Statisztikus tanulás az idegrendszerben

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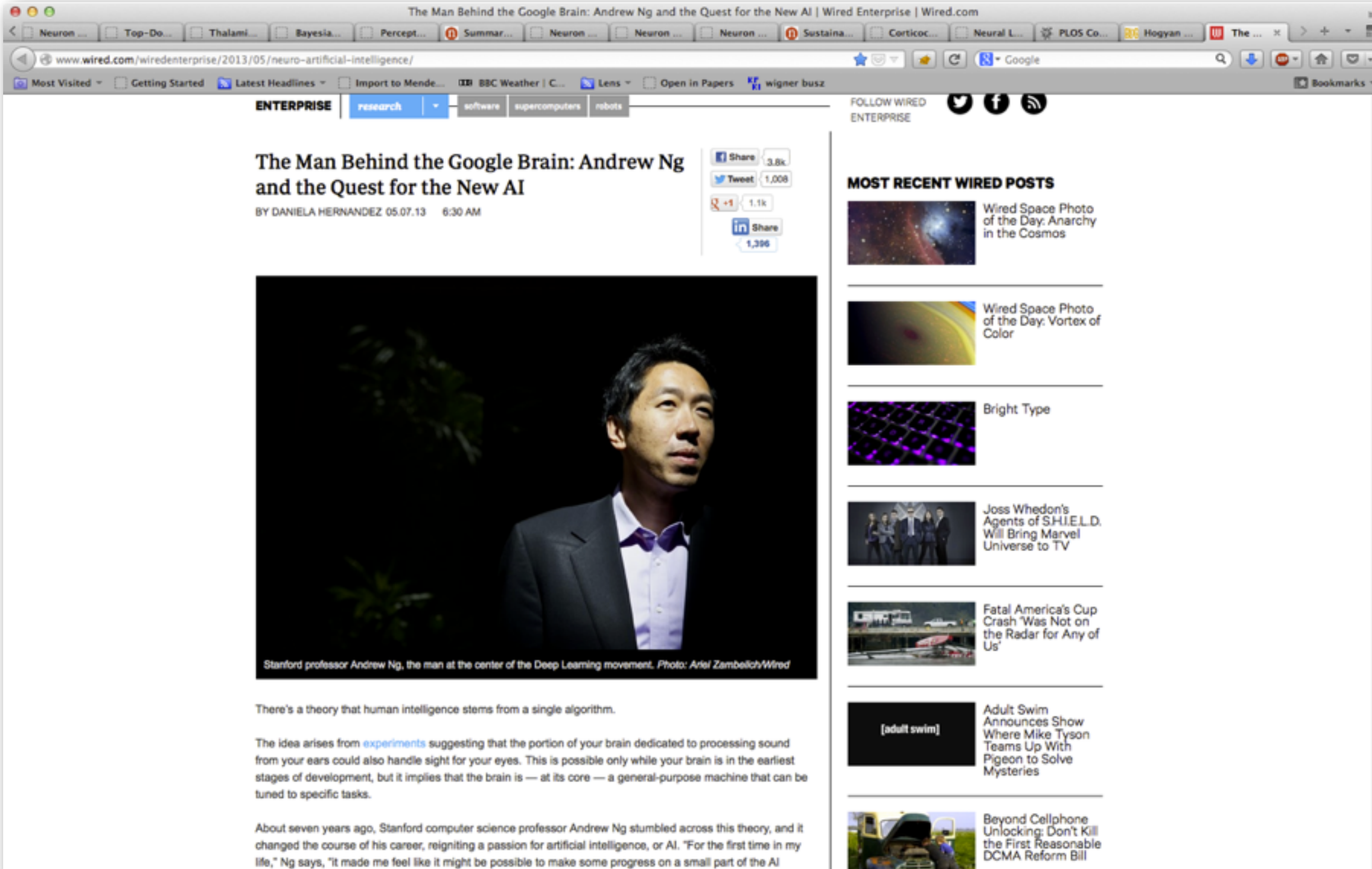

The Man Behind the Google Brain: Andrew Ng and the Quest for the New AI | Wired Enterprise | Wired.com

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The Man Behind the Google Brain: Andrew Ng and the Quest for the New AI

BY DANIELA HERNANDEZ 05.07.13 6:30 AM

Stanford professor Andrew Ng, the man at the center of the Deep Learning movement. *Photo: Ariel Zambelich/Wired*

There's a theory that human intelligence stems from a single algorithm.

The idea arises from experiments suggesting that the portion of your brain dedicated to processing sound from your ears could also handle sight for your eyes. This is possible only while your brain is in the earliest stages of development, but it implies that the brain is — at its core — a general-purpose machine that can be tuned to specific tasks.

About seven years ago, Stanford computer science professor Andrew Ng stumbled across this theory, and it changed the course of his career, reigniting a passion for artificial intelligence, or AI. "For the first time in my life," Ng says, "it made me feel like it might be possible to make some progress on a small part of the AI

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RECAP: Bayesian inference

$$\mathrm{P}\left(\mathbf{z} \,|\, \mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x} \,|\, \mathbf{z}\right) \mathrm{P}\left(\mathbf{z}\right)$$

RECAP: Bayesian inference

$$\mathrm{P}\left(\mathbf{z} \,\middle|\, \mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x} \,\middle|\, \mathbf{z}\right) \mathrm{P}\left(\mathbf{z}\right)$$

stimulus

RECAP: Bayesian inference

$$\mathrm{P}\left(\mathbf{z}\,|\,\mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x}\,|\,\mathbf{z}\right)\mathrm{P}\left(\mathbf{z}\right)$$

stimulus

x

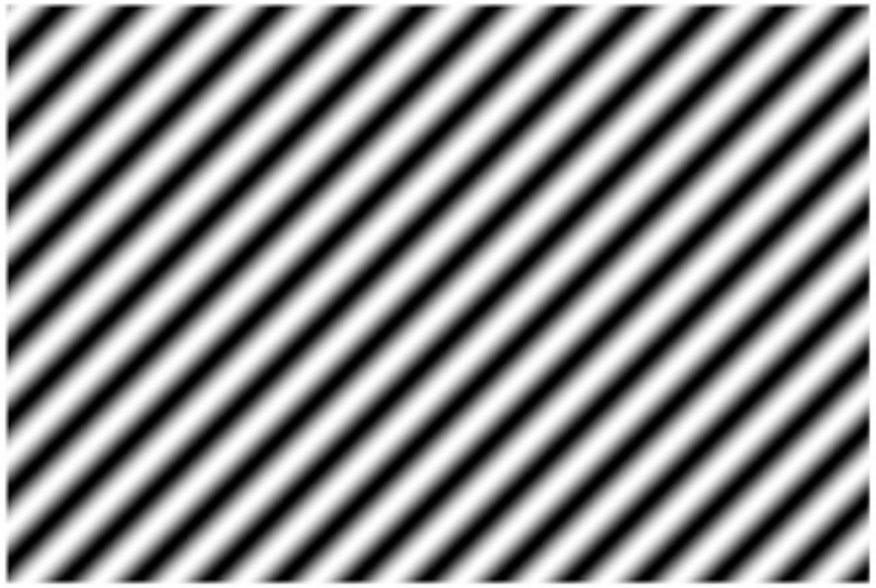
RECAP: Bayesian inference

$$\mathrm{P}\left(\mathbf{z}\,|\,\mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x}\,|\,\mathbf{z}\right)\mathrm{P}\left(\mathbf{z}\right)$$

percept/
inference

stimulus

x



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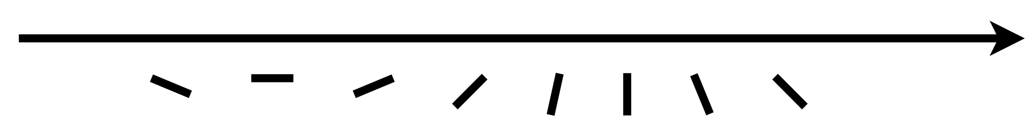


z

RECAP: Bayesian inference

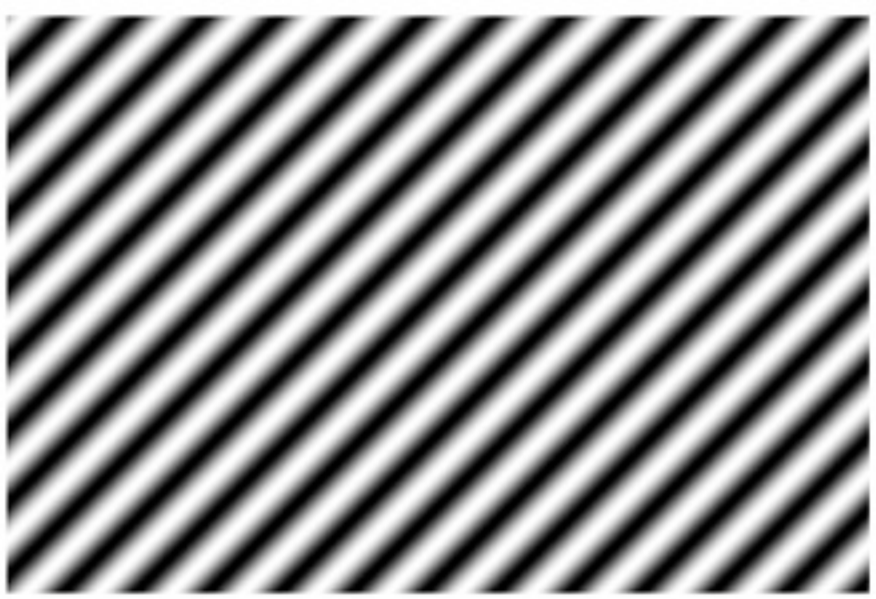
posterior probability

$$\mathrm{P}\left(\mathbf{z}\,|\,\mathbf{x}\right) \propto \mathrm{P}\left(\mathbf{x}\,|\,\mathbf{z}\right)\mathrm{P}\left(\mathbf{z}\right)$$

percept/
inference

stimulus

x



z

RECAP: Bayesian inference

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percept/
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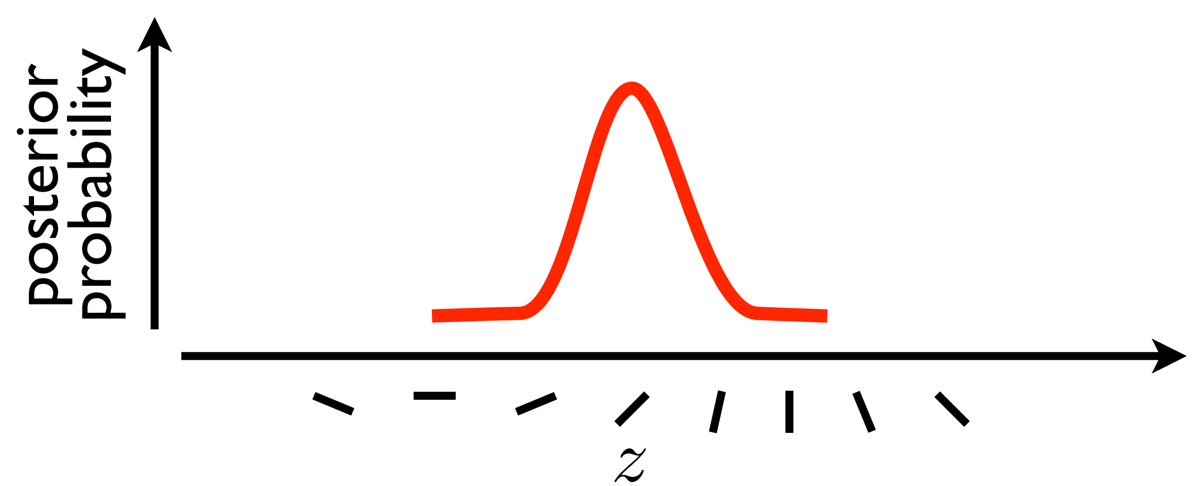
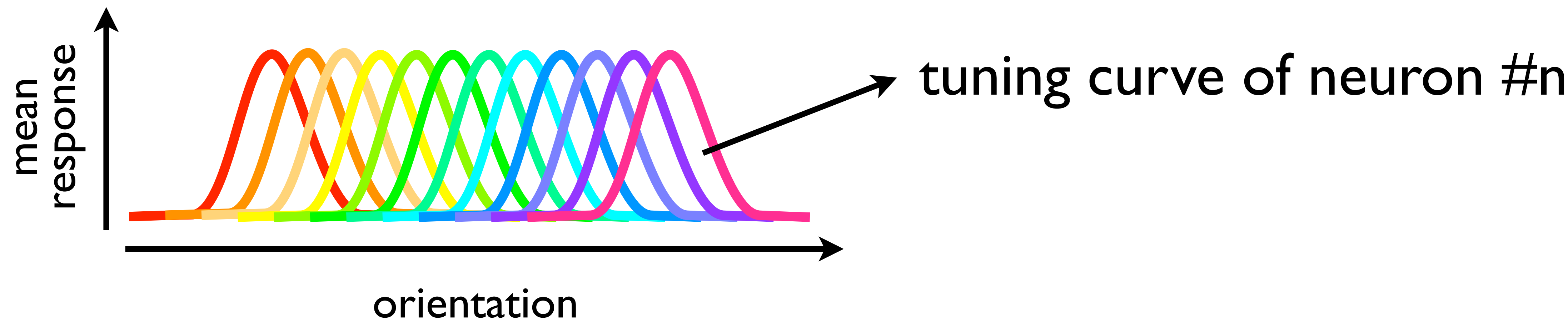


RECAP: Probabilistic Population Code



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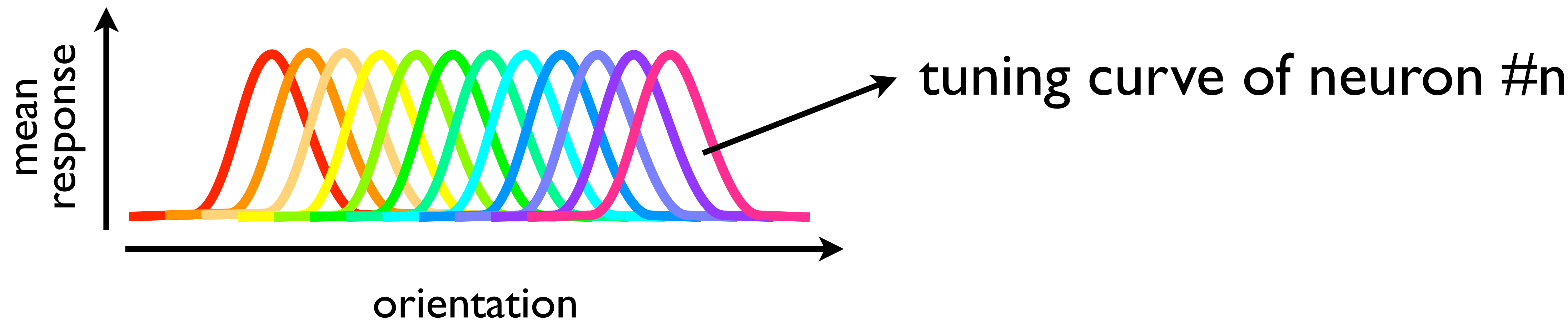

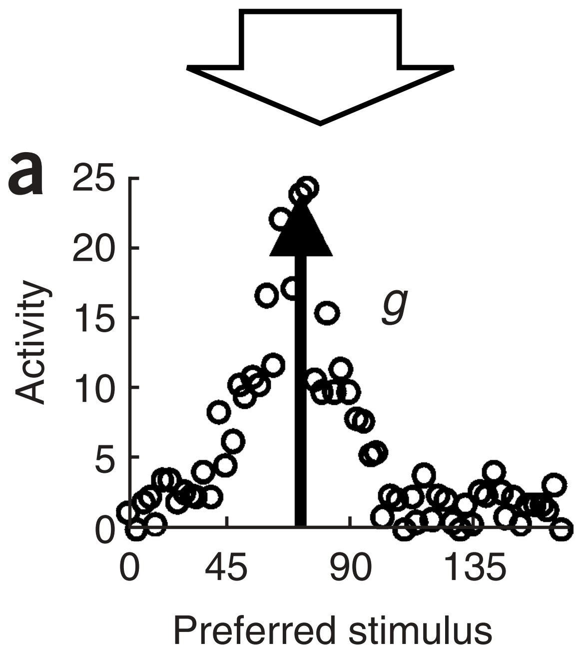

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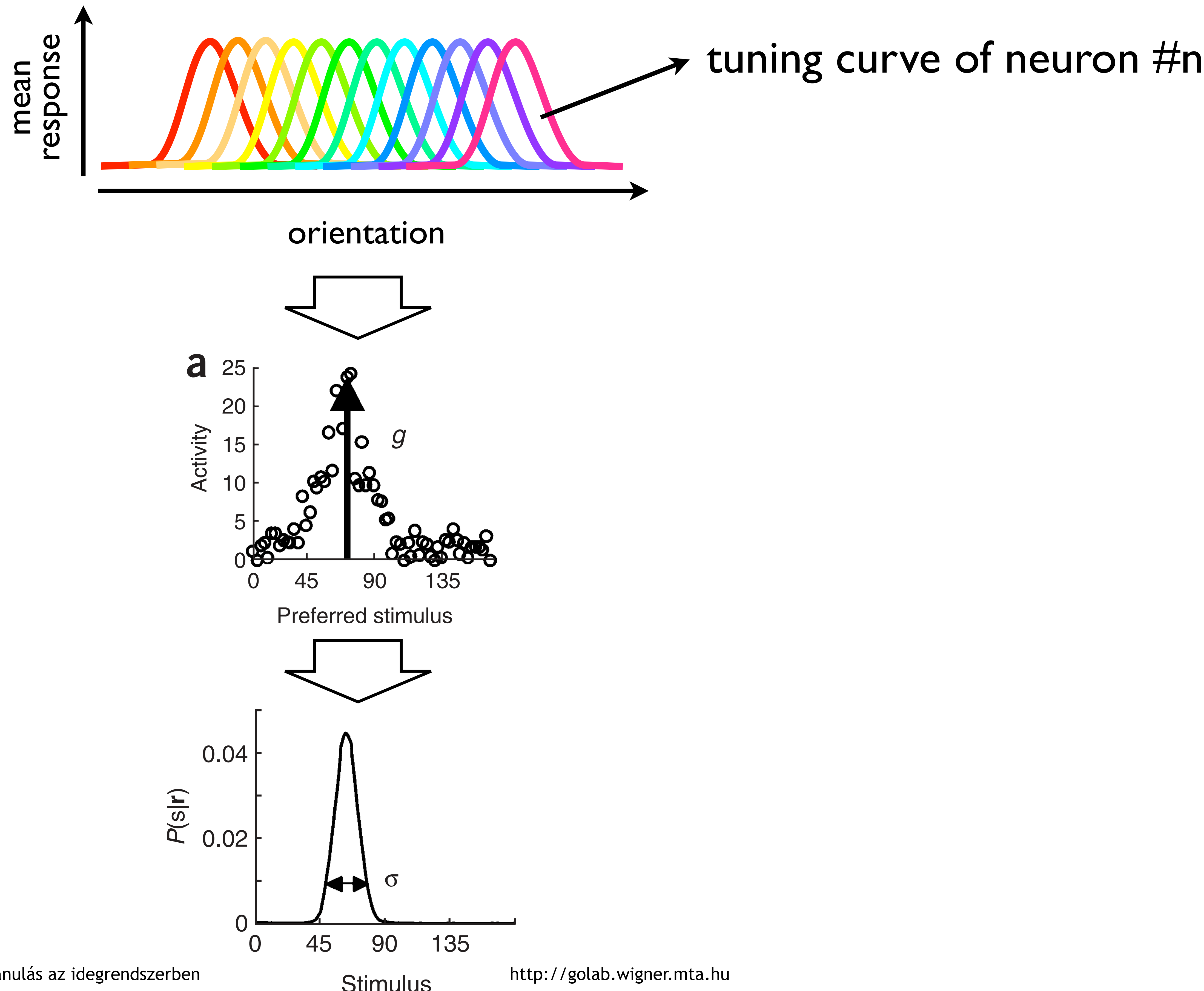

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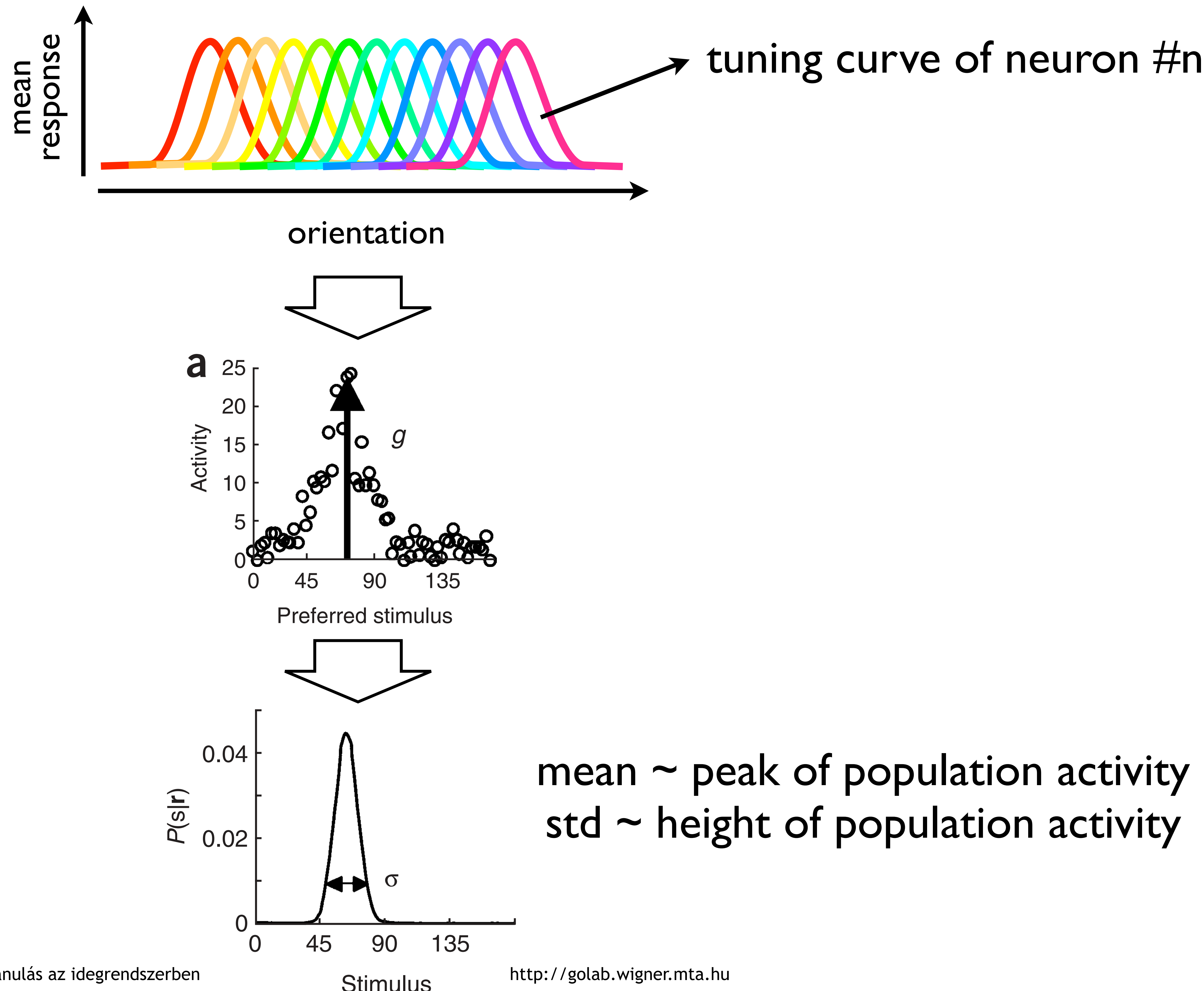
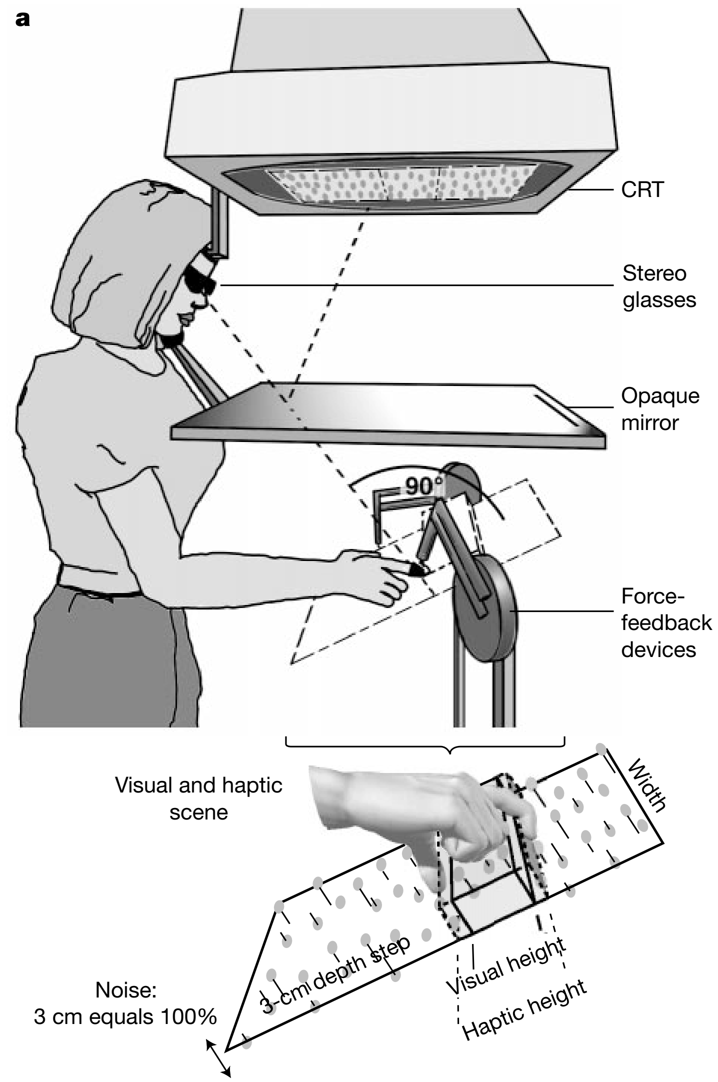


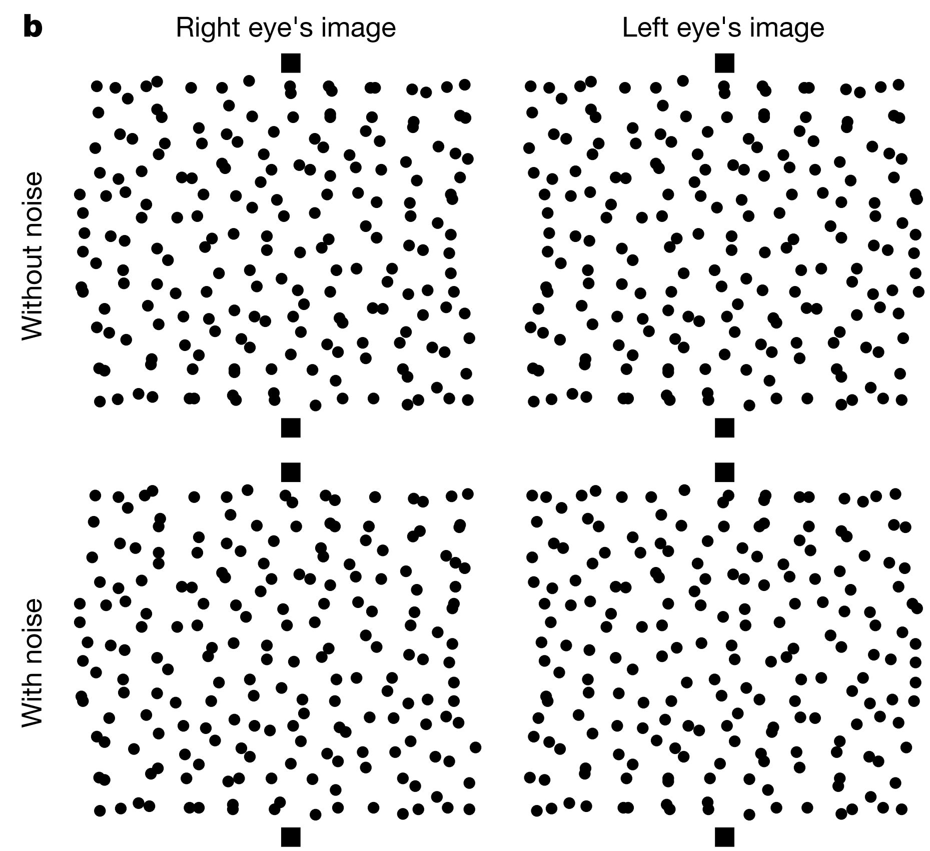
mean ~ peak of population activity
std ~ height of population activity

Miért érdekes a Bayes szabály?

Emberi viselkedés utal arra, hogy ilyesfajta komputációk jelen vannak

Multi-szenzoros integráció
Ernst & Banks, Nature (2002)





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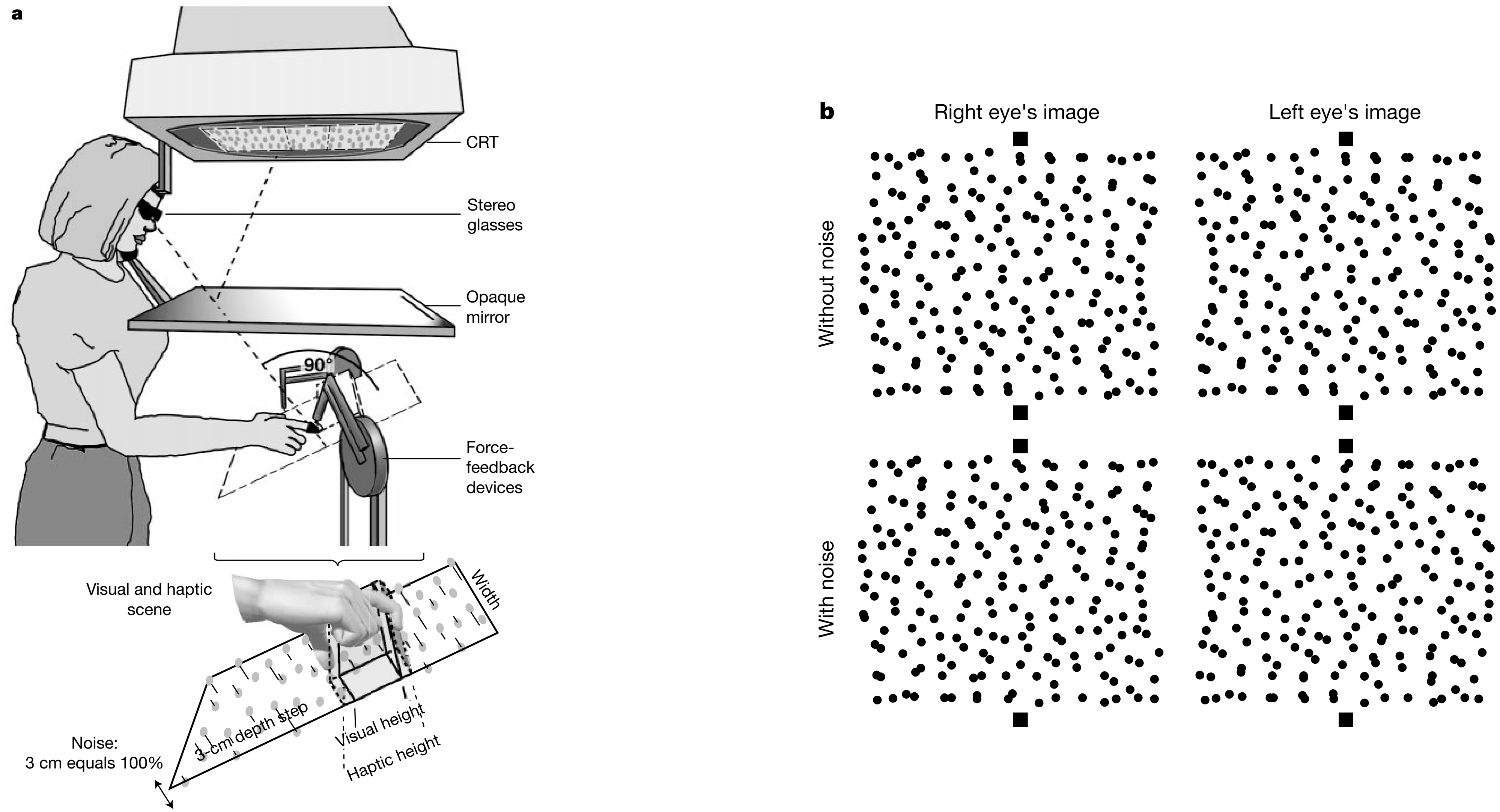

- Két információforrás:
  - vizuális
  - haptikus/tapintás
- Egy inferálandó változó: magasság

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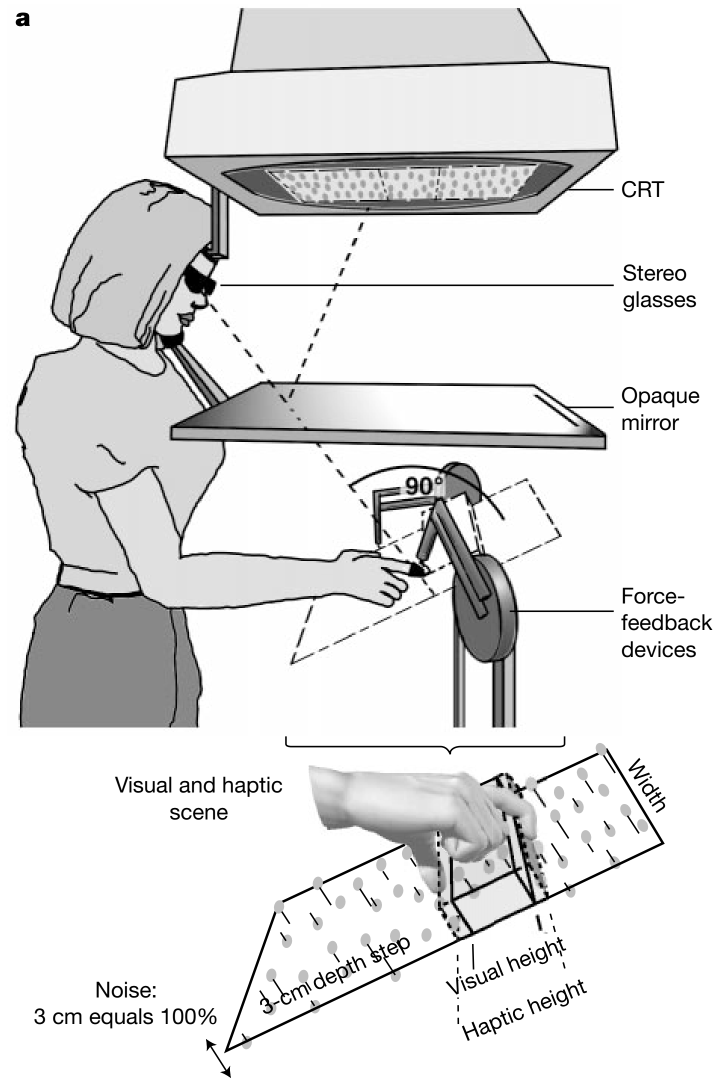
Ernst & Banks, Nature (2002)



