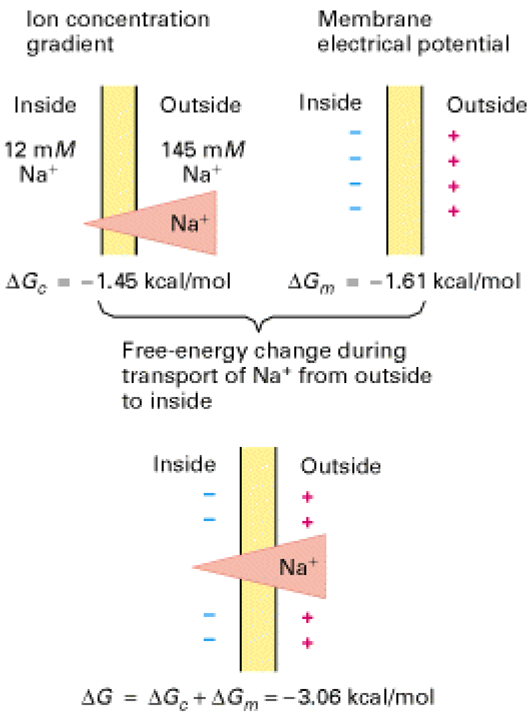
Origin of the resting membrane potential



Nernst-Planck equation:
ionic flux (current) as a function of the electro-chemical potentials

Nernst-equation:
relationship between potential difference and ion concentration in equlibrium for one sort of ion

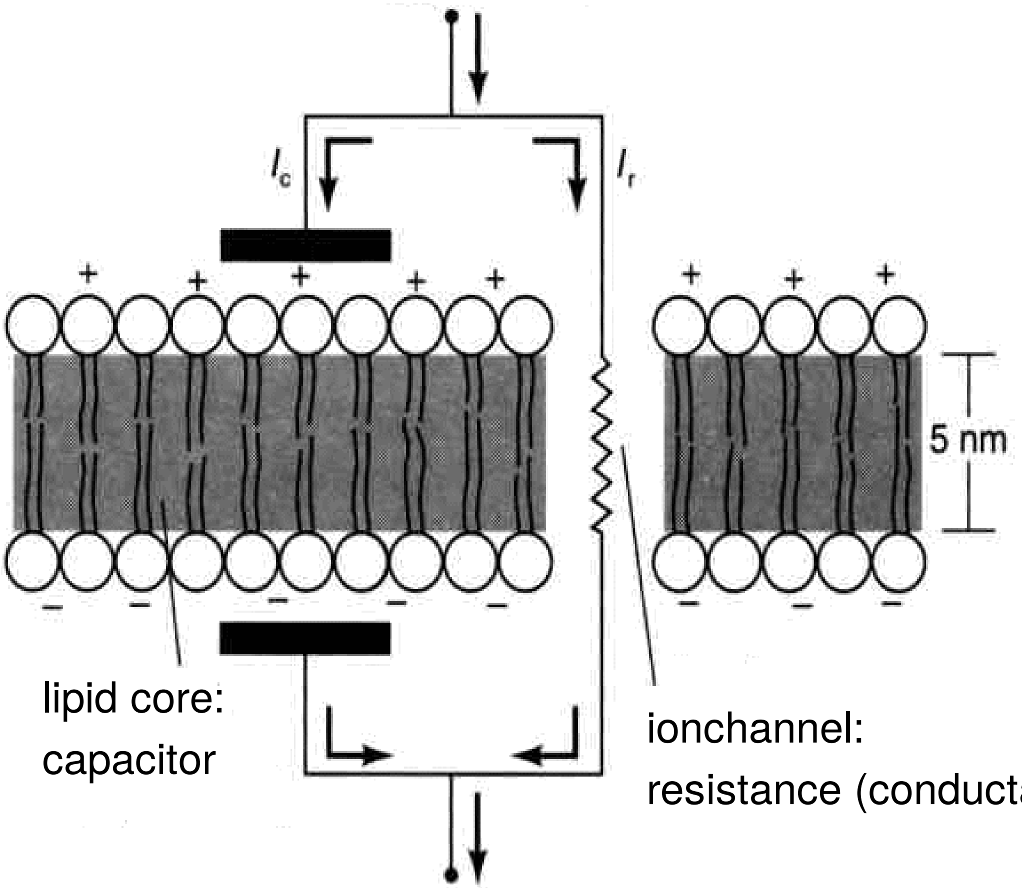
$$E = V_{IC} - V_{EC} = \frac{RT}{zF} \ln \frac{[C]_{EC}}{[C]_{IC}}$$

