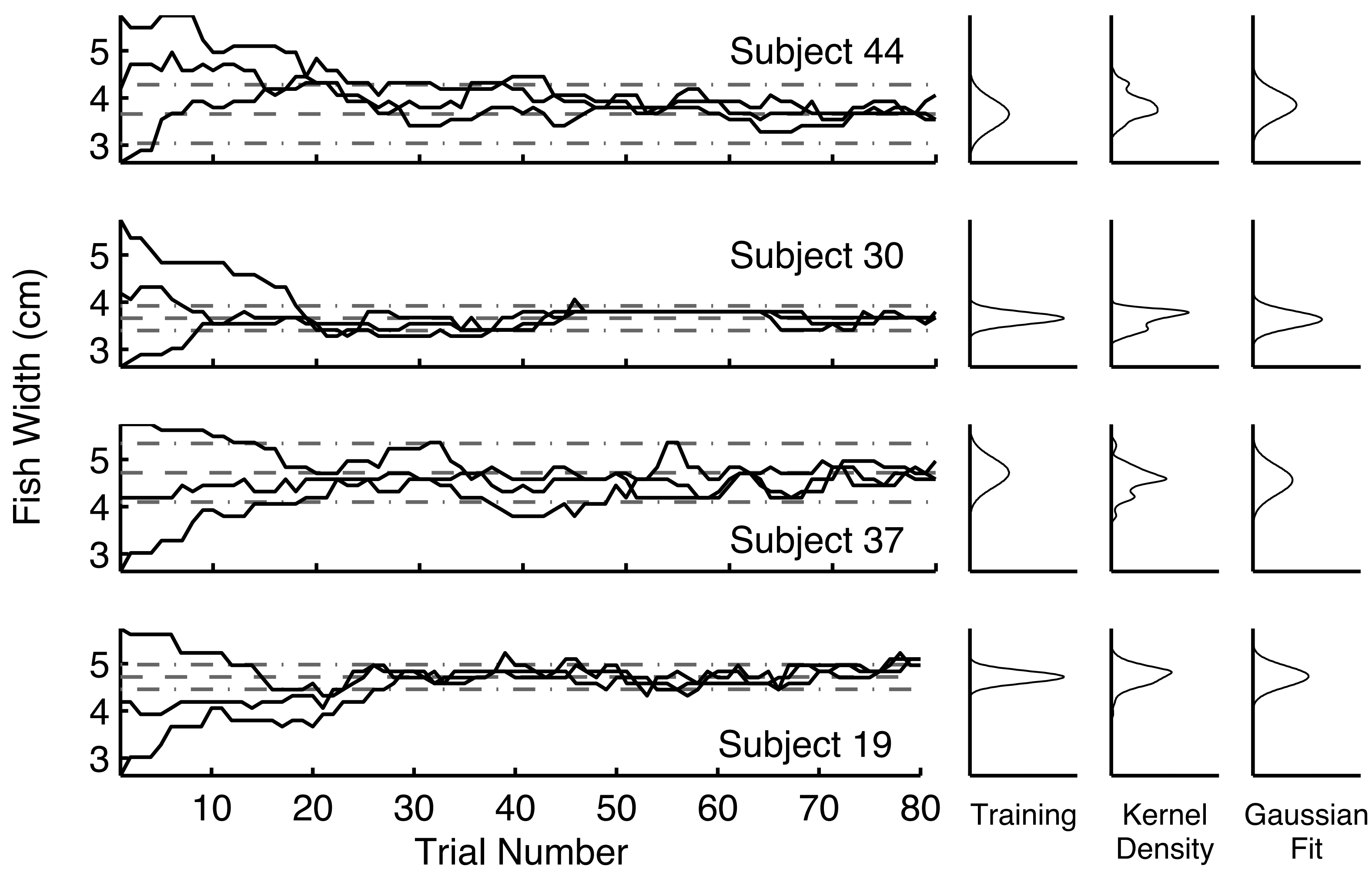


RECAP: representing priors

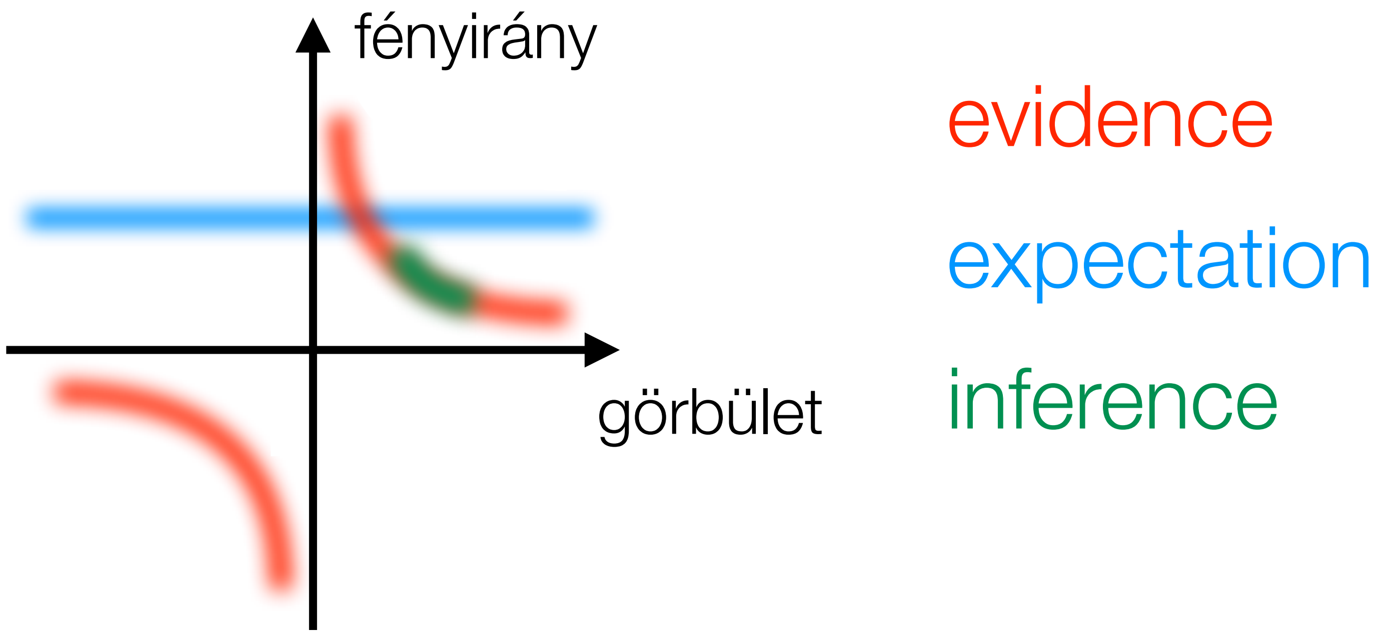
Barker dynamics:

$$A(x^{*}, x) = p(x^{*}) / (p(x^{*}) + p(x))$$





RECAP: role of priors



$$P(\text{feature} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{feature}) \times P(\text{feature})$$