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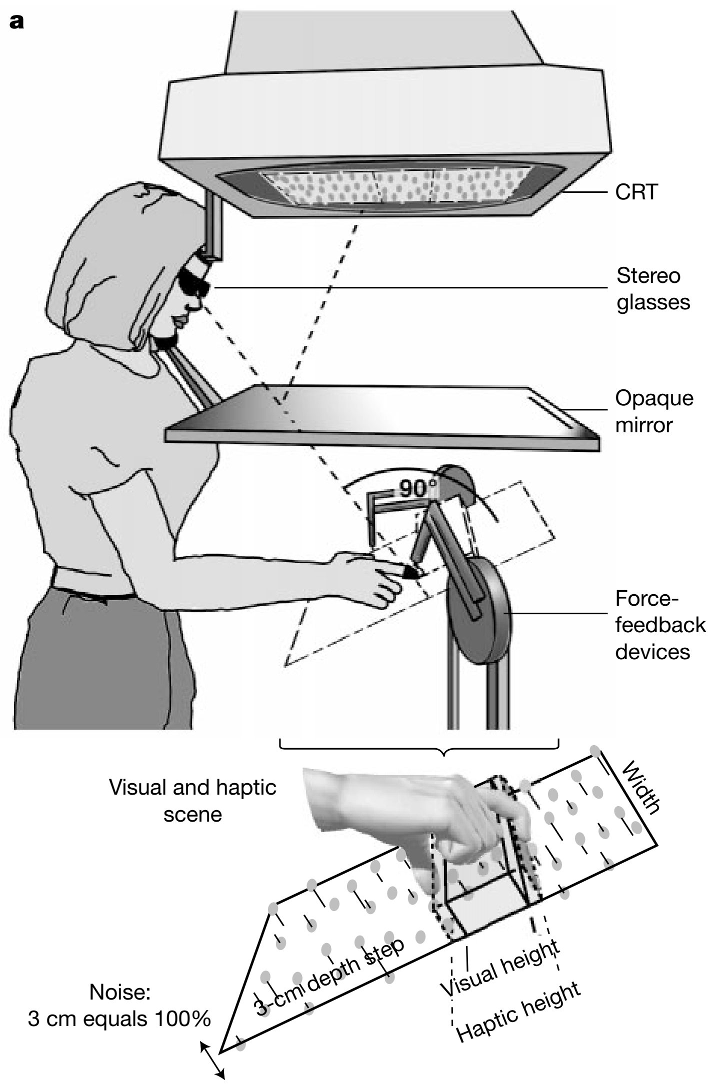

$$P\left(h|\mathrm{data}\right) = P\left(\mathrm{data}|h\right)P\left(h\right)$$

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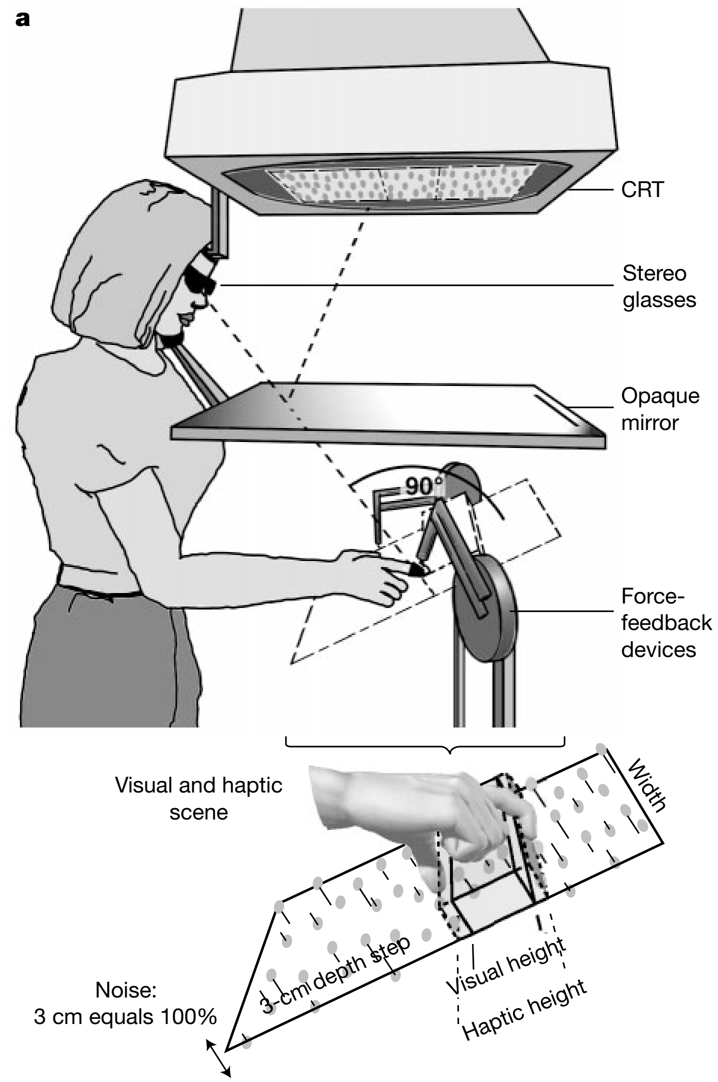

$$P(h|\mathrm{data}) = P(\mathrm{data}|h)\,P(h)$$
$$= P(h_{\mathrm{visual}}|h)\,P(h_{\mathrm{haptic}}|h)\,P(h)$$

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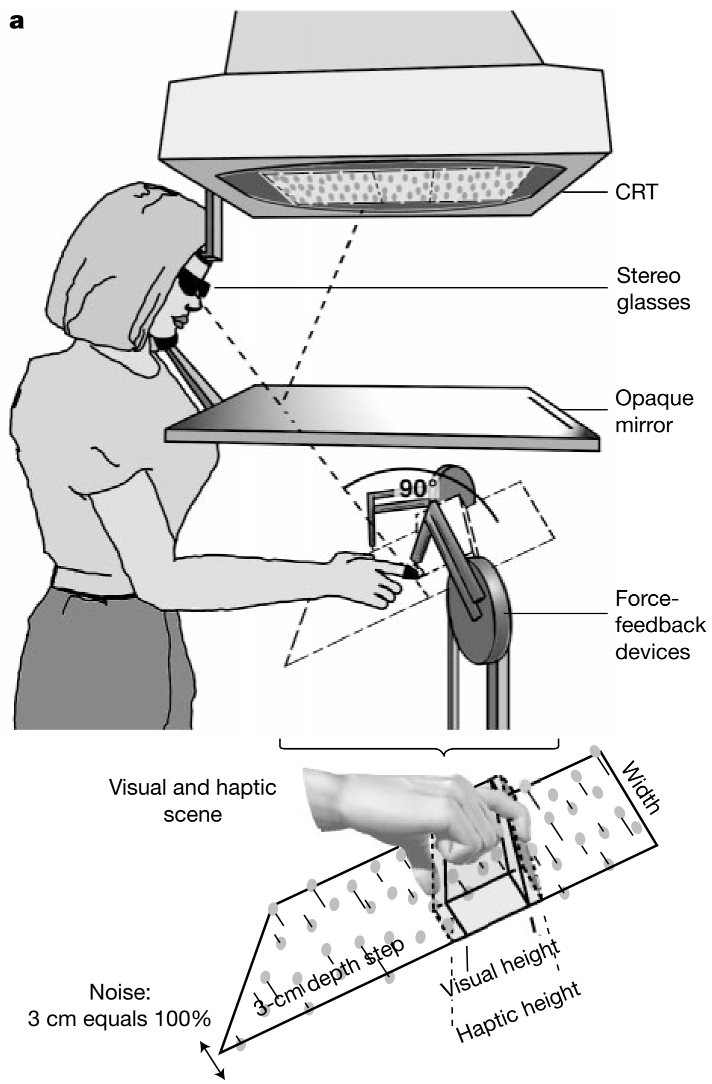

$$P\left(h|\mathrm{data}\right) = P\left(\mathrm{data}|h\right)P\left(h\right)$$
$$= P\left(h_{\mathrm{visual}}|h\right)P\left(h_{\mathrm{haptic}}|h\right)P\left(h\right)$$

Gauss-zaj

Gauss-zaj

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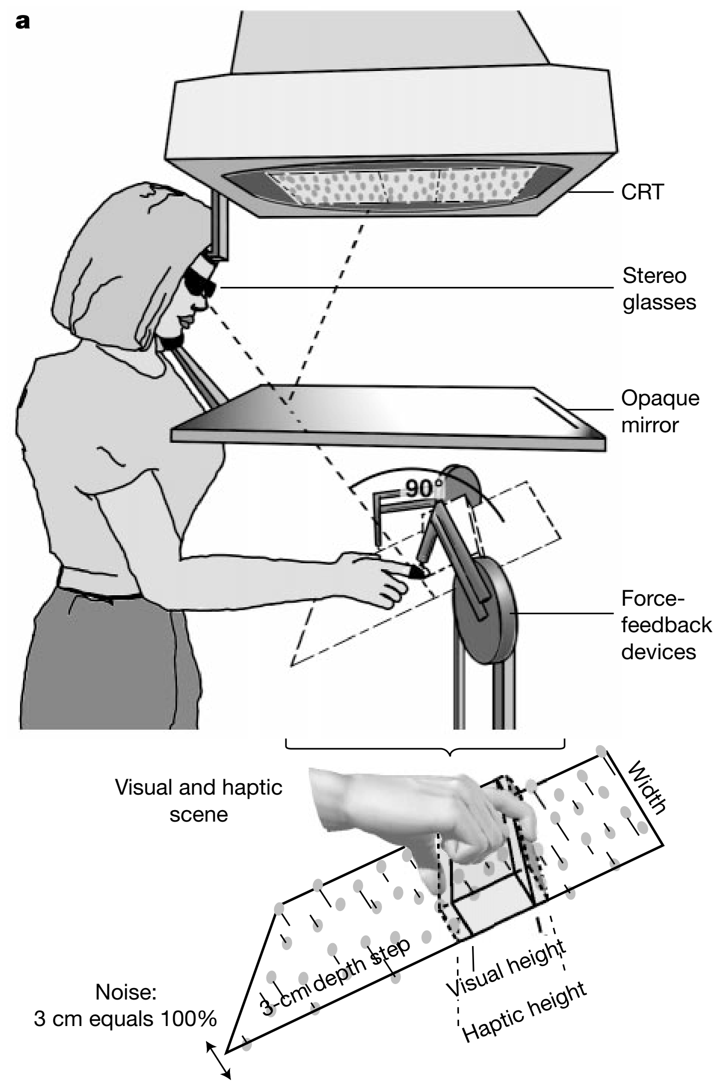

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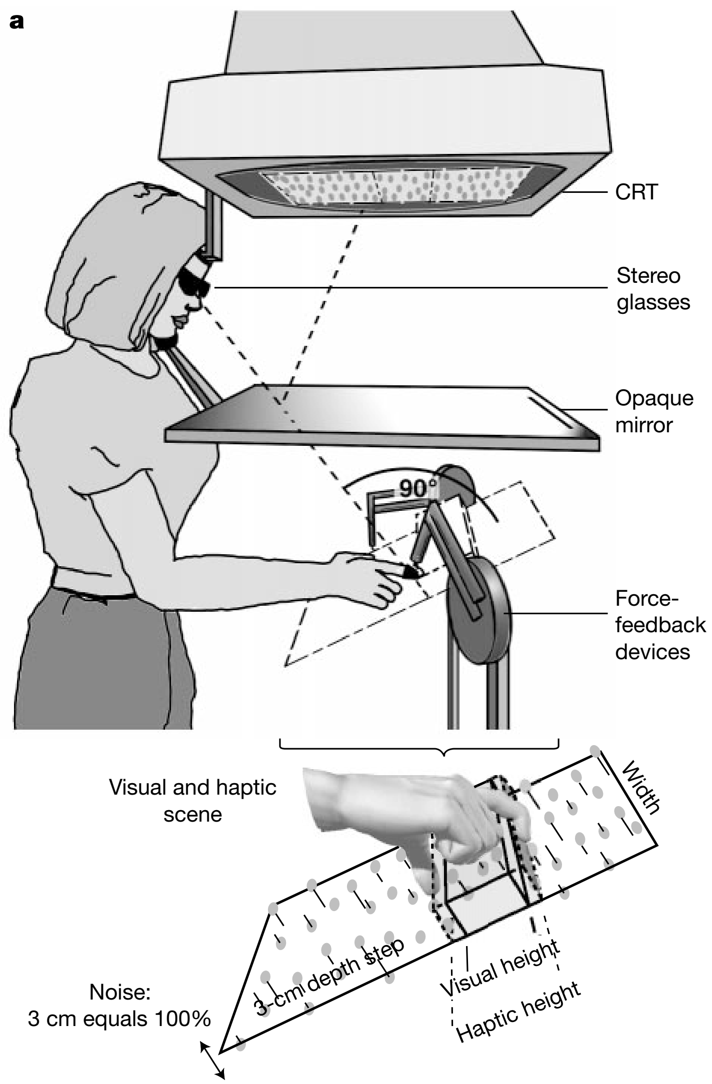

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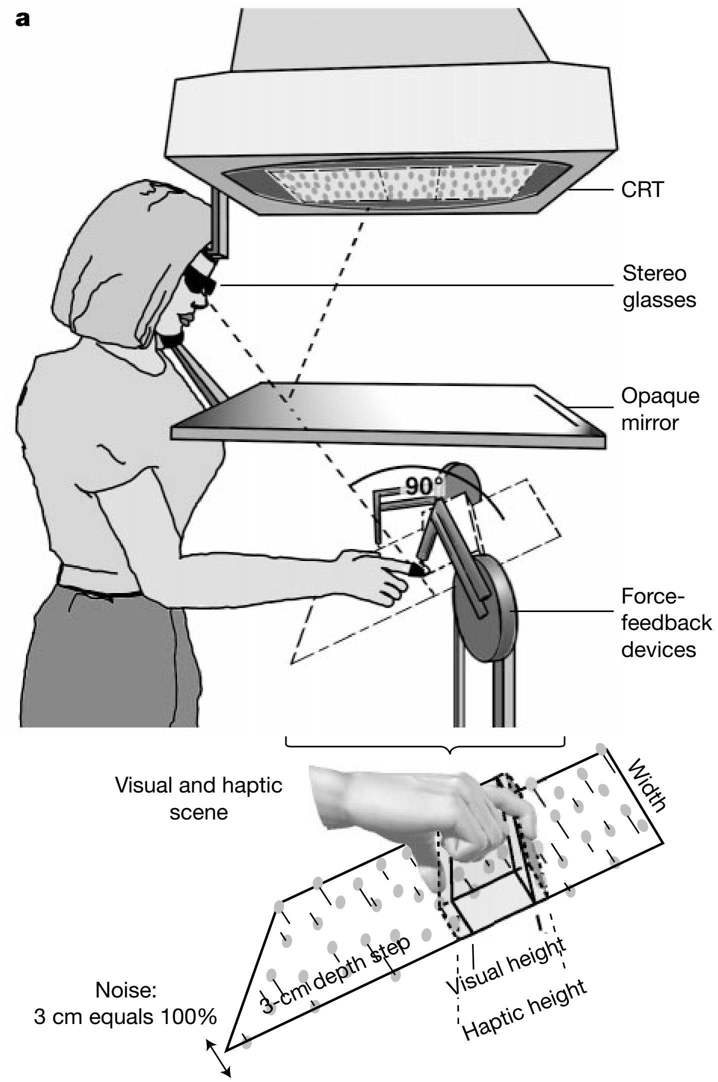

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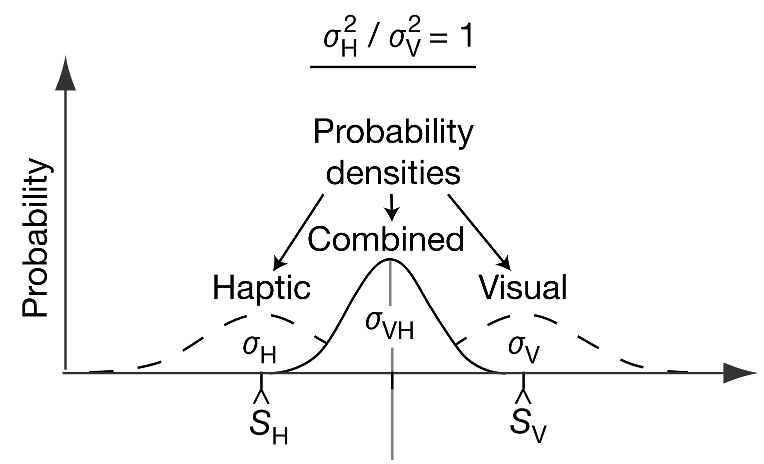

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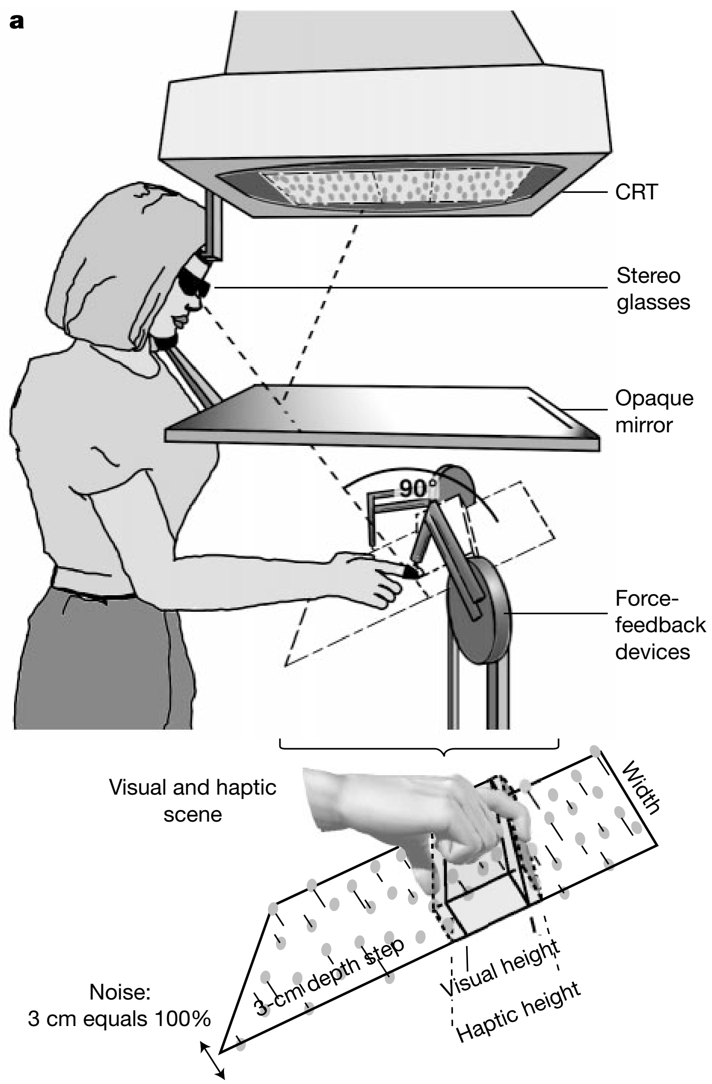

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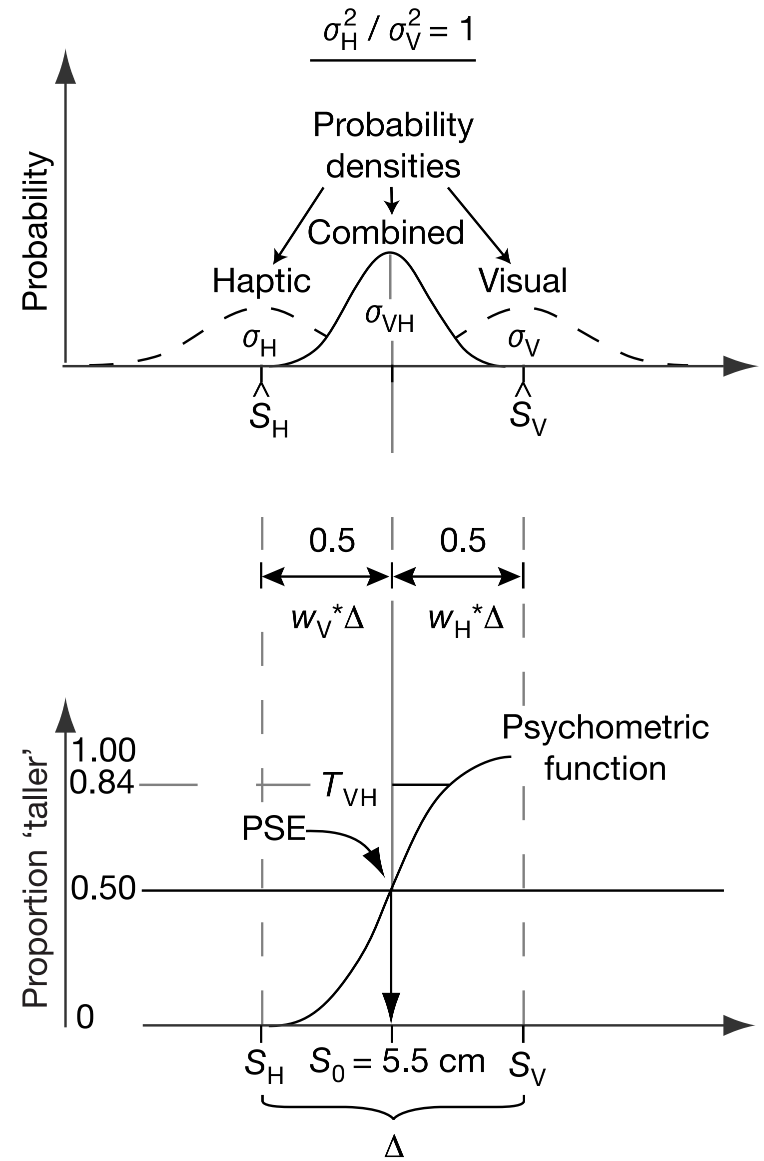

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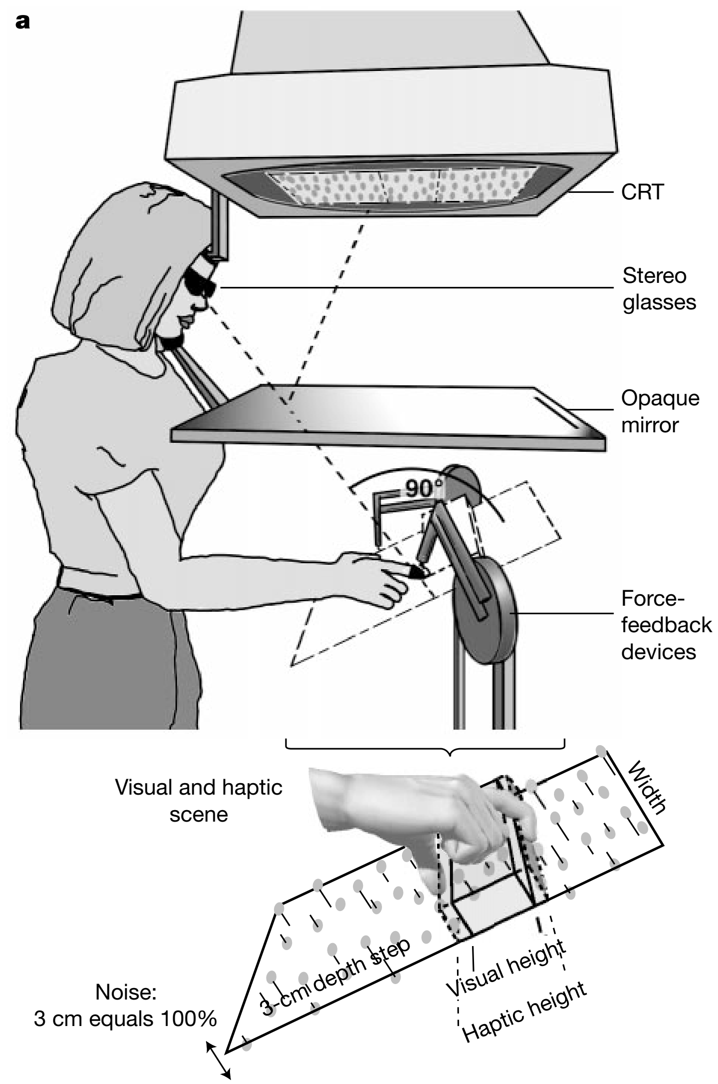

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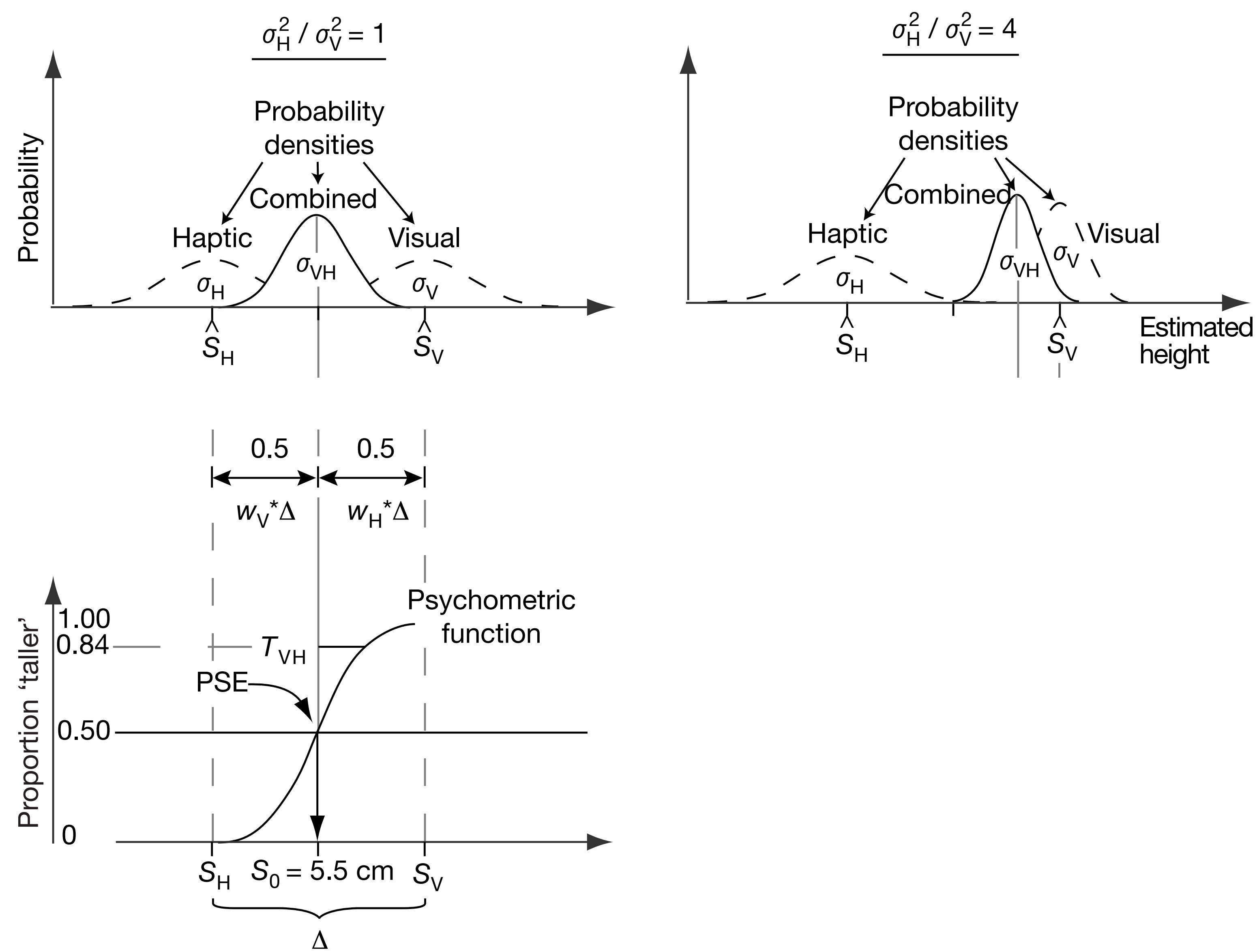

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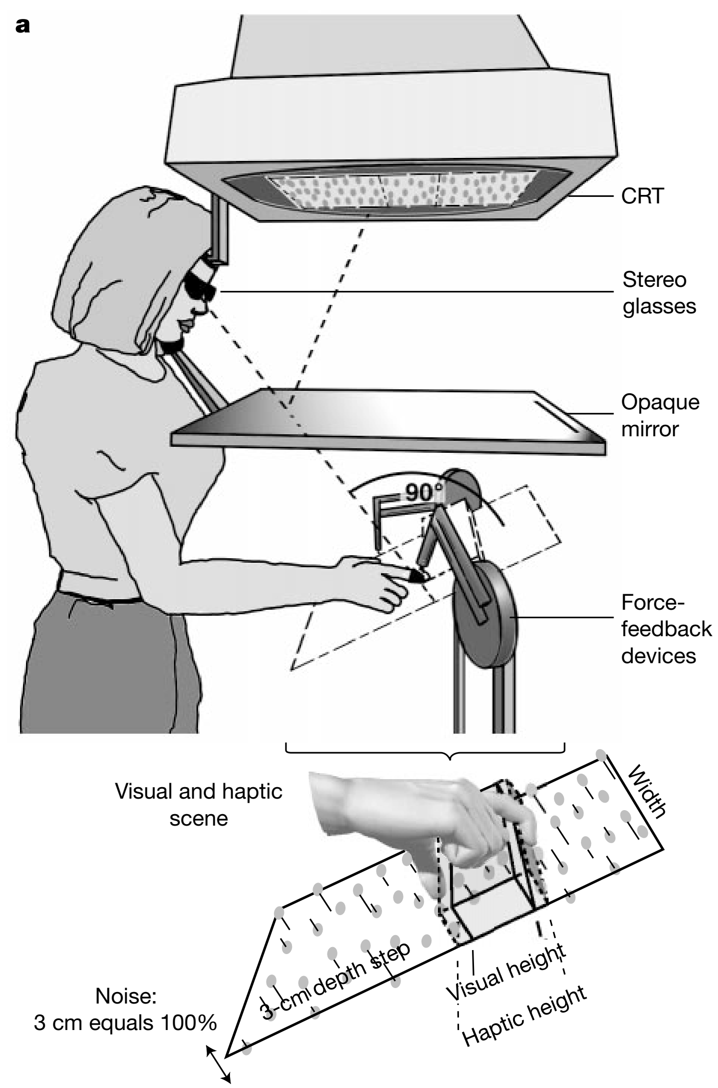

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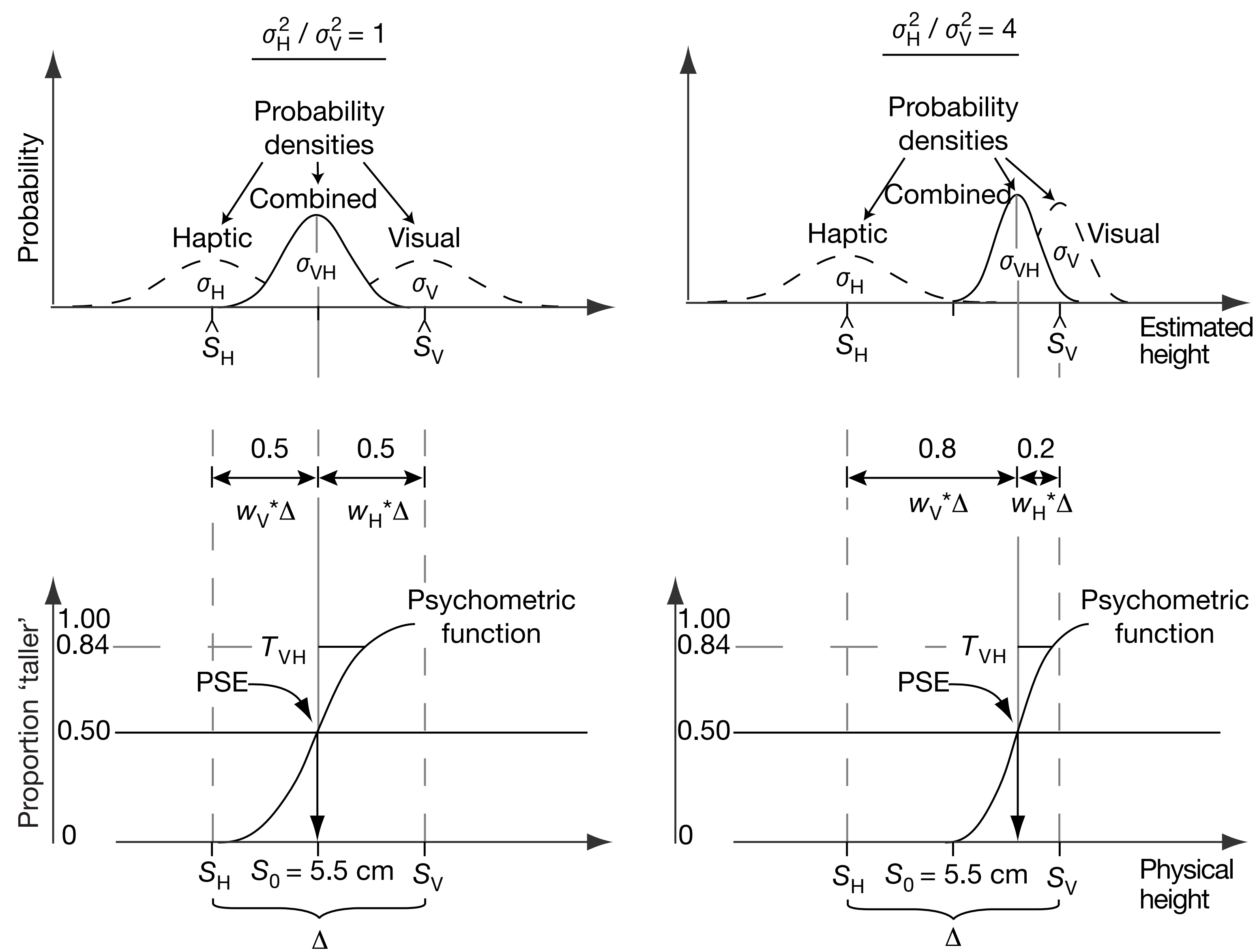

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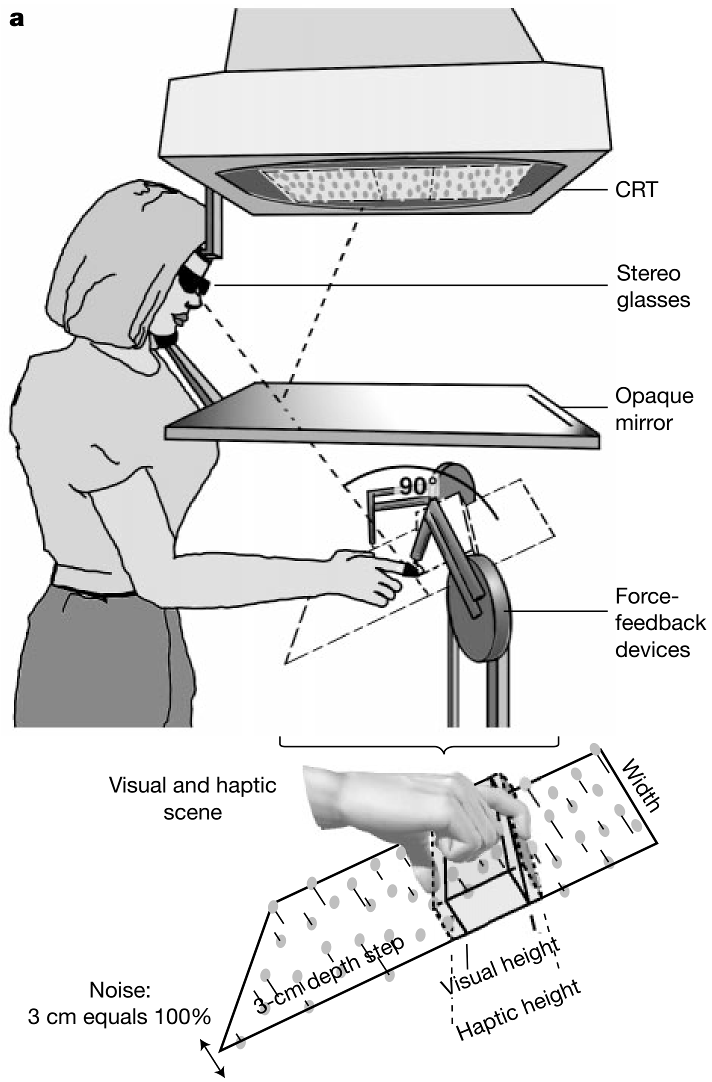