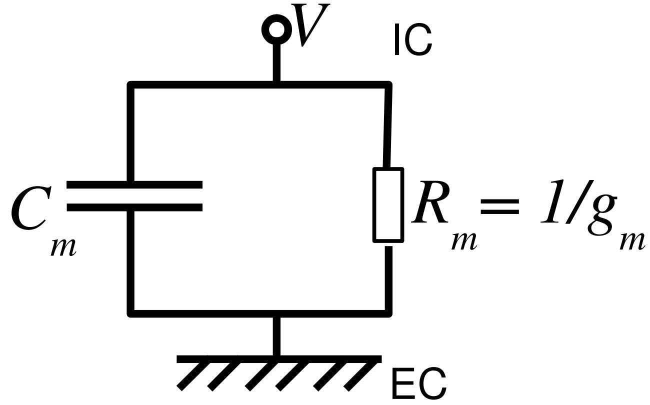
Goldman-Hodgkin-Katz-equation (GHK):
resting membrane potential as a function of the ion concentrations and permeabilities

more, independently moveing ions
constant electric force intensity throug the membrane

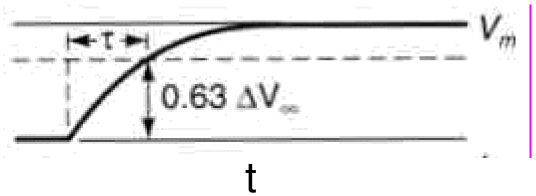
$$V_{rest} = \frac{RT}{F} \ln \frac{P_K[K^+]_{EC} + P_{Na}[Na^+]_{EC} + P_{Cl}[Cl^-]_{IC}}{P_K[K^+]_{IC} + P_{Na}[Na^+]_{IC} + P_{Cl}[Cl^-]_{EC}}$$

Basics of the conductance-based models

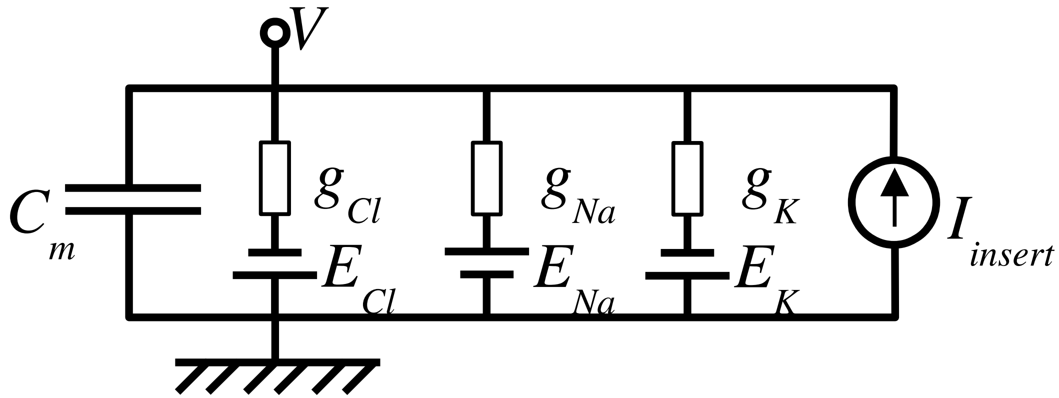




$$\underbrace{C_m \frac{dV(t)}{dt}}_{\text{capacitice current}} = \underbrace{-\frac{V(t)}{R_m} = -V(t)\, g_m}_{\text{resistive or conductive current}}$$



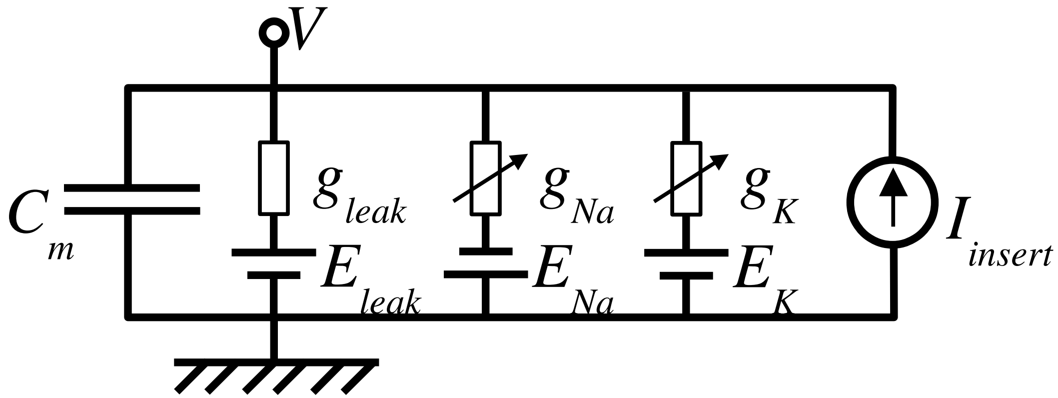
Model with parallel conductances

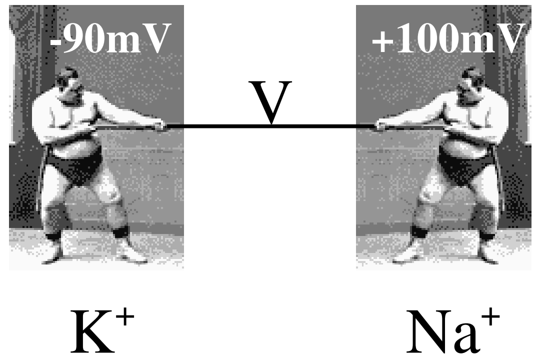


Equation for current equlibrium:

$$C_m \frac{dV(t)}{dt} = g_{Cl}\left(E_{Cl} - V(t)\right) + \underbrace{g_{Na}\underbrace{\left(\underset{\textbf{\textit{Nernst-potential}}}{E_{Na}} - V(t)\right)}_{\text{driving force}}}_{\text{single ion current}} + g_K\left(E_K - V(t)\right) + I_{insert}(t)$$