posterior: inference

likelihood: evidence

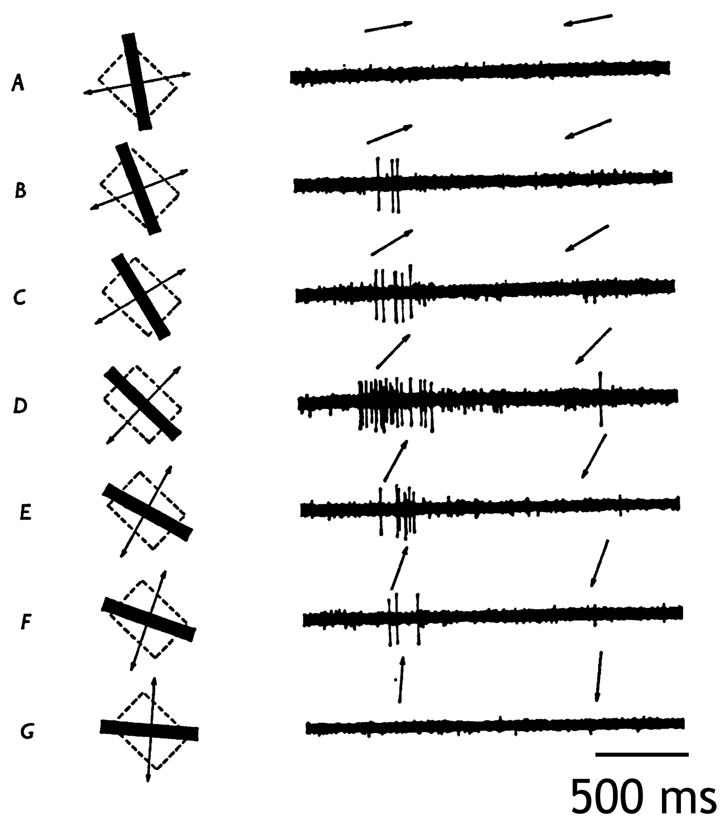
prior : expectations

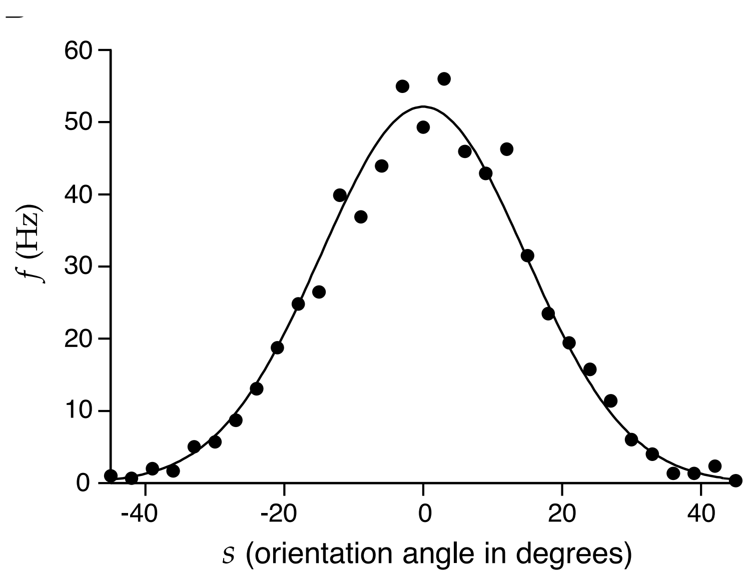
features $\rightsquigarrow$ neurons

Neurális válaszok

Neurális válaszok

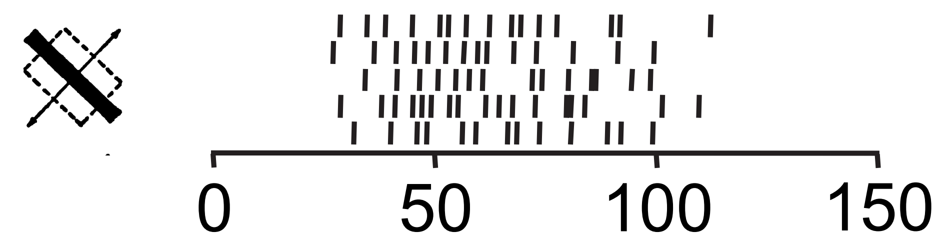
V1 characteristic response





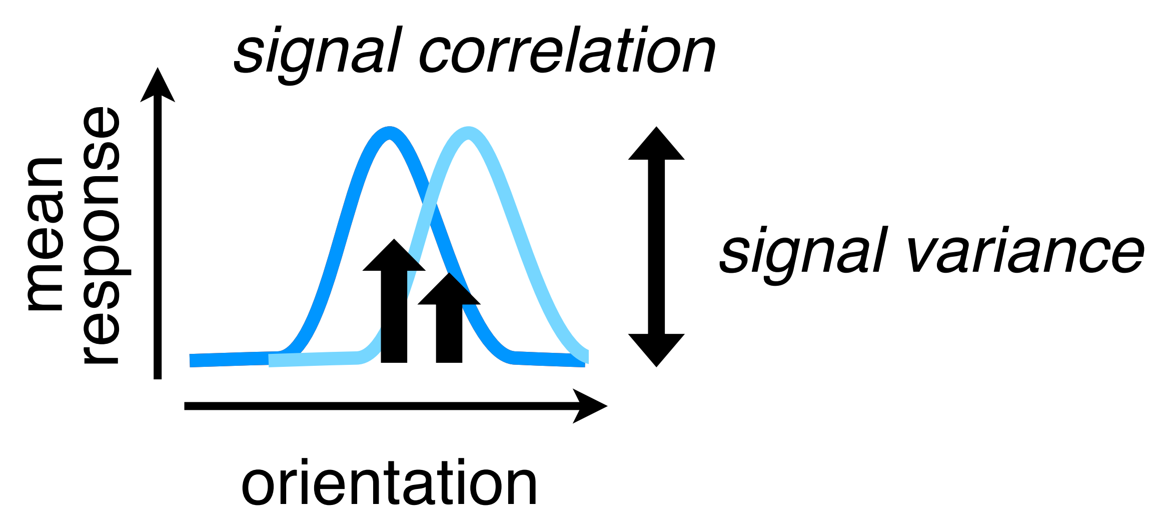
Hubel & Wiesel, J Physiol 1968

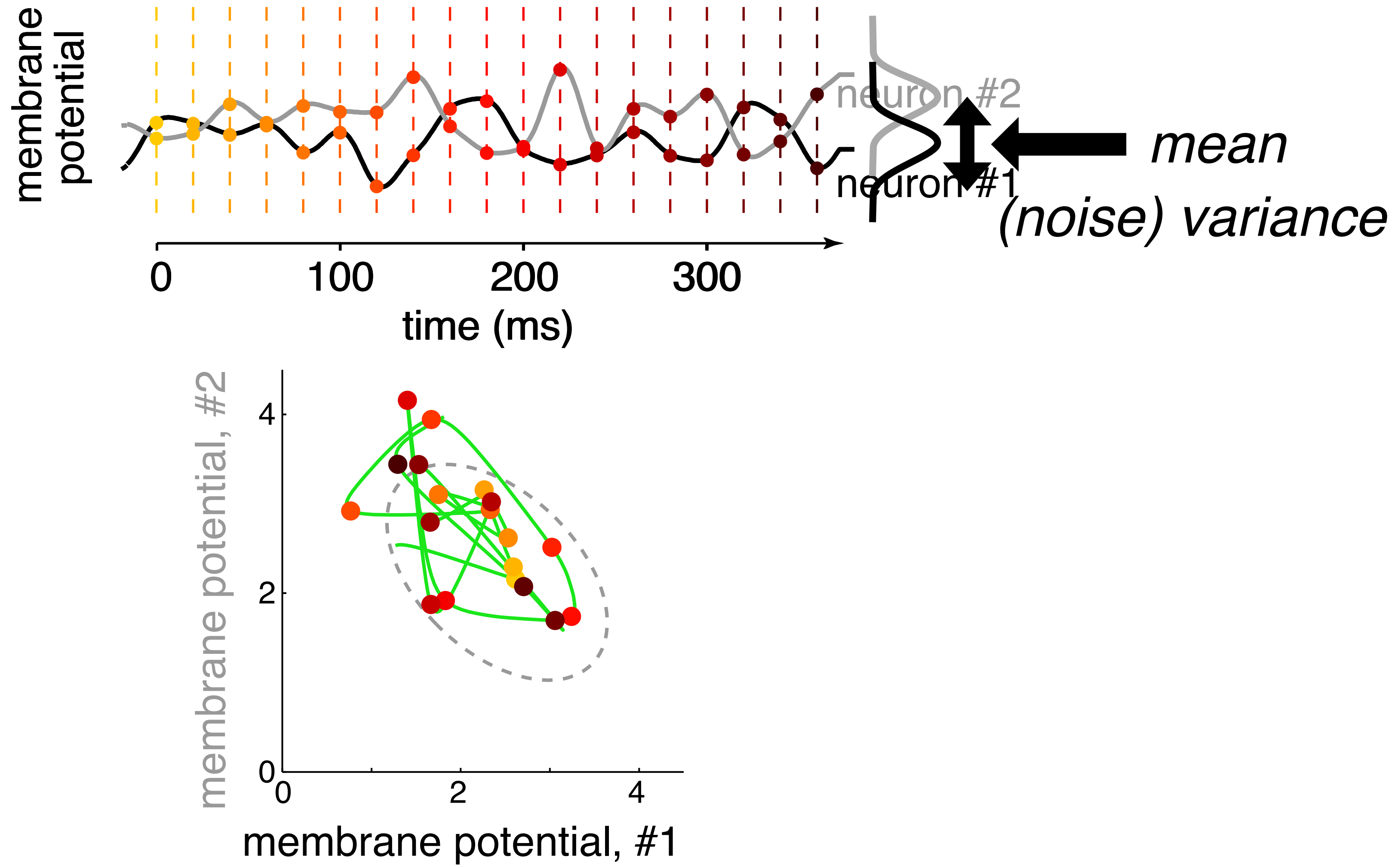
V1 spike train variability



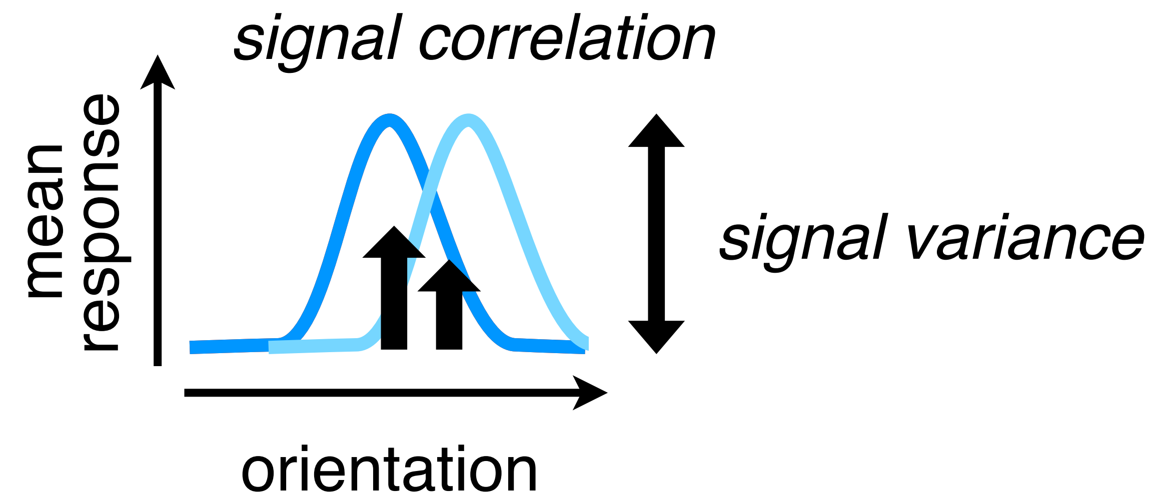
Gur & Snodderly, Cereb Cortex 2006

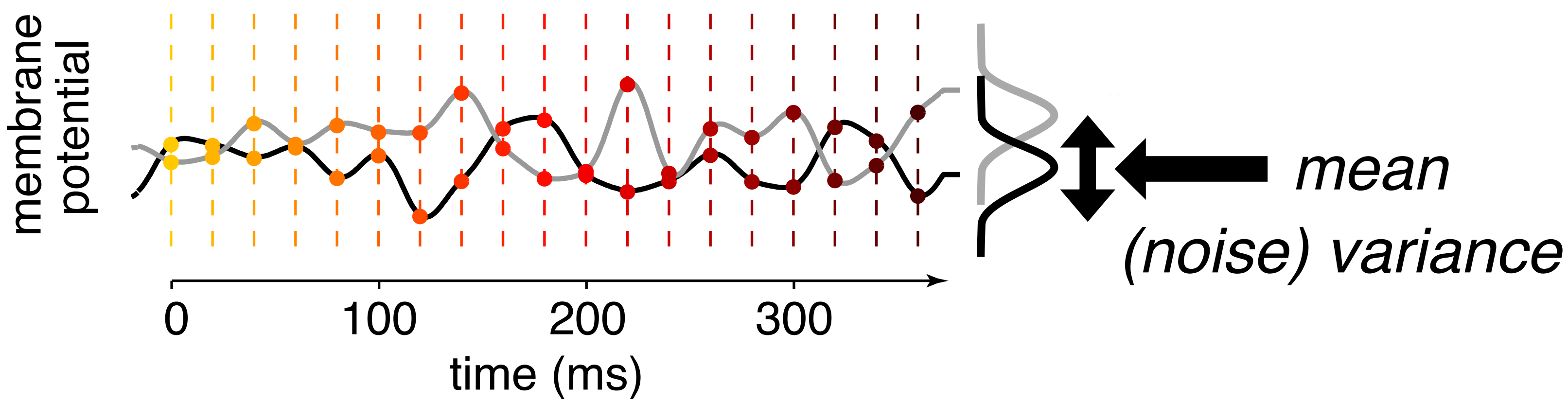
response statistics



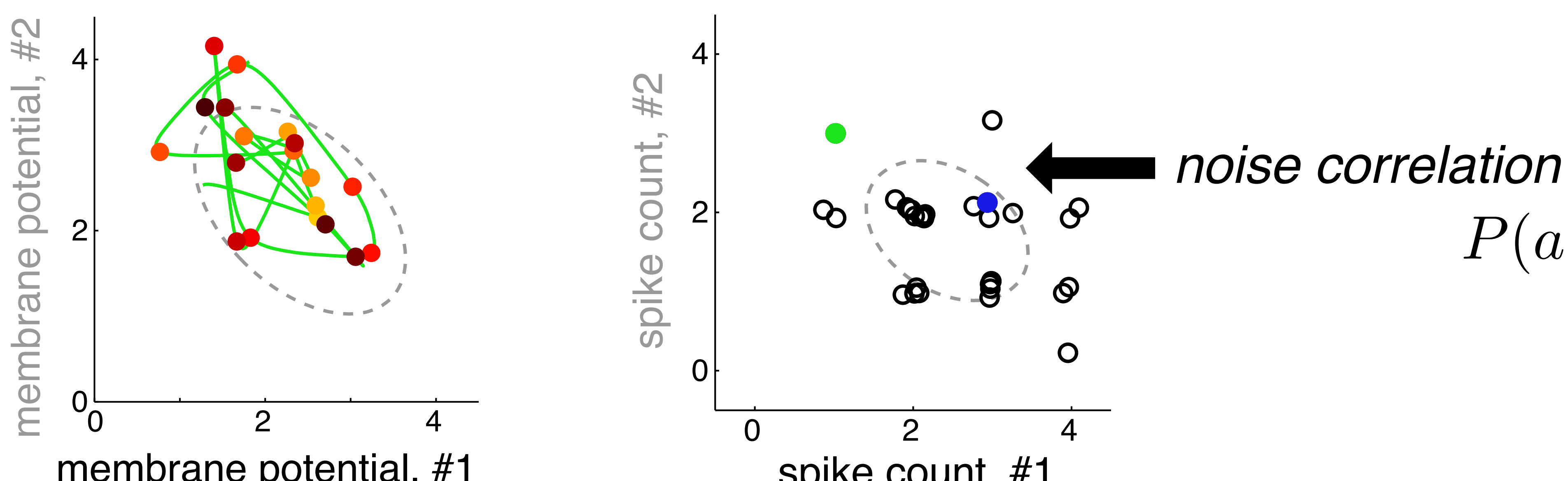


response statistics

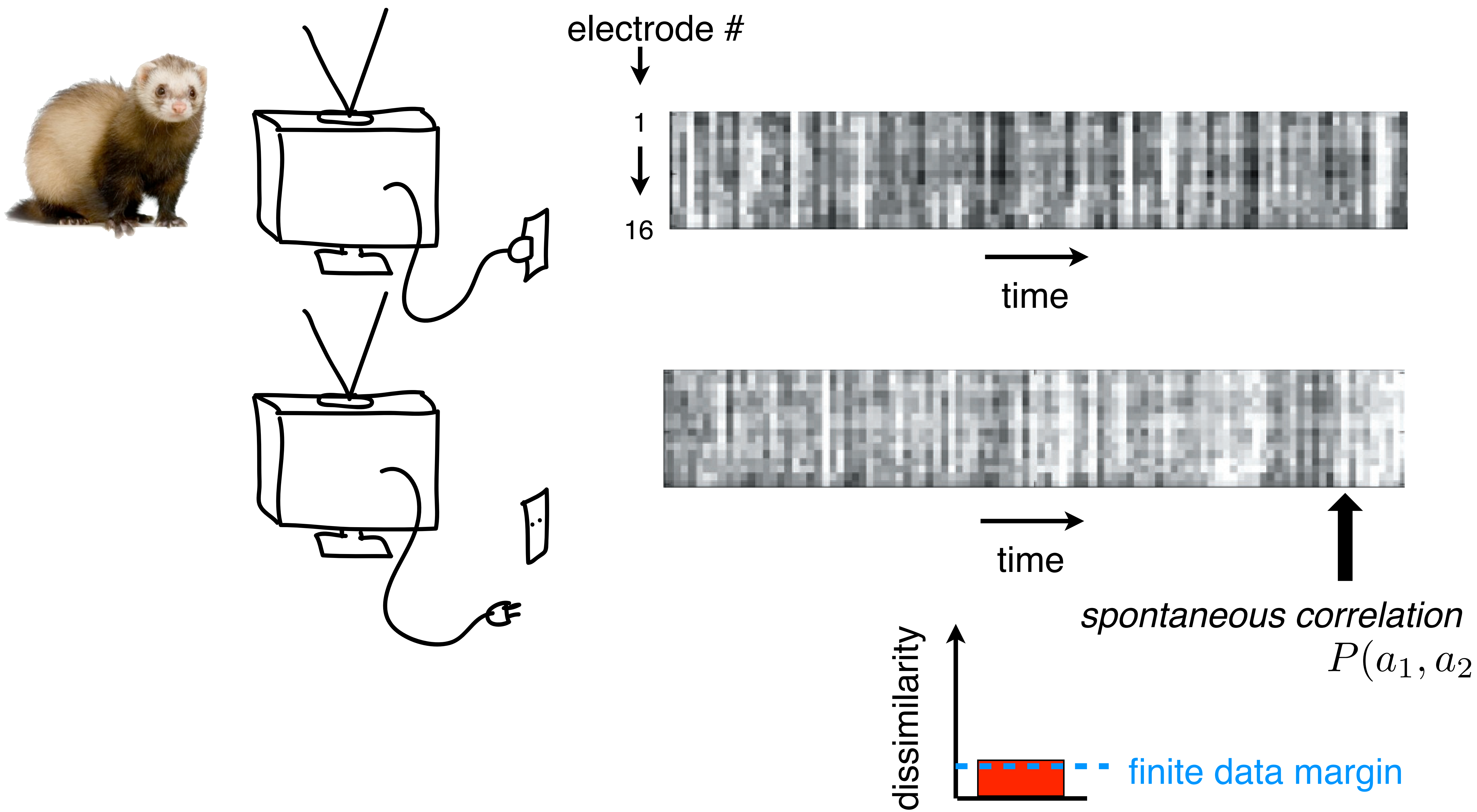




$$P(a_1 \mid \text{stimulus})$$



$$P(a_1, a_2 \mid \text{stimulus})$$


electrode #
1
16
time
time
spontaneous correlation
$P(a_1, a_2)$
dissimilarity
finite data margin

Unsupervised learning

Input: $x_1, x_2, \ldots, x_t$ összefoglaló néven: adat - vizuális, auditoros, szöveg

Gól: $\mathrm{P}\left(\mathbf{x}\right)$

(Reinforcement learning:

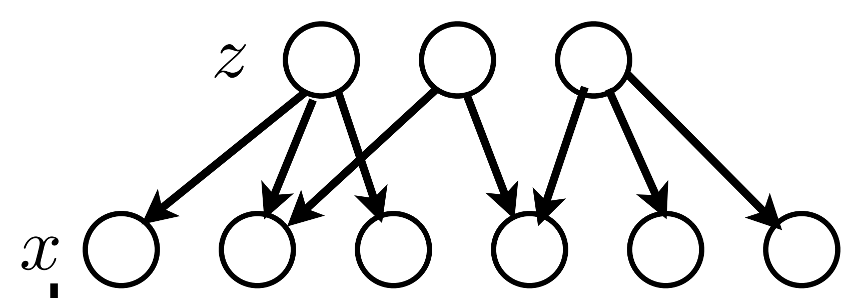
Input: $\{x_1, y_1\}, \{x_2, y_2\}, \ldots, \{x_t, y_t\}$

Gól: $\mathrm{P}\left(\mathbf{x} \,|\, \mathbf{y}\right)$)

$\mathrm{P}\left(\mathbf{x}\right)$ Bonyolult!

Miért is?

Egyszerűsítés: $\mathrm{P}\left(\mathbf{x}\right) = \mathrm{P}\left(\mathbf{x} \,|\, \mathbf{z}\right) \mathrm{P}\left(\mathbf{z}\right)$



- az adatot a “z”-k terében reprezentáljuk
- kategorizáció, dimenzió redukció
- általánosabban a feladat: predikció, döntéshozatal, kommunikáció

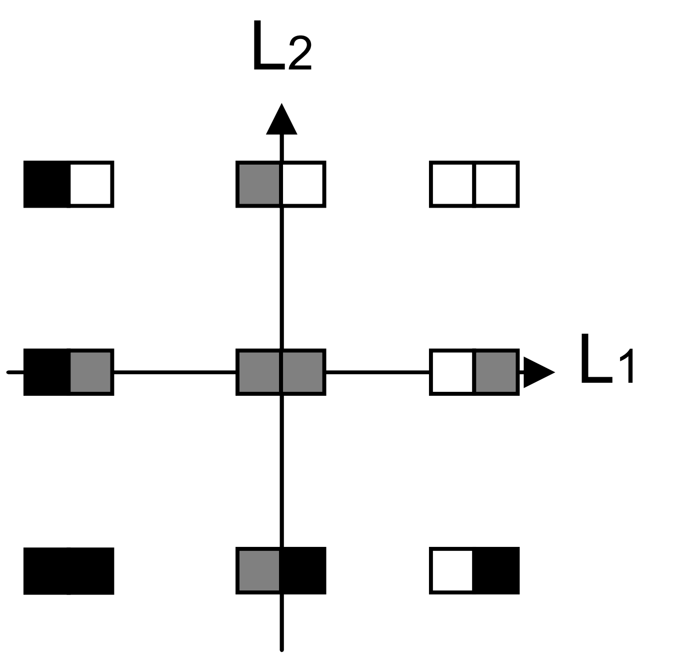
Lineáris modellek

$$\mathrm{P}\left(x \mid z\right) = \mathrm{Normal}\left(x; z, \theta\right) = C \exp\left(\left(x - Az\right)^T \Sigma^{-1} \left(x - Az\right)\right)$$

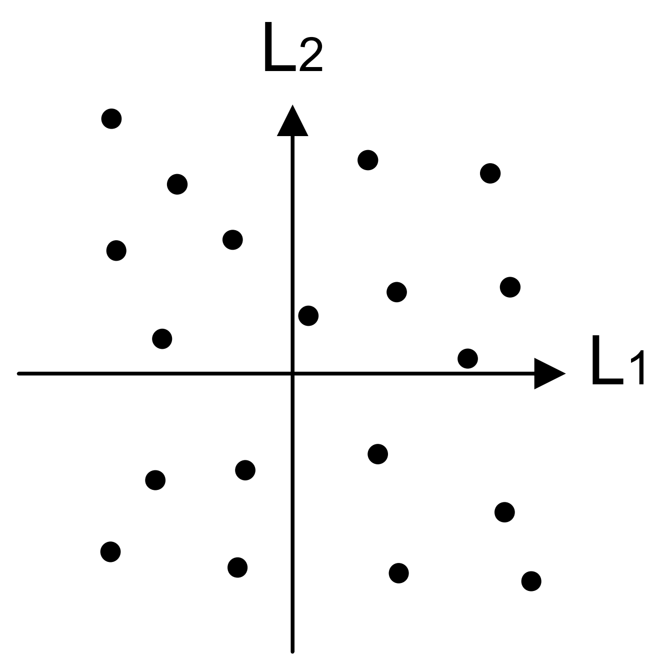
$$x = \mathbf{A} \cdot z + \epsilon$$

PCA

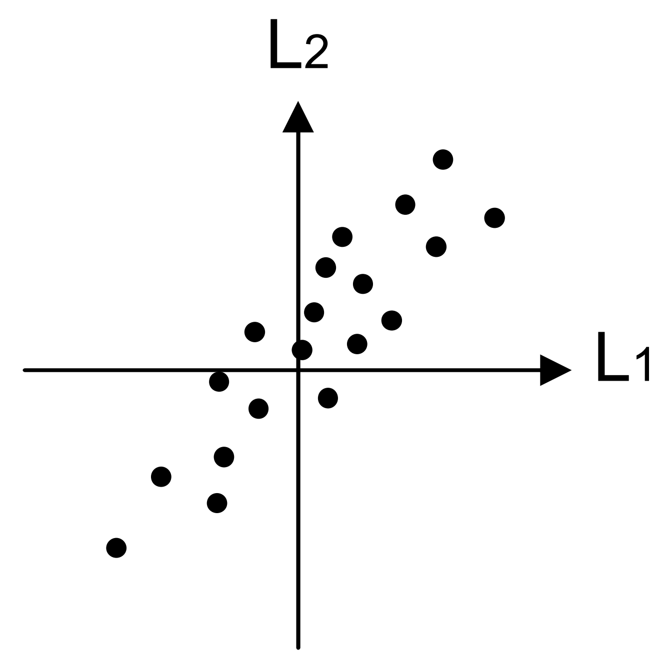
- A oszlopvektorai ortogonalisak
- D(x) = D(z)
- Izotróp zaj



State space of two pixel images



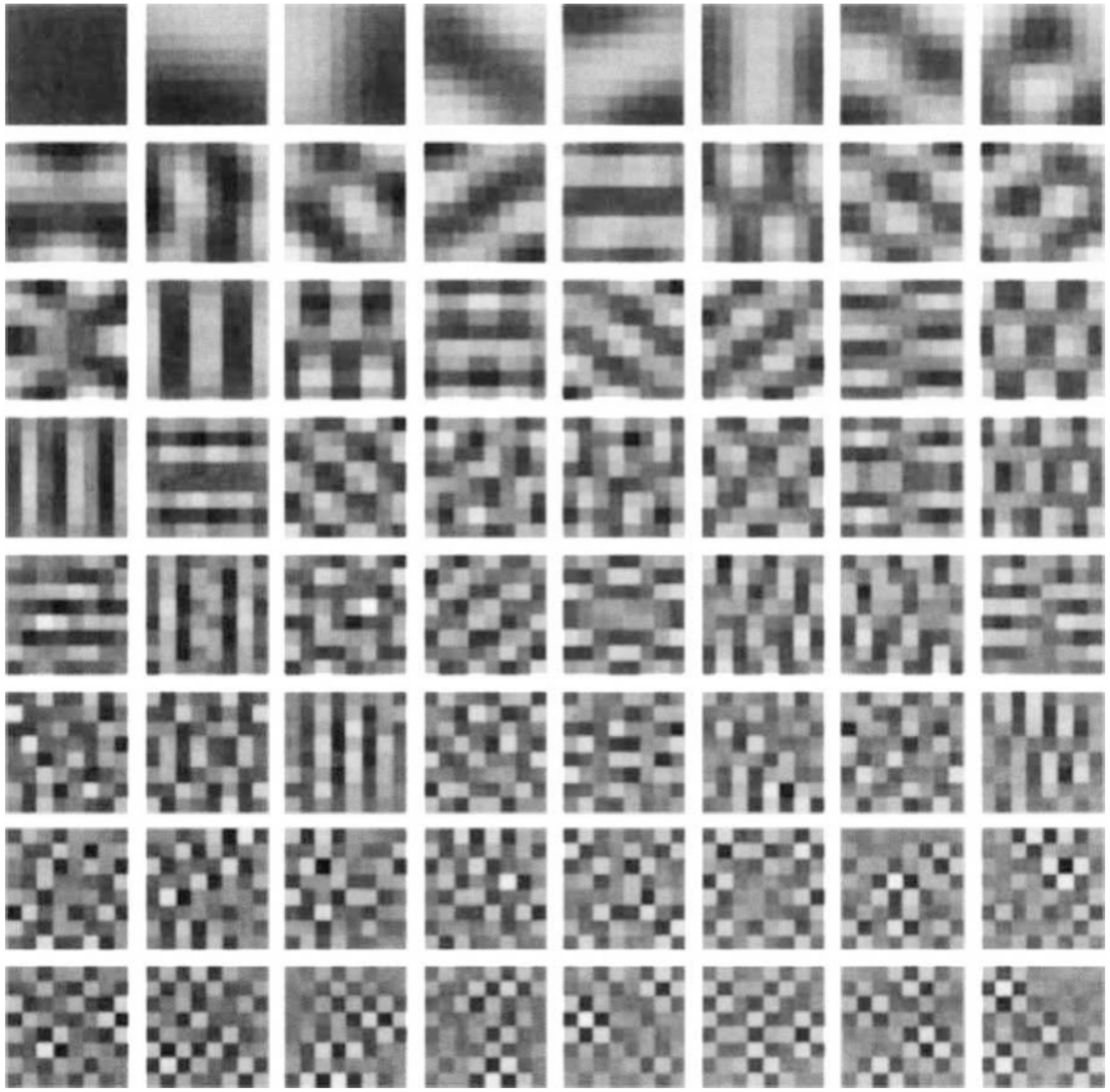
Random images



Structured images

PCA tulajdonságok

- Kompakt kódot eredményez
- Egy adatponért leírásáért általában a teljes hálózat felel