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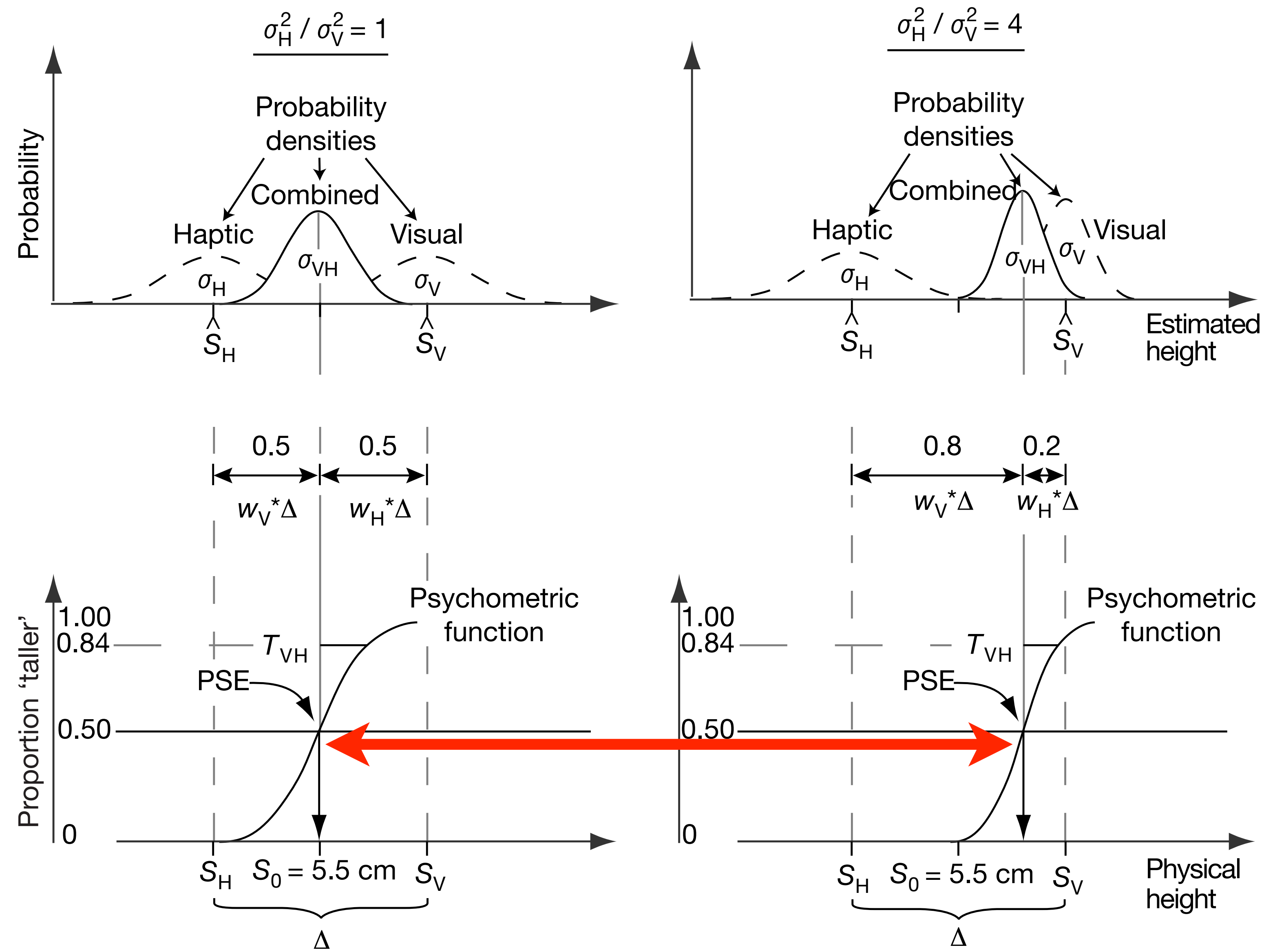

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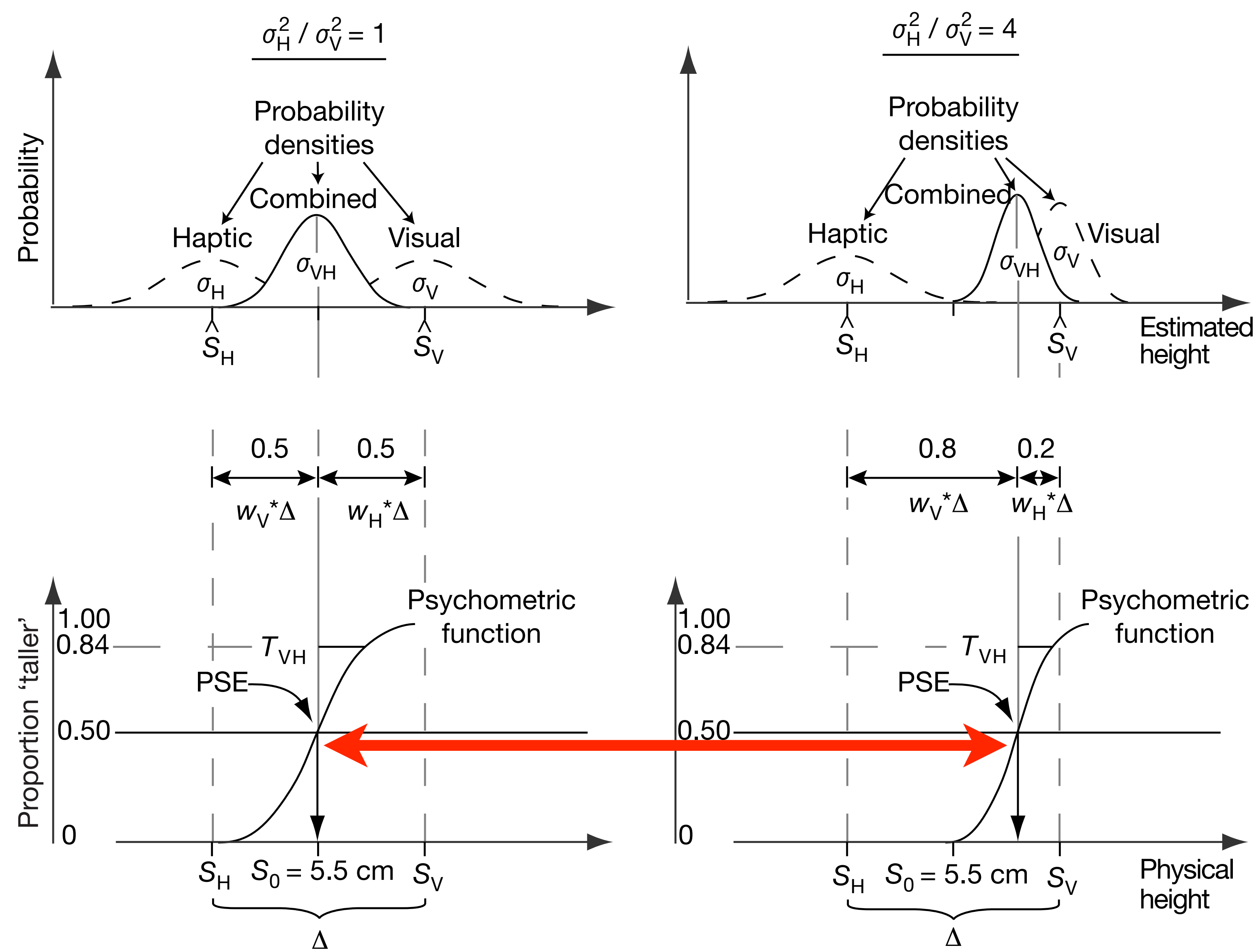

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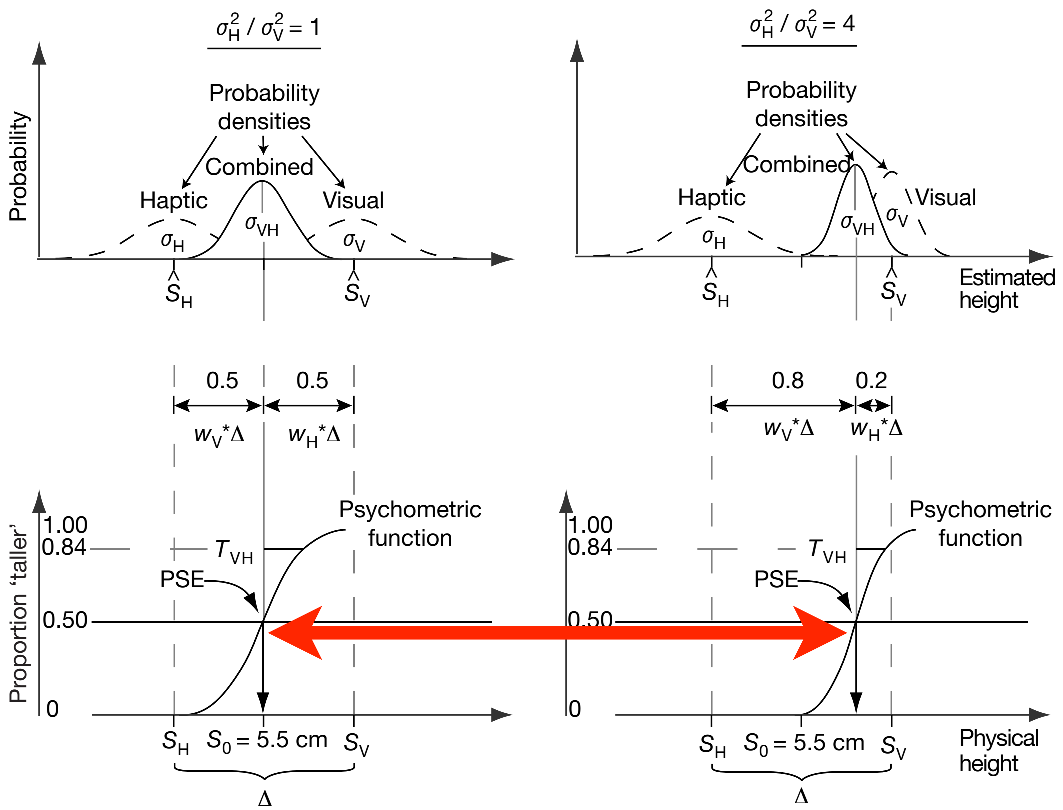

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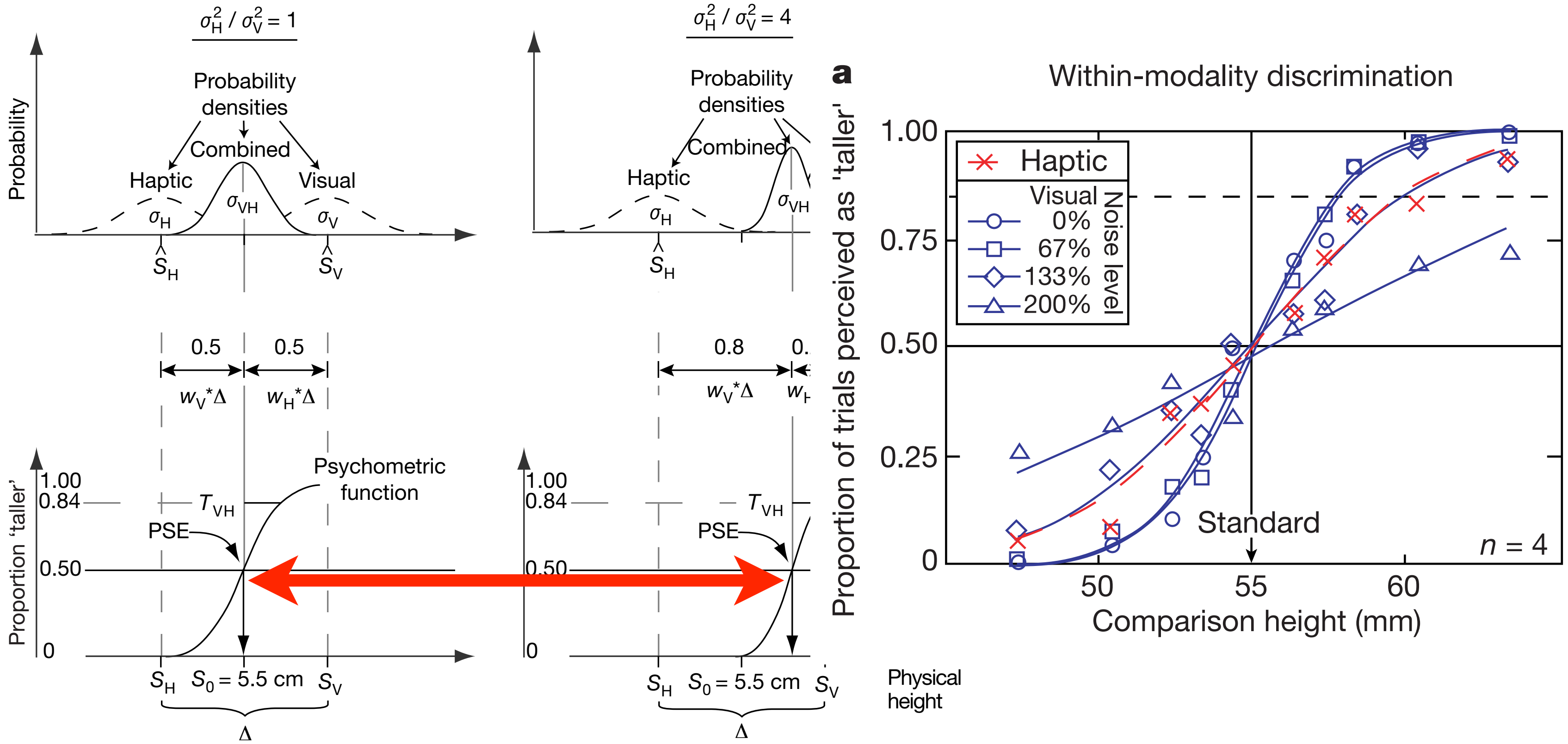

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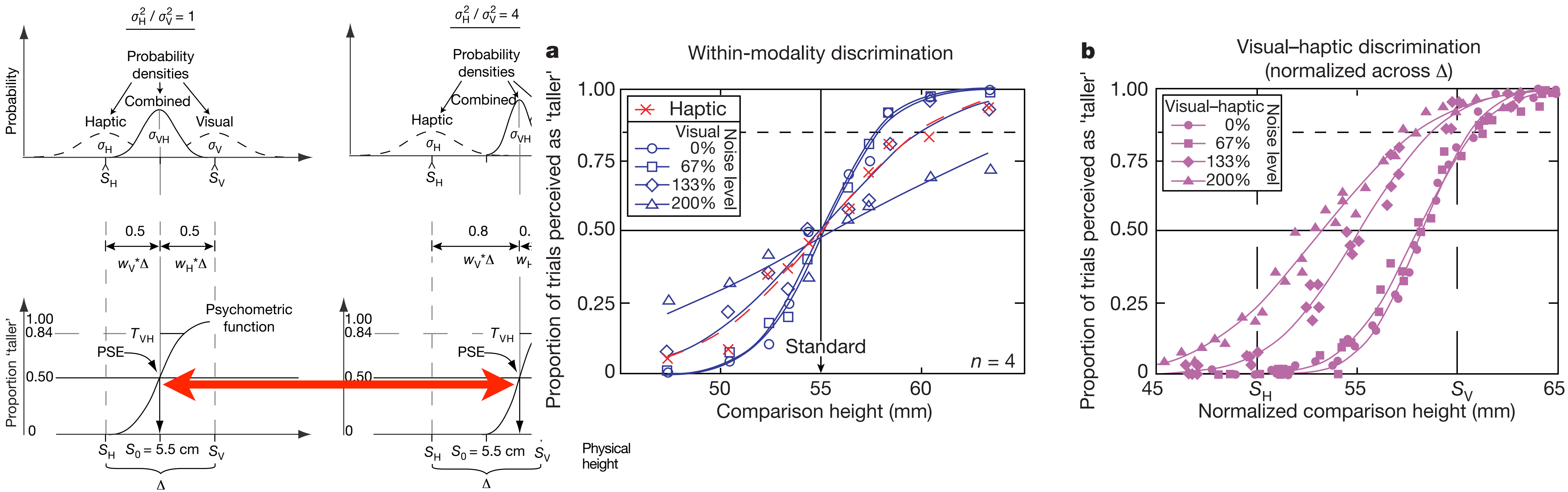

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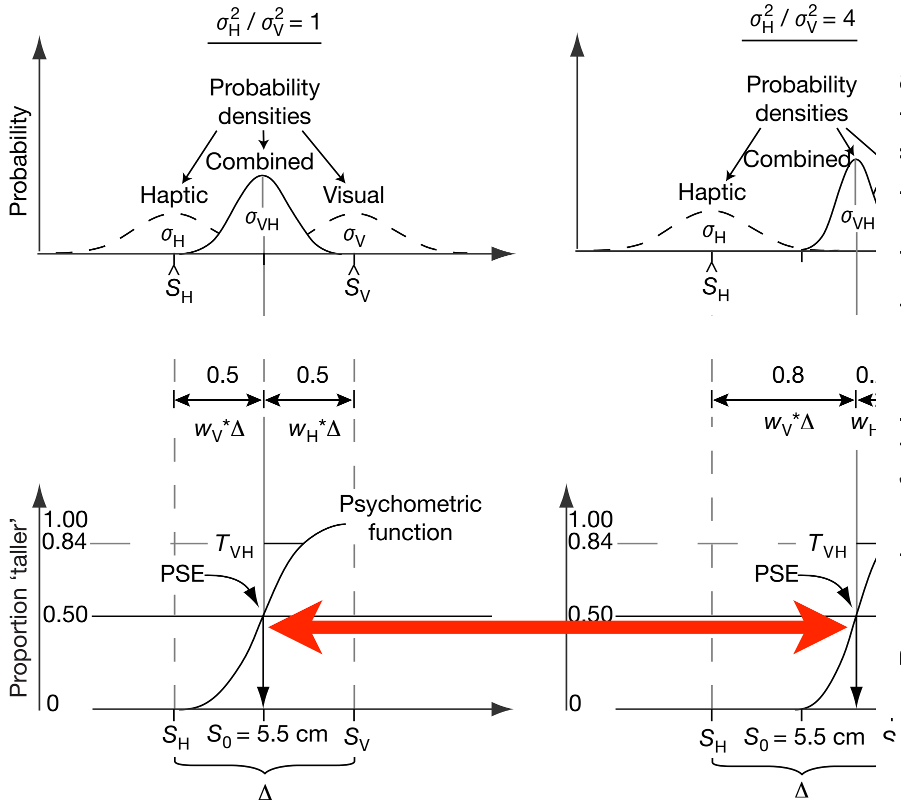

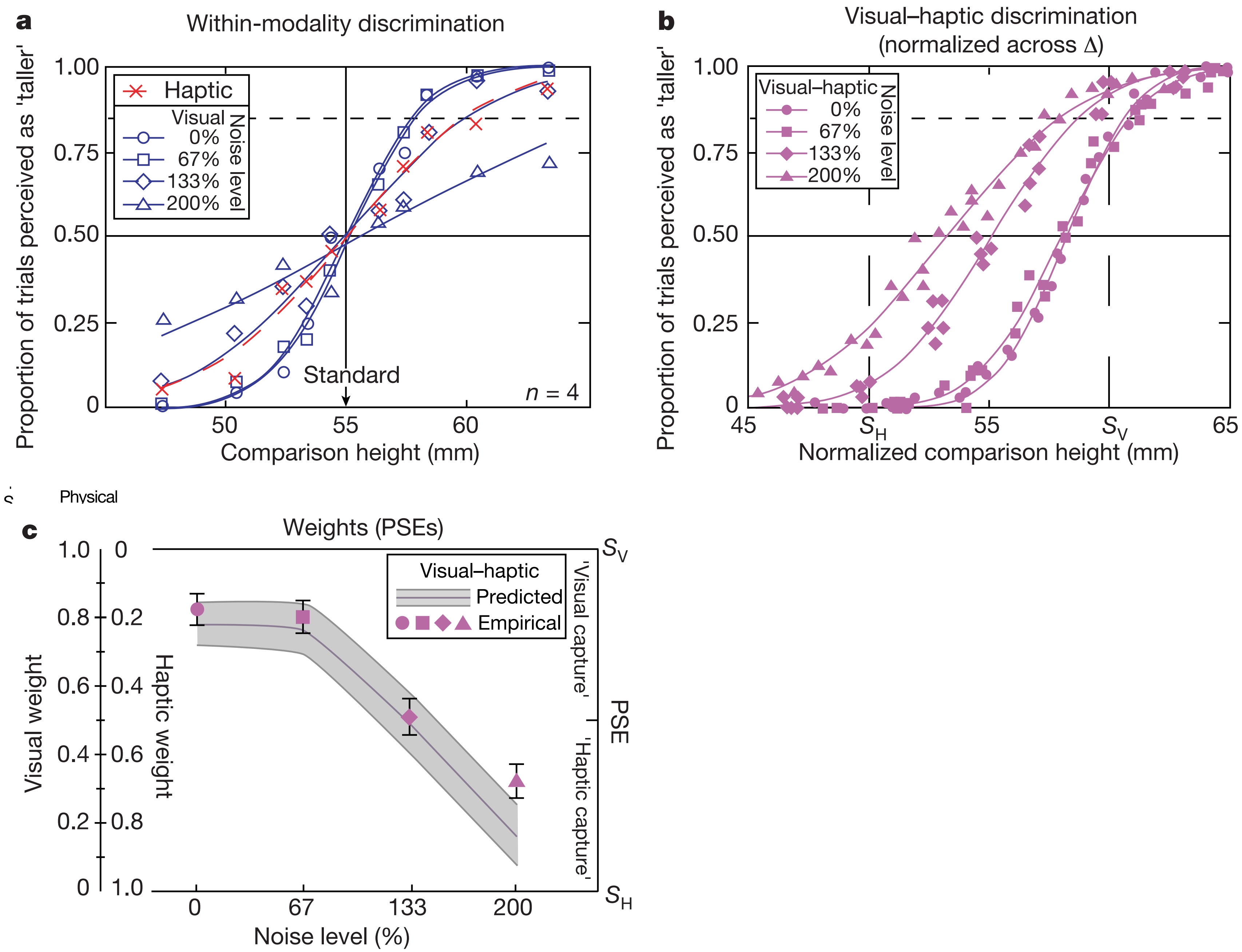

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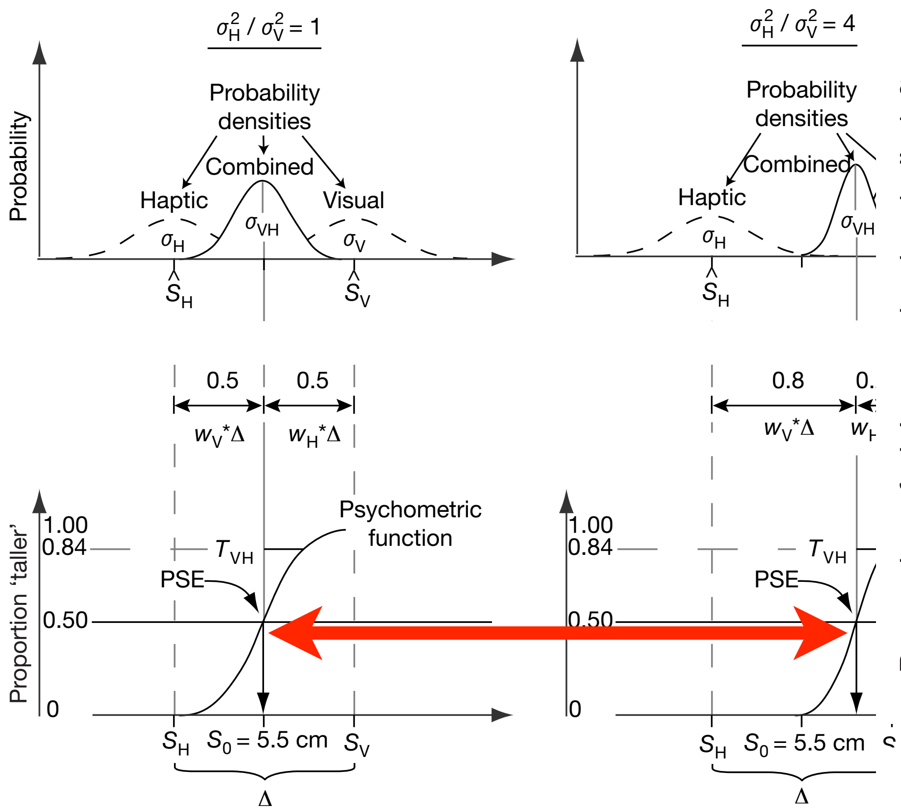
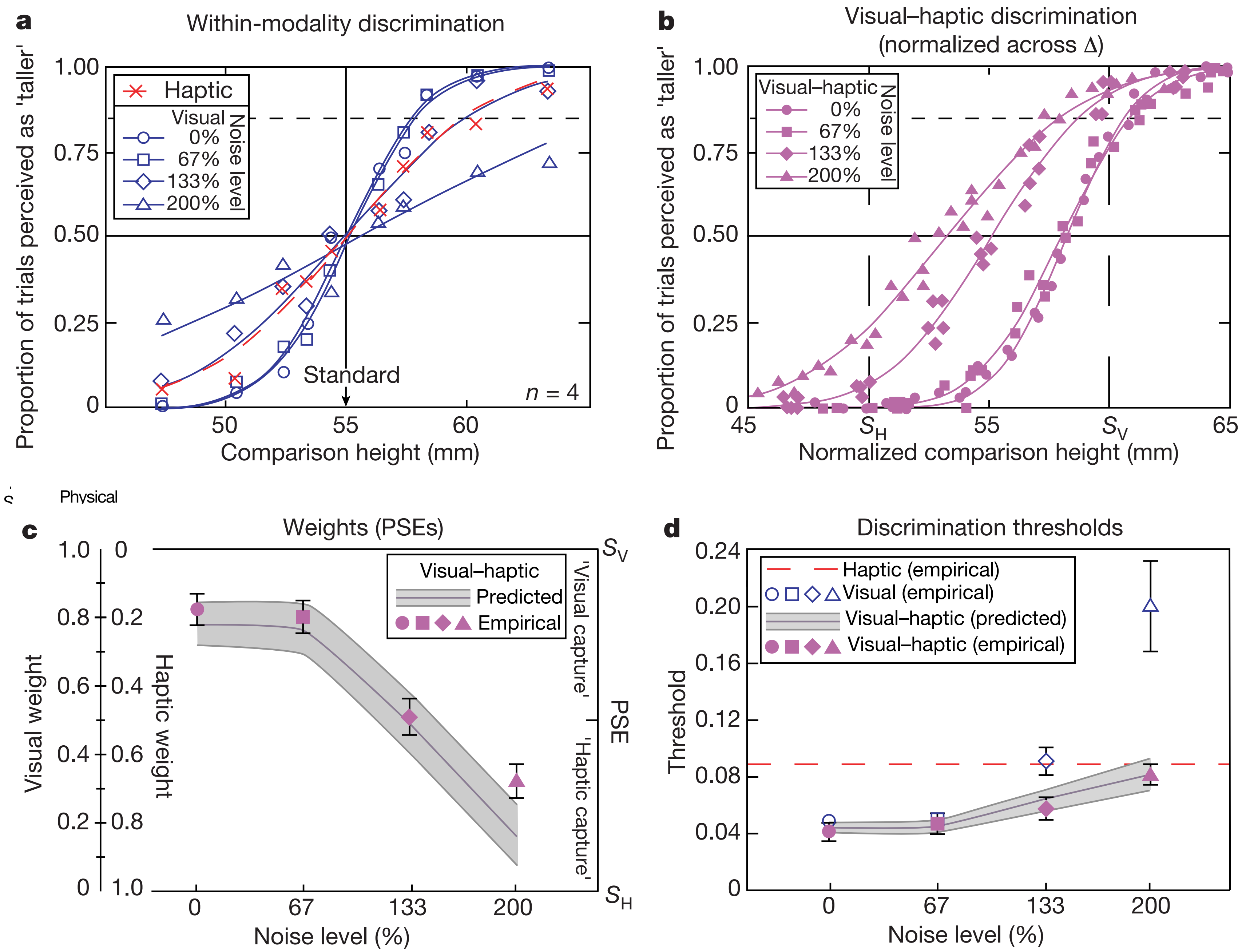
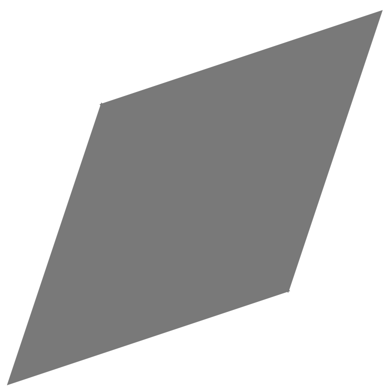




Illúziók, mint Bayes-i komputációk

Weiss, Simoncelli & Adelson (2002)



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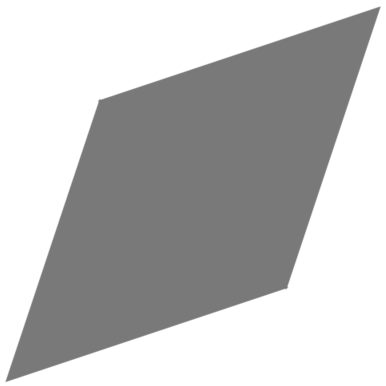

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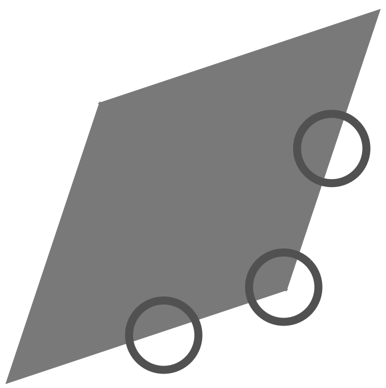

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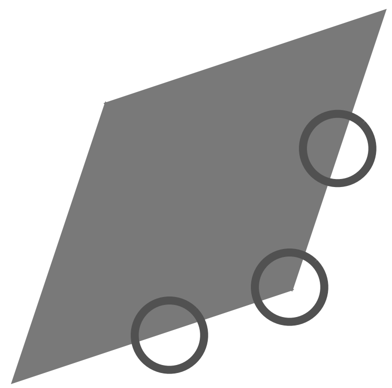

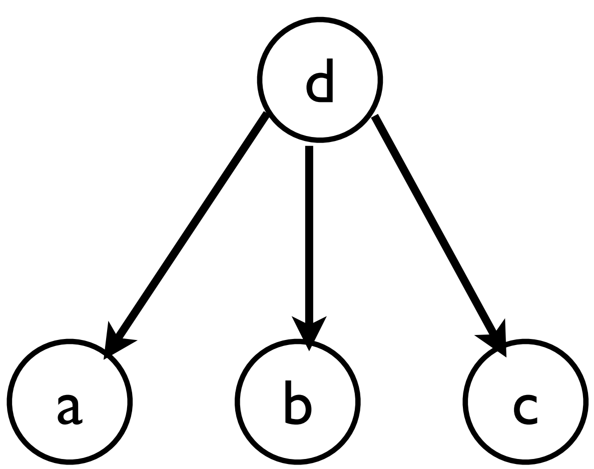

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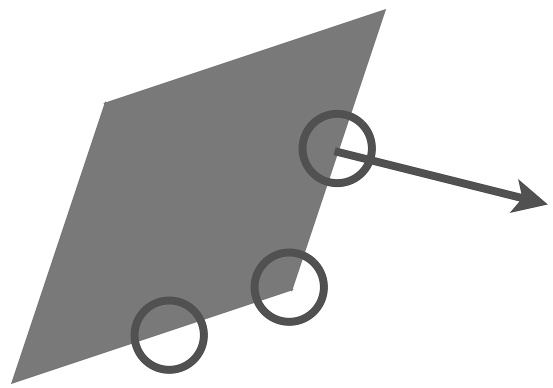

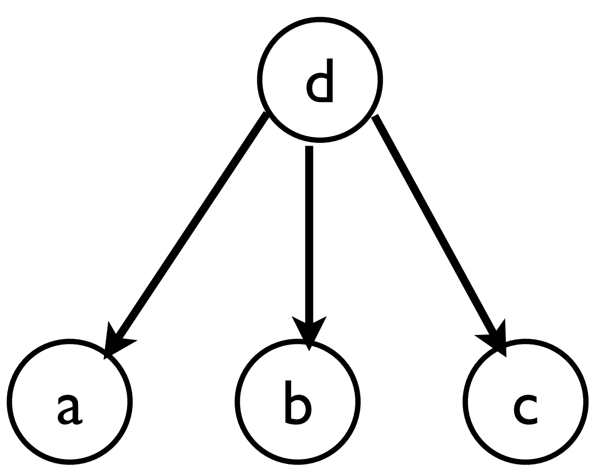

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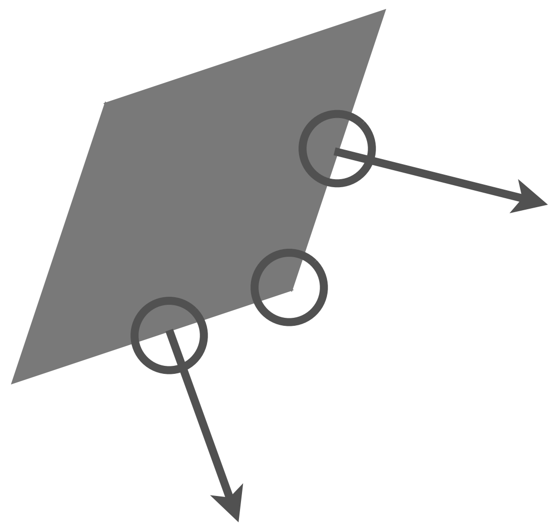

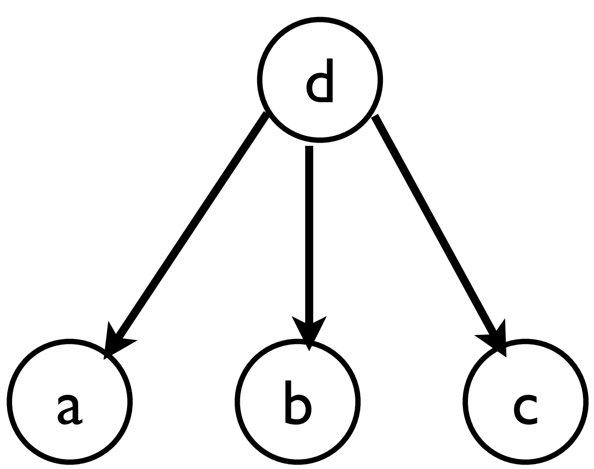

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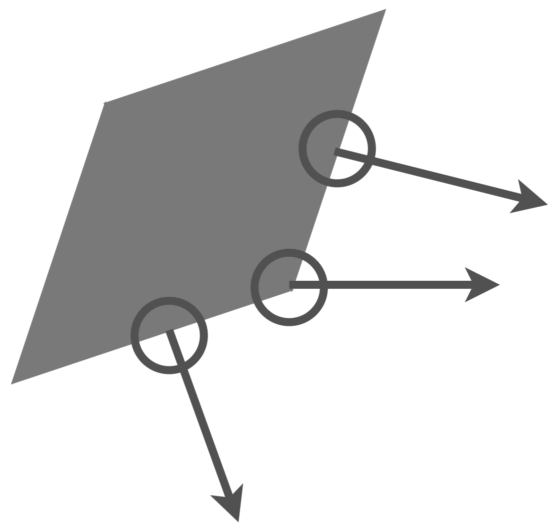

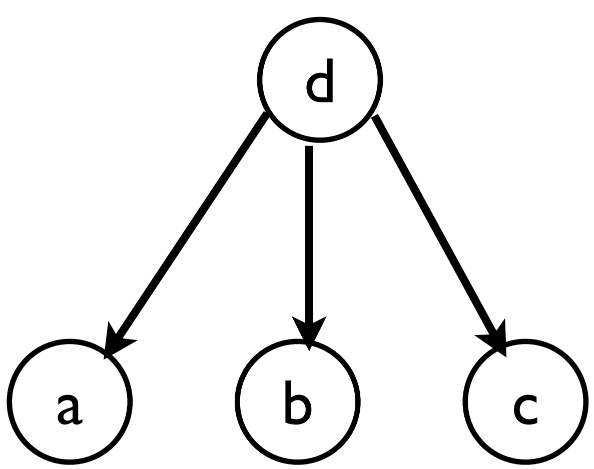

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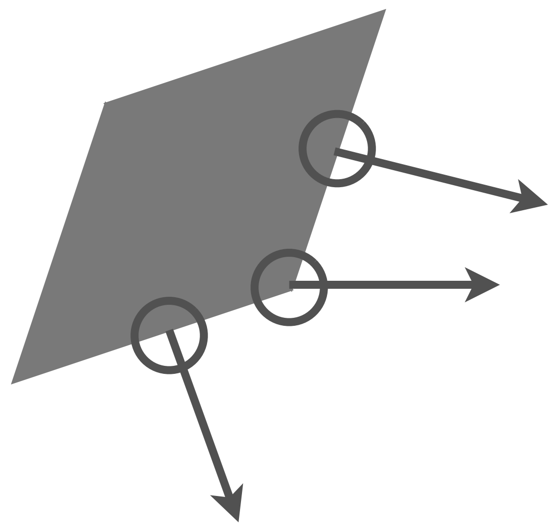
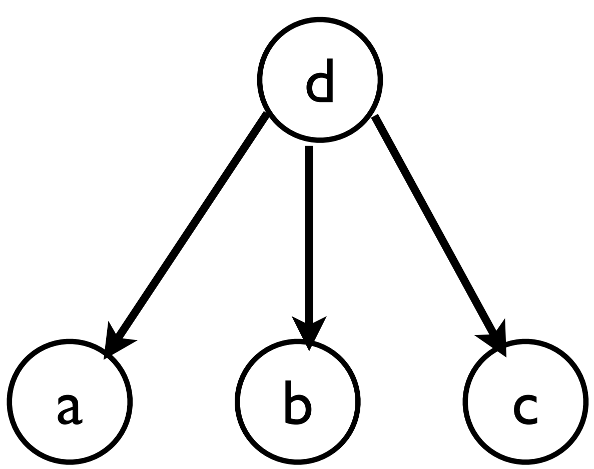


Milyen mozgásokkal konzisztens?
(mik a stimulus alapján a valószínű mozgások?)



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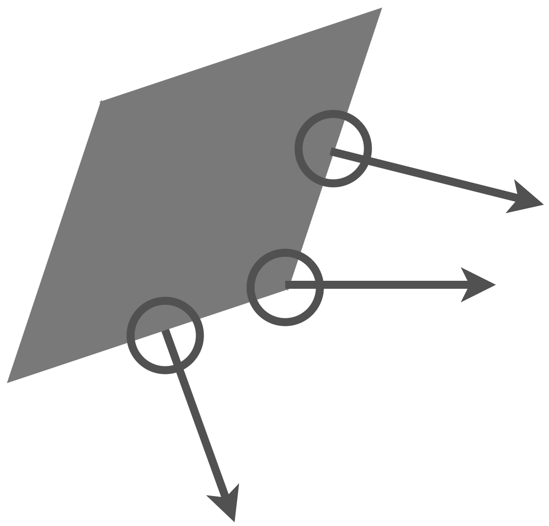

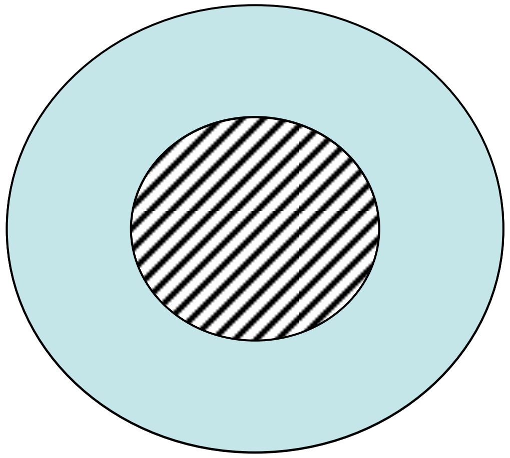

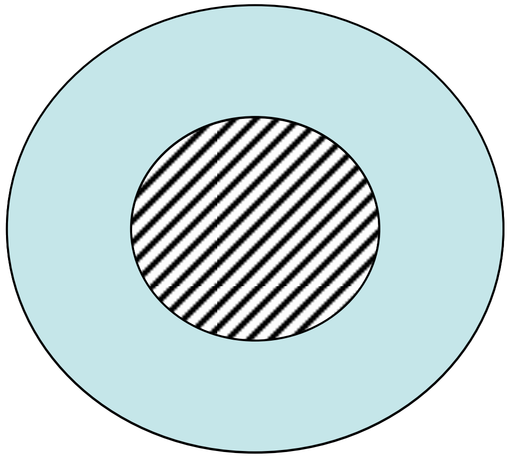

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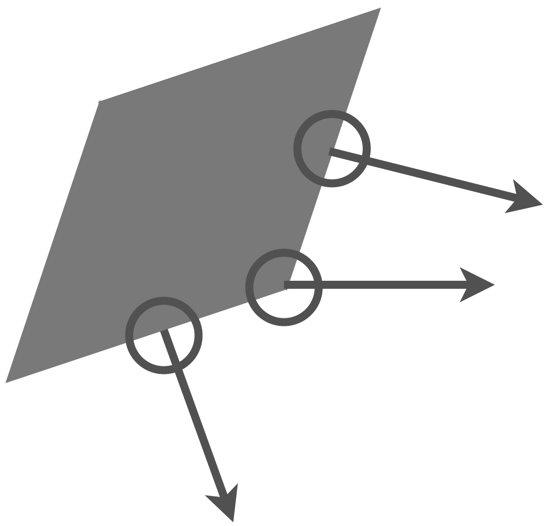

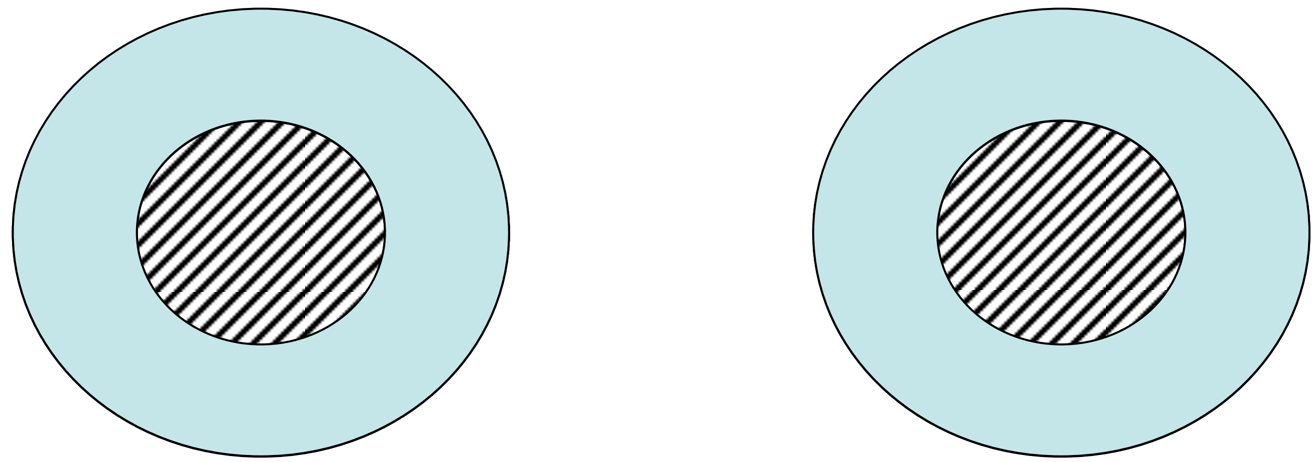

Milyen mozgásokkal konzisztens?
(mik a stimulus alapján a valószínű mozgások?)

Illúziók, mint Bayes-i komputációk

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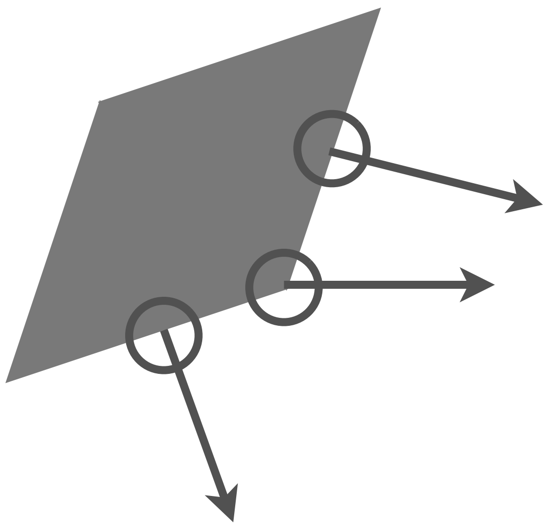

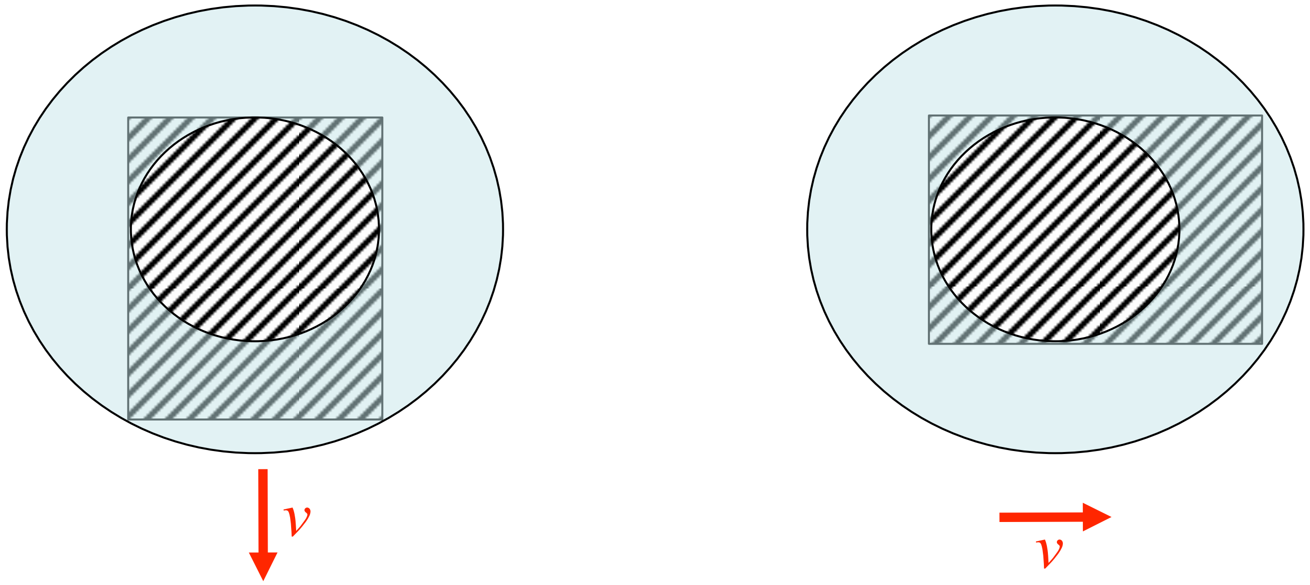

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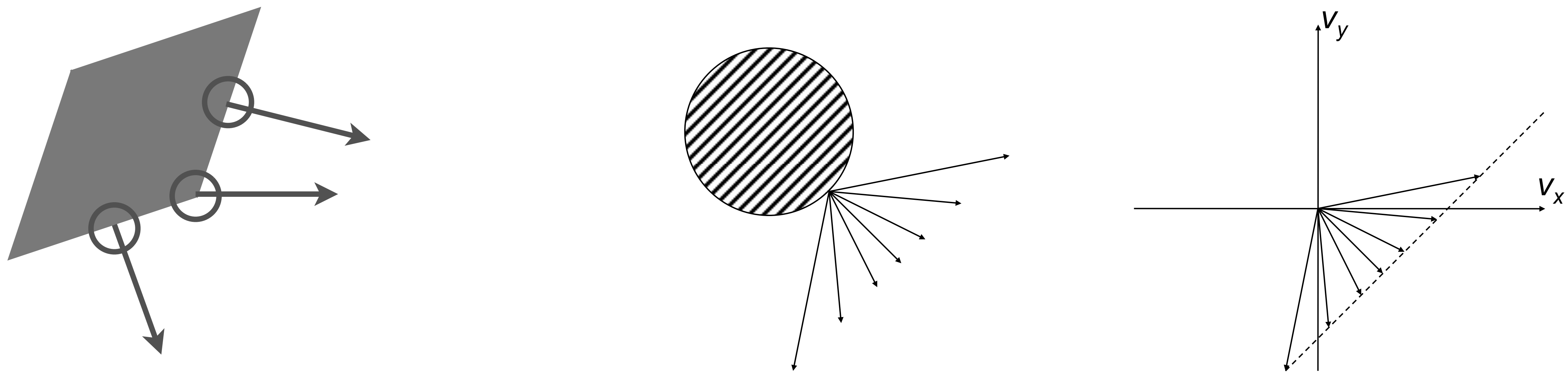

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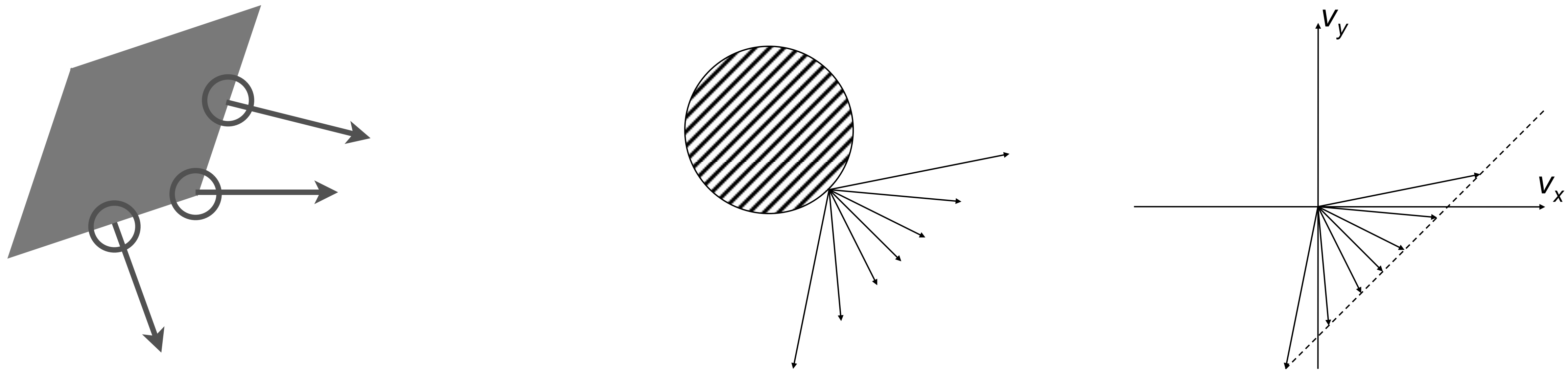

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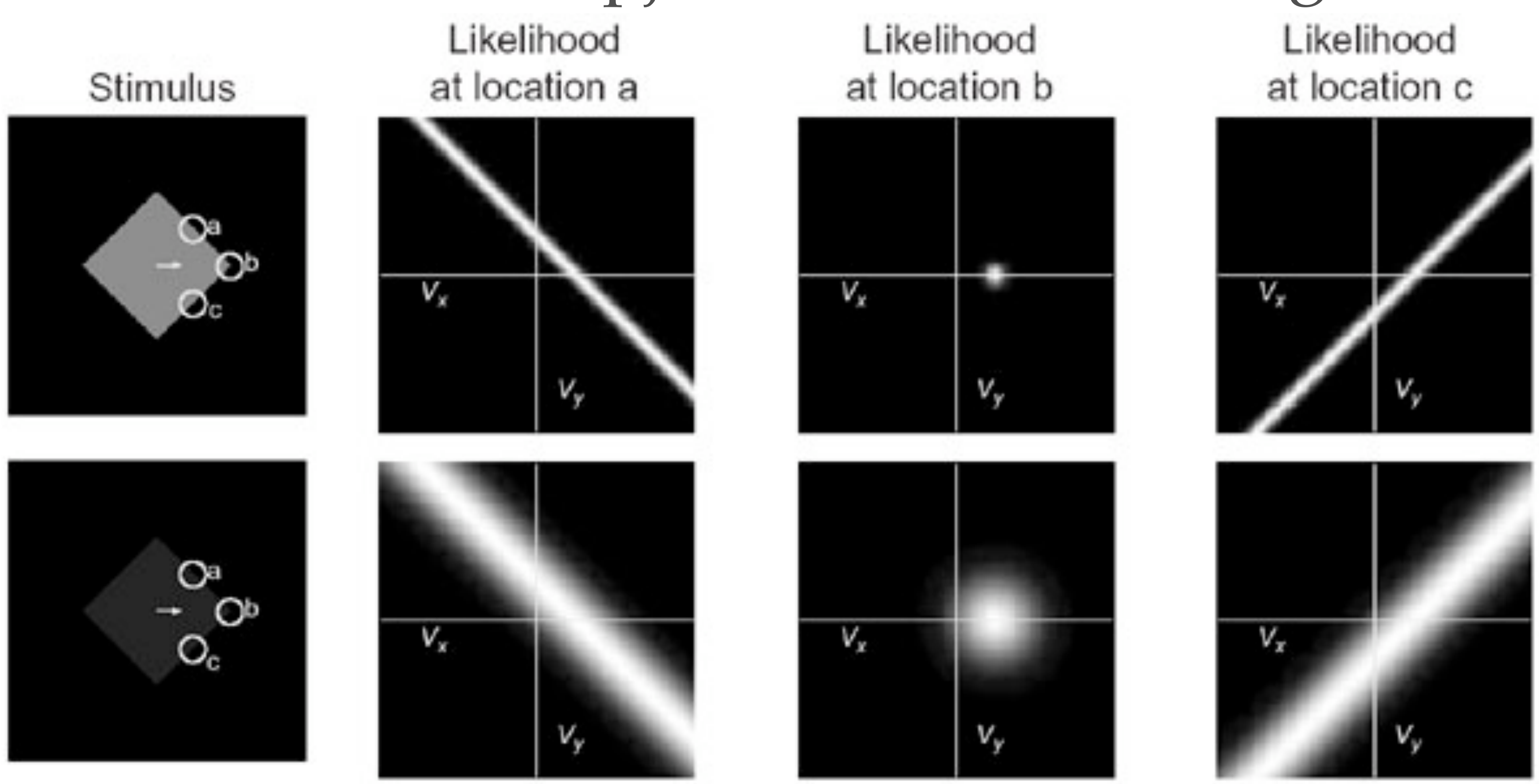

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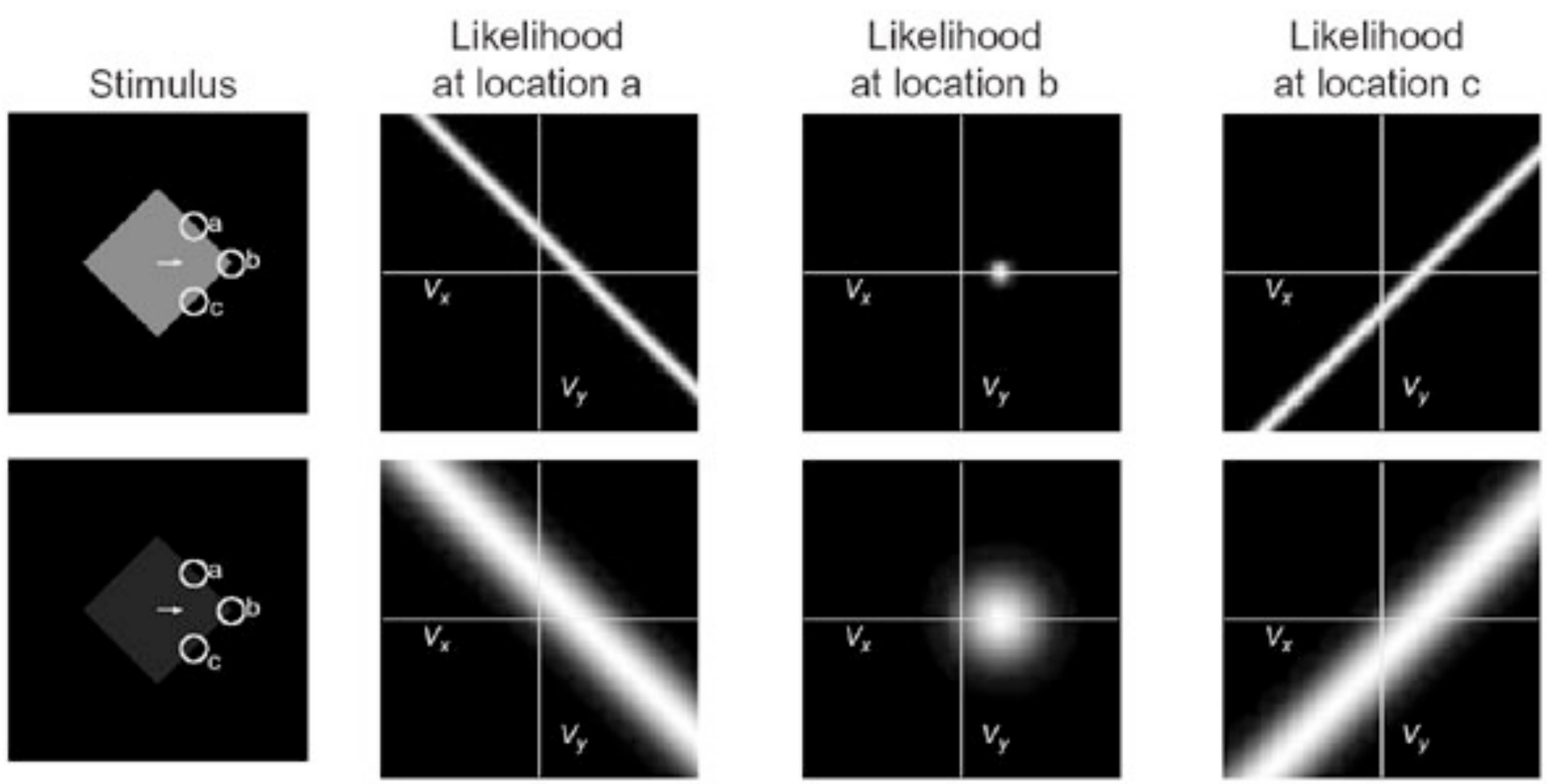

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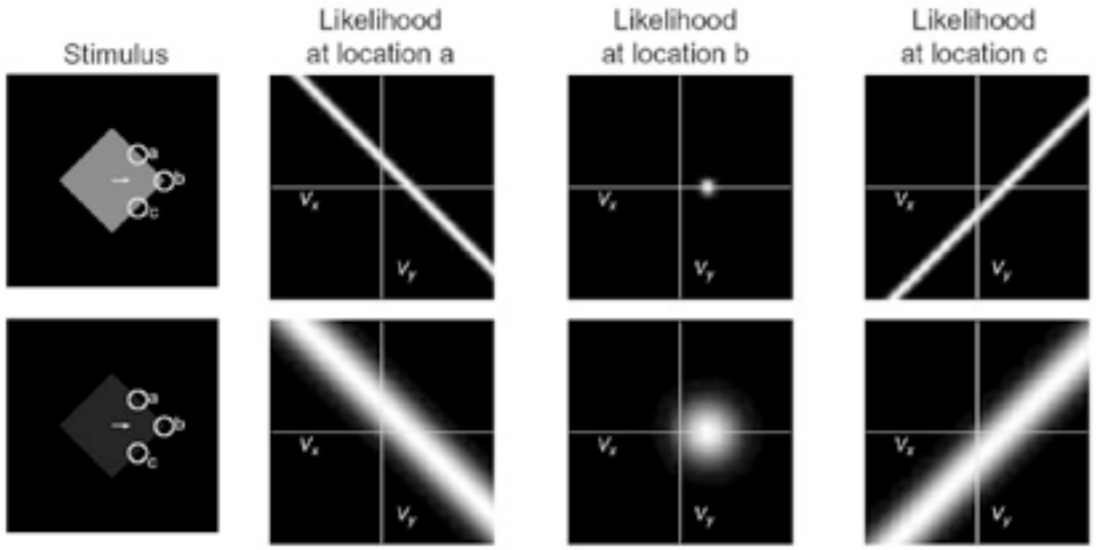

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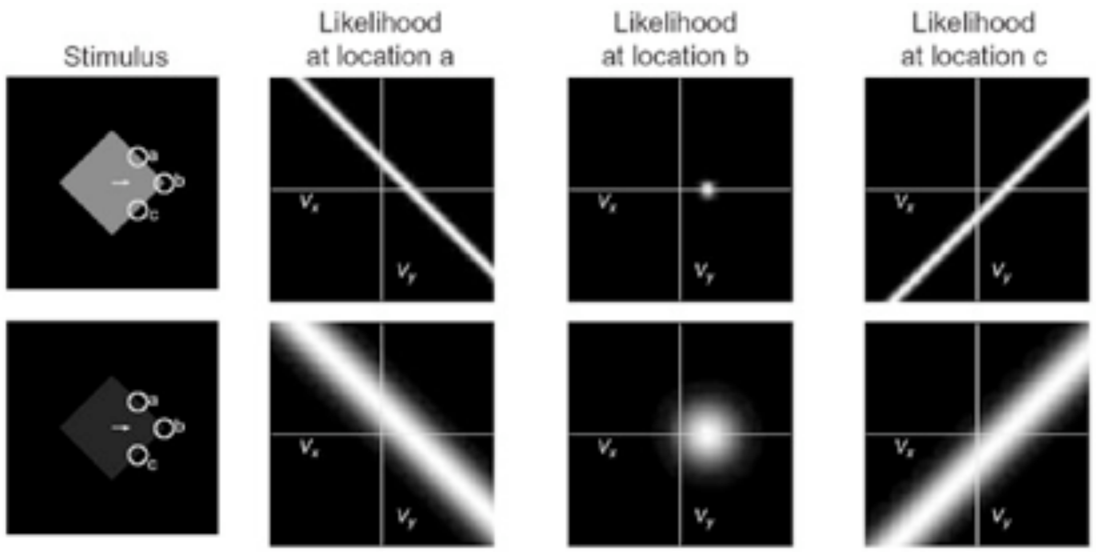

'Ideális megfigyelő analízis'

Illúziók, mint Bayes-i komputációk

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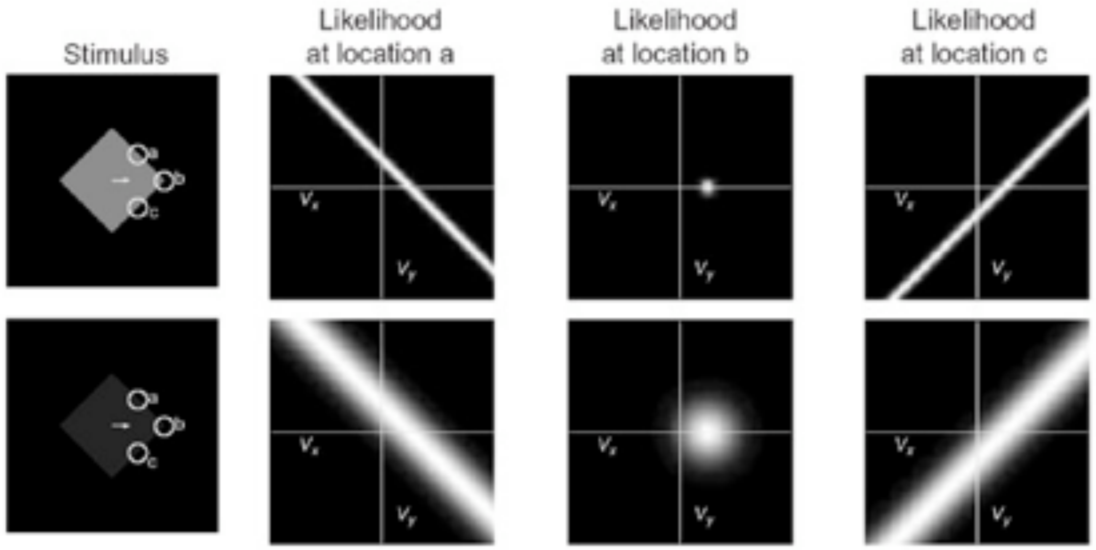

'Ideális megfigyelő analízis'

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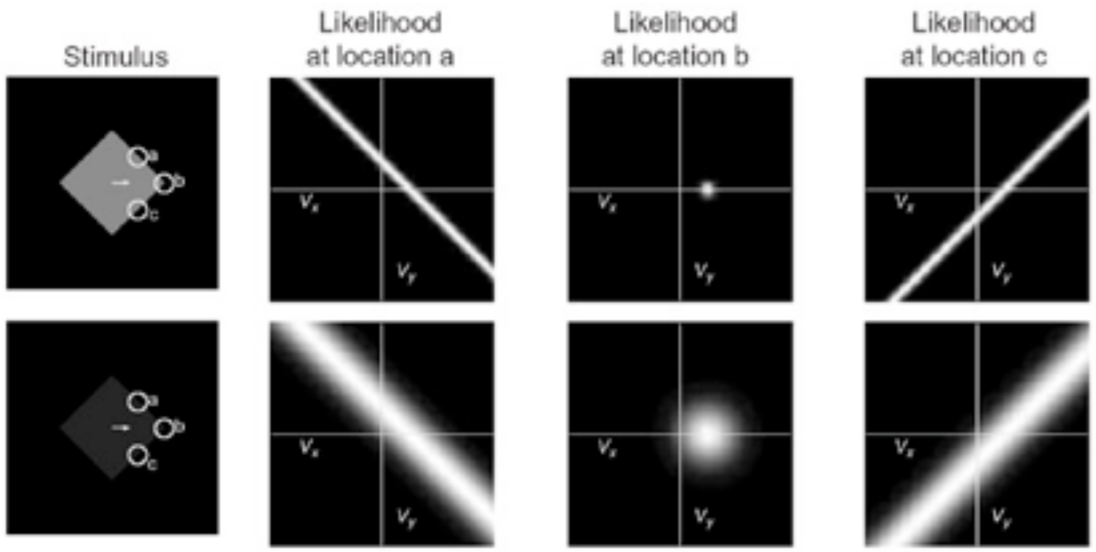

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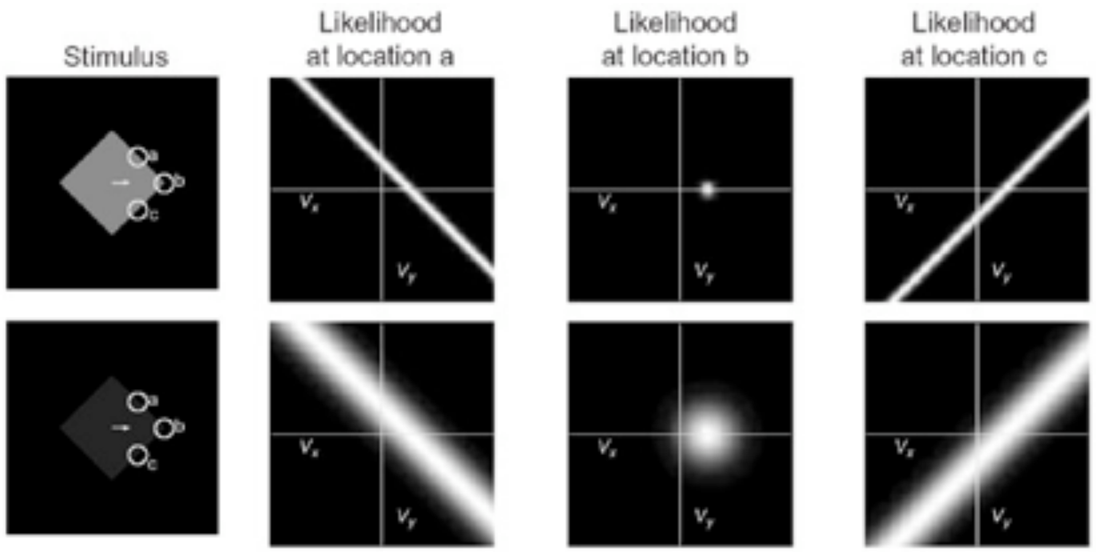

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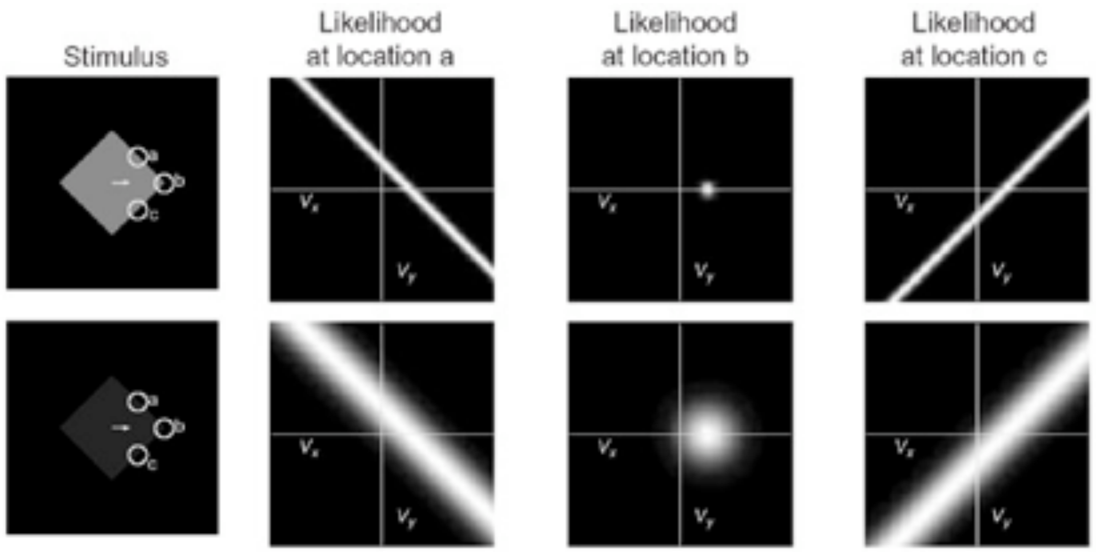