Sparse kódolás, ICA

$$x = \mathbf{A} \cdot z + \epsilon$$

- "z"-k függetlenek
- y priorja "ritka"(P(z))

Komputációs kritériumok:

- Hiteles rekonstrukció
  költség egy adatpontra (képre):

$$\mathrm{cost}_1 = \left(x - \sum_i A_i' \cdot z_i \right)^2$$

- Kis "energiafelhasználás (kevés szimultán aktiv neuron)
  további költség a kód "ritkasága":

$$\mathrm{cost}_2 = -\sum_i S\left(\frac{z_i}{\sigma}\right)$$

  S a Gauss-nál nagyobb kurtózissal bíró eloszlás

- teljes költség (~energia):

$$E = -\mathrm{cost}_1 - \lambda \mathrm{cost}_2$$

Sparse kód tanulása: E-M

Algoritmus:

- Itáráció EM lépésekkel
- Random kezdeti feltételek
- Adott konnektivitási mátrixnál az aktiviások segítségével a költség minimalizálása
- Adott aktivitásokkal a költség minimalizálása a súlyok adaptálásával

Adott konnektivitási mátrix esetén a legjobb aktivitások megtalalása:

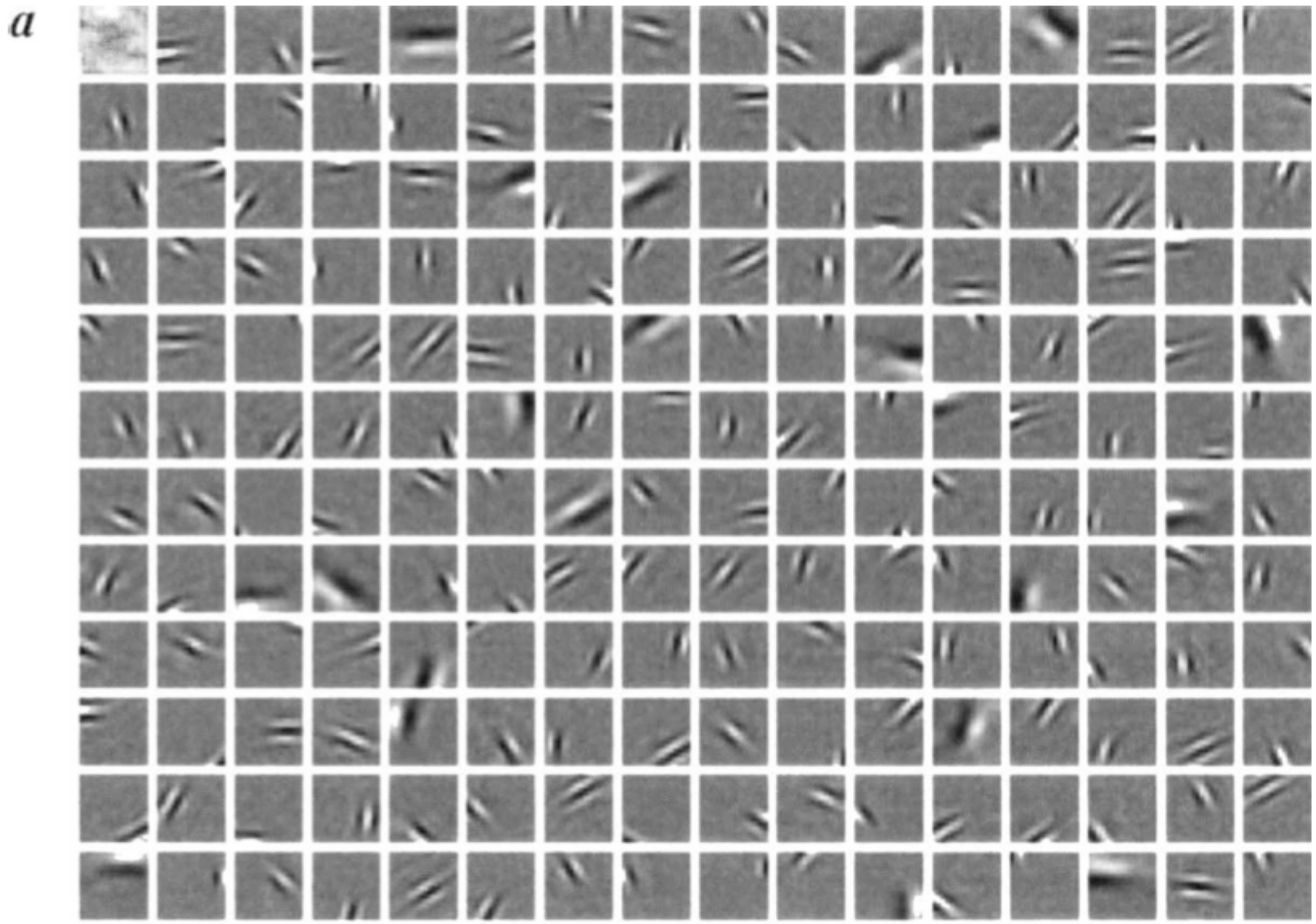
$$\dot{z}_i = \mathbf{A}_i x_t - \sum_j \mathbf{A}_i' \mathbf{A}_j z_j - \frac{\lambda}{\sigma} S' \left(\frac{z_i}{\sigma} \right)$$

Adott konnektivitási aktivációk esetén a legjobb súlyok megtalalása:

$$\Delta A_i = \eta \left\langle a_i \left[x - \hat{x} \right] \right\rangle_t$$

Sparse kódolás: eredmény

tréningezés természetes képekkel

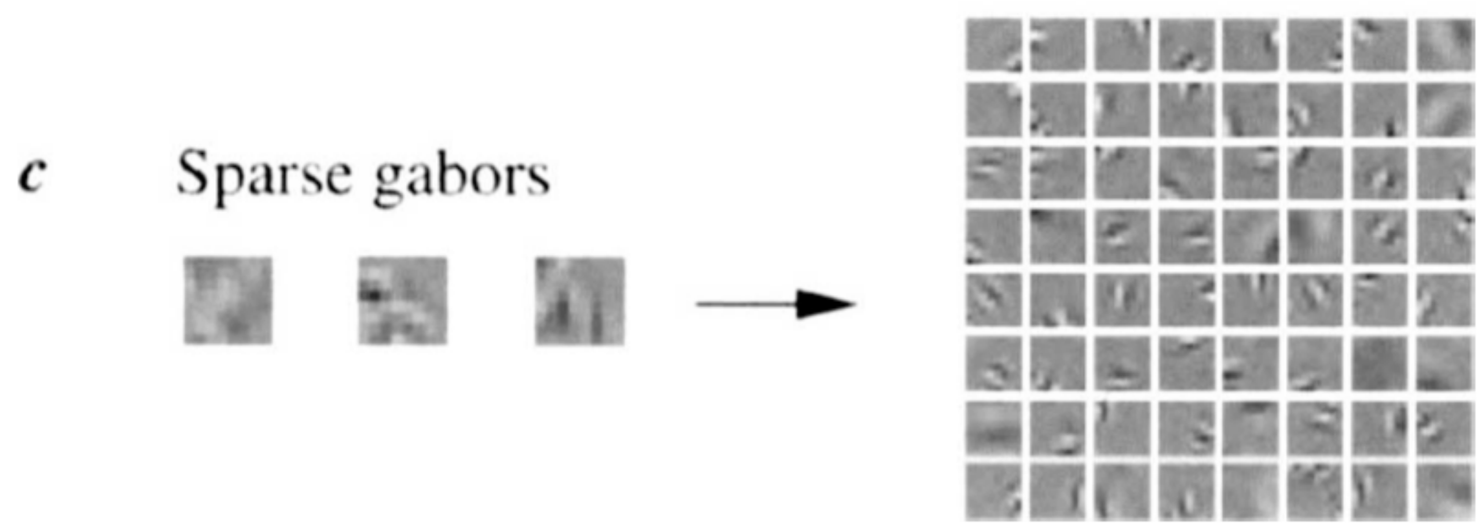


Olshausen & Field '96

A kialakult bázis:

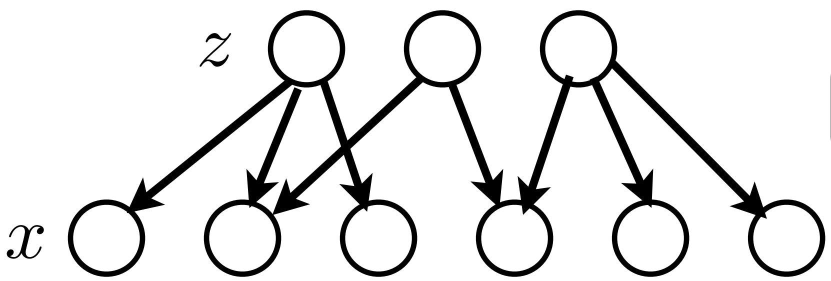
- irányított
- térbeli sávszűrést valósít meg
- lokalizált

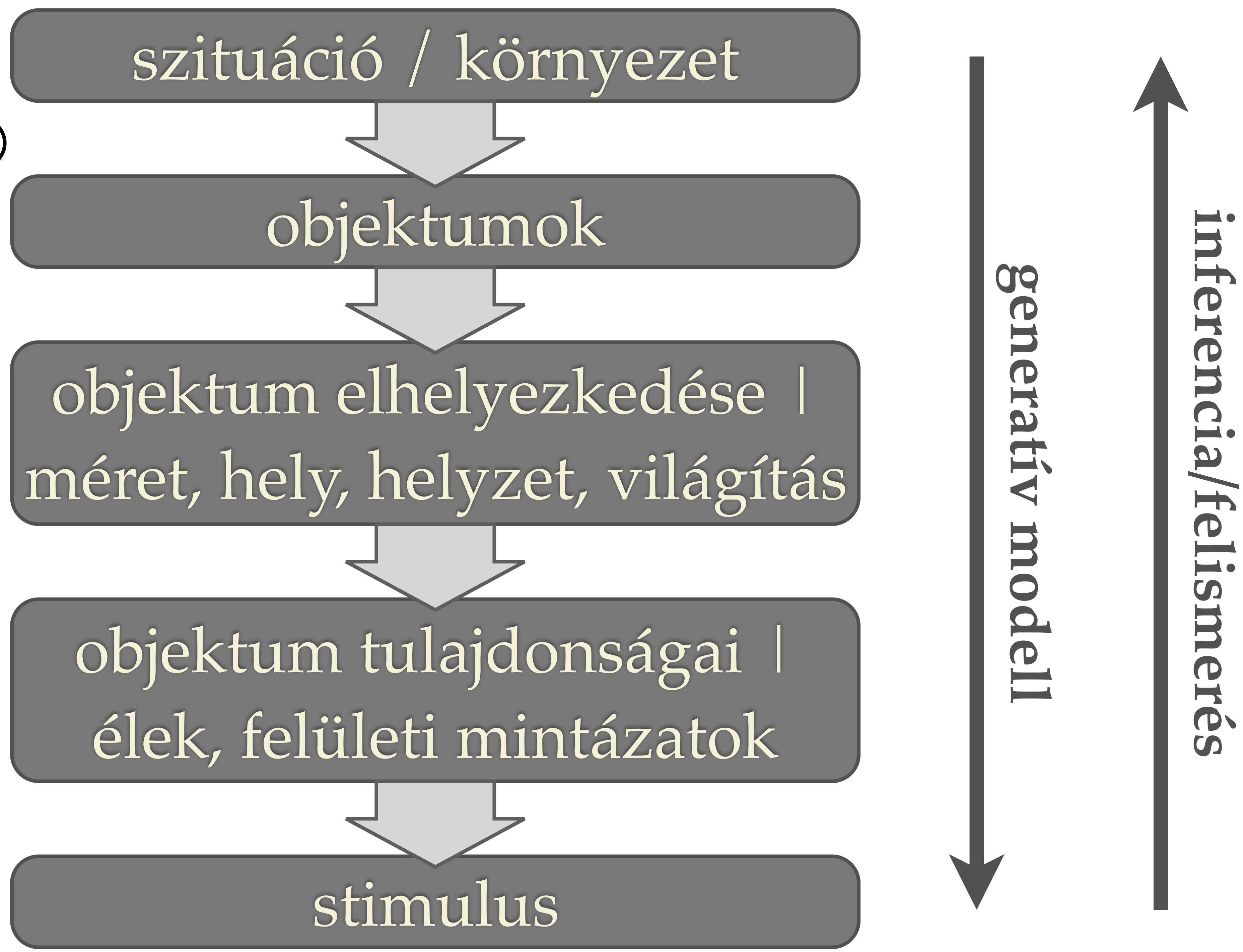
Tanulás és stimulus statisztika



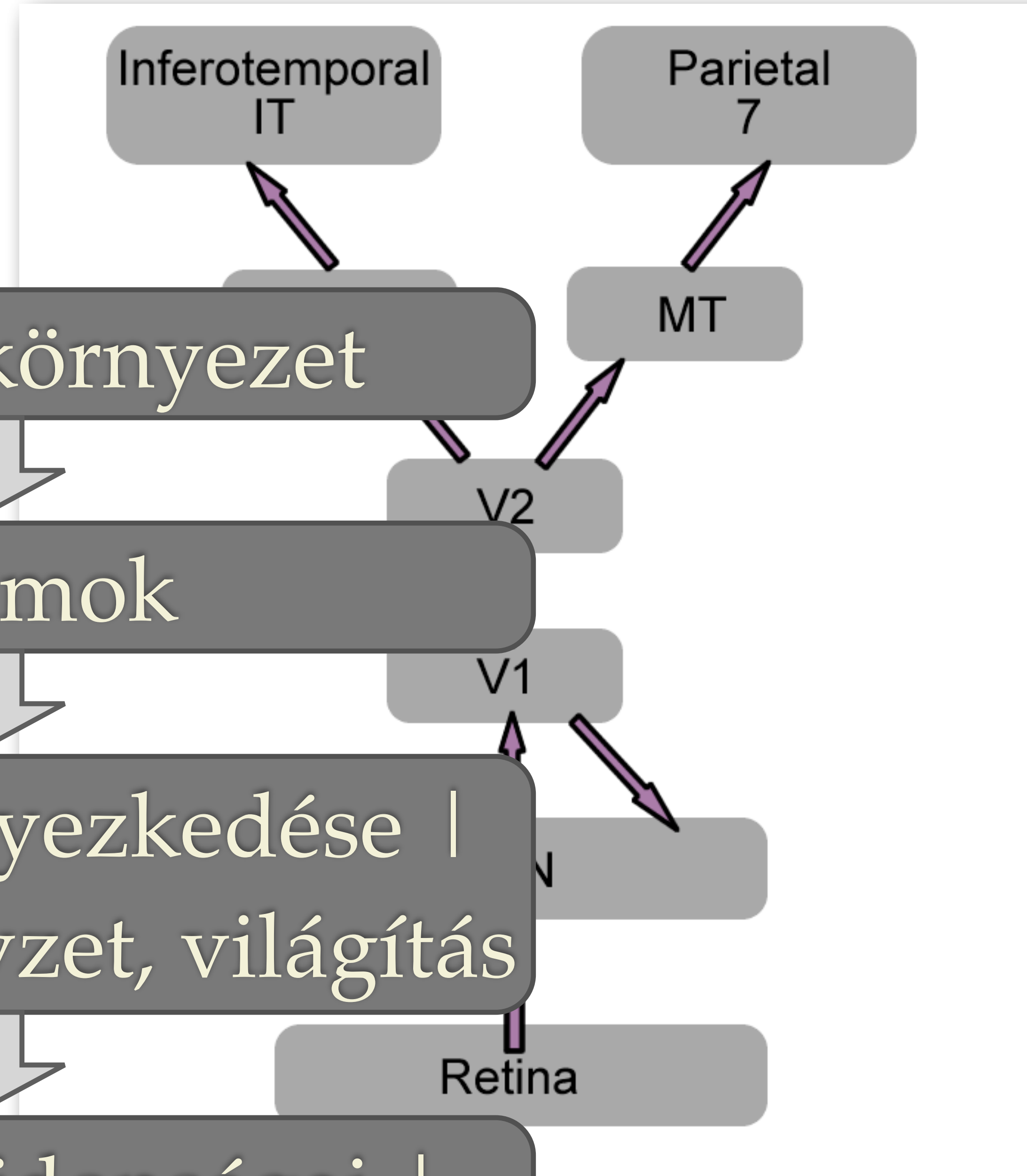
Generatív/rekogniciós modell

$$\mathrm{P}(\mathbf{x}) = \mathrm{P}(\mathbf{x} \,|\, \mathbf{z})\, \mathrm{P}(\mathbf{z})$$





Generatív/rekogniciós modell



szituáció / környezet

objektumok

objektum elhelyezkedése | méret, hely, helyzet, világítás

objektum tulajdonságai | élek, felületi mintázatok

stimulus

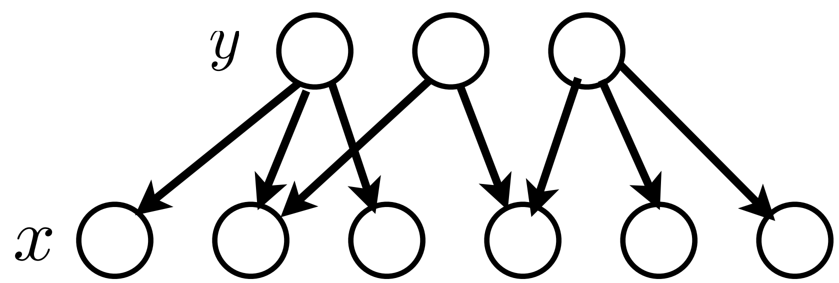
Modell definíció -> rekogníció:

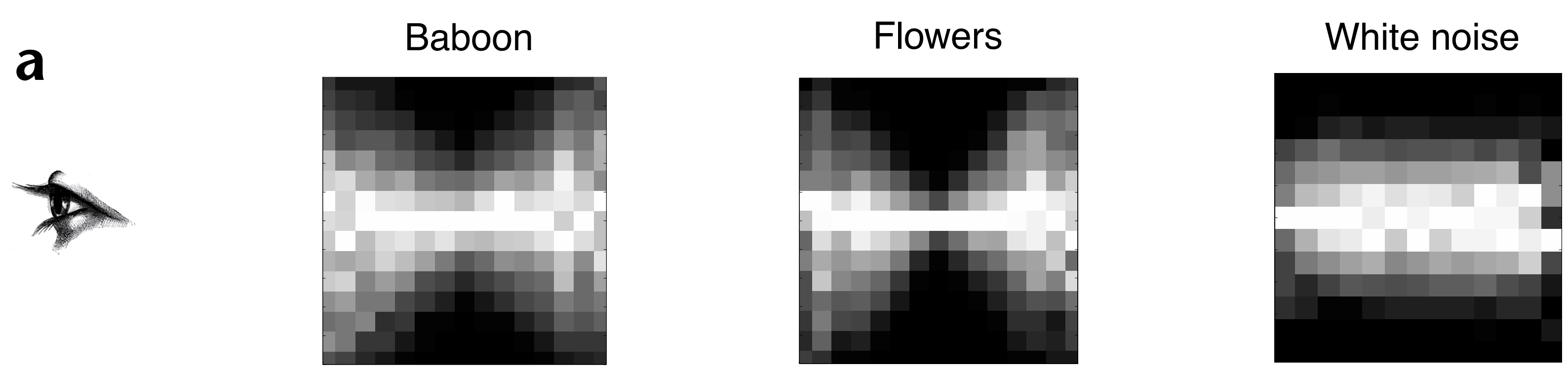
$$\mathrm{P}\left(x\,|z\right)$$

Inferencia igénye -> rekogníció:

$$\mathrm{P}\left(z\,|x\right)$$

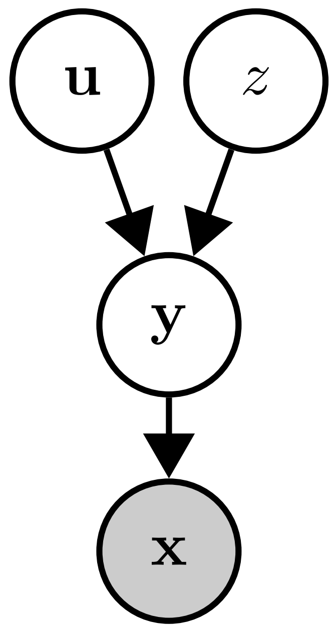
Independens komponensek





Schwartz & Simoncelli, 2001

Gaussian Scale Mixtures

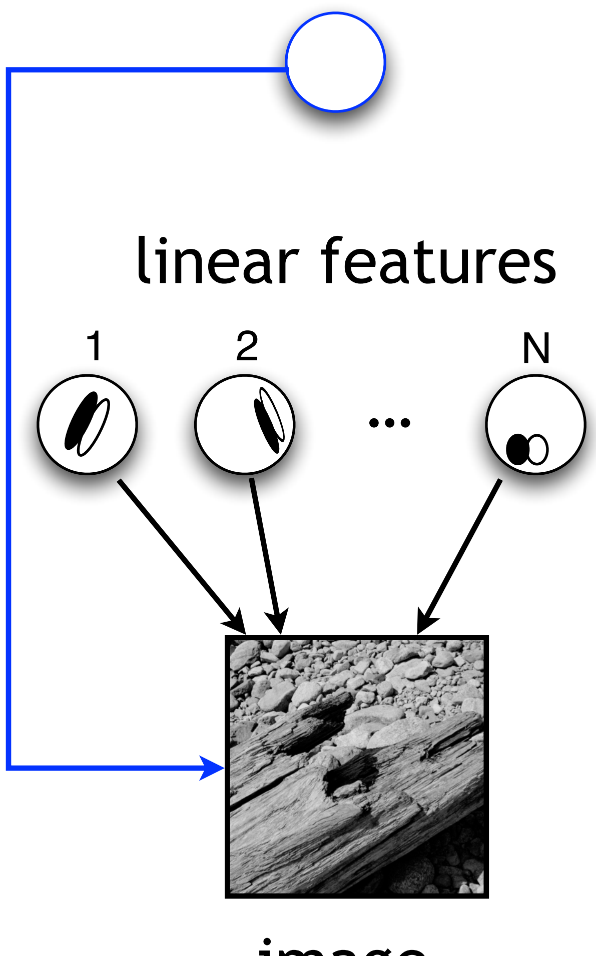


$$P(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_x^2 \mathbf{I})$$

$$\mathbf{y} = z\,\mathbf{u}$$

$$P(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$

$$P(z) = \mathrm{Gamma}(z; k, \theta)$$



$$\text{image} = \text{contrast} \times \left(a_1\, \text{feature}_1 + a_2\, \text{feature}_2 + \ldots + a_N\, \text{feature}_N + \text{noise} \right)$$

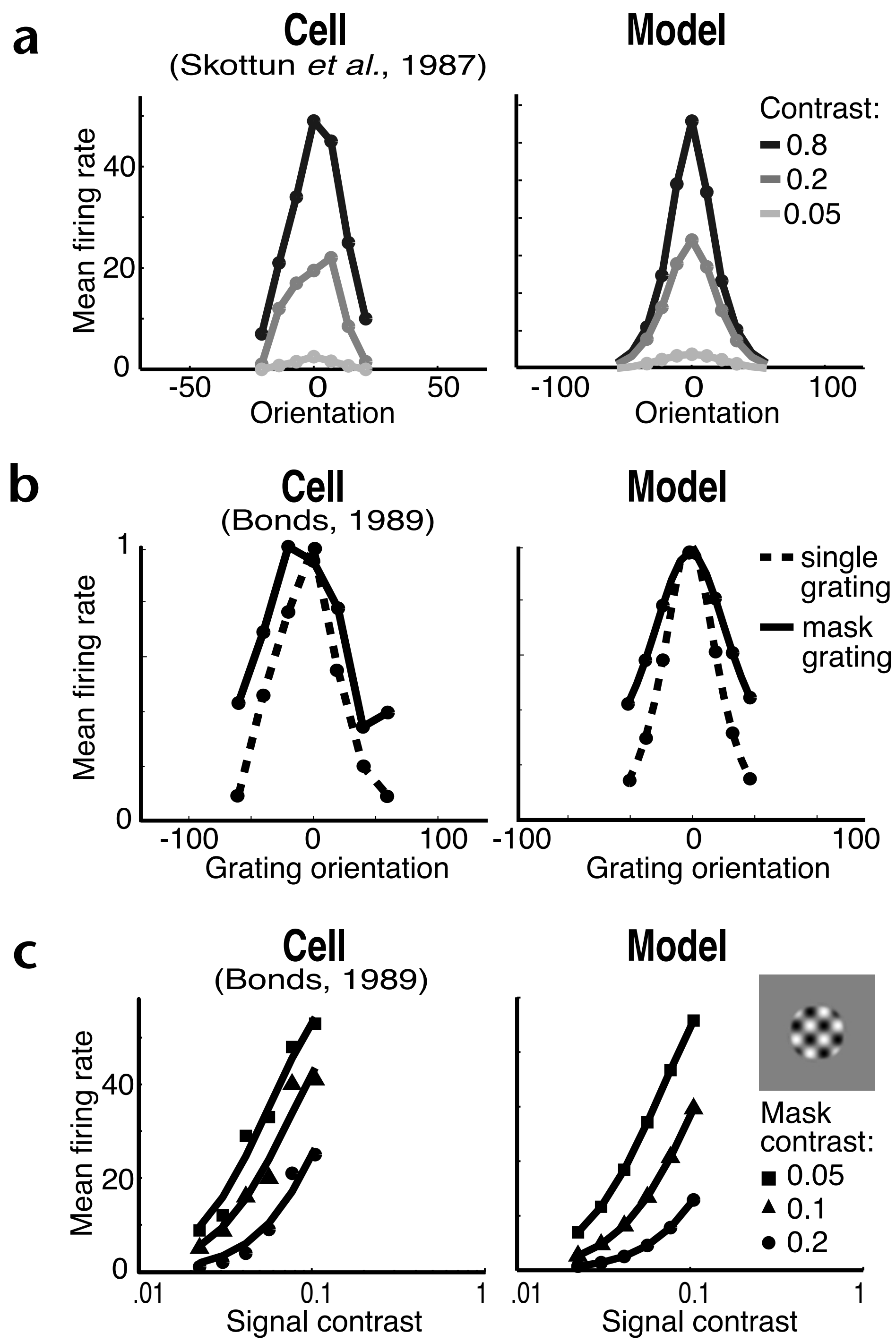
$$var\,(L_1 | L_2) = wL_2^2 + \sigma^2$$

$$R_1 = \frac{L_1^2}{wL_2^2 + \sigma^2}$$

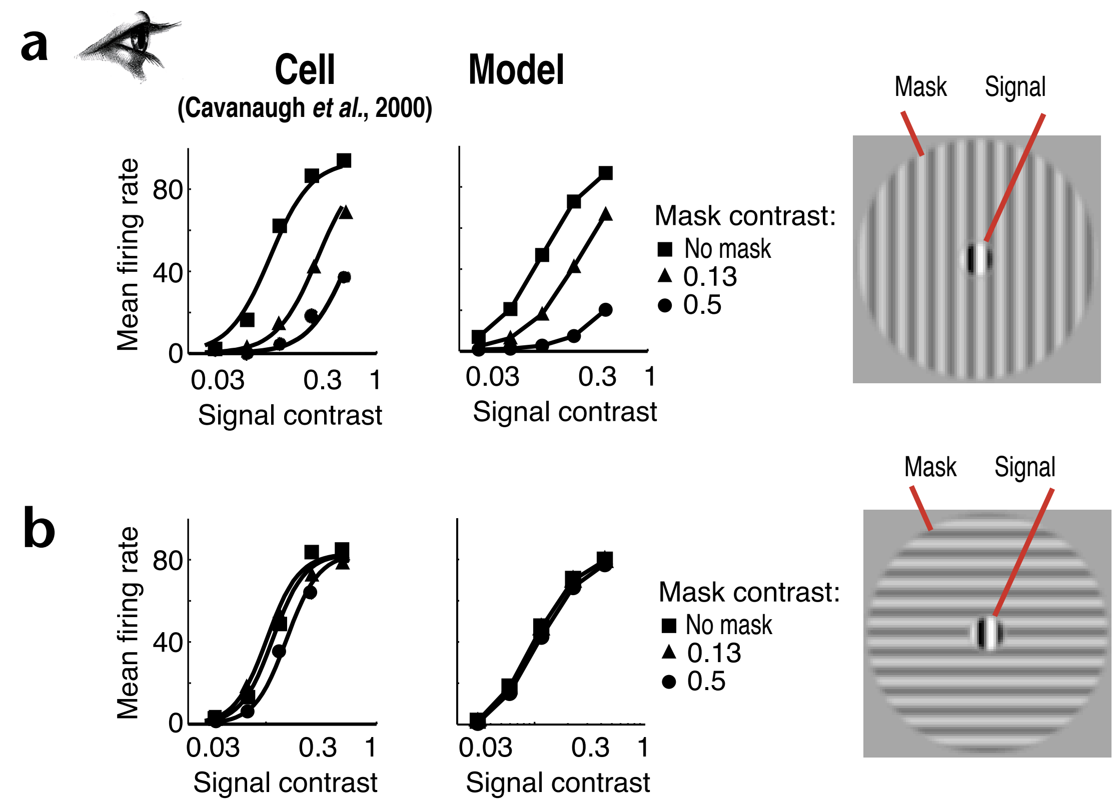
$$var\,(L_i | \{L_j, j \in N_i\}) = \sum w_{ji} L_j^2 + \sigma^2$$

$$R_i = \frac{L_i^2}{\sum_j w_{ji} L_j^2 + \sigma^2}$$

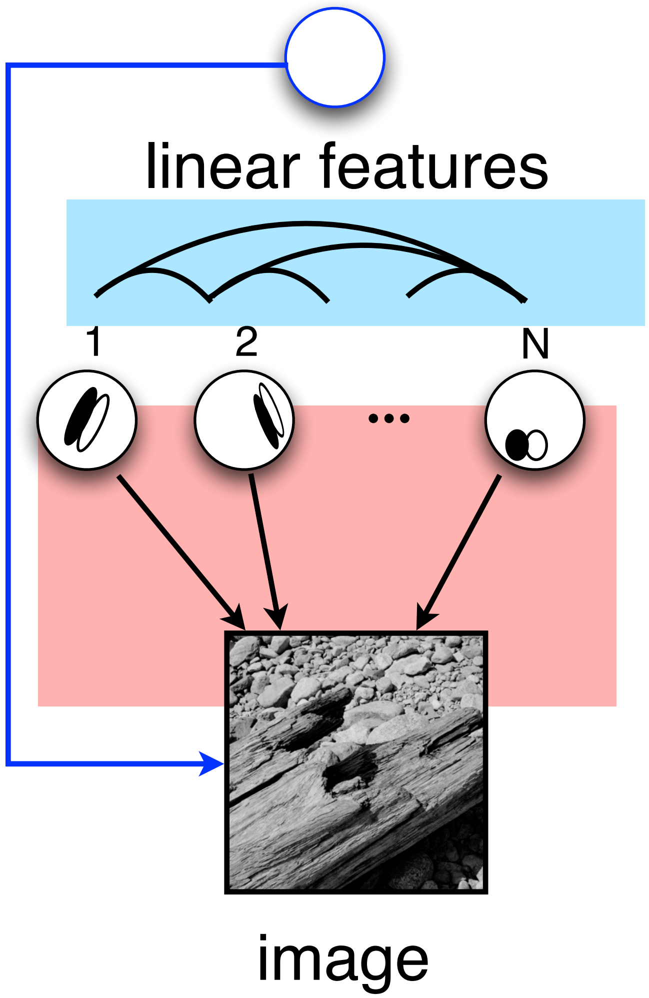
Neurális adatok és GSM



Schwartz & Simoncelli, 2001



Bayesian inference



expectation

neural activities

[a_1, a_2, ..., a_N]

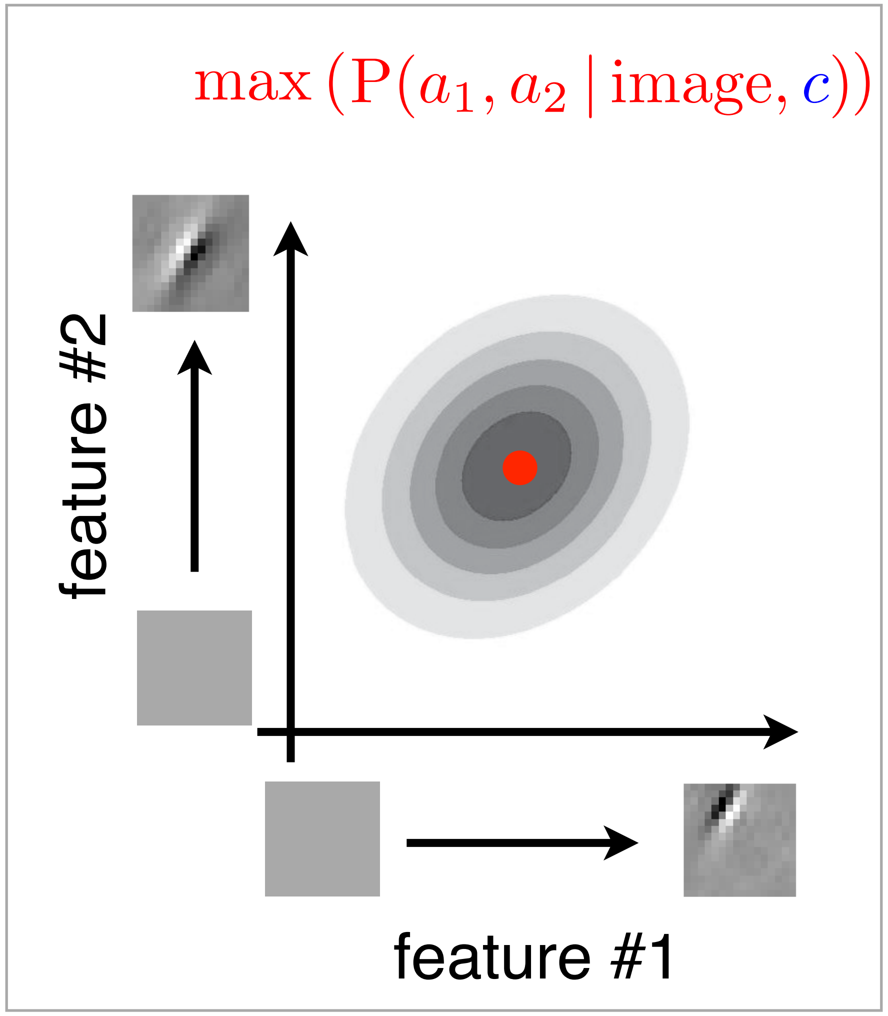
evidence

Demonstrated efficiency in:

- ★ pattern-completion
- ★ compression
- ★ denoising

the parametric form of both
evidence and expectation is determined by
natural image statistics

mean responses



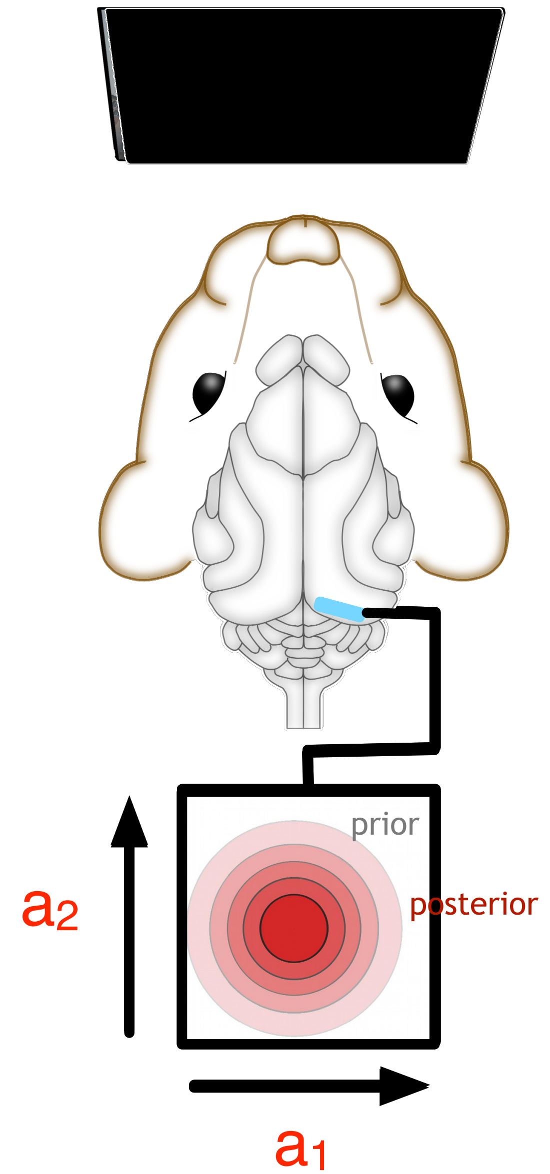
traditional theories
e.g. Olshausen & Field, Nature 1996,
Schwartz & Simoncelli, Nat Neurosci 2001

mean response $\rightsquigarrow$ maximum a posteriori inference

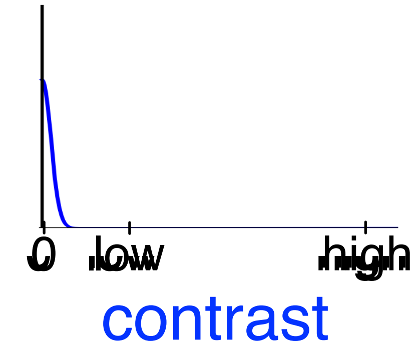
roadmap

- image model
- consequence of the representation of prior
- stimulus-dependence of variability
- stimulus dependence of covariability of multiple neurons

$$\underbrace{P(a_1, a_2, \ldots, a_N \mid \text{image}, c)}_{\text{posterior}} \propto \underbrace{P(a_1, a_2, \ldots, a_N \mid c)}_{\text{prior}} \times \underbrace{P(\text{image} \mid a_1, a_2, \ldots, a_N, c)}_{\text{sensory evidence}}$$