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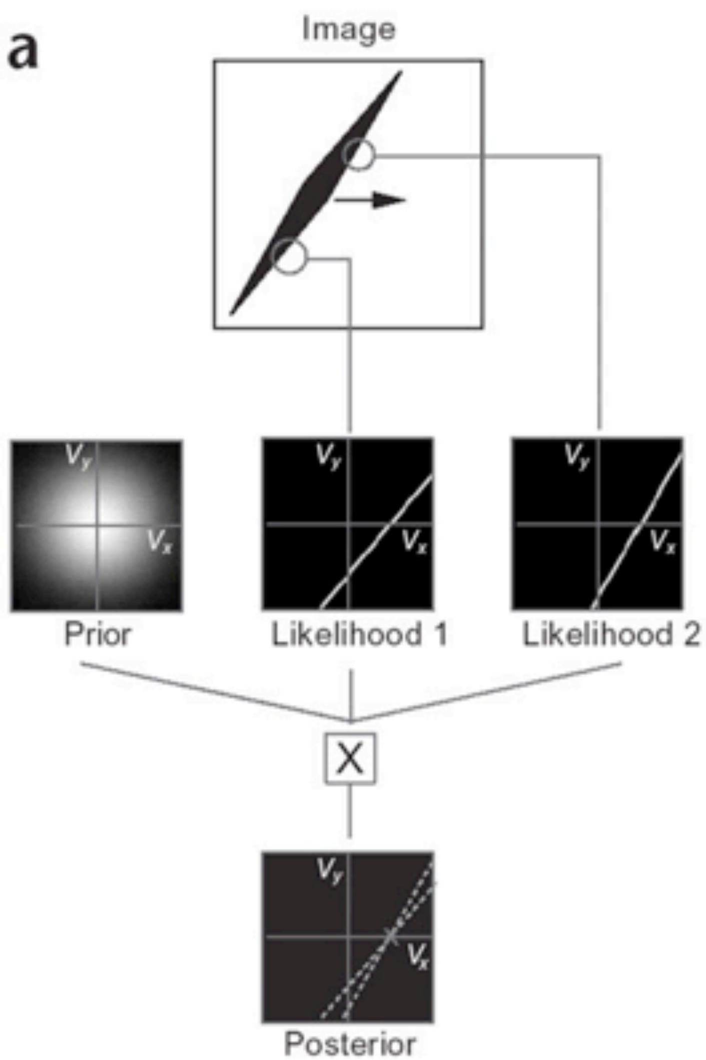

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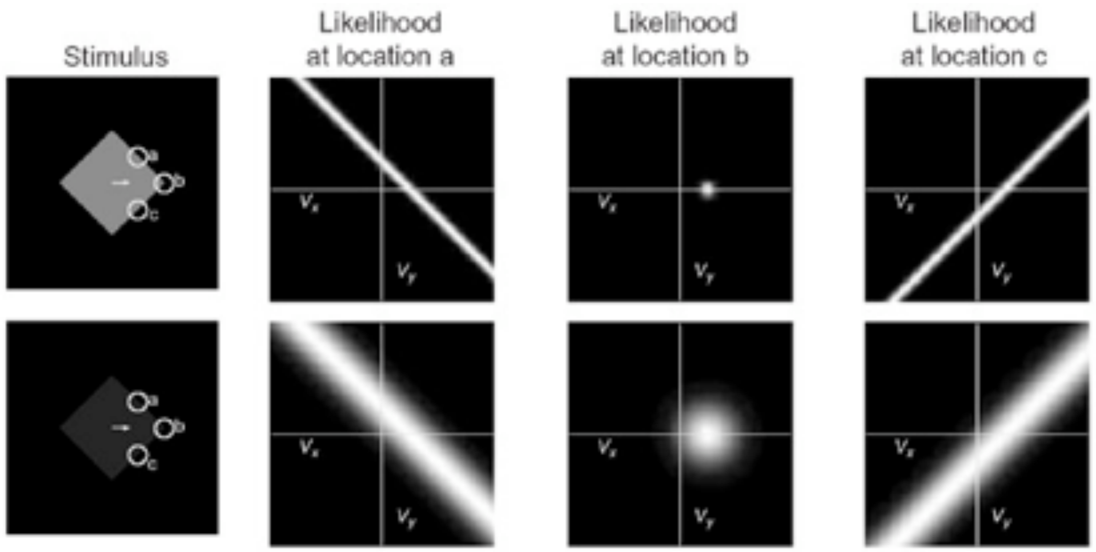

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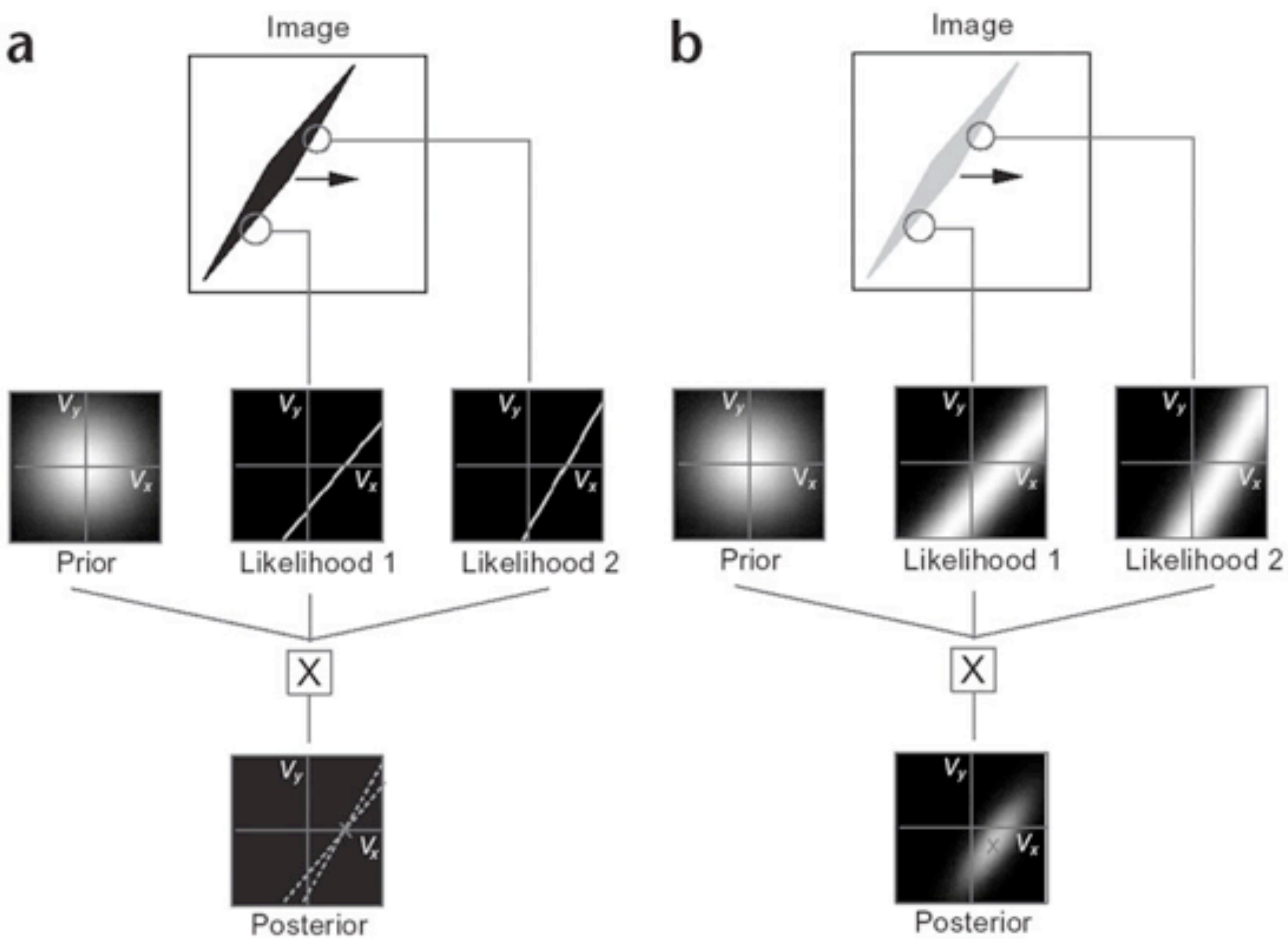

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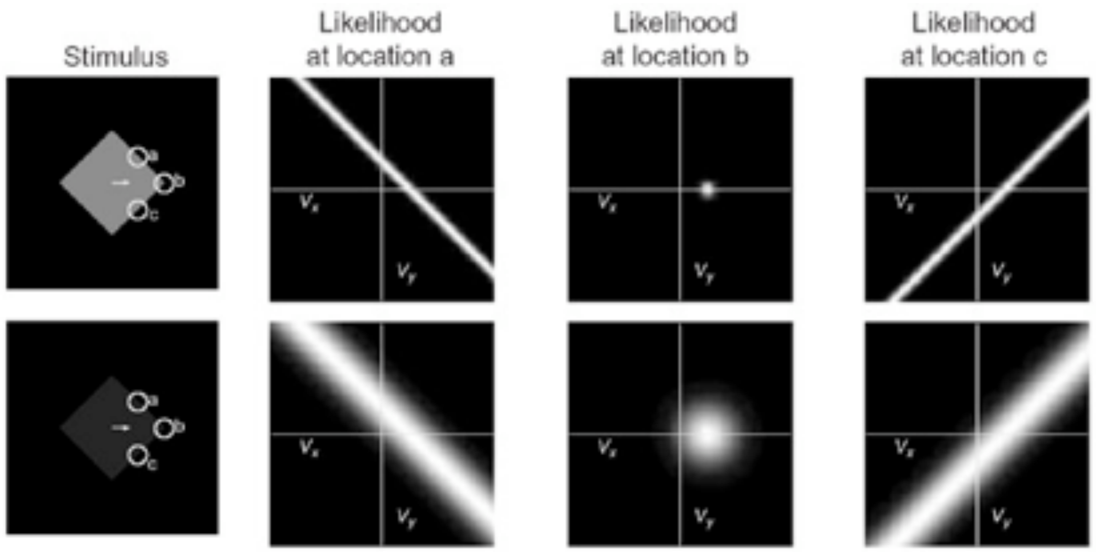

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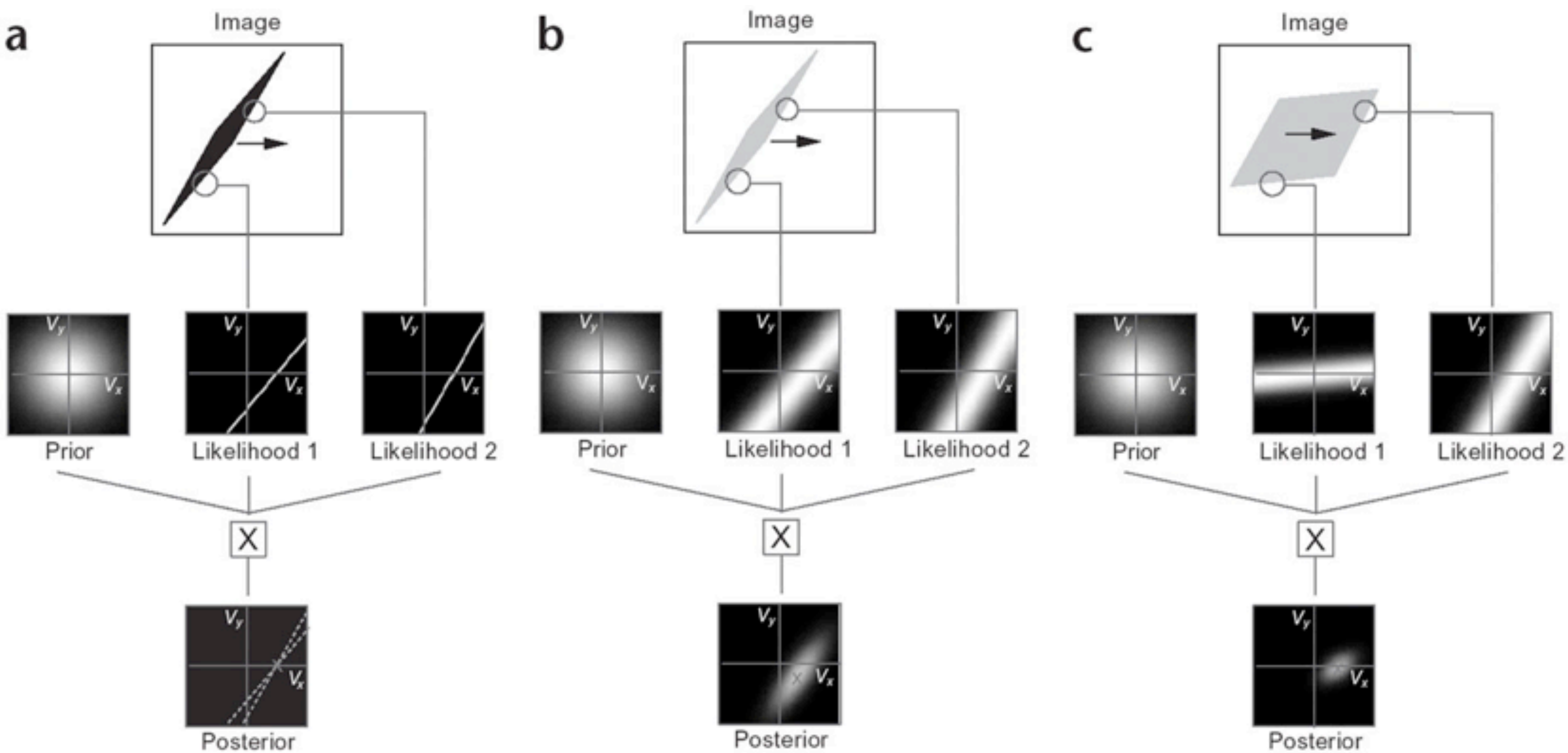
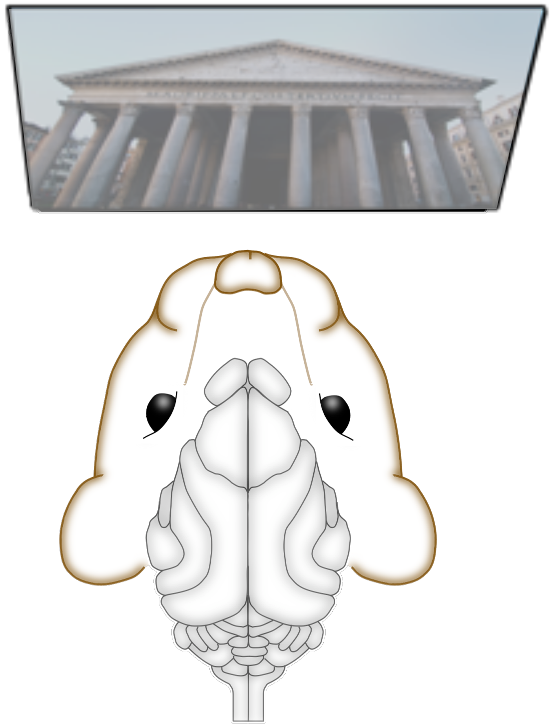
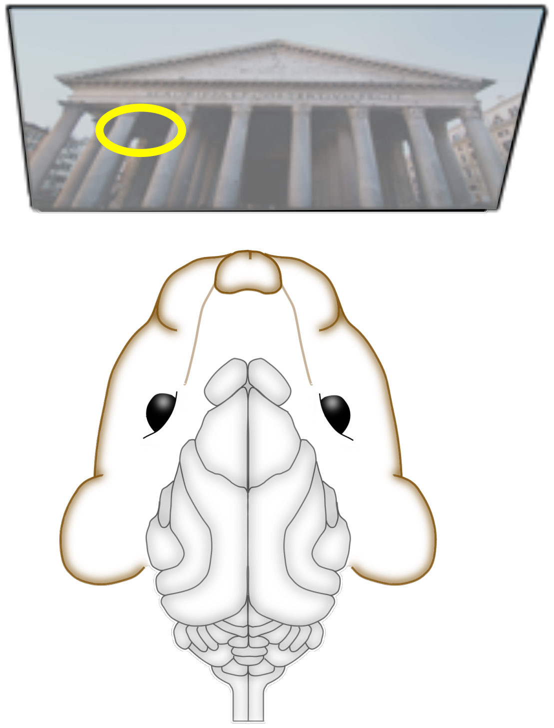
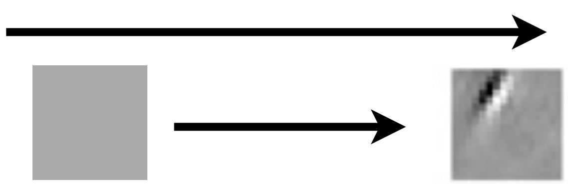




uncertainty in inferences

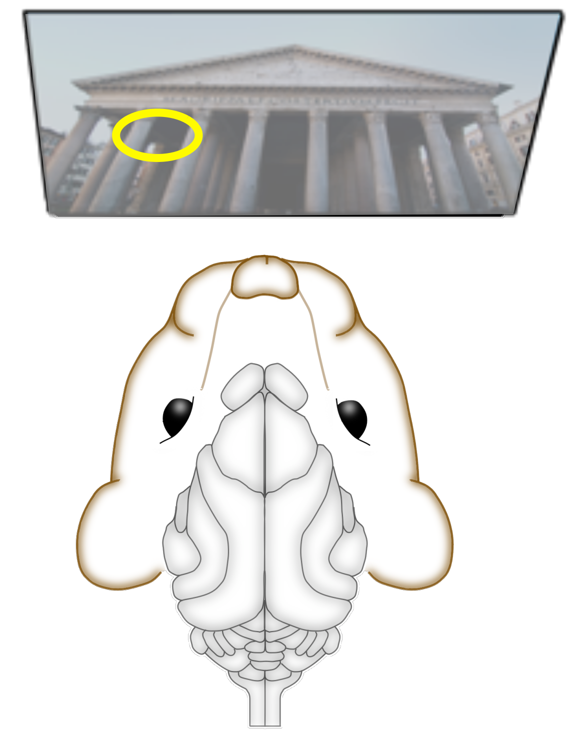


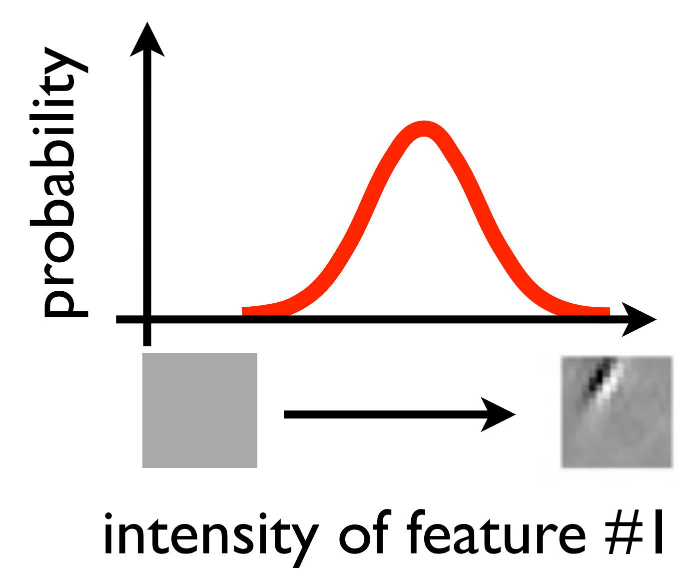


intensity of feature #1

uncertainty in inferences

every possible feature intensity is associated with a probability





uncertainty in inferences

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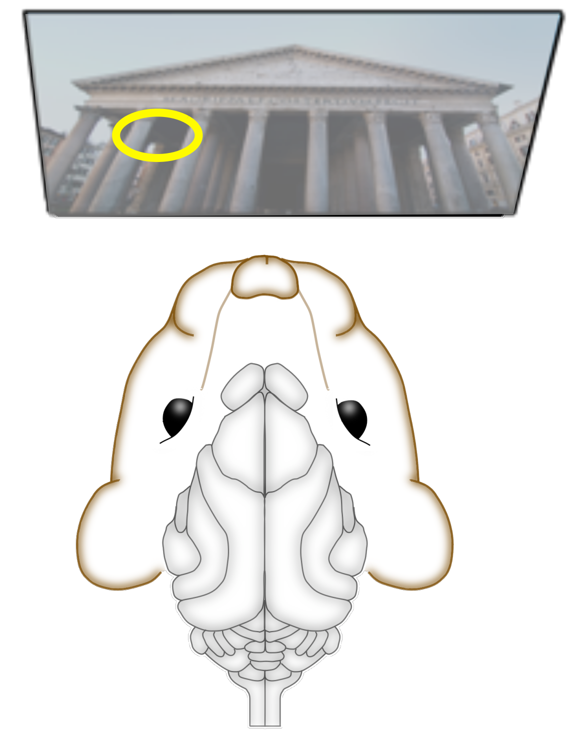
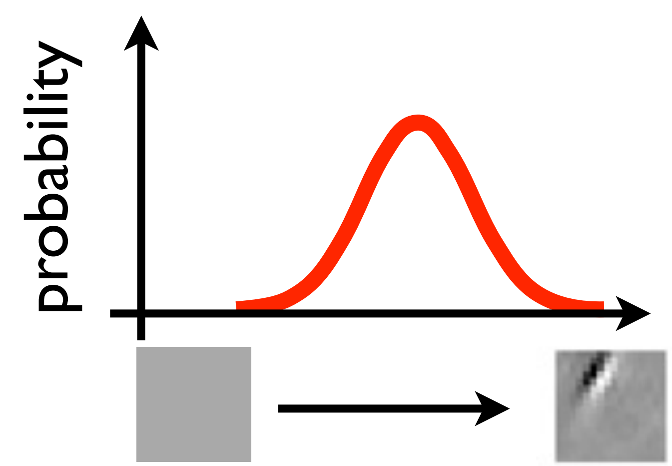
neural activity represents feature intensity





intensity of feature #1

activity of neuron #1

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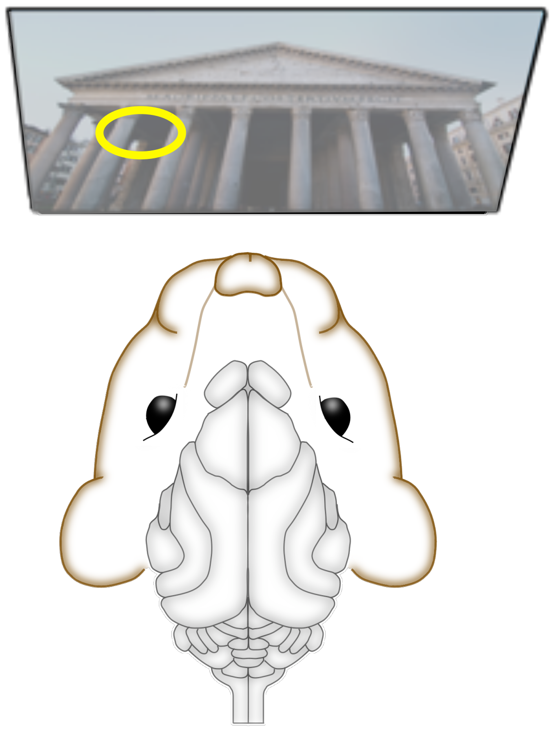
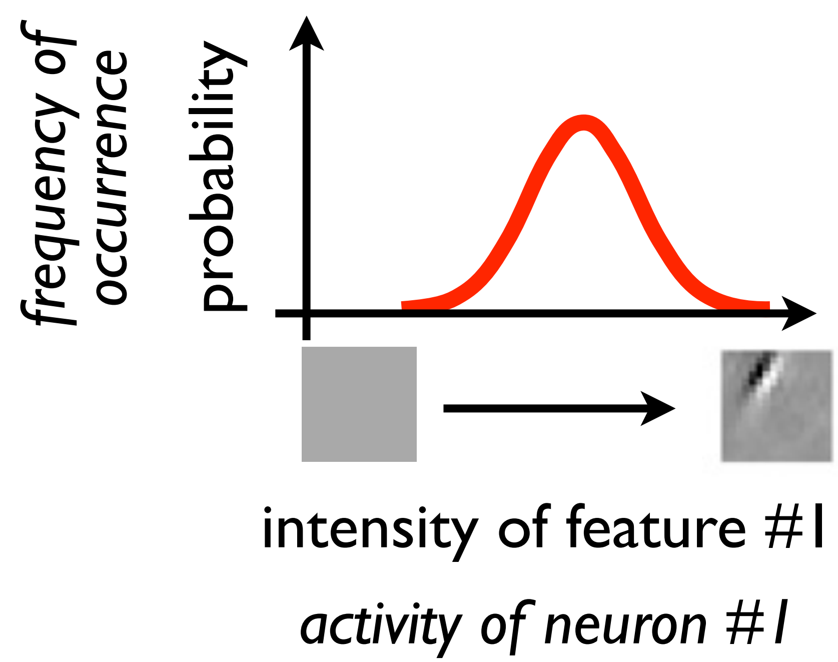
occurrence frequency of activity
~
probability of intensity

Hoyer & Hyvarinen, NIPS 2003, Lee & Mumford 2003





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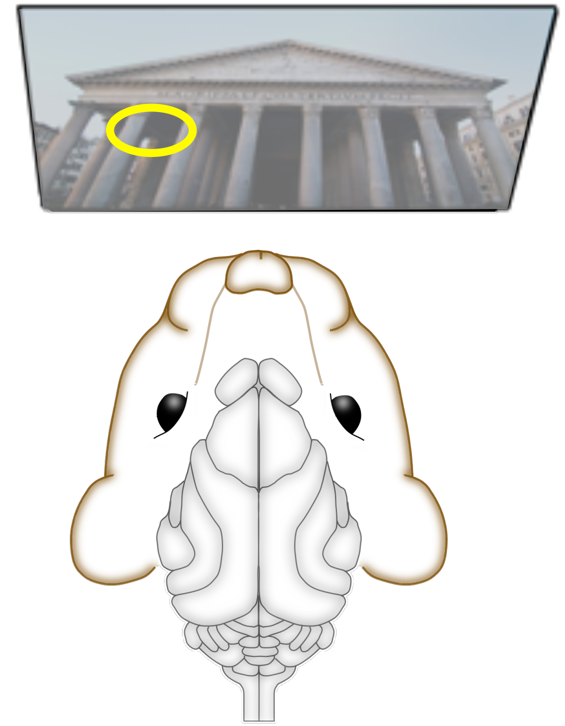

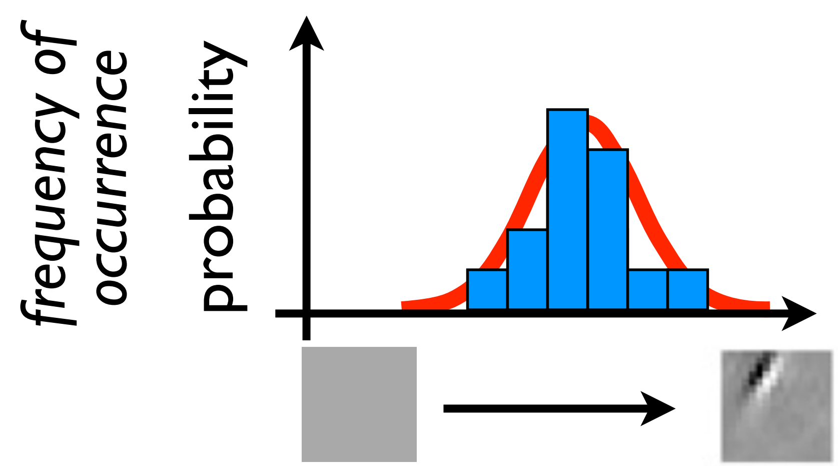

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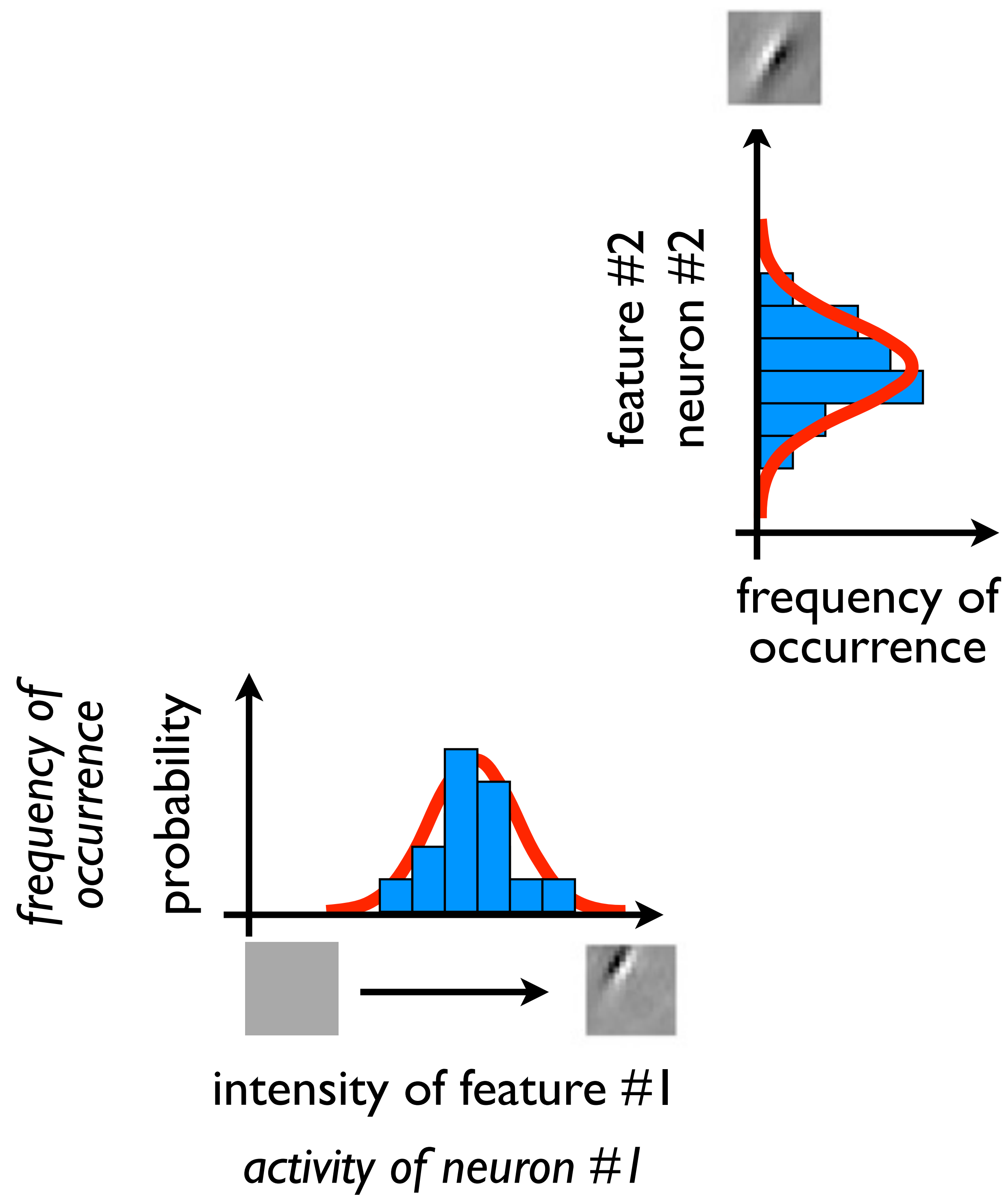

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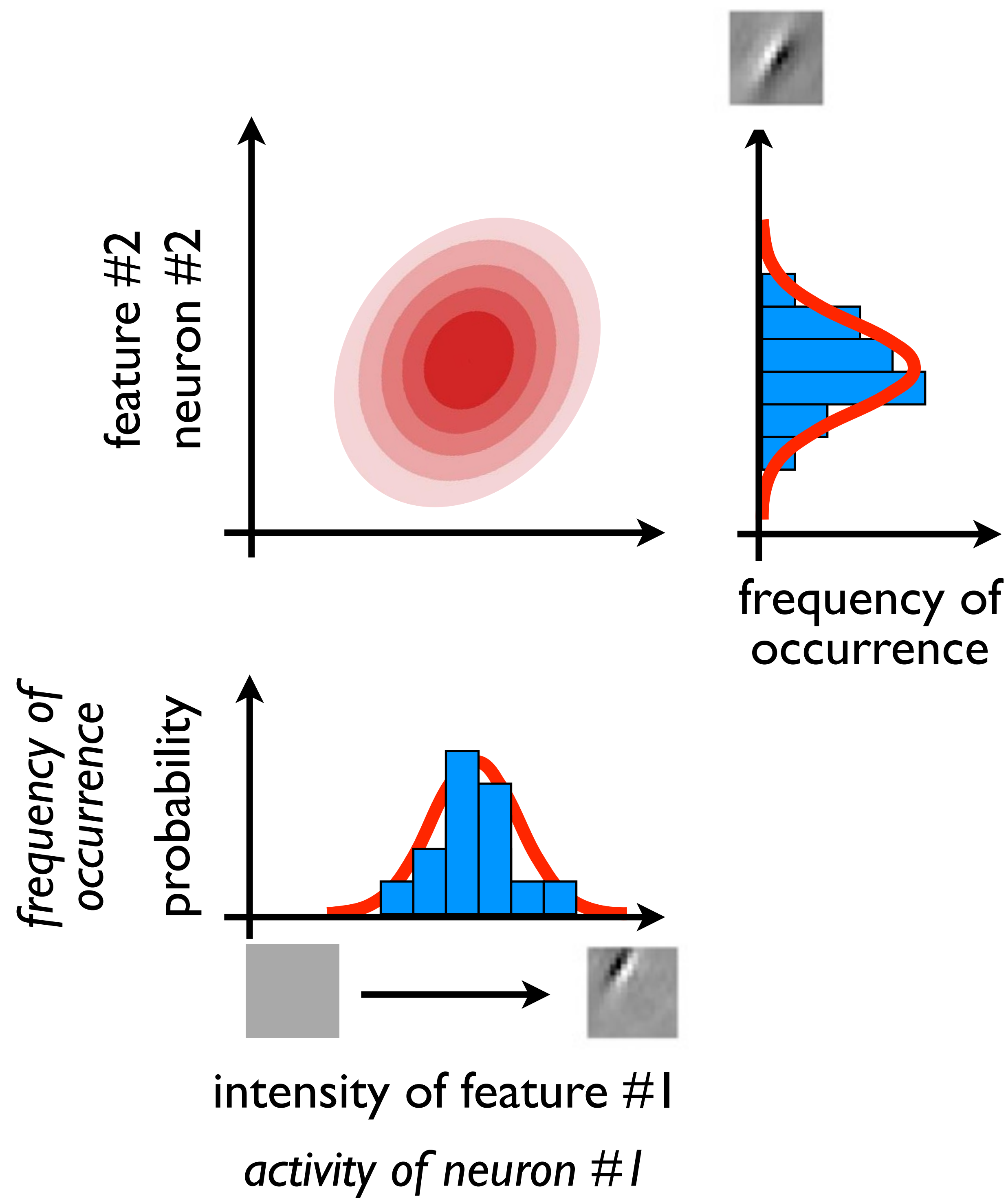

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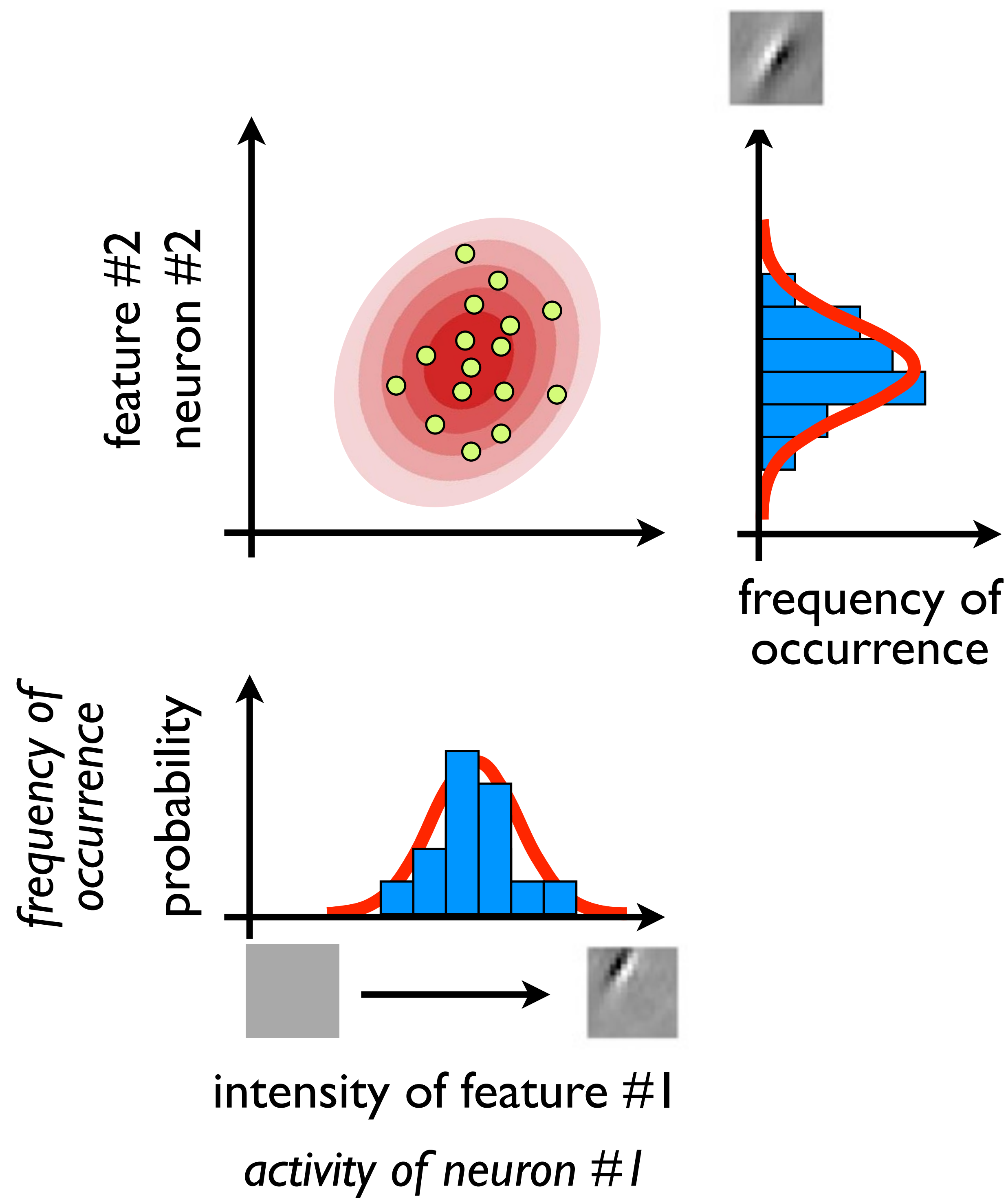
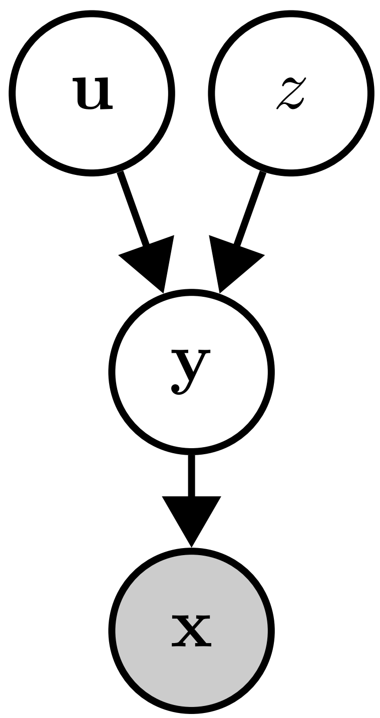


Gaussian Scale Mixtures ismét



$$
\begin{aligned}
\mathrm{P}(\mathbf{x}|\mathbf{y}) &= \mathcal{N}\big(\mathbf{x}; \mathbf{A}\mathbf{y}, \sigma_{\mathrm{x}}^2 \mathbf{I}\big) \\
\mathbf{y} &= z\,\mathbf{u} \\
\mathrm{P}(\mathbf{u}) &= \mathcal{N}(\mathbf{u}; \mathbf{0}, \mathbf{C}) \\
\mathrm{P}(z) &= \mathrm{Gamma}(z; k, \theta)
\end{aligned}
$$

deterministic inference
receptive field properties *Olshausen & Field, Nature 1996*
contrast invariance *Schwartz & Simoncelli, Nat Neurosci 2001*
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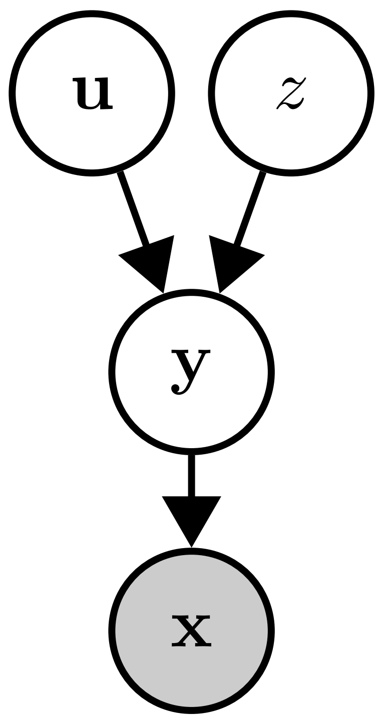

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$$P\left(\mathbf{z} \mid \mathbf{x}\right) \propto P\left(\mathbf{x} \mid \mathbf{z}\right) P\left(\mathbf{z}\right) \quad ?$$

Inference and learning

Inference

$$
\begin{aligned}
\mathrm{P}(\mathbf{u}|z,\mathbf{x}) &\propto \mathrm{P}(\mathbf{u})\ \mathrm{P}(\mathbf{x}|\mathbf{u},z) \\
&\propto \mathcal{N}(\mathbf{u};\mathbf{0},\mathbf{C})\ \mathcal{N}(\mathbf{u};\mathbf{m},\mathbf{D}) \\
&= \mathcal{N}(\mathbf{u};\boldsymbol{\mu}(z,\mathbf{x})\,,\boldsymbol{\Sigma}(z))
\end{aligned}
$$

with

$$
\boldsymbol{\mu}(z,\mathbf{x}) = \boldsymbol{\Sigma}(z)\left(\mathbf{C}^{-1}\mathbf{0}+\mathbf{D}^{-1}\mathbf{m}\right) \qquad = \frac{z}{\sigma_{\mathrm{x}}^2}\,\boldsymbol{\Sigma}(z)\,\mathbf{A}^{\mathsf{T}}\mathbf{x}
$$

$$
\boldsymbol{\Sigma}(z) = \left(\mathbf{C}^{-1}+\mathbf{D}^{-1}\right)^{-1} \qquad = \left(\mathbf{C}^{-1}+\frac{z^2}{\sigma_{\mathrm{x}}^2}\,\mathbf{A}^{\mathsf{T}}\mathbf{A}\right)^{-1}
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Inference and learning

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$$\mathrm{P}(\mathbf{x}|z) = \mathcal{N}\left(\mathbf{x};\ \mathbf{0},\sigma_{\mathrm{x}}^2\,\mathbf{I}+z^2\,\mathbf{A}\,\mathbf{C}\,\mathbf{A}^{\mathsf{T}}\right)$$

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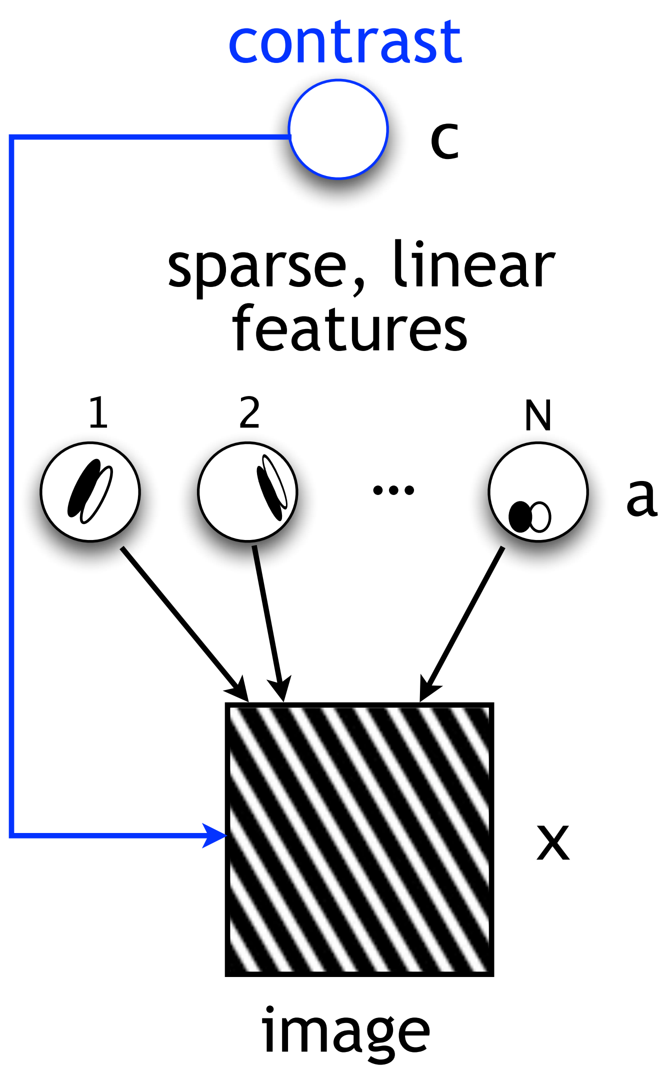
Learning

- Only the covariance matrix $\mathbf{C}$ is learned
- Projective fields (A) are learned by sparse coding
- EM is used

Prior and posterior

Stimulus onset quenches neural variability



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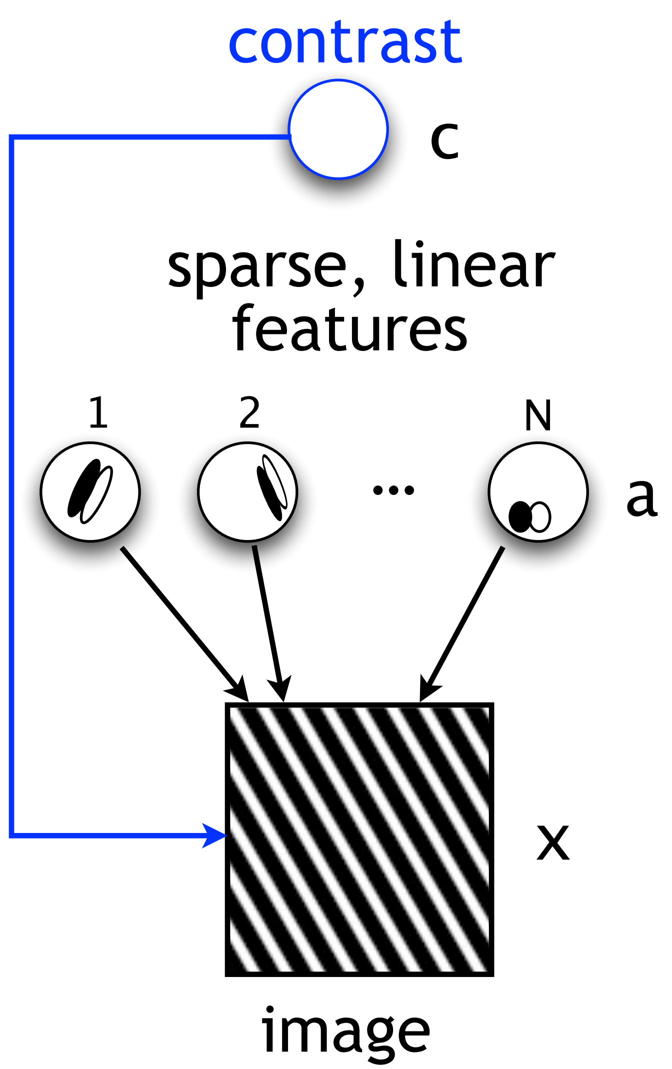

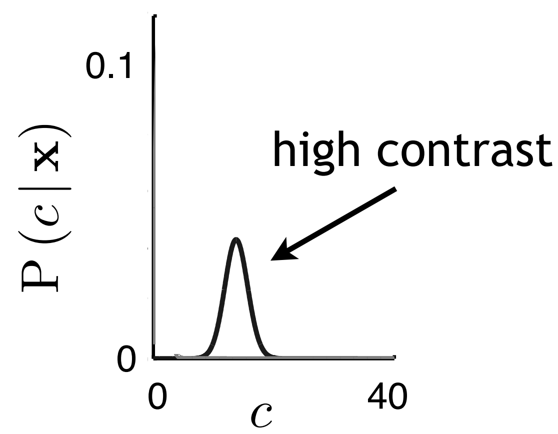

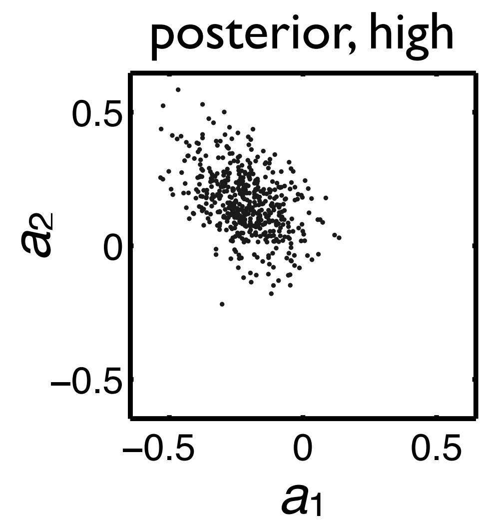

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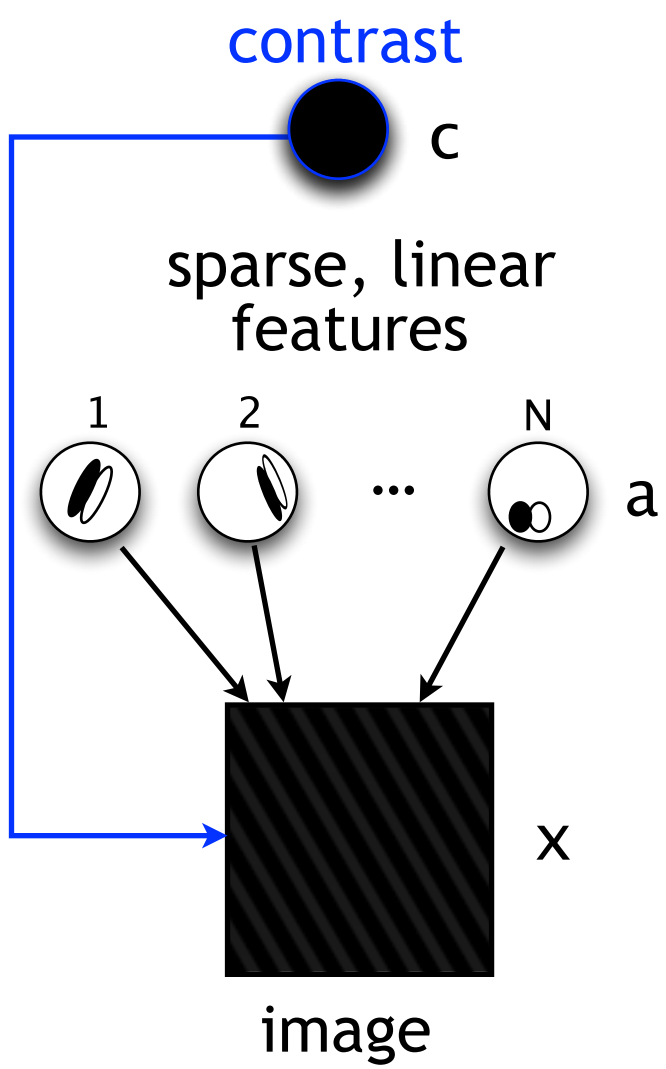

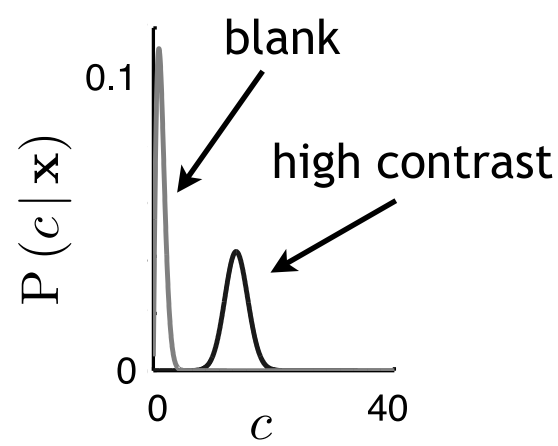

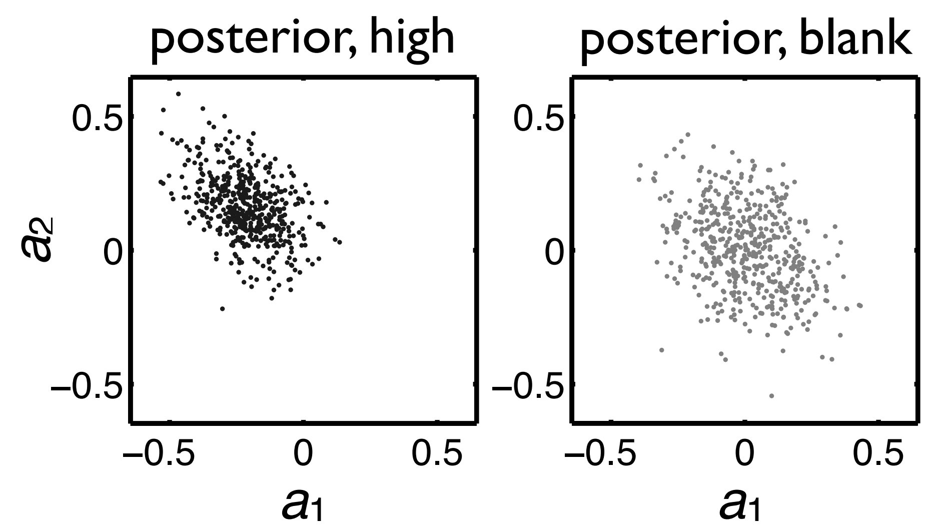

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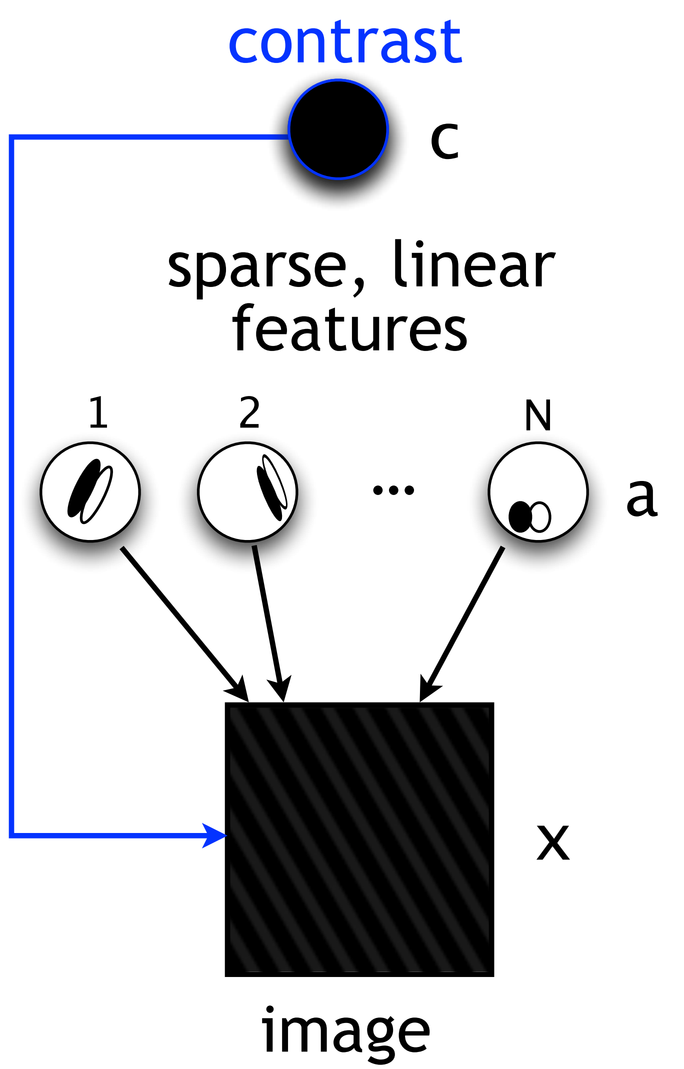
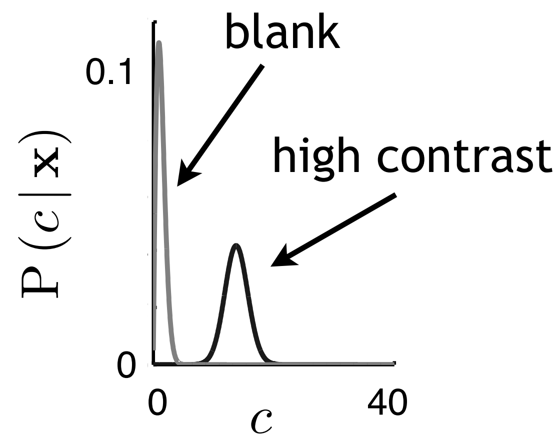
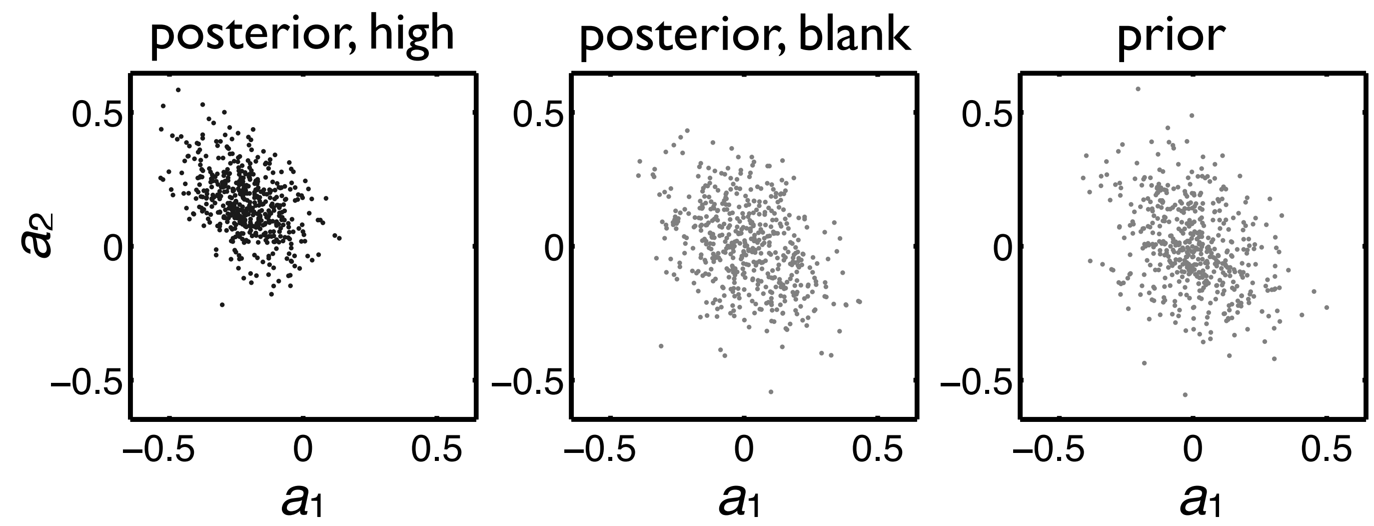
Stimulus onset quenches neural variability

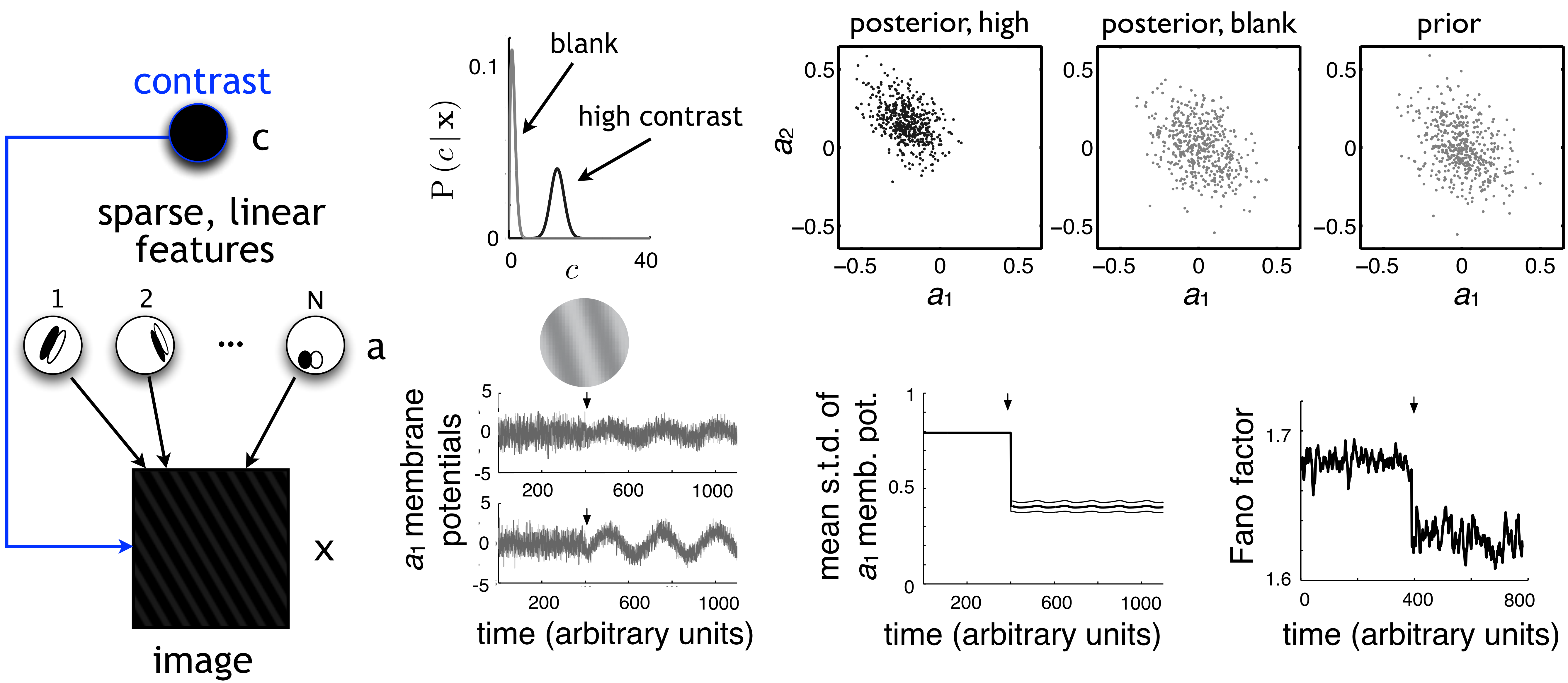






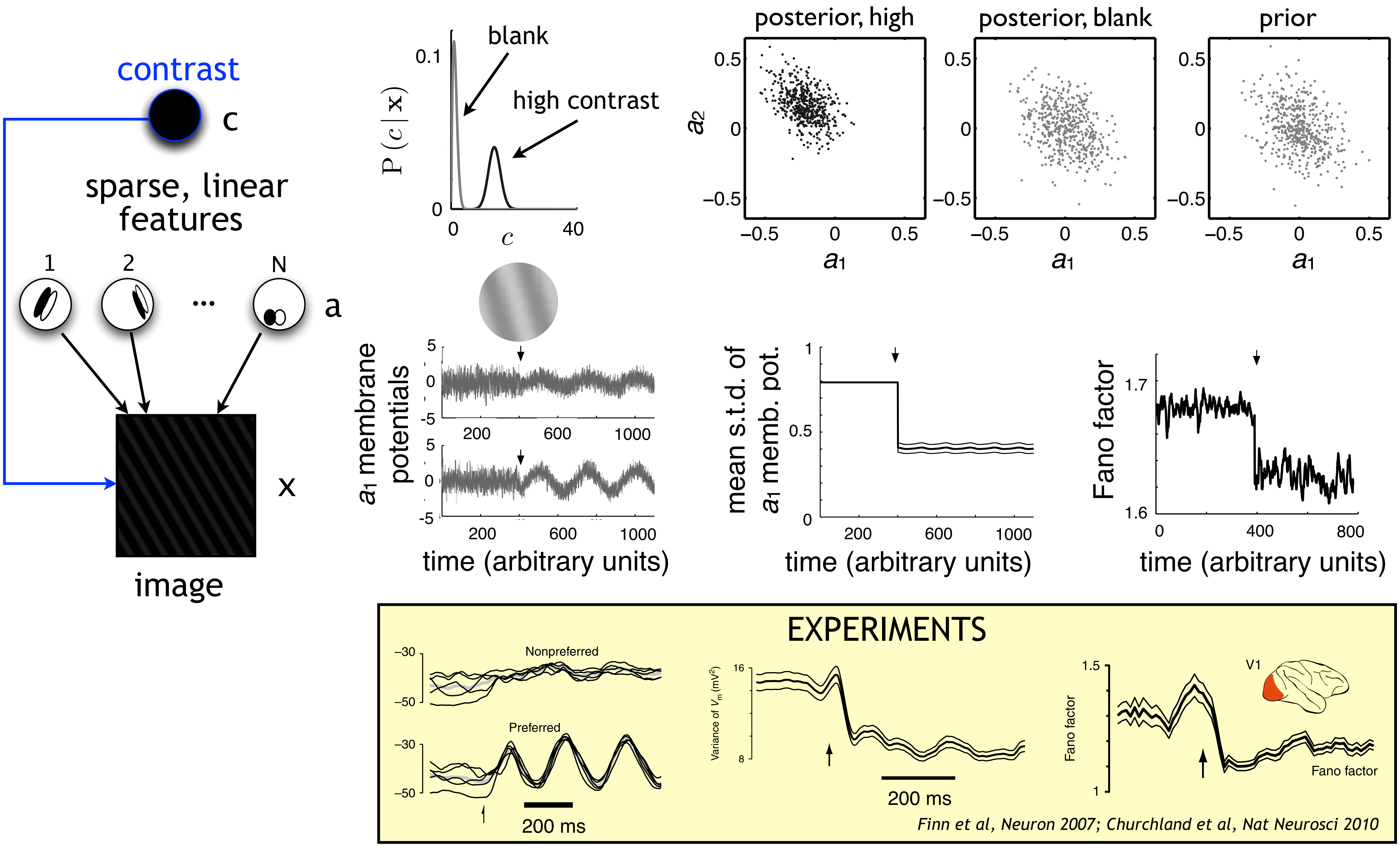
Prior and posterior

Stimulus onset quenches neural variability



Prior and posterior

Stimulus onset quenches neural variability



Bayes-i inferencia GSM-ben

deterministic inference
receptive field properties *Olshausen & Field, Nature 1996*
contrast invariance *Schwartz & Simoncelli, Nat Neurosci 2001*
extra-classical receptive fields *Schwartz & Simoncelli, Nat Neurosci 2001*
psychophysics *Schwartz & Dayan, JVis 2009*

Bayes-i inferencia GSM-ben

deterministic inference	Bayesian inference + sampling	
	mean	(co-)variance
receptive field properties *Olshausen & Field, Nature 1996*	***reproduces results with deterministic inference***	variability at stimulus onset *Churchland et al, Nat Neurosci 2010*
contrast invariance *Schwartz & Simoncelli, Nat Neurosci 2001*		contrast dependence of membrane potential variance *Finn et al, Neuron 2007*
extra-classical receptive fields *Schwartz & Simoncelli, Nat Neurosci 2001*		contrast dependence of firing rate covariability *Smith & Kohn, J Neurosci 2005*
psychophysics *Schwartz & Dayan, J Vis 2009*		dependence on stimulus statistics *Berkes et al, Science 2011*

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

↓

percept

↑

visual stimulus

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

$\mathbf{y}^*$
objects, features

percept

visual stimulus

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

$\mathrm{P}^*(\mathbf{y})$

$\mathbf{y}^*$
objects, features

percept

visual stimulus

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

$\mathrm{P}^*(\mathbf{y})$

$\mathbf{y}^*$
objects, features

percept

$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$

$\mathbf{x}$
visual stimulus

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

$\mathrm{P}^*(\mathbf{y})$

$\mathbf{y}^*$
objects, features

$\mathbf{y}$
percept

$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$

$\mathbf{x}$
visual stimulus

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

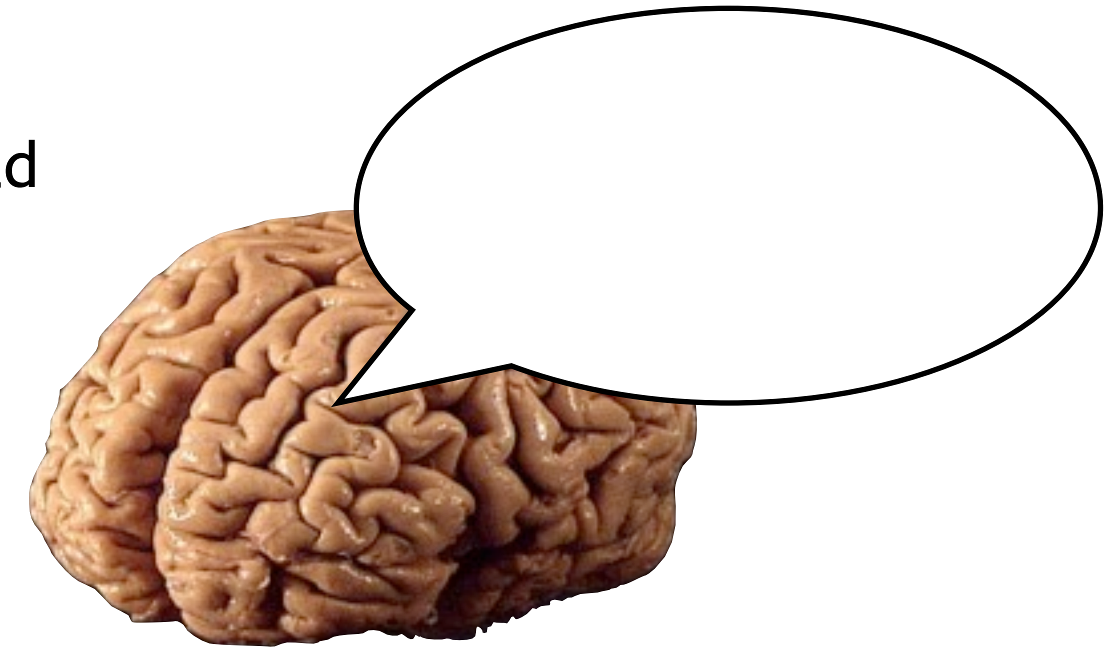
$\mathrm{P}^*(\mathbf{y})$ $\downarrow$ $\mathbf{y}^*$
objects, features

$\mathrm{P}_\theta(\mathbf{y})$ $\downarrow$ $\mathbf{y}$
percept

$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$ $\searrow$

$\mathrm{P}_\theta(\mathbf{x}|\mathbf{y})$ $\uparrow$

$\mathbf{x}$
visual stimulus



STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

$\mathrm{P}^*(\mathbf{y})$

$\mathbf{y}^*$
objects, features

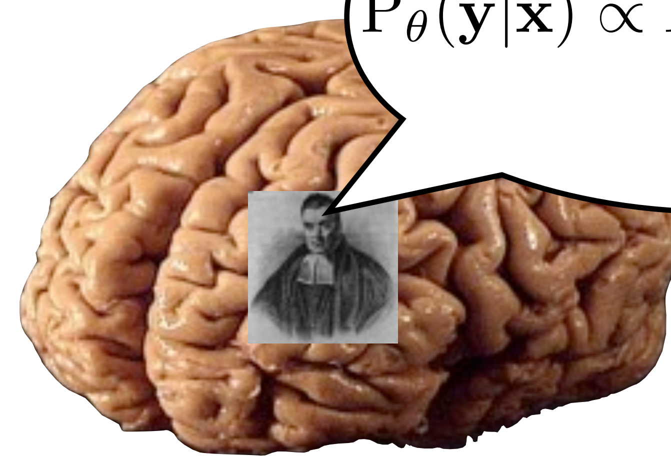
$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$