$P(\text{contrast}|\text{image})$



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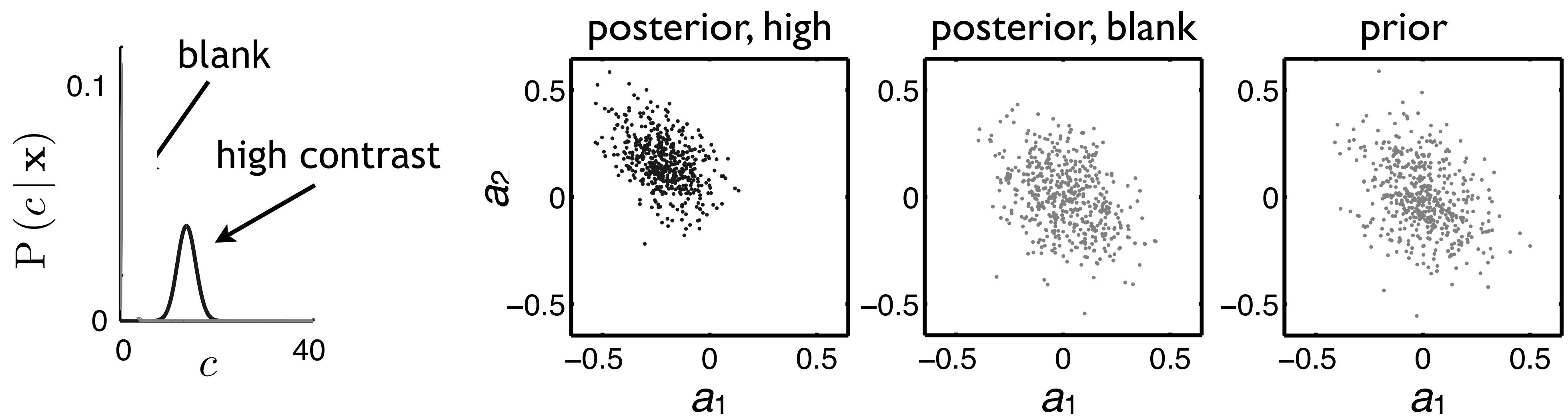
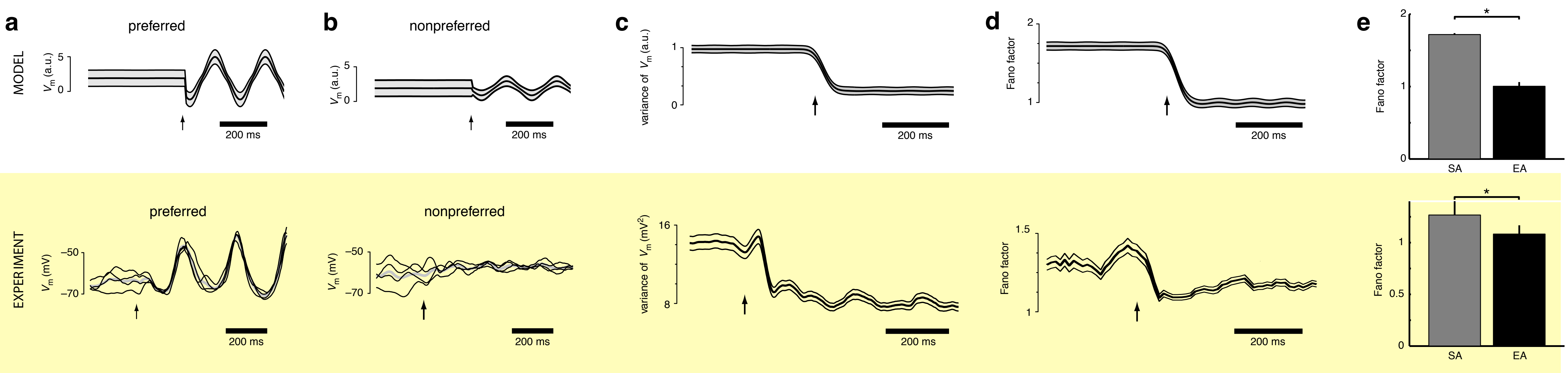
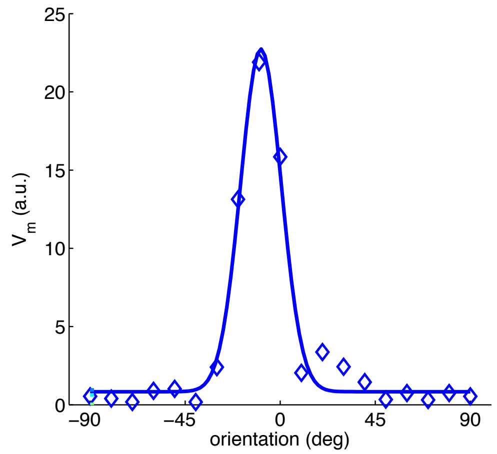
Stimulus onset quenches neural variability

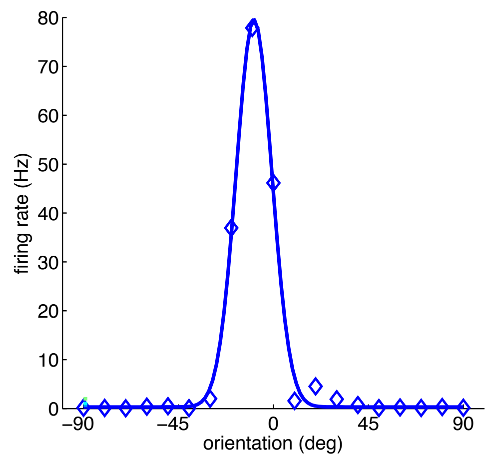


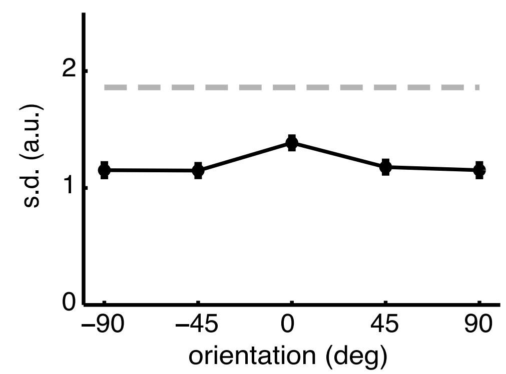


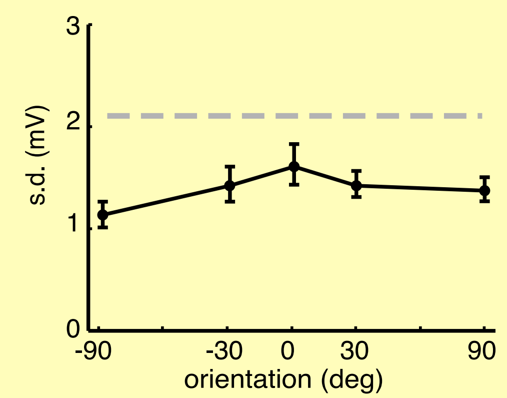
Orientation-dependence of response statistics

- orientation has a big impact on response mean
- however, no change in uncertainty is expected
- no significant change in variance is expected in membrane potential



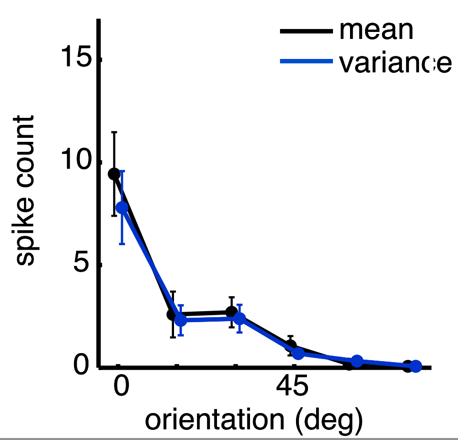


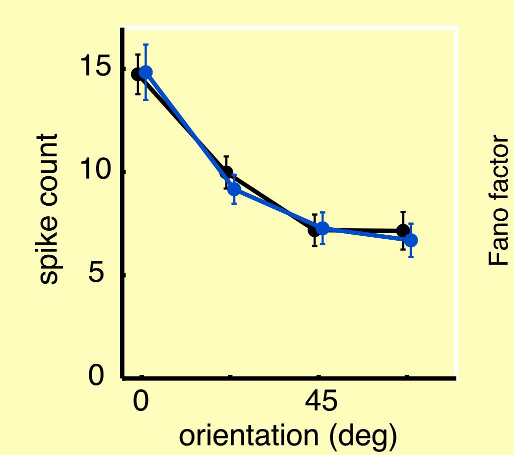


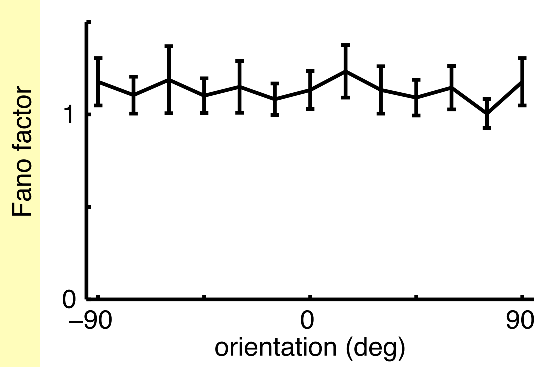


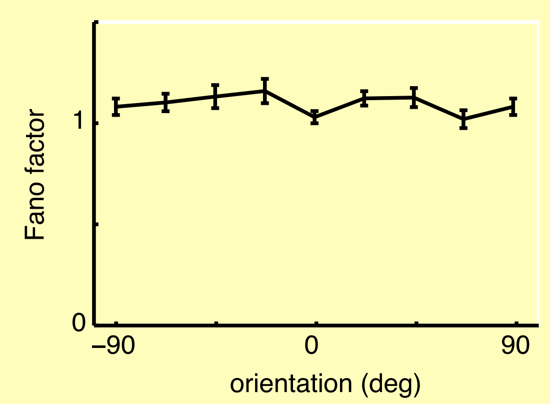
Finn et al, Neuron 2007

- spike count variance increases with firing rate
- Fano factor is still expected to be independent of orientation



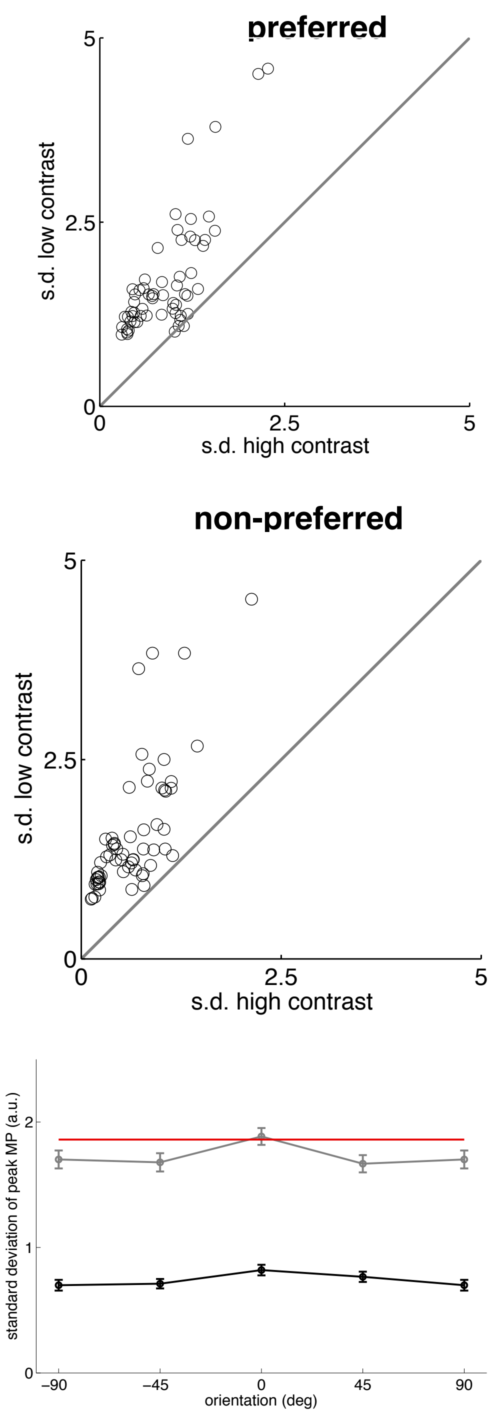


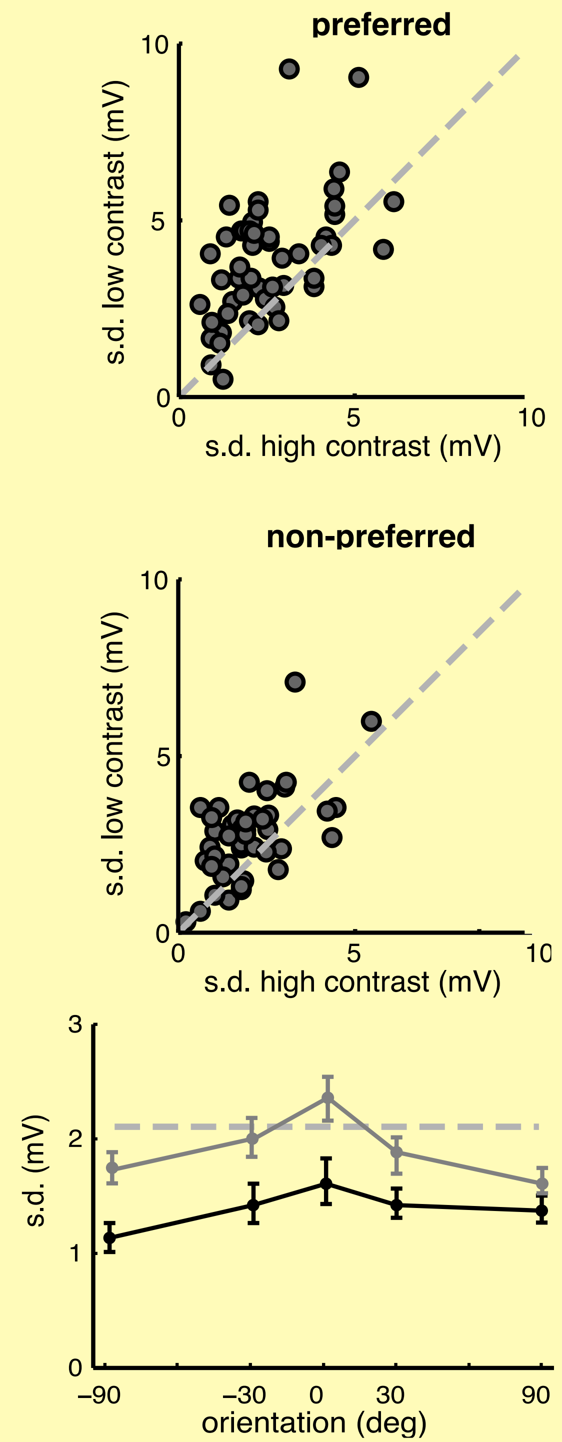




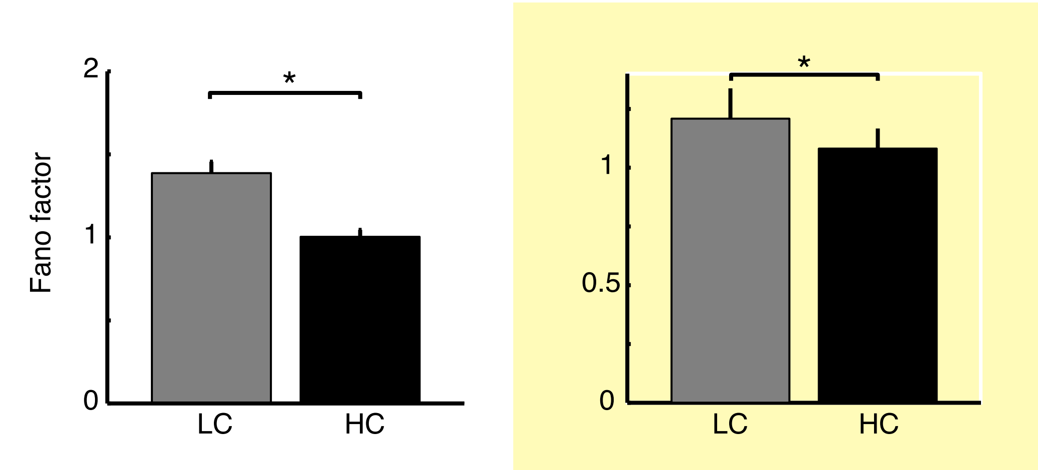
Contrast-dependence of response statistics