$\mathrm{P}_\theta(\mathbf{y})$

$\mathbf{y}$
percept

$\mathrm{P}_\theta(\mathbf{x}|\mathbf{y})$

$\mathbf{x}$
visual stimulus



$$\mathrm{P}_\theta(\mathbf{y}|\mathbf{x}) \propto \mathrm{P}_\theta(\mathbf{x}|\mathbf{y})\,\mathrm{P}_\theta(\mathbf{y})$$

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

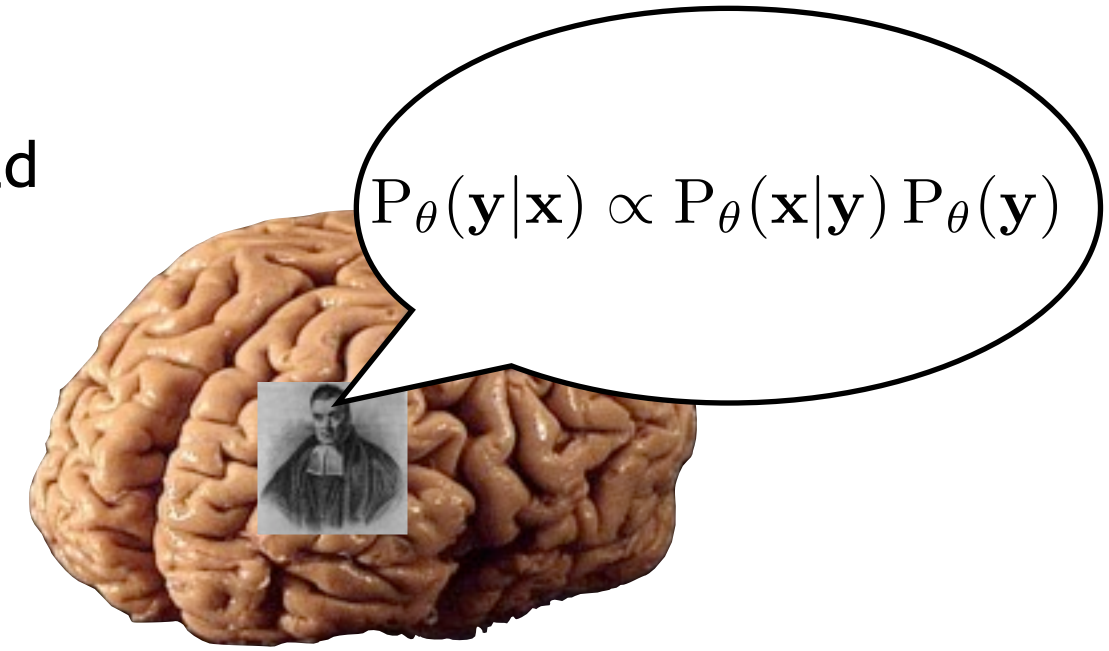
$\mathrm{P}^*(\mathbf{y})$ ↓ $\mathbf{y}^*$
objects, features

$\mathrm{P}_\theta(\mathbf{y})$ ↓ $\mathbf{y}$
percept

$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$ ↘

↑ $\mathrm{P}_\theta(\mathbf{x}|\mathbf{y})$

$\mathbf{x}$
visual stimulus



HOW DO WE KNOW IF AN INTERNAL MODEL IS OPTIMAL?

$$\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} \quad = \quad \mathrm{P}_\theta(\mathbf{y})$$

STATISTICALLY OPTIMAL INTERNAL MODELS?

prior knowledge
about the visual world

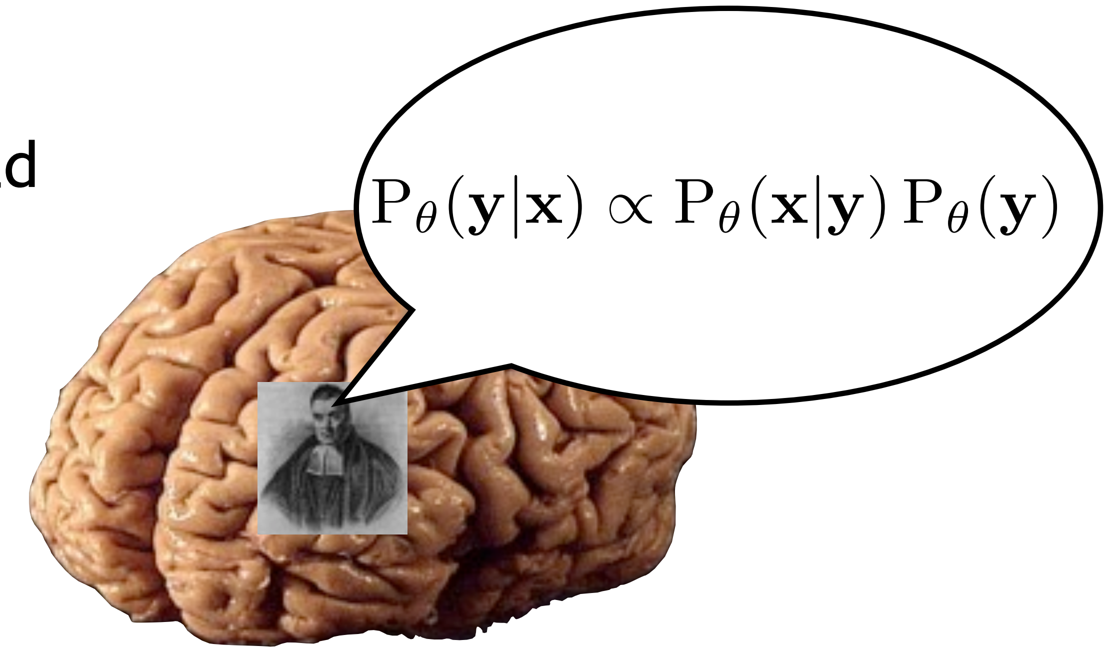
$\mathrm{P}^*(\mathbf{y})$ $\mathrm{P}_\theta(\mathbf{y})$

$\mathbf{y}^*$
objects, features

$\mathbf{y}$
percept

$\mathrm{P}^*(\mathbf{x}|\mathbf{y})$ $\mathrm{P}_\theta(\mathbf{x}|\mathbf{y})$

$\mathbf{x}$
visual stimulus



HOW DO WE KNOW IF AN INTERNAL MODEL IS OPTIMAL?

"average inferences" "prior expectations"

$$\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} \quad = \quad \mathrm{P}_\theta(\mathbf{y})$$

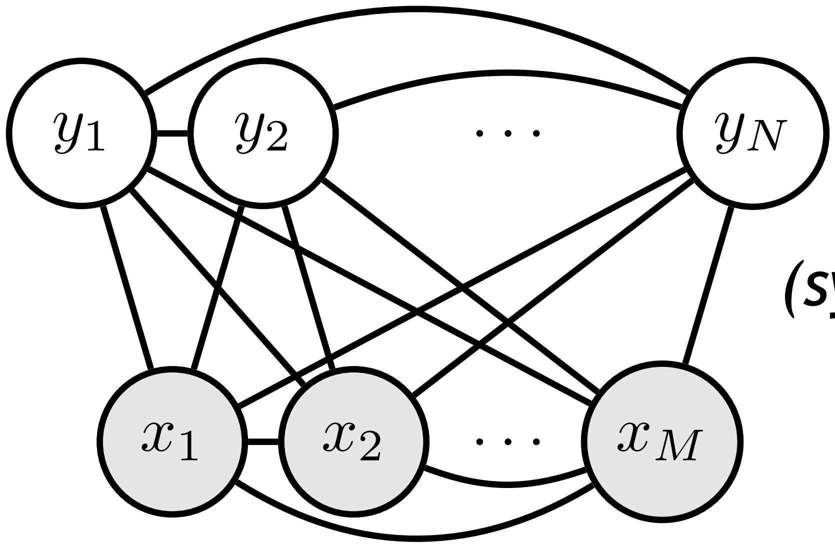
for natural stimulus ensemble

$$\mathrm{P}^*(\mathbf{x}) = \int \mathrm{P}^*(\mathbf{y})\,\mathrm{P}^*(\mathbf{x}|\mathbf{y})\,d\mathbf{y}$$

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$

observed variables
(retina)
$\mathbf{x}$

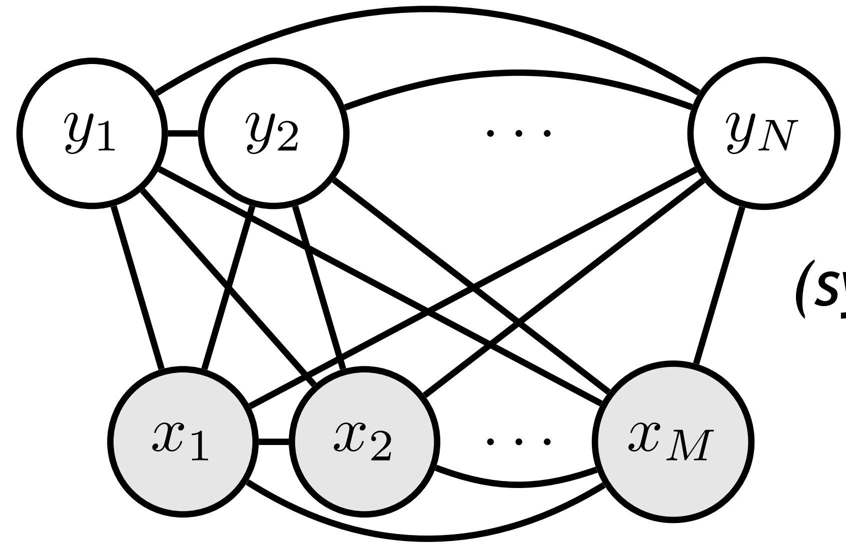


parameters
(synaptic efficacies, etc)
θ

$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} = \mathrm{P}_\theta(\mathbf{y})}$$

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$



parameters
(synaptic efficacies, etc)
θ

observed variables
(retina)
$\mathbf{x}$

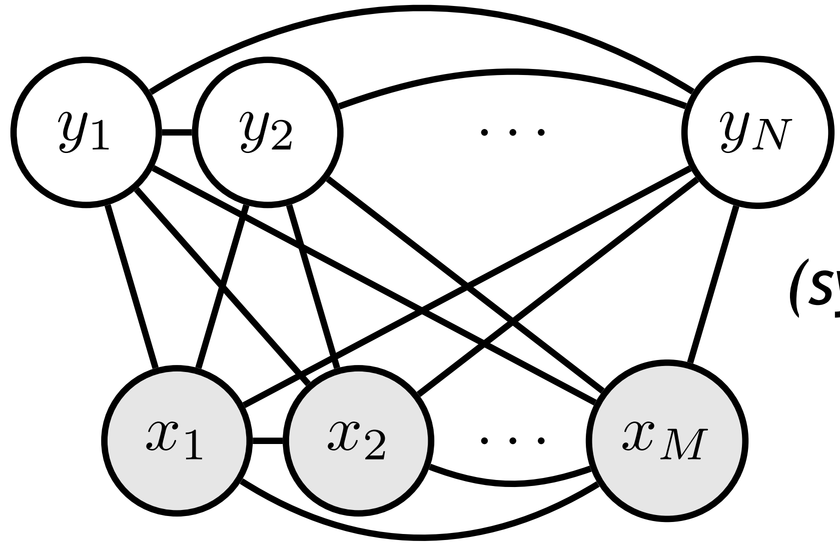
$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} = \mathrm{P}_\theta(\mathbf{y})}$$

evoked
activity

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$

observed variables
(retina)
$\mathbf{x}$



parameters
(synaptic efficacies, etc)
θ

$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} = \mathrm{P}_\theta(\mathbf{y})}$$

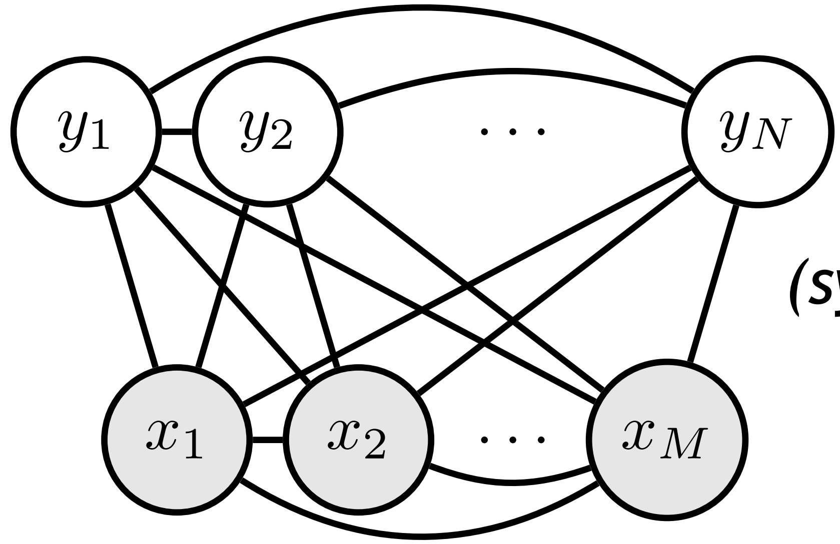
evoked
activity

spontaneous
activity

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$

observed variables
(retina)
$\mathbf{x}$



parameters
(synaptic efficacies, etc)
θ

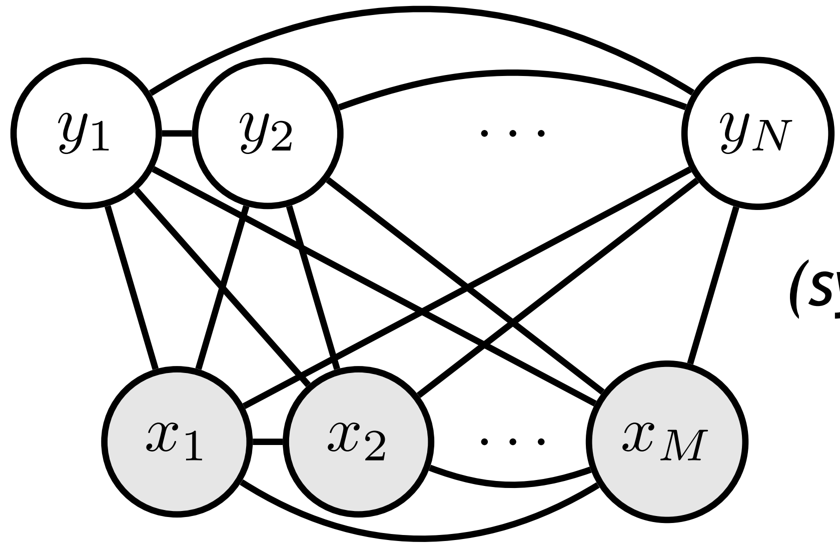
$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} = \mathrm{P}_\theta(\mathbf{y})}$$

evoked activity — stimulus ensemble — spontaneous activity

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$

observed variables
(retina)
$\mathbf{x}$



parameters
(synaptic efficacies, etc)
θ

$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} \stackrel{?}{=} \mathrm{P}_\theta(\mathbf{y})}$$

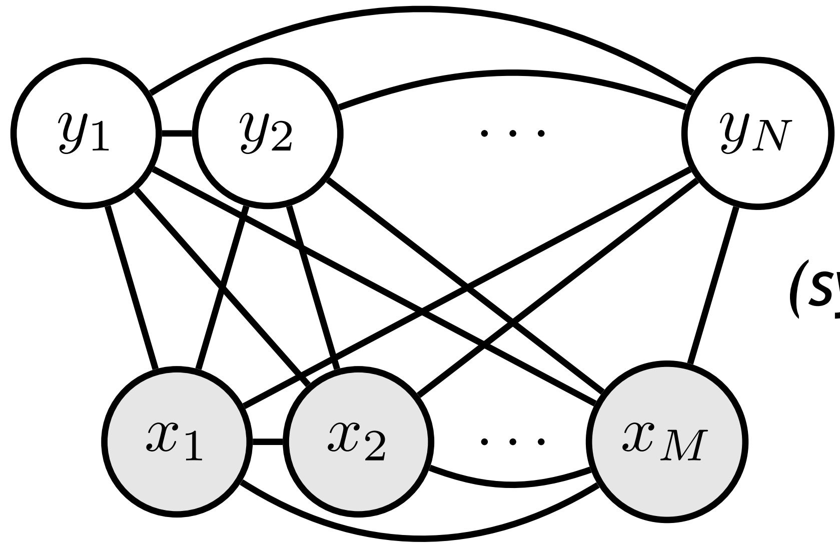
evoked activity

stimulus ensemble

spontaneous activity

APPLICATION TO VISUAL CORTEX

latent variables
(visual cortex)
$\mathbf{y}$



parameters
(synaptic efficacies, etc)
θ

observed variables
(retina)
$\mathbf{x}$

KL divergence

$$\boxed{\int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*(\mathbf{x})\ d\mathbf{x} \overset{?}{=} \mathrm{P}_\theta(\mathbf{y})}$$

evoked activity

stimulus ensemble

spontaneous activity

RECORDINGS

RECORDINGS

awake behaving ferrets

RECORDINGS

awake behaving ferrets
aged P29 (eye opening) — P151 (mature visual system)

RECORDINGS

awake behaving ferrets
aged P29 (eye opening) — P151 (mature visual system)
multi-unit recordings from layers 2/3 of V1

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multi-unit recordings from layers 2/3 of V1
16 electrodes with 200 μm spacing

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multi-unit recordings from layers 2/3 of V1
16 electrodes with 200 μm spacing

conditions

- **spontaneous**

darkness

$\mathrm{S}(\mathbf{y}) = \mathrm{P}_\theta(\mathbf{y})$

RECORDINGS

awake behaving ferrets
aged P29 (eye opening) – P151 (mature visual system)
multi-unit recordings from layers 2/3 of V1
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conditions

- **spontaneous**

darkness

$$\mathrm{S}(\mathbf{y}) = \mathrm{P}_\theta(\mathbf{y})$$

- **evoked**

natural image movies

$$\mathrm{M}(\mathbf{y}) = \int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*_{\mathrm{movie}}(\mathbf{x})\ d\mathbf{x}$$

RECORDINGS

awake behaving ferrets
aged P29 (eye opening) – P151 (mature visual system)
multi-unit recordings from layers 2/3 of V1
16 electrodes with 200 μm spacing

conditions

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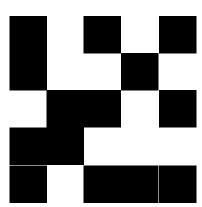

dynamic block noise

$$\mathrm{N}(\mathbf{y}) = \int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*_{\mathrm{noise}}(\mathbf{x})\ d\mathbf{x}$$

RECORDINGS

awake behaving ferrets
aged P29 (eye opening) – P151 (mature visual system)
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conditions

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darkness

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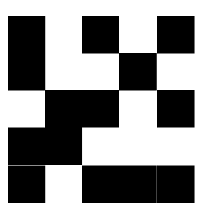

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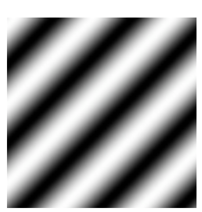
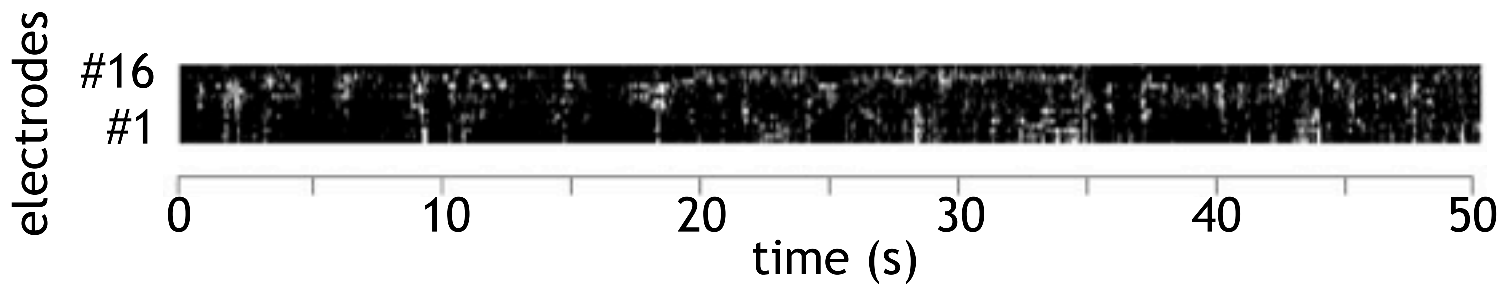


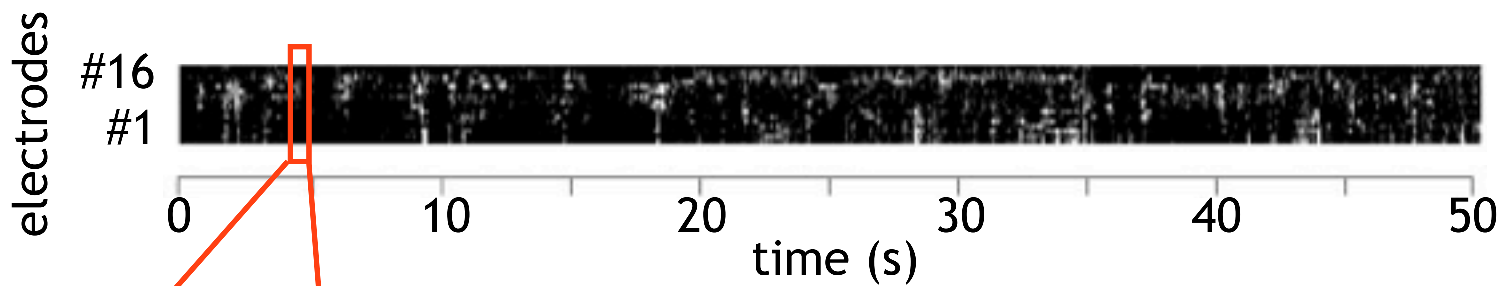
drifting full-field gratings

$$\mathrm{G}(\mathbf{y}) = \int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*_{\mathrm{grating}}(\mathbf{x})\ d\mathbf{x}$$

DATA ANALYSIS



DATA ANALYSIS



discretisation and binarisation

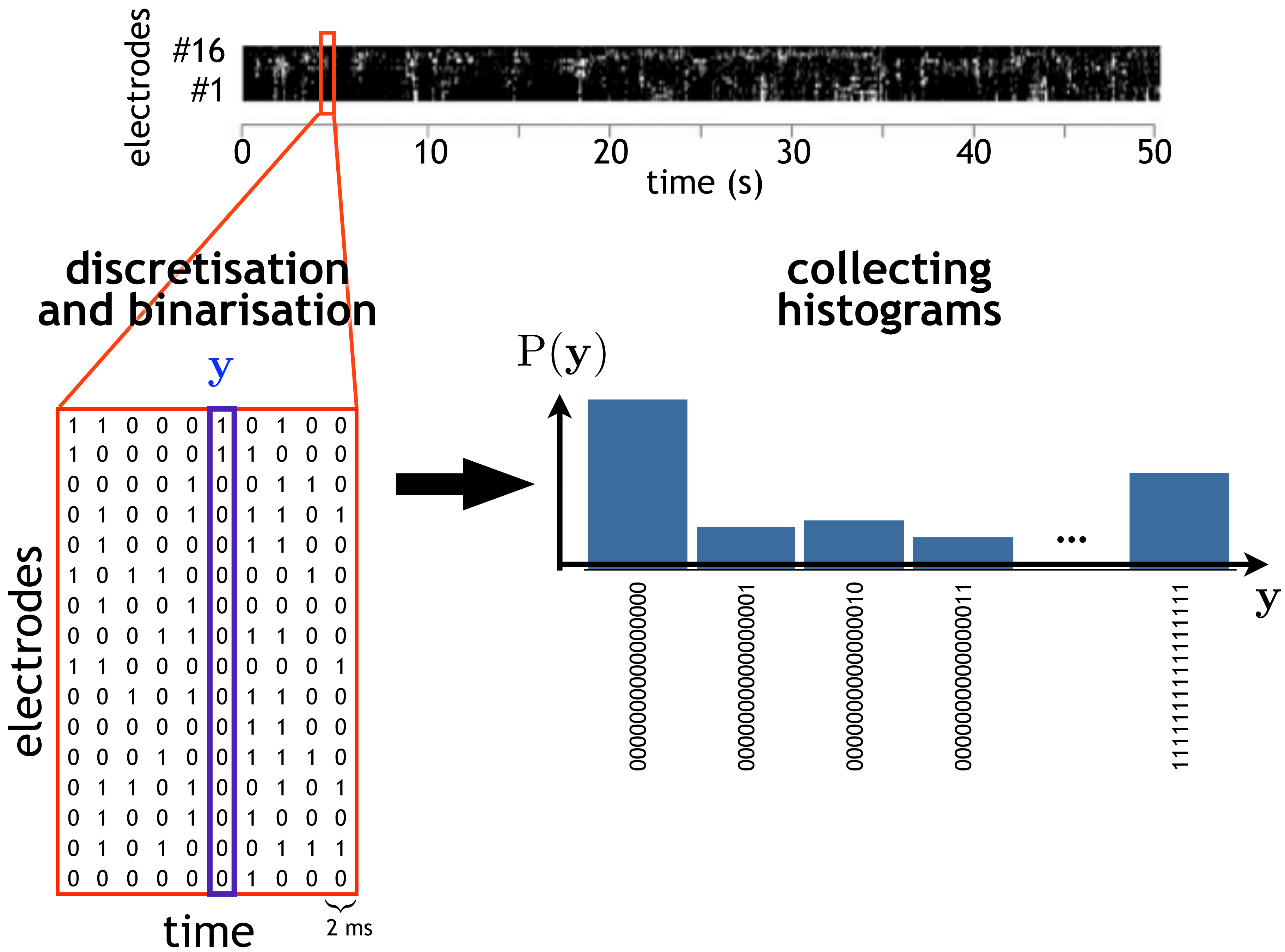
electrodes

					y				
1	1	0	0	0	1	0	1	0	0
1	0	0	0	0	1	1	0	0	0
0	0	0	0	1	0	0	1	1	0
0	1	0	0	1	0	1	1	0	1
0	1	0	0	0	0	1	1	0	0
1	0	1	1	0	0	0	0	1	0
0	1	0	0	1	0	0	0	0	0
0	0	0	1	1	0	1	1	0	0
1	1	0	0	0	0	0	0	0	1
0	0	1	0	1	0	1	1	0	0
0	0	0	0	0	0	1	1	0	0
0	0	0	1	0	0	1	1	1	0
0	1	1	0	1	0	0	1	0	1
0	1	0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	1	1	1
0	0	0	0	0	0	1	0	0	0

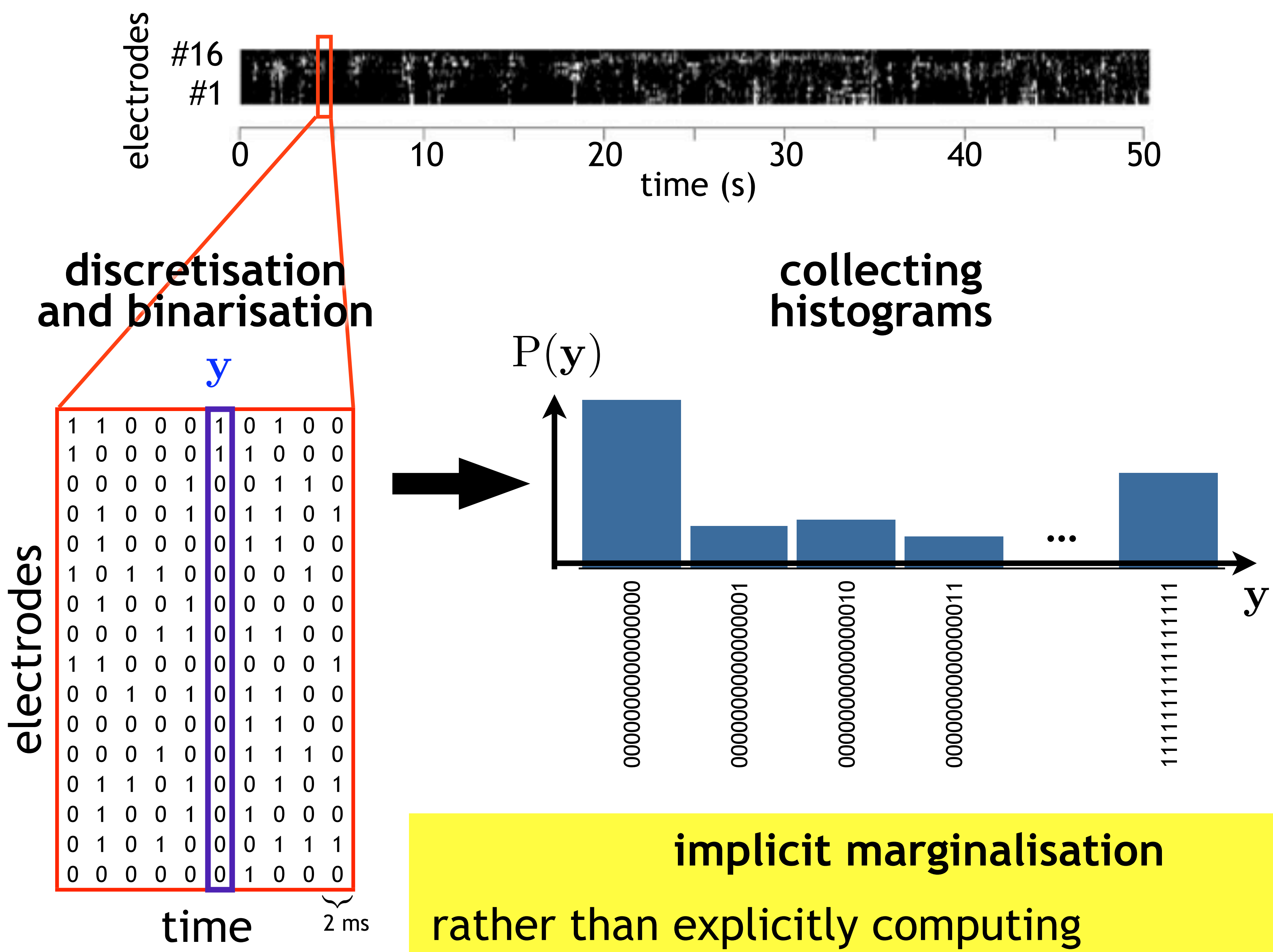
time

$\underbrace{\quad}_{\text{2 ms}}$

DATA ANALYSIS



DATA ANALYSIS



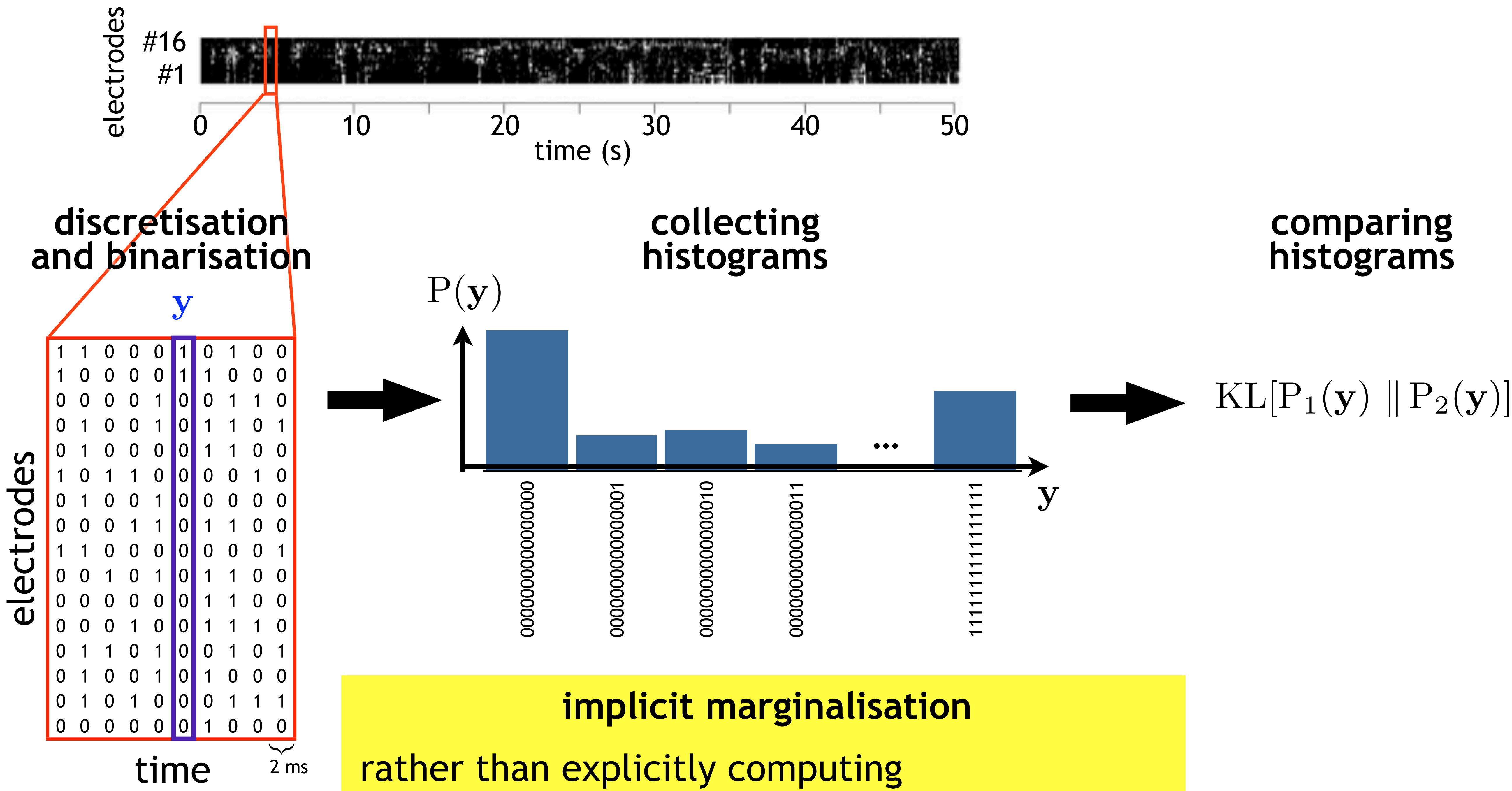
implicit marginalisation

rather than explicitly computing

$$\mathrm{P}_{\mathrm{evoked}}(\mathbf{y}) = \int \mathrm{P}_\theta(\mathbf{y}|\mathbf{x})\ \mathrm{P}^*_{\mathrm{stim}}(\mathbf{x})\ d\mathbf{x}$$

we collect evoked activity under each condition without regard to the actual ongoing stimulus