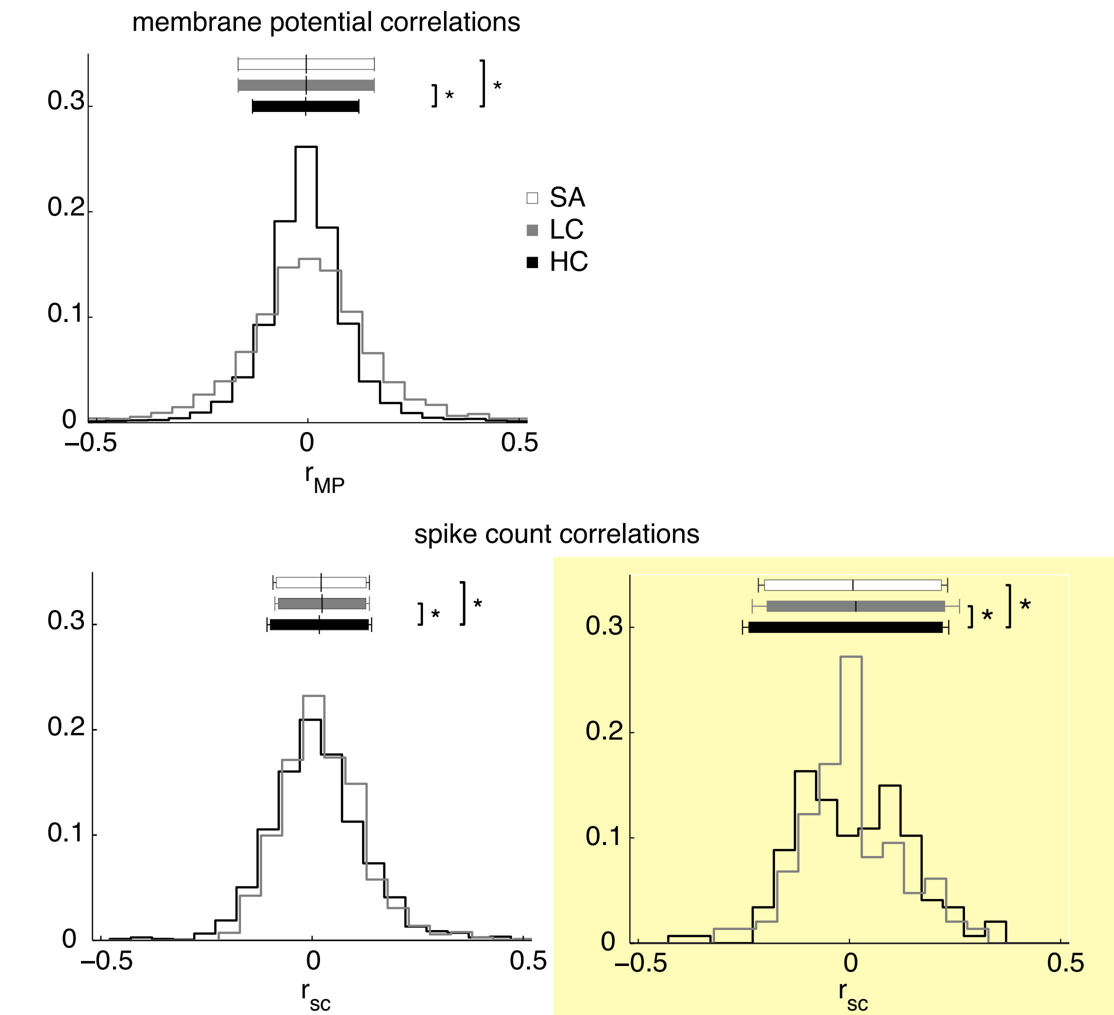
- contrast has fundamental effect on mean: *decreased* contrast results in *decreased* mean
- decreased *contrast* results in *increased* uncertainty





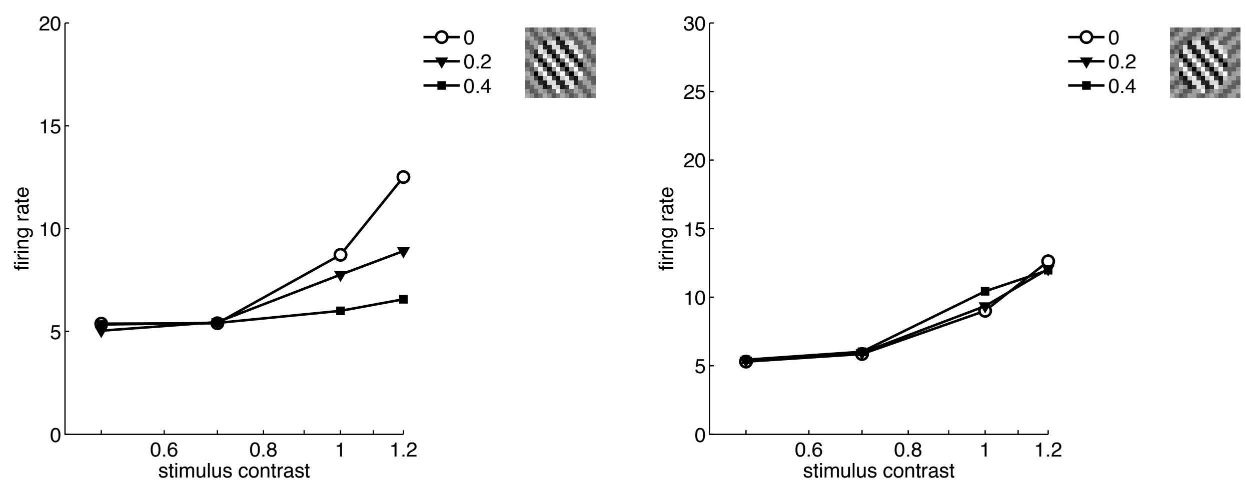
Finn et al, Neuron 2007

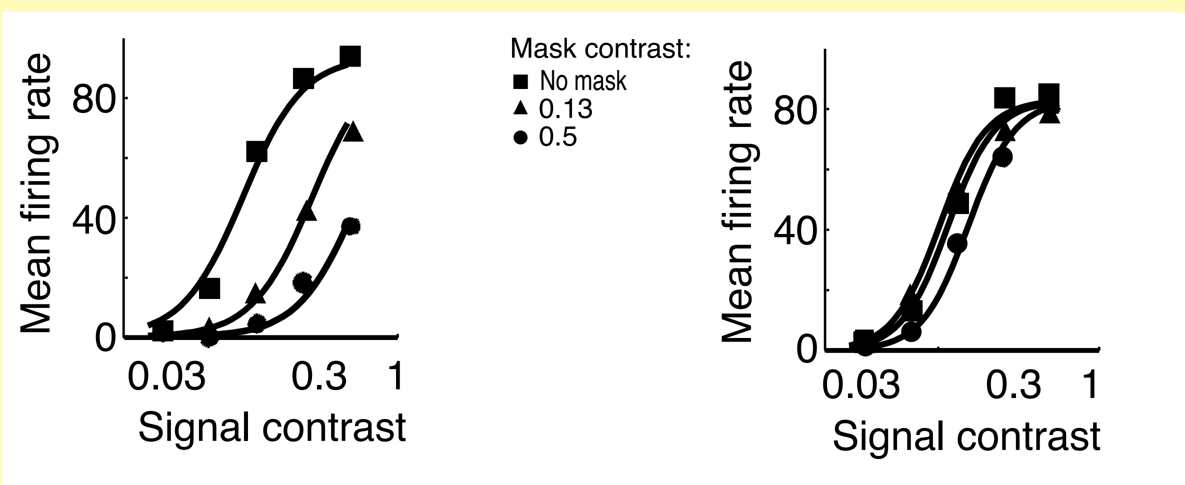




Non-classical RF dependence of response statistics

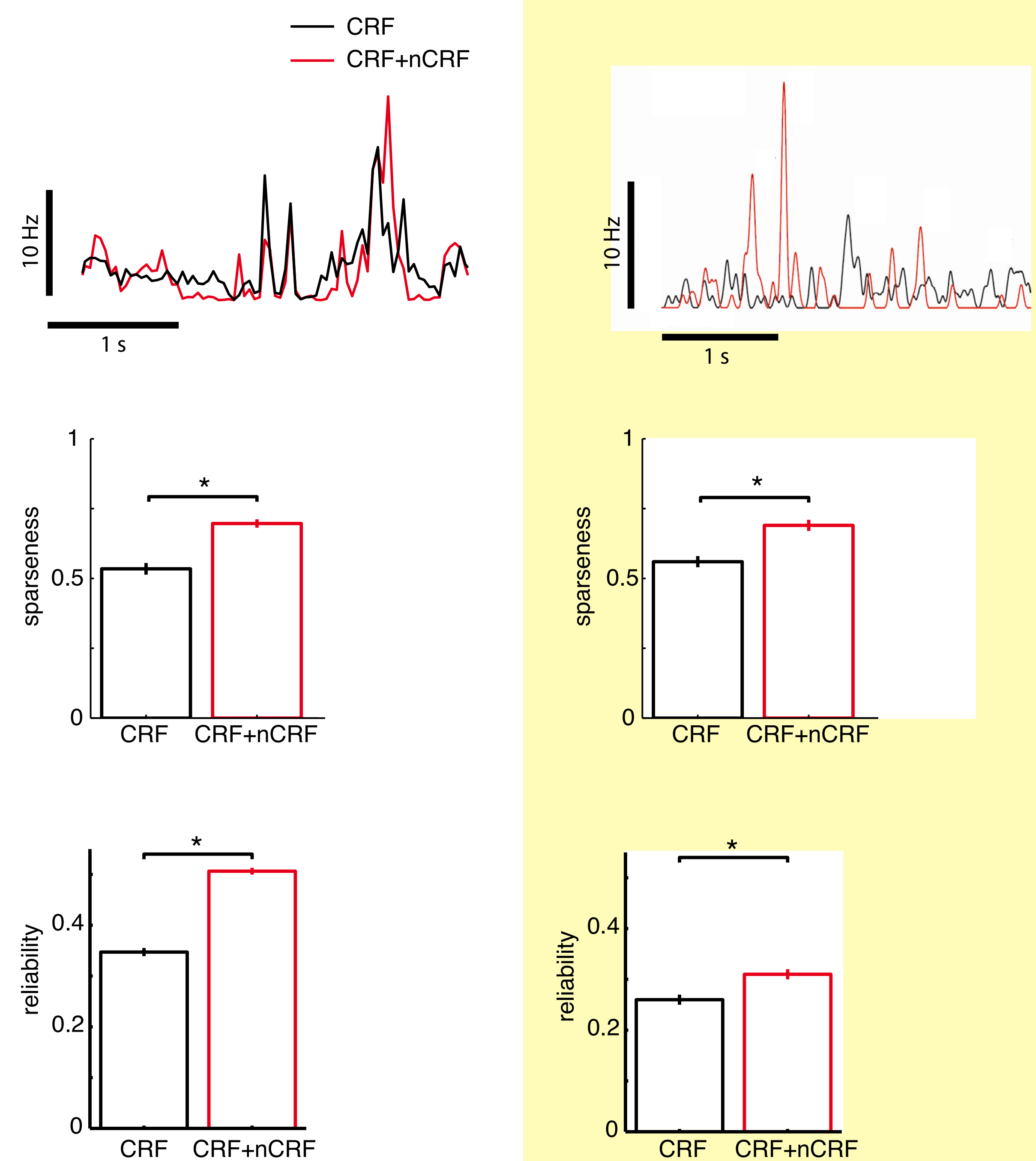
- non-linear interaction between with-receptive field and extra-receptive field stimulation





Cavanagh, 2000

- uncertainty is affected by extra information



Haider et al, Neuron 2010

roadmap

- image model
- consequence of the representation of prior
- stimulus-dependence of variability
- stimulus dependence of covariability of multiple neurons

Learning and correlations structure

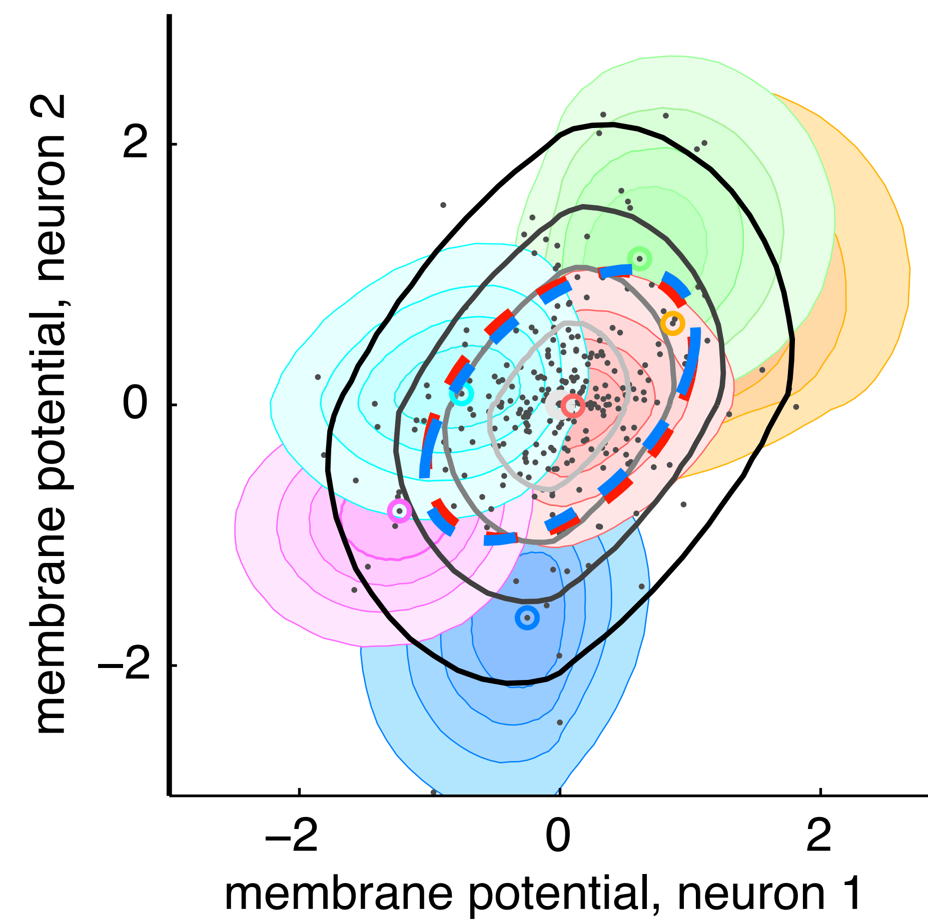
$$P(\text{responses} \mid \text{stimulus}) \propto P(\text{stimulus} \mid \text{responses}) \times P(\text{responses})$$

prior correlation

posterior correlation

signal correlation

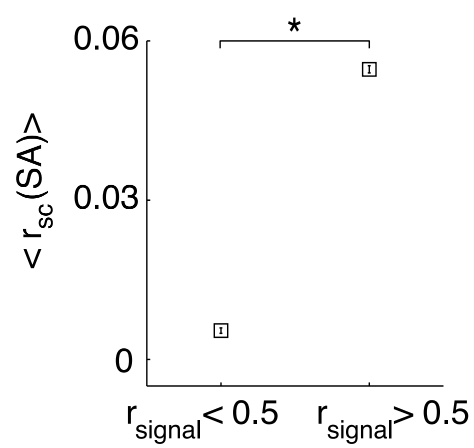
$$C^* \approx \frac{1}{T}\left(\sum_t \boldsymbol{\Sigma}(t) + \sum_t \mu(t)\,\mu^{\mathsf{T}}(t)\right)$$

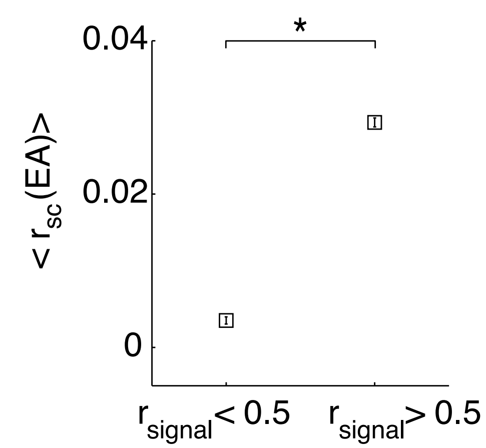


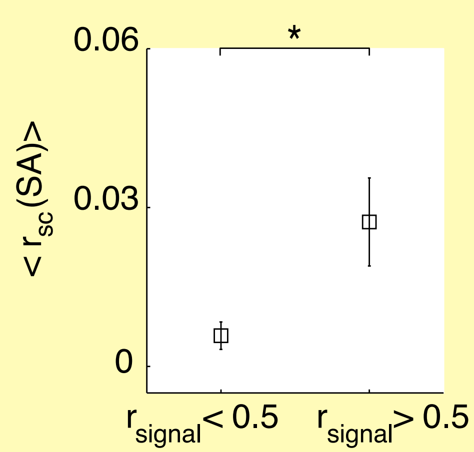
Relationship between various forms of correlations