DATA ANALYSIS



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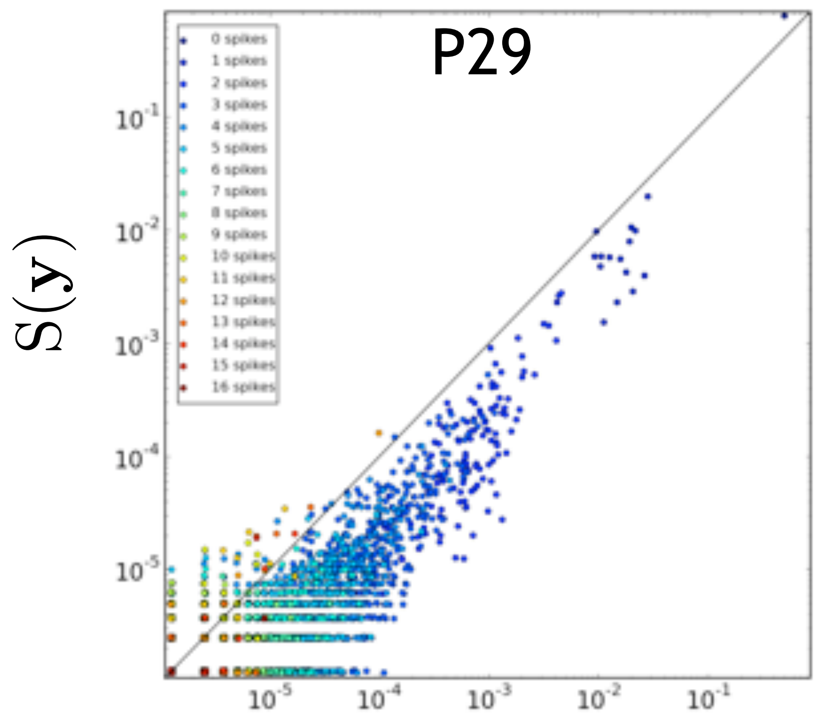
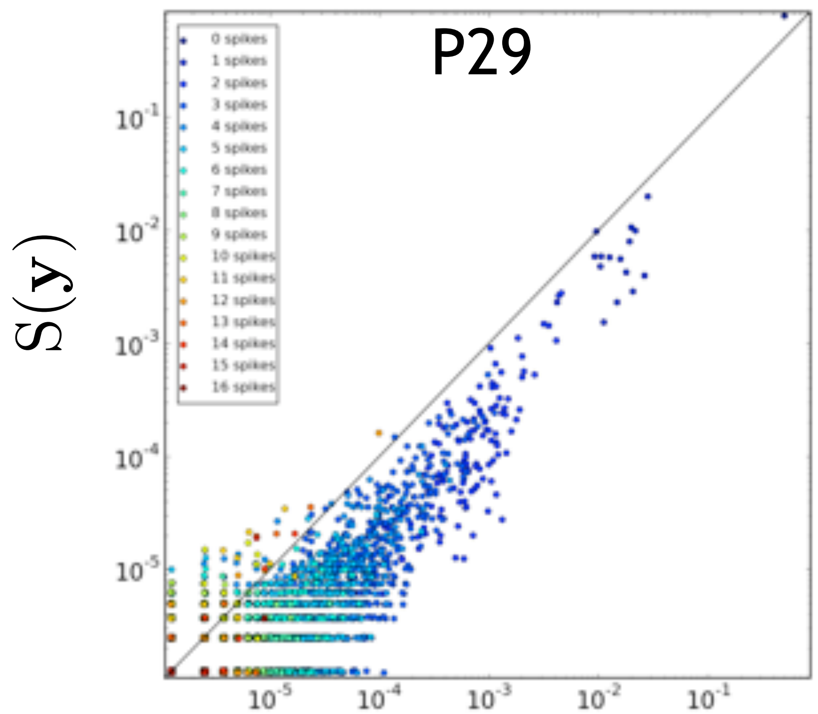
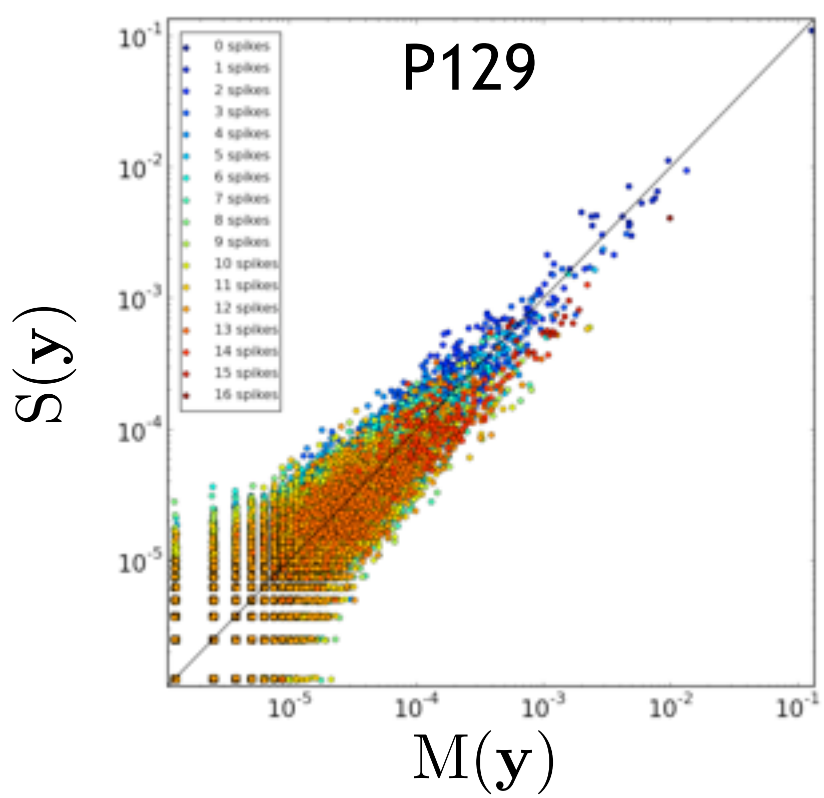
DEVELOPMENTAL CHANGES

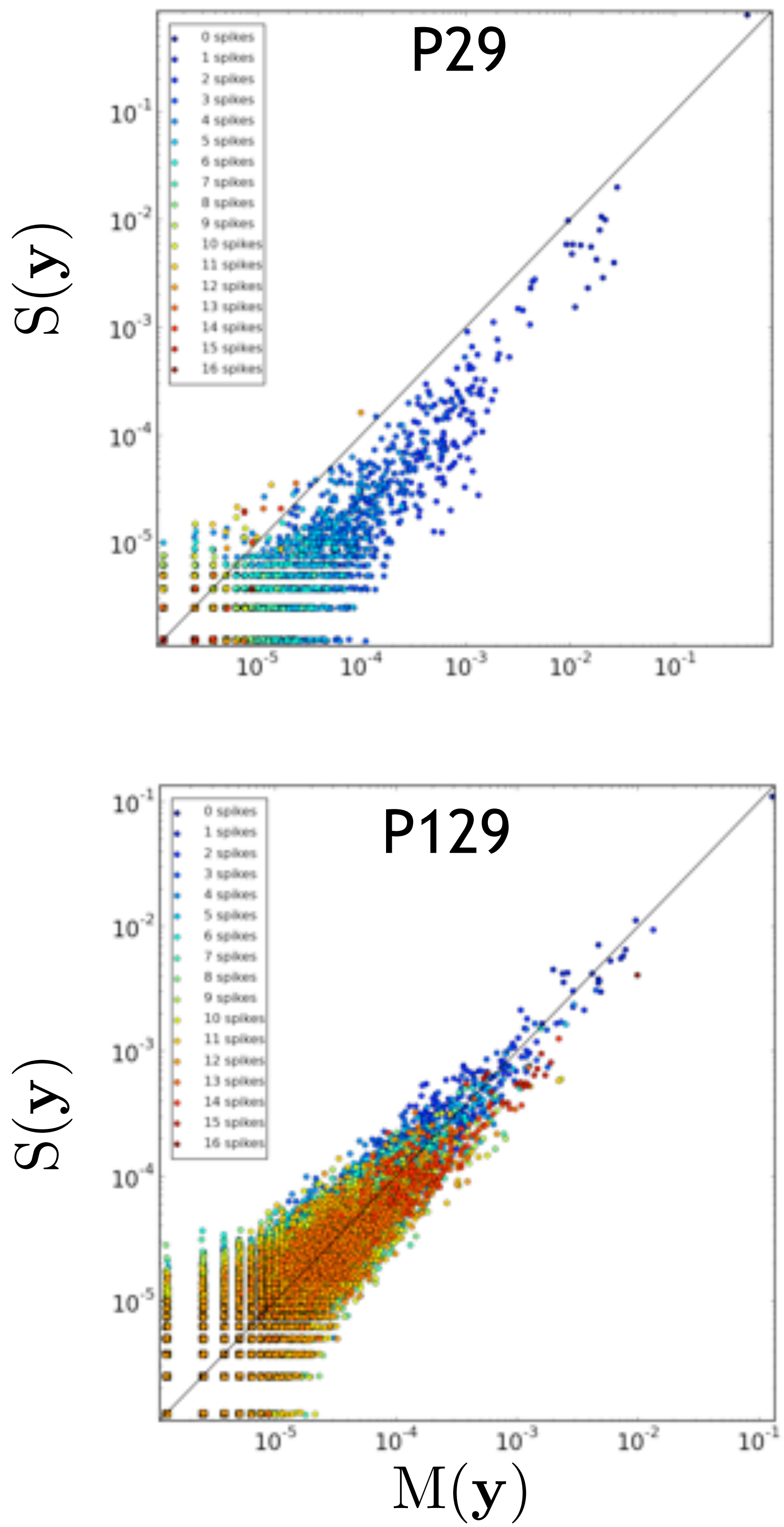


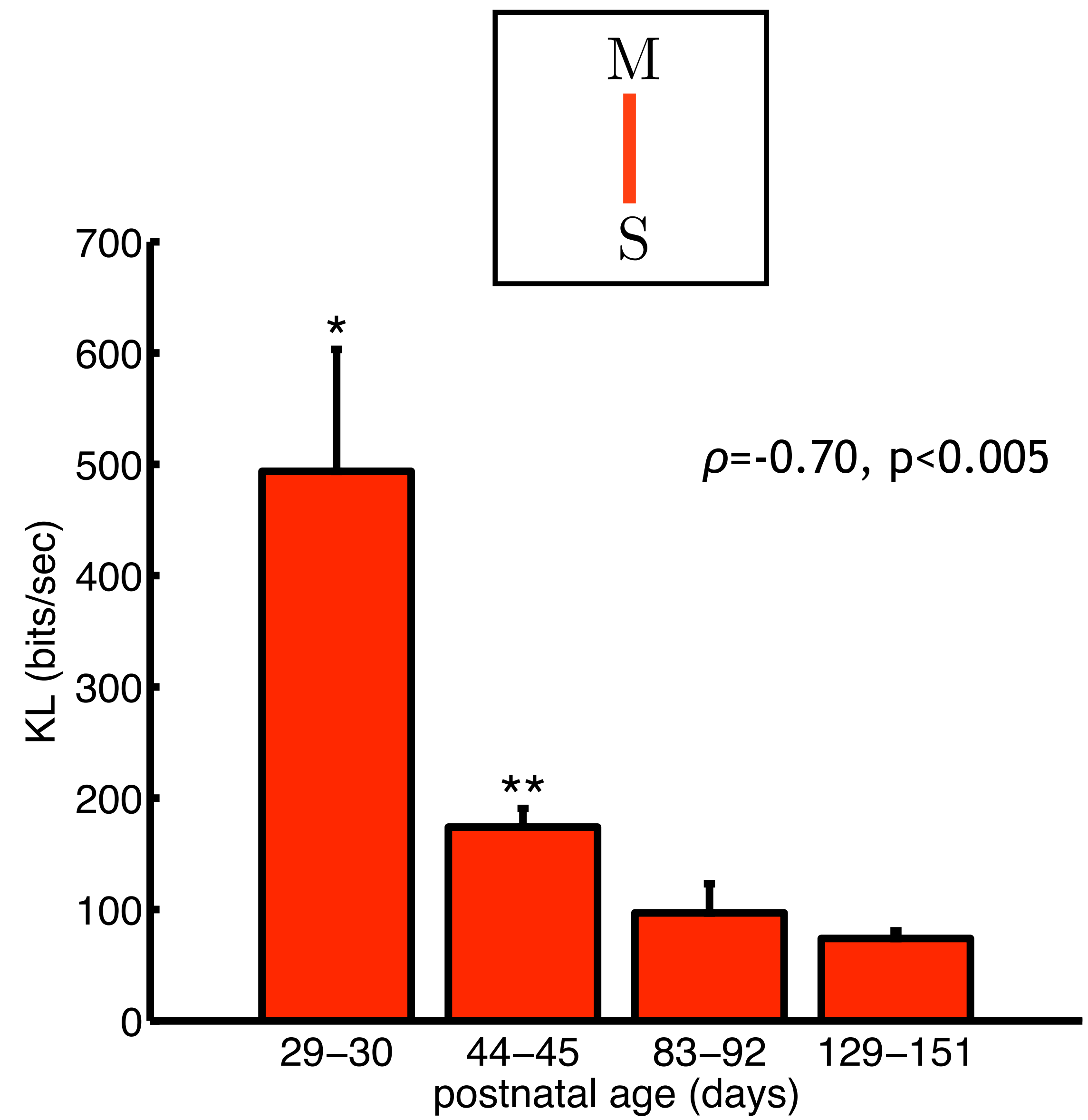
DEVELOPMENTAL CHANGES





DEVELOPMENTAL CHANGES





NON-NATURAL STIMULUS ENSEMBLES

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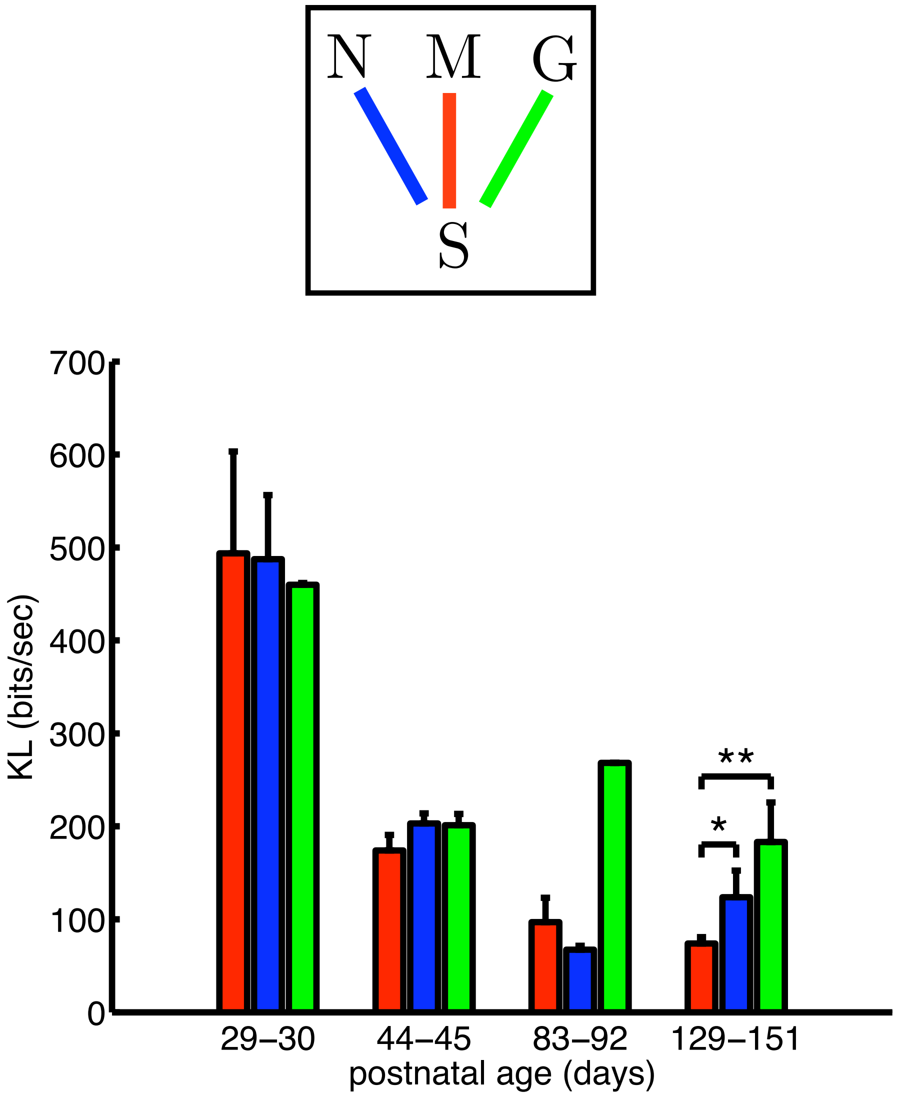

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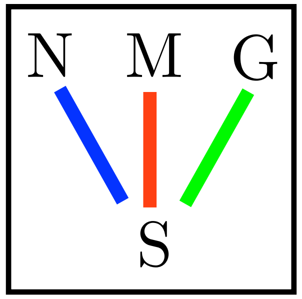

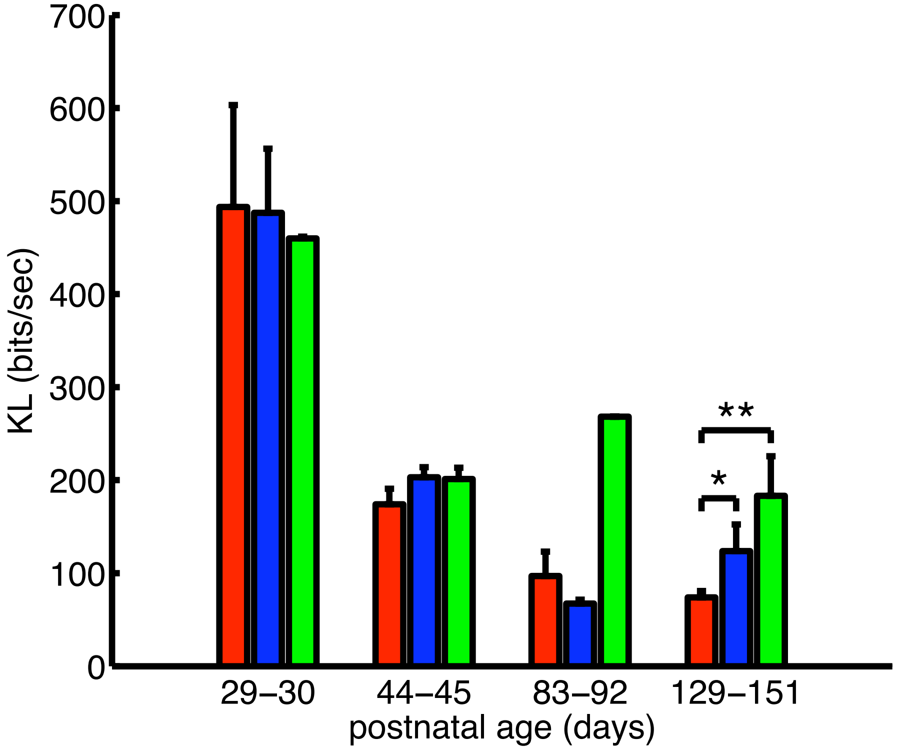

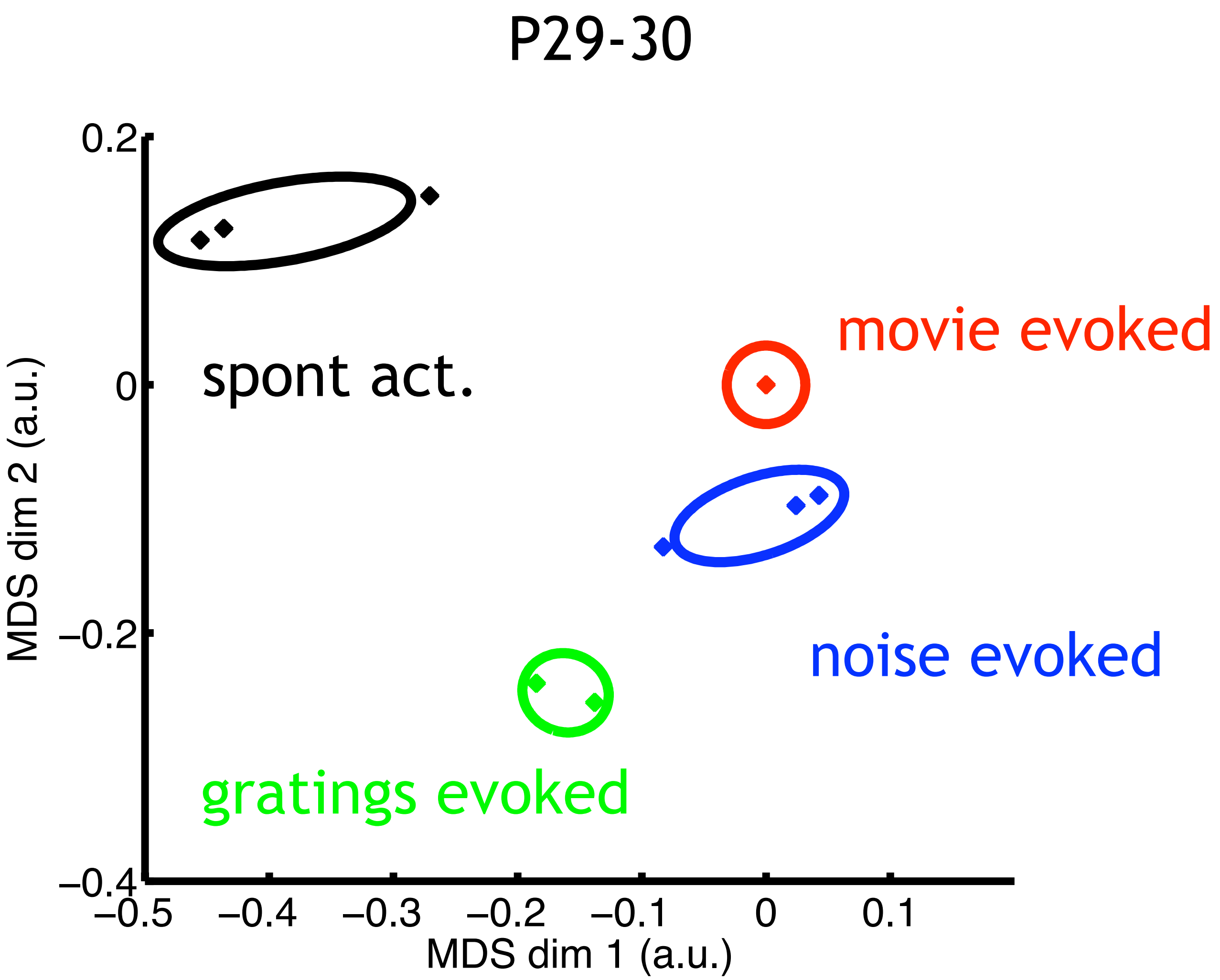

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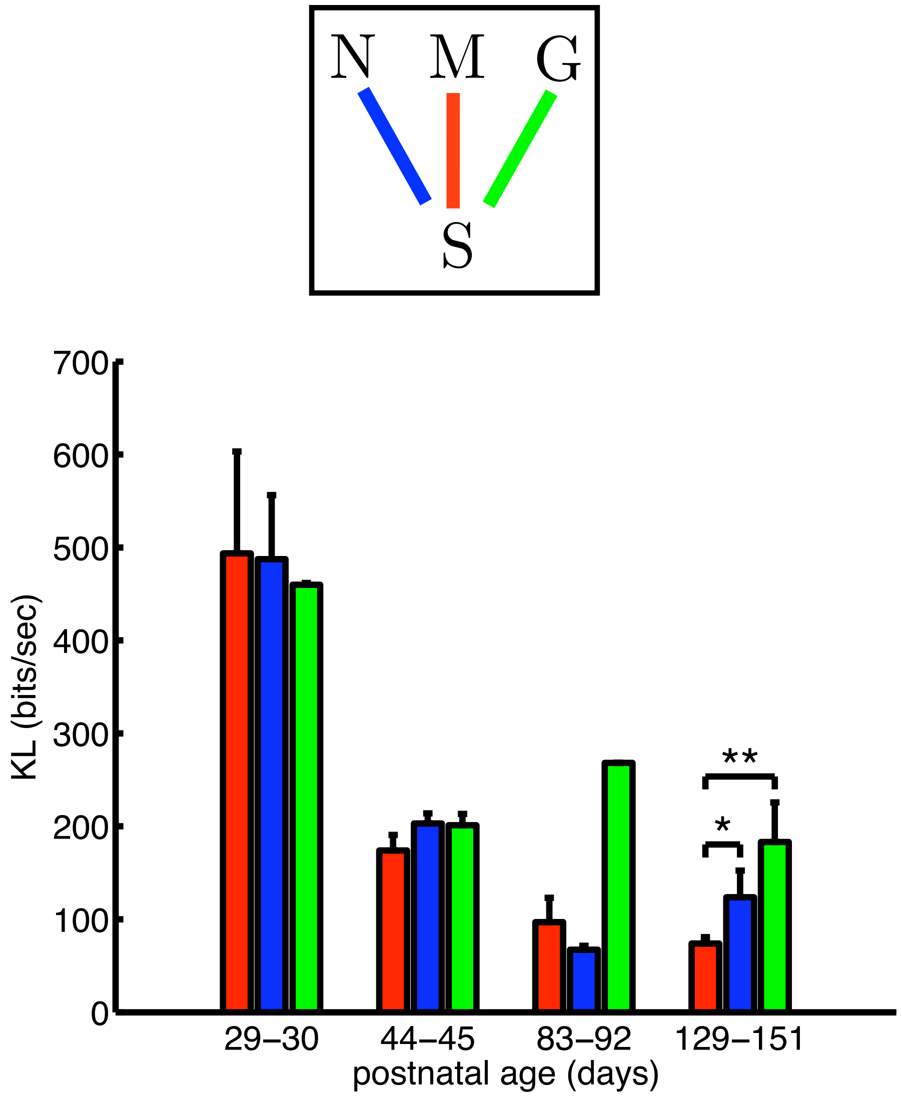

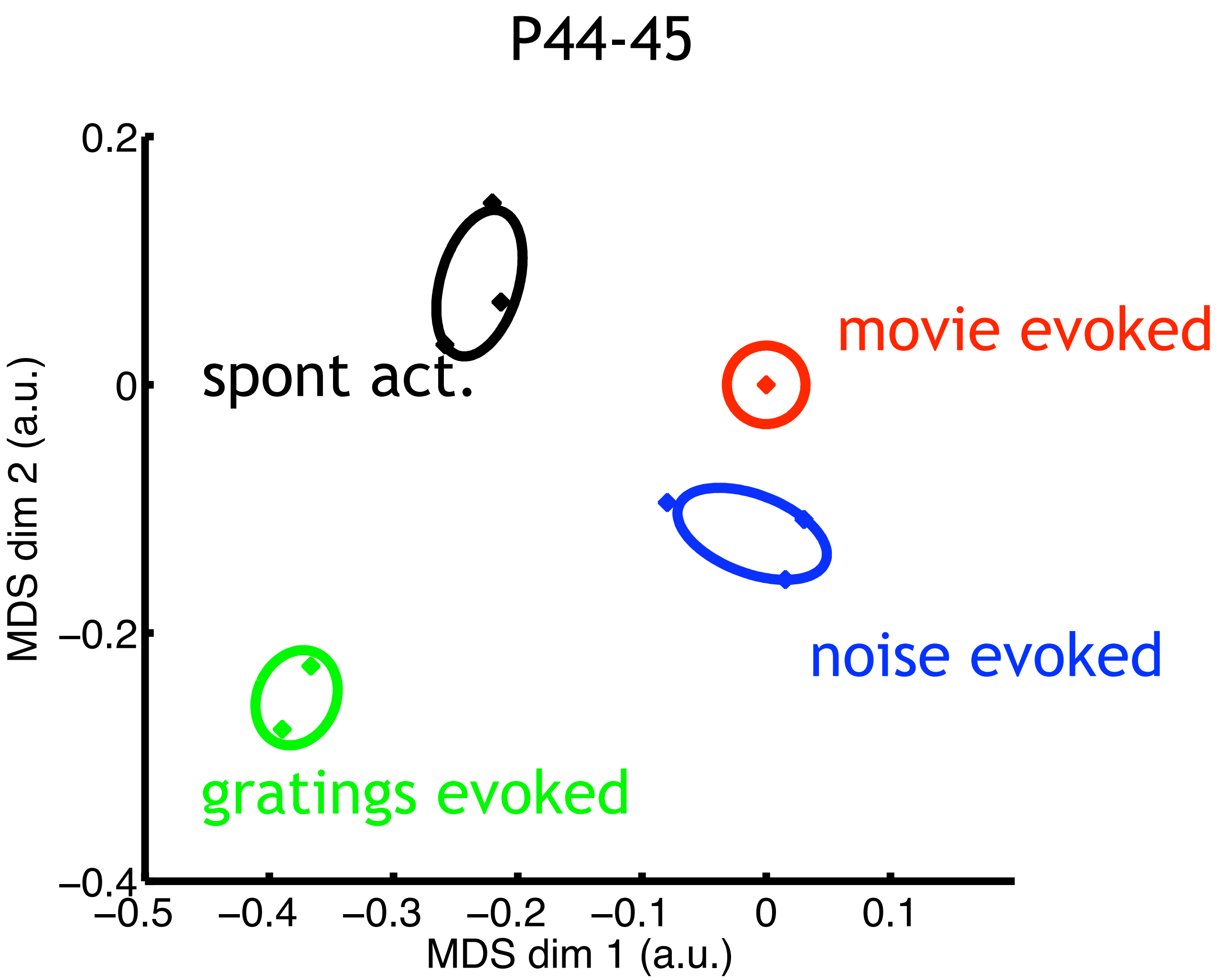

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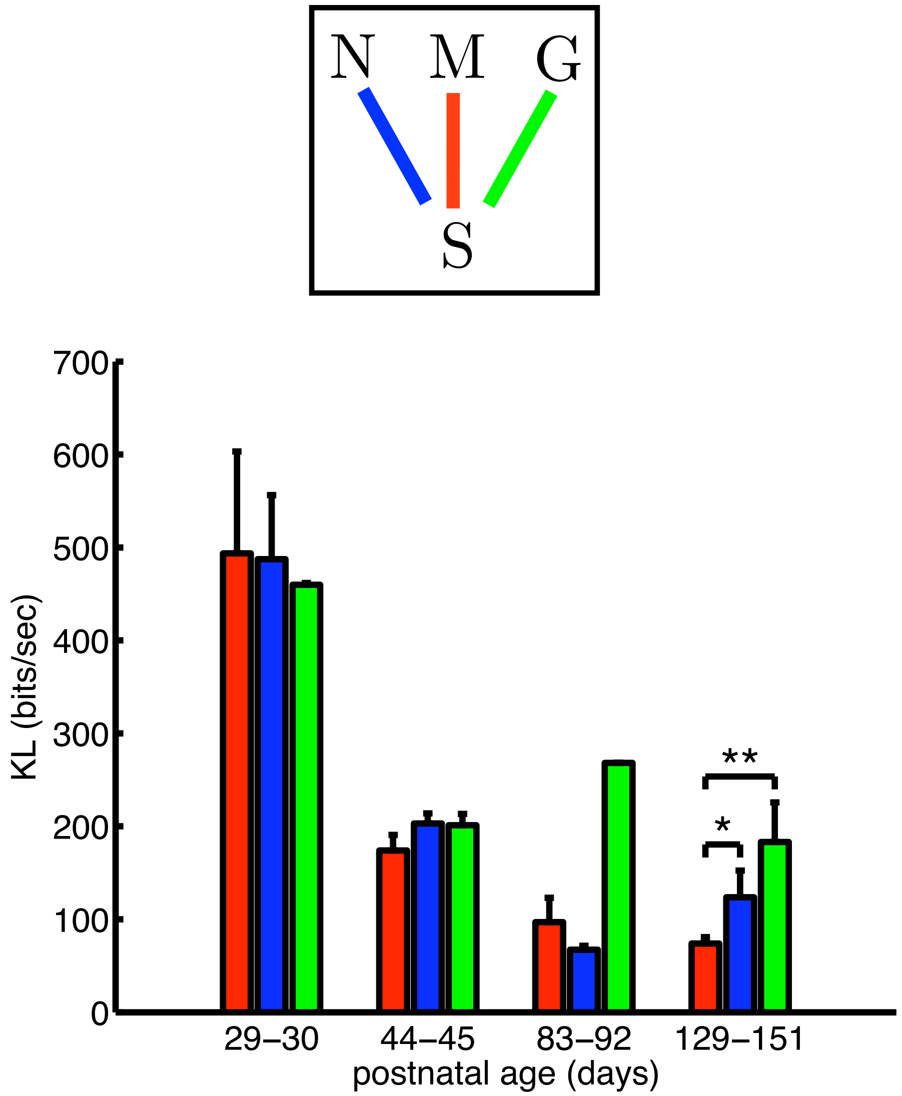

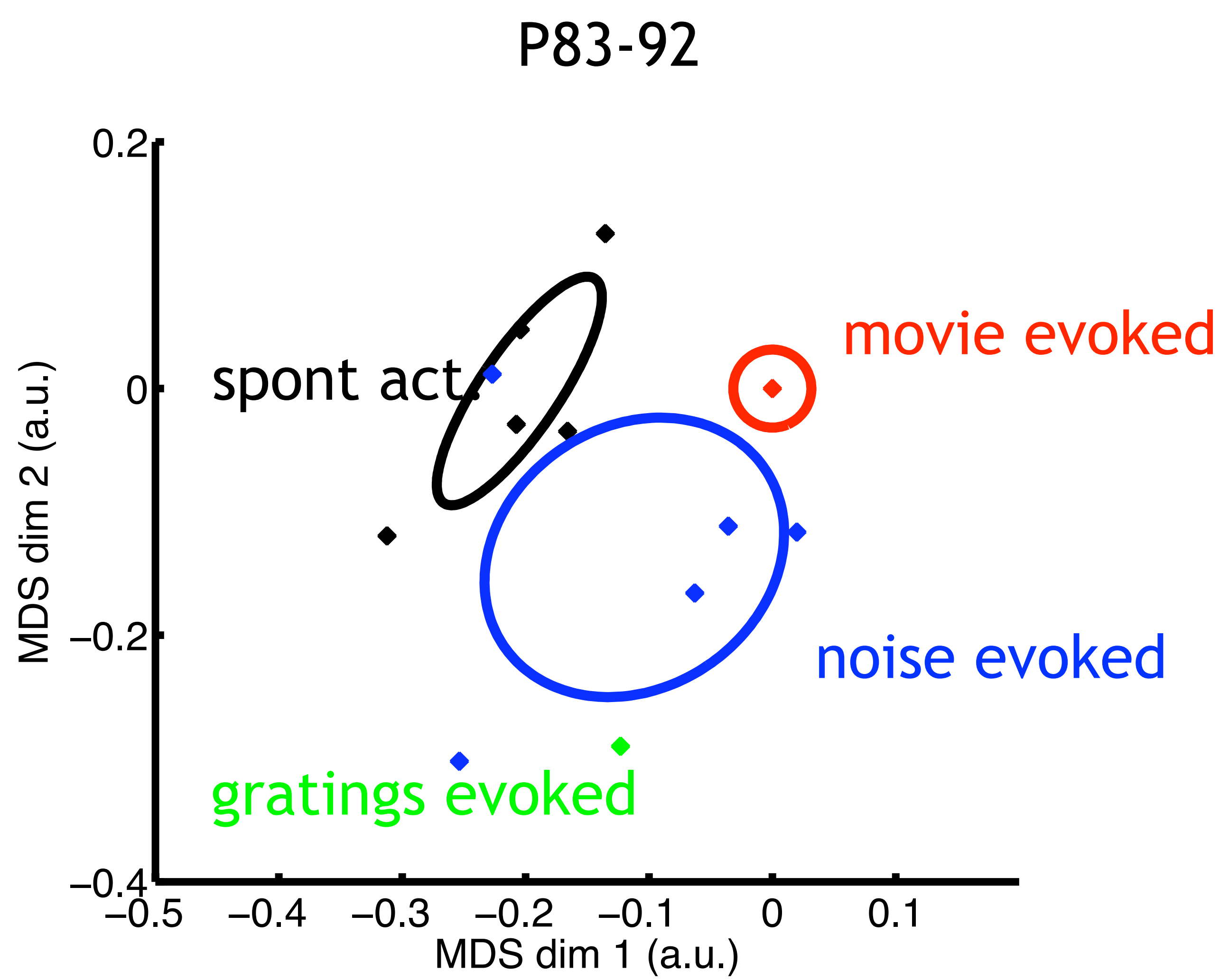

NON-NATURAL STIMULUS ENSEMBLES