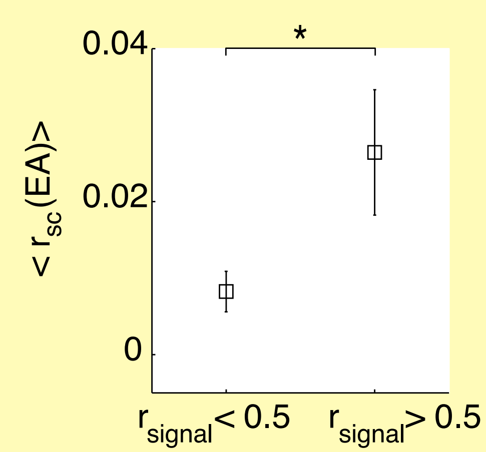
signal vs. spontaneous correlation

signal vs. noise correlation

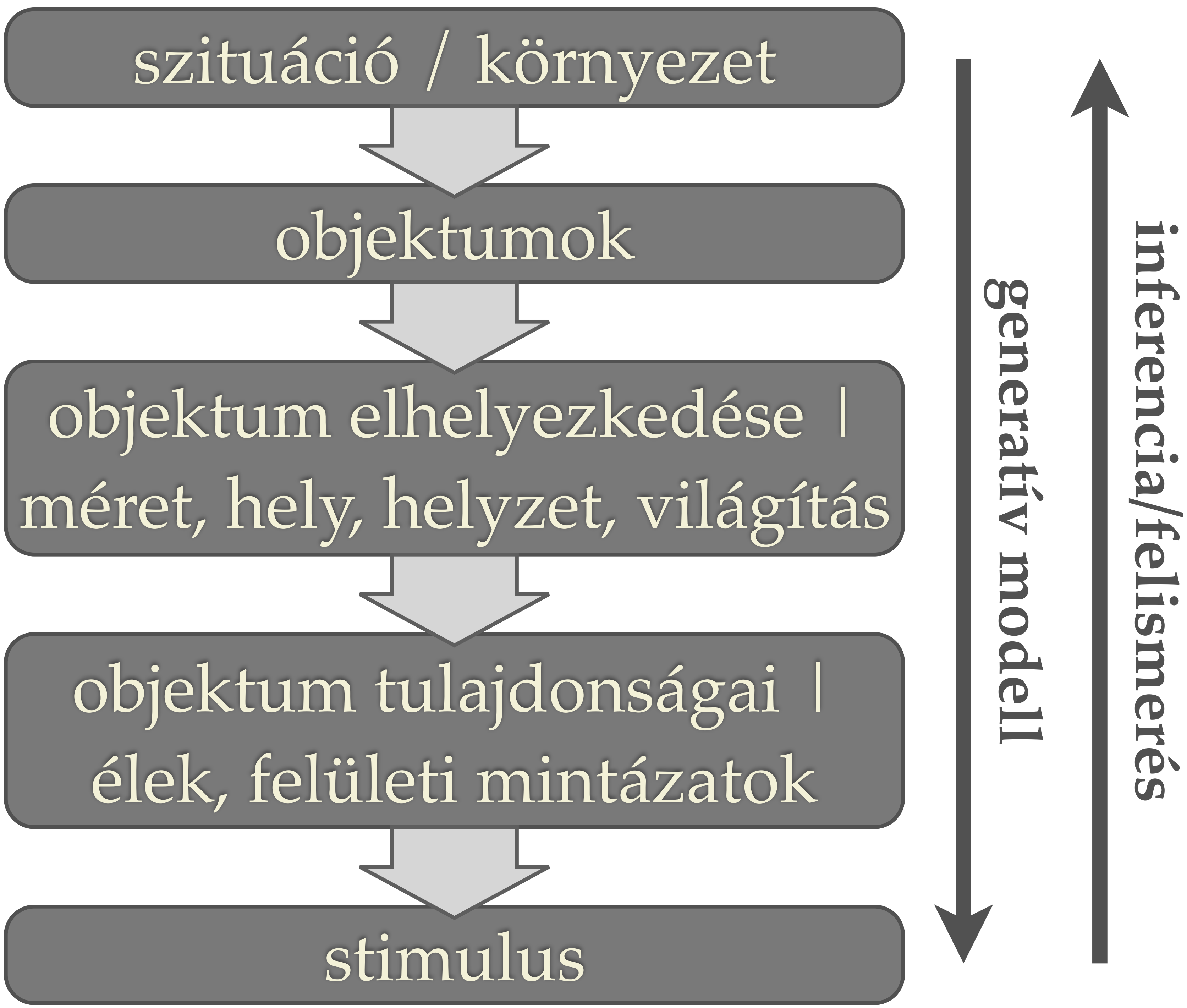


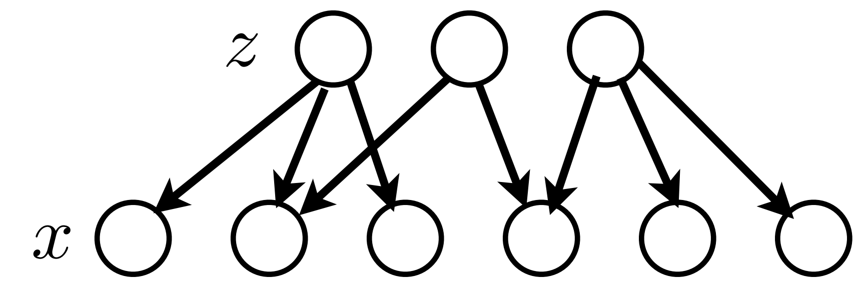






Bayes inferencia





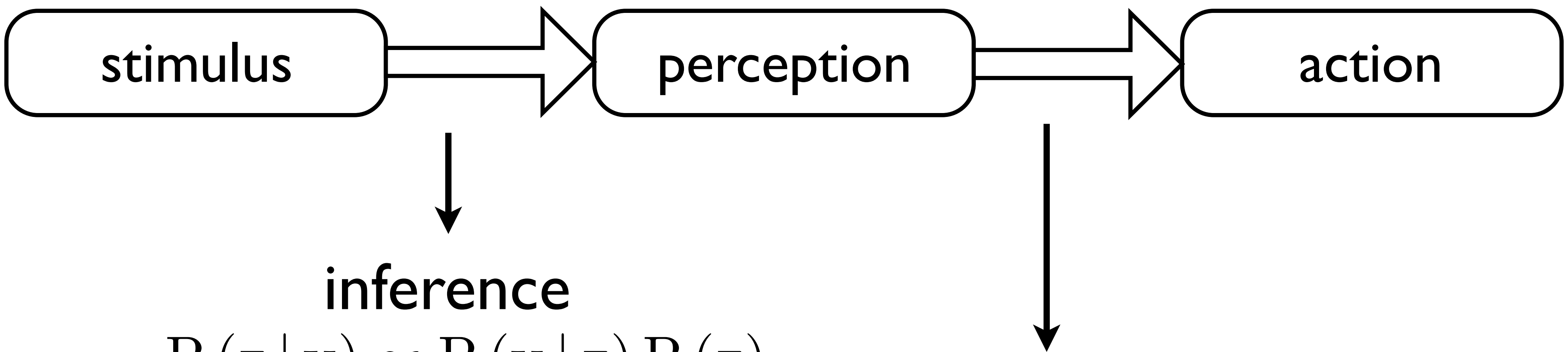
$$\mathrm{P}(\mathbf{z} \mid \mathbf{x}) \propto \mathrm{P}(\mathbf{x} \mid \mathbf{z})\, \mathrm{P}(\mathbf{z})$$

Eddig arra koncentráltunk, hogy mi a legvalószínűbb aktivitás

Ez a maximum a posteriori becslés (MAP)

Bayes inferencia

Miért érdekes a poszterior eloszlás?



inference

$$\mathrm{P}\left(\mathbf{z} \,|\, \mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x} \,|\, \mathbf{z}\right) \mathrm{P}\left(\mathbf{z}\right)$$

decision making

$$R(a) = \int dz\, U(z, a)\, \mathrm{P}(\mathbf{z} \,|\, \mathbf{x})$$

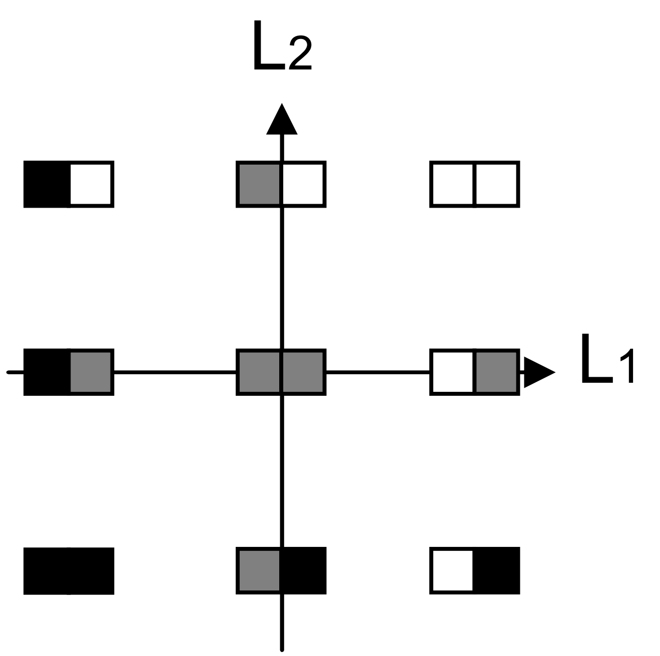
Lineáris modellek

$$\mathrm{P}\left(x\,|\,z\right) = \mathrm{Normal}\left(x; z, \theta\right) = C \exp\left(\left(x - Az\right)^T \Sigma^{-1} \left(x - Az\right)\right)$$

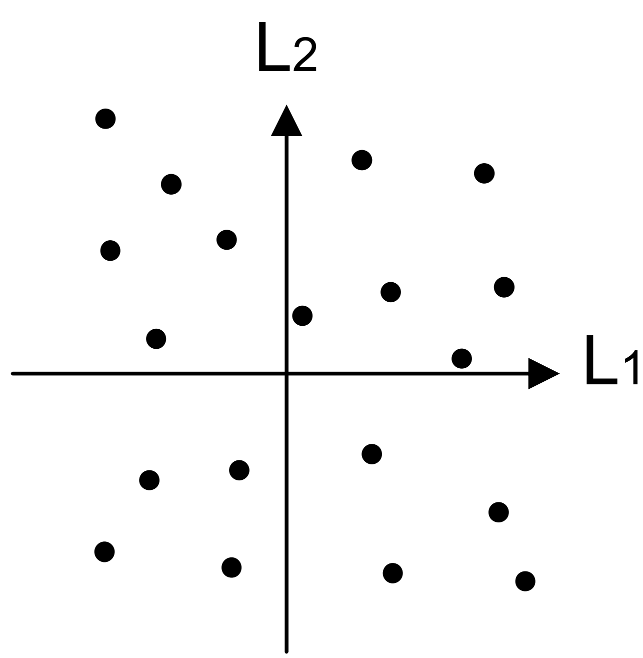
$$x = \mathbf{A} \cdot z + \epsilon$$

PCA

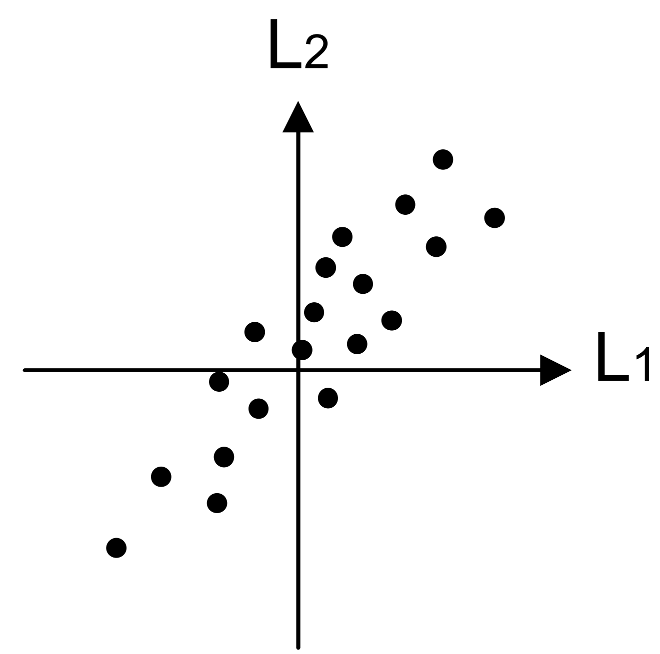
- A oszlopvektorai ortogonalisak
- D(x) = D(z)
- Izotróp zaj



State space of two pixel images



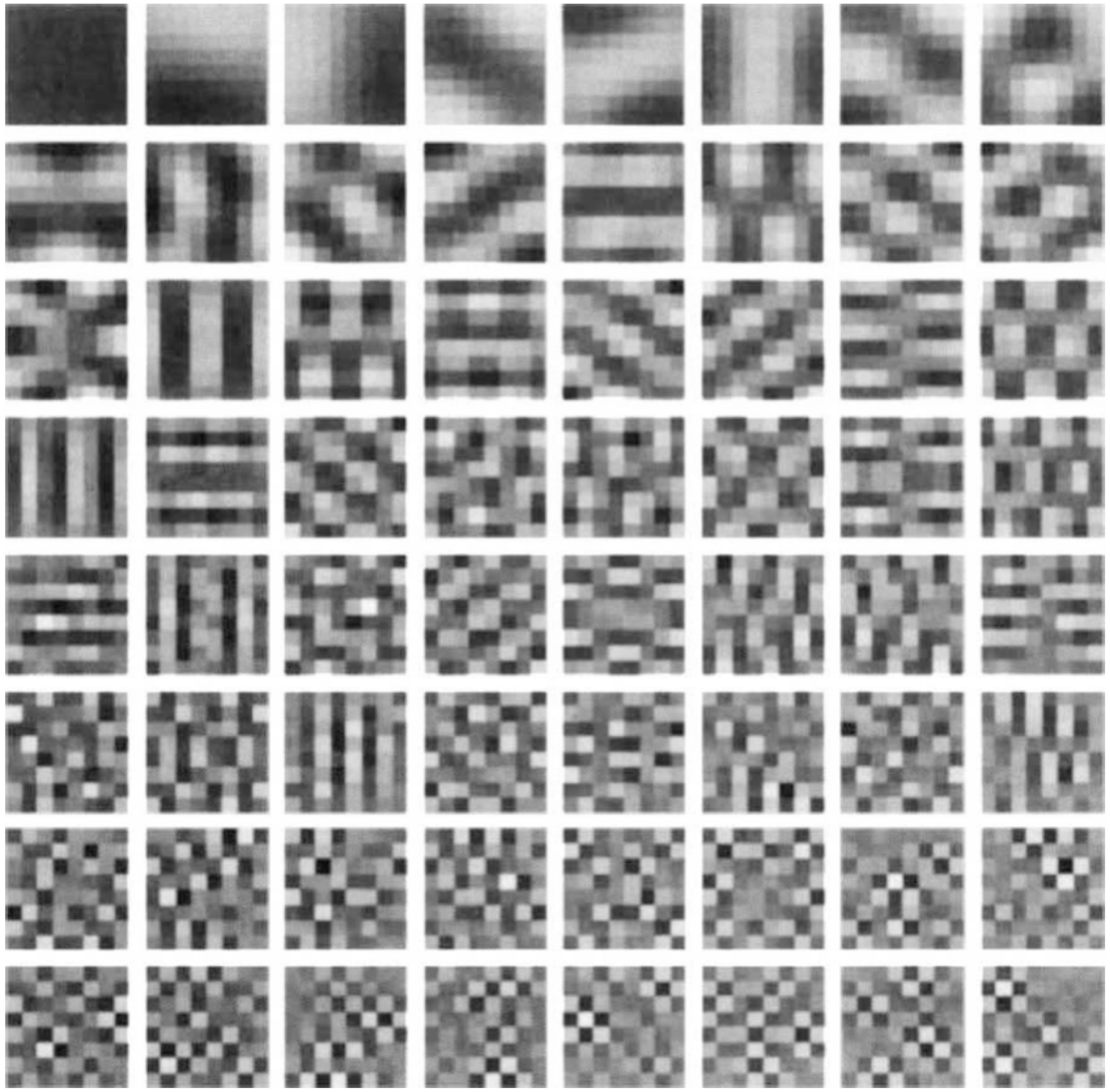
Random images



Structured images

PCA tulajdonságok

- Kompakt kódot eredményez
- Egy adatponért leírásáért általában a teljes hálózat felel



Sparse kódolás, ICA

$$x = \mathbf{A} \cdot z + \epsilon$$

- “z”-k függetlenek
- y priorja “ritka”(P(z))

Komputációs kritériumok:

- Hiteles rekonstrukció
  költség egy adatpontra (képre):
  $$\text{cost}_1 = \left(x - \sum_i A'_i \cdot z_i \right)^2$$
- Kis “energiafelhasználás (kevés szimultán aktiv neuron)
  további költség a kód “ritkasága”:
  $$\text{cost}_2 = -\sum_i S\left(\frac{z_i}{\sigma}\right)$$
  S a Gauss-nál nagyobb kurtózissal bíró eloszlás
- teljes költség (~energia):
  $$E = -\text{cost}_1 - \lambda \text{cost}_2$$

Sparse kód tanulása: E-M

Algoritmus:

- Itáráció EM lépésekkel
- Random kezdeti feltételek
- Adott konnektivitási mátrixnál az aktiviások segítségével a költség minimalizálása
- Adott aktivitásokkal a költség minimalizálása a súlyok adaptálásával

Adott konnektivitási mátrix esetén a legjobb aktivitások megtalalása:

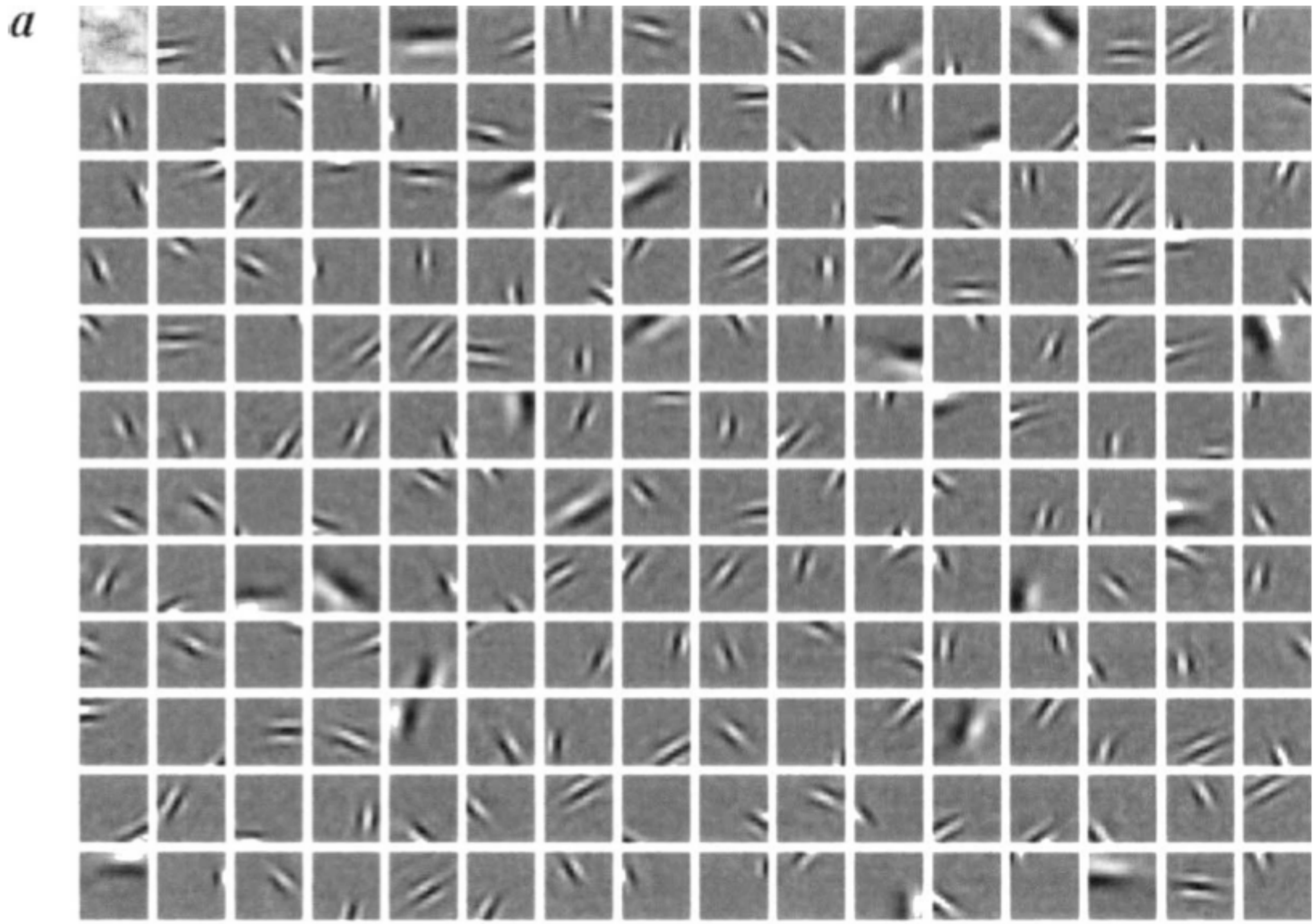
$$\dot{z}_i = \mathbf{A}_i x_t - \sum_j \mathbf{A}_i' \mathbf{A}_j z_j - \frac{\lambda}{\sigma} S' \left(\frac{z_i}{\sigma} \right)$$

Adott konnektivitási aktivációk esetén a legjobb súlyok megtalalása:

$$\Delta A_i = \eta \left\langle a_i \left[x - \hat{x} \right] \right\rangle_t$$

Sparse kódolás: eredmény

tréningezés természetes képekkel

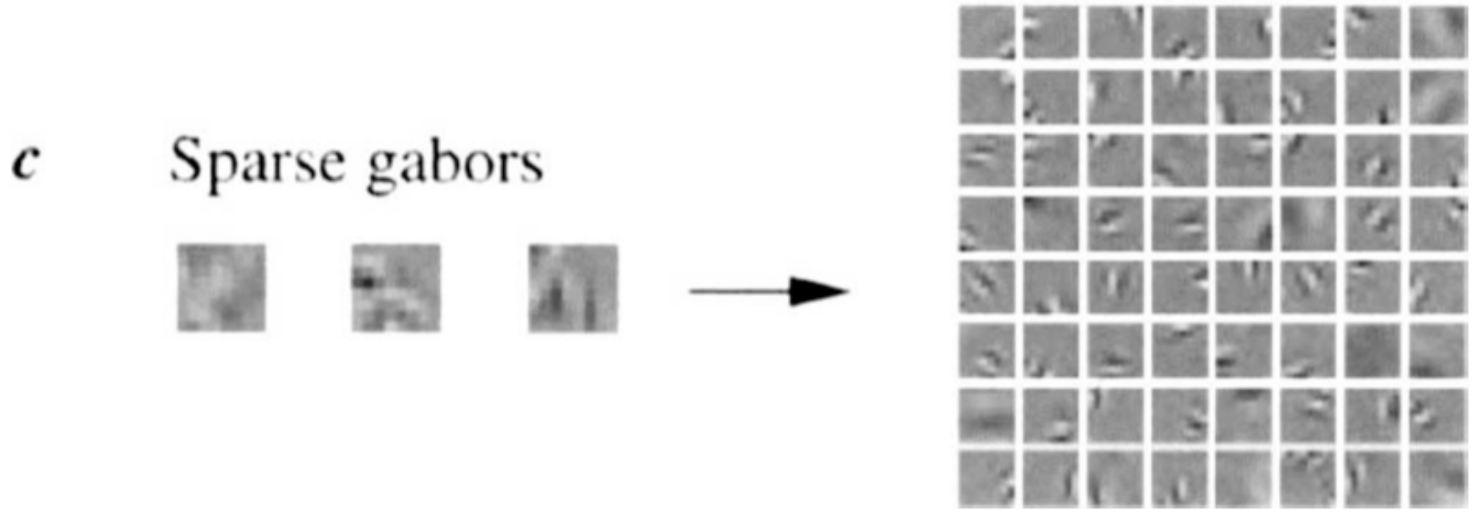


Olshausen & Field '96

A kialakult bázis:

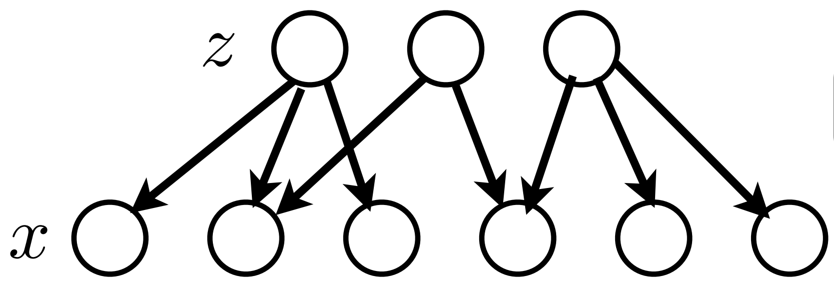
- irányított
- térbeli sávszűrést valósít meg
- lokalizált

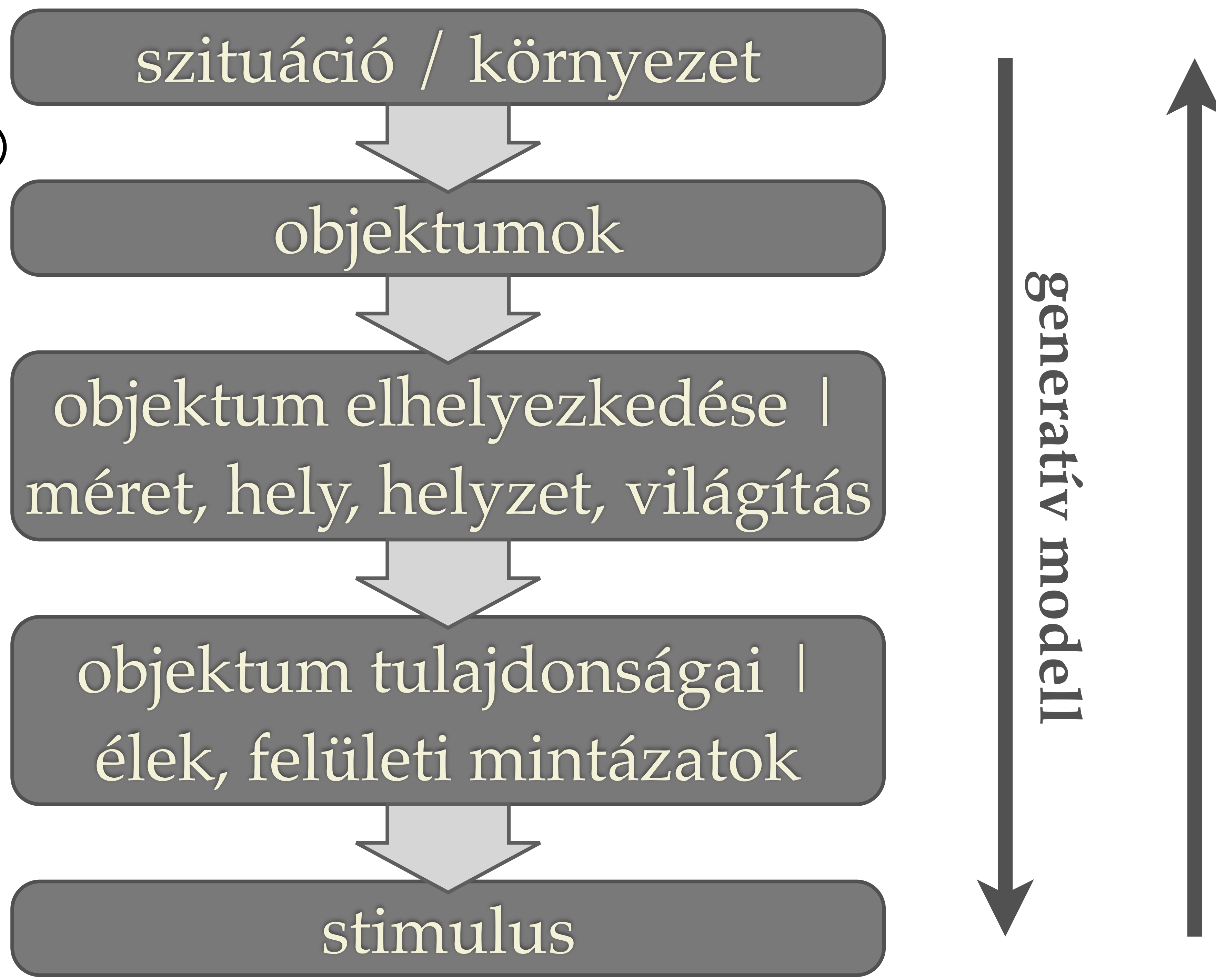
Tanulás és stimulus statisztika



Generatív/rekogniciós modell

$$\mathrm{P}\left(\mathbf{x}\right) = \mathrm{P}\left(\mathbf{x}\,|\,\mathbf{z}\right)\mathrm{P}\left(\mathbf{z}\right)$$





Generatív/rekogniciós modell

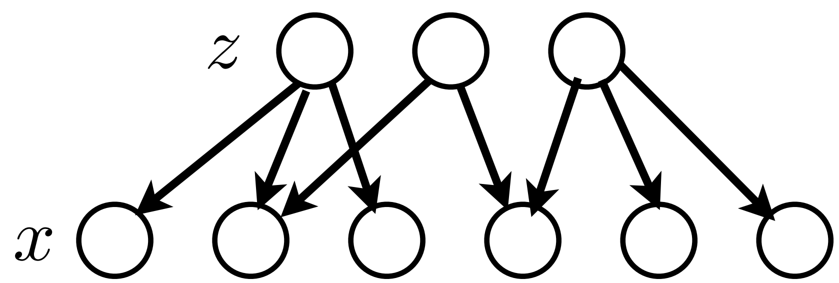
Modell definíció -> rekogníció:

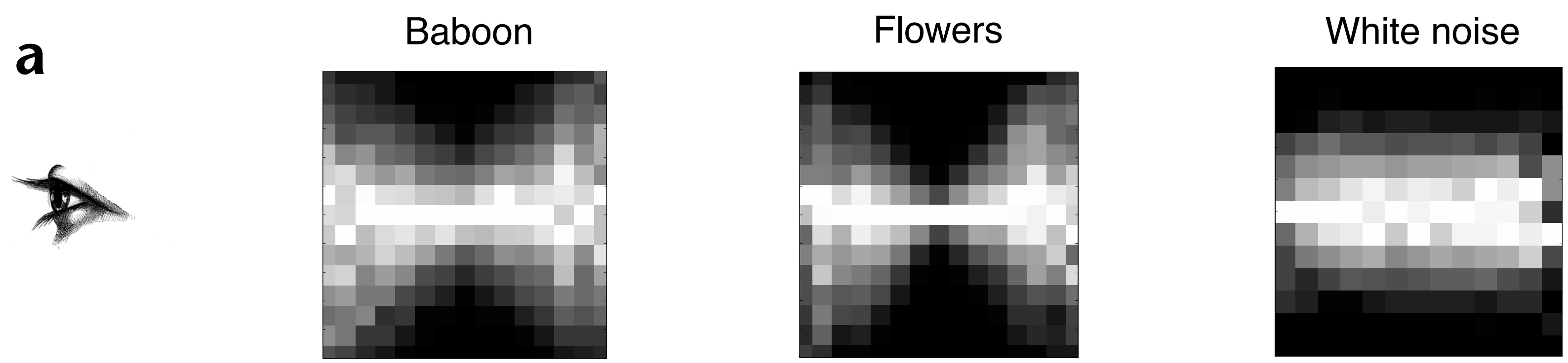
$$\mathrm{P}\left(x\,|z\right)$$

Inferencia igénye -> rekogníció:

$$\mathrm{P}\left(z\,|x\right)$$

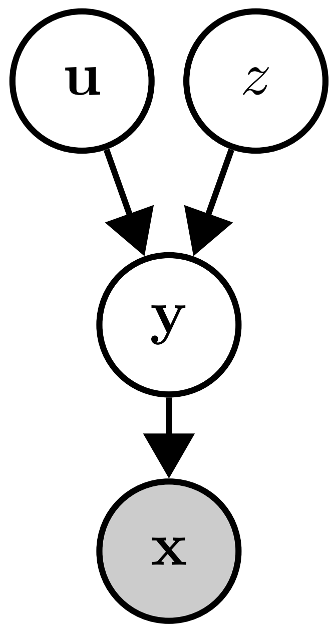
Independens komponensek





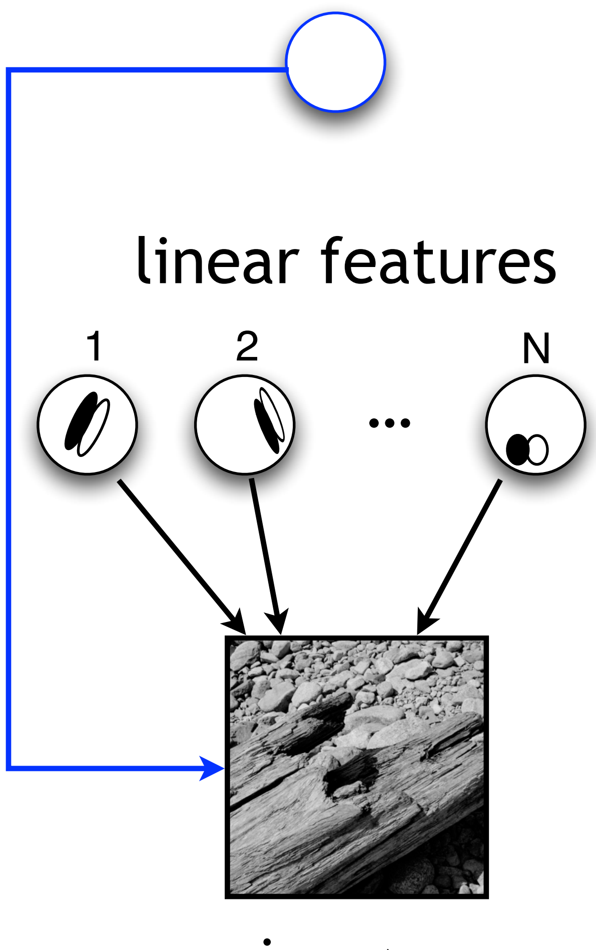
Schwartz & Simoncelli, 2001

Gaussian Scale Mixtures



$$\mathrm{P}(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_{\mathrm{x}}^2\mathbf{I})$$

$$\mathbf{y} = z\,\mathbf{u}$$
$$\mathrm{P}(\mathbf{u}) = \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C})$$
$$\mathrm{P}(z) = \mathrm{Gamma}(z; k, \theta)$$



$$\text{image} = \text{contrast} \times \left(a_1\,\text{feature}_1 + a_2\,\text{feature}_2 + \ldots + a_N\,\text{feature}_N + \text{noise} \right)$$

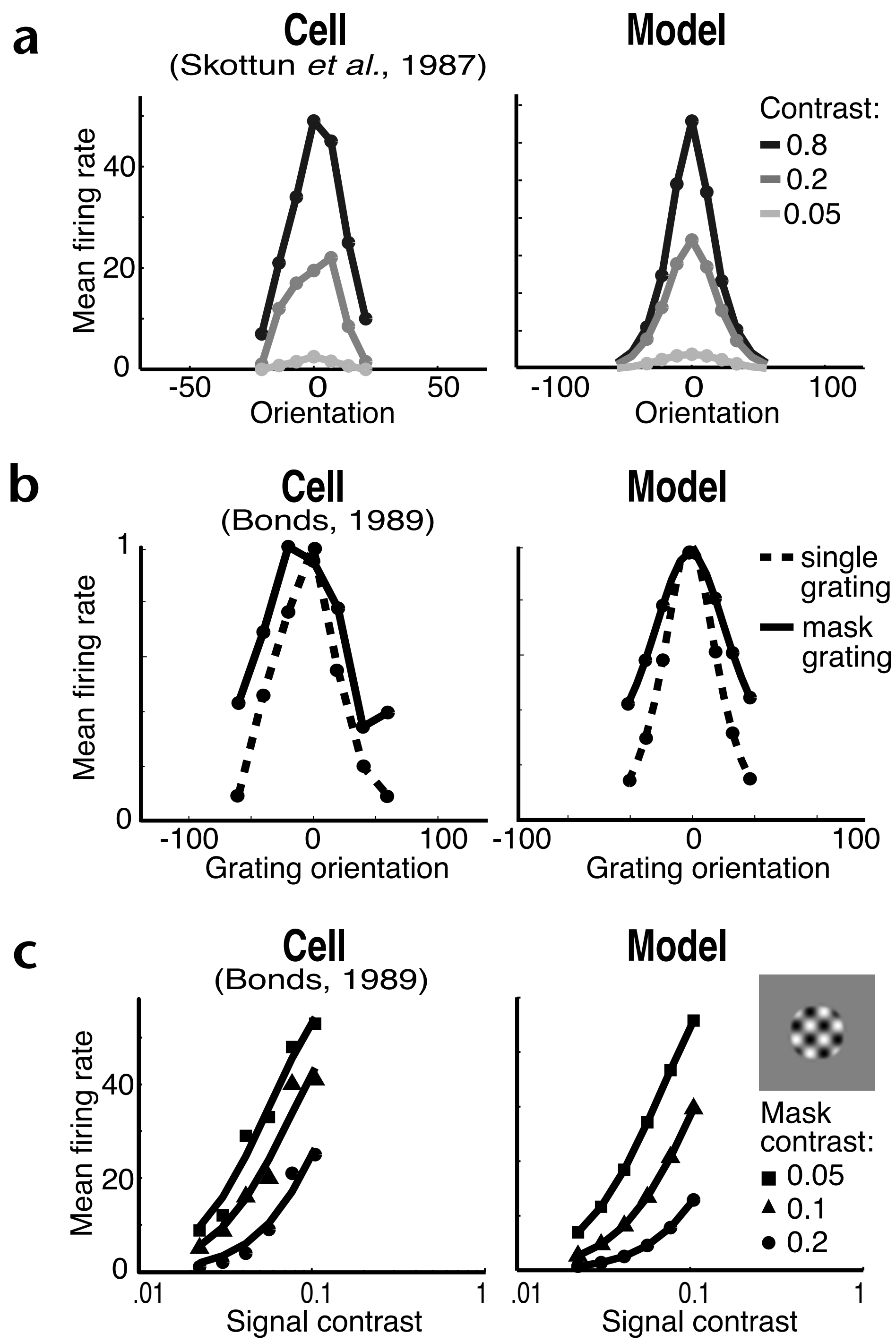
$$var\,(L_1|\,L_2) = wL_2^2 + \sigma^2$$

$$\mathrm{R}_1 = \frac{L_1^2}{wL_2^2 + \sigma^2}$$

$$var\,(L_i|\{L_j, j \in N_i\}) = \sum \mathrm{w}_{ji}\mathrm{L}_j^2 + \sigma^2$$

$$R_i = \frac{L_i^2}{\sum_j \mathrm{w}_{ji}\mathrm{L}_j^2 + \sigma^2}$$

Neurális adatok és GSM



Schwartz & Simoncelli, 2001