The Hodgkin-Huxley model / 1





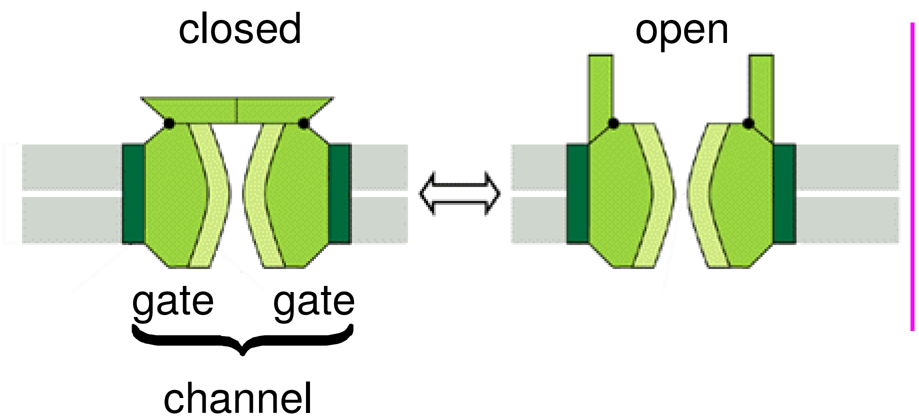
Equation for current equlibrium:

$$C_m \frac{dV(t)}{dt} = \underbrace{g_{leak}\left(E_{leak} - V(t)\right)}_{\text{Leak current (mainly Cl}^-\text{)}} + g_{Na}(t)\left(E_{Na} - V(t)\right) + g_K(t)\left(E_K - V(t)\right) + I_{insert}(t)$$

Equations for ionic currents:

$$g_{Na}(t) = \bar{g}_{Na} \cdot m^3(t) \cdot h(t)$$

$$g_K(t) = \bar{g}_K \cdot n^4(t)$$



The Hodgkin-Huxley model / 2

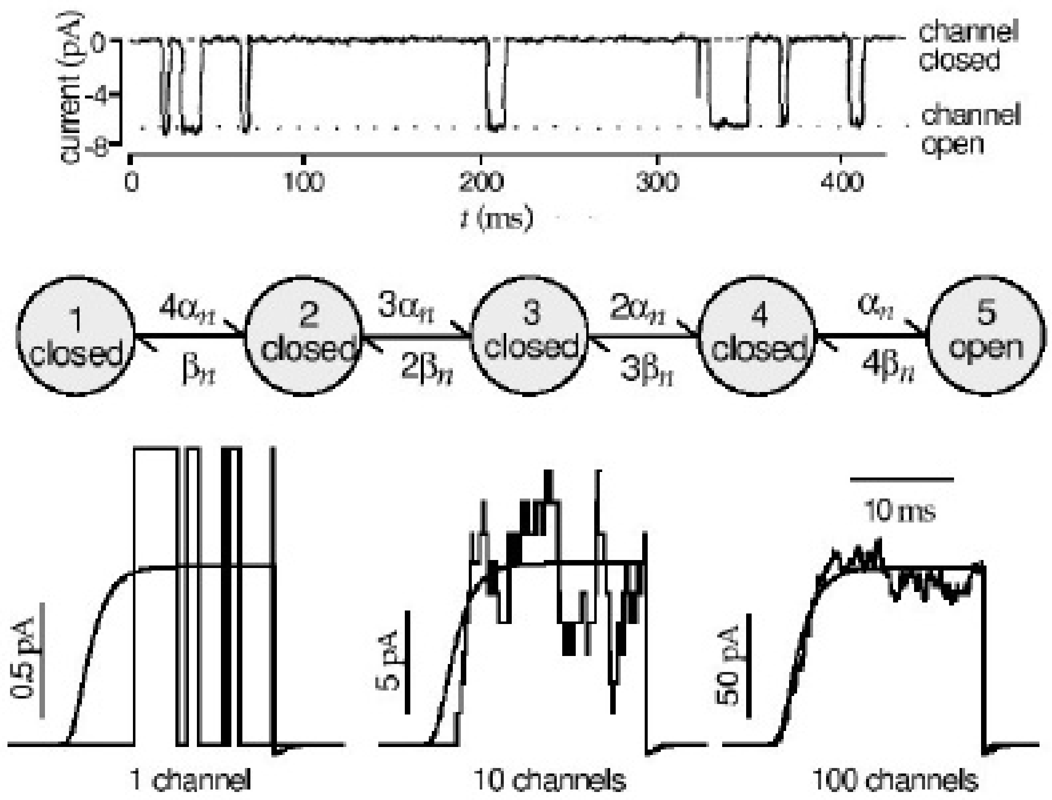
What is most important in the HH model: voltage dependent gating kinetics

$$\underset{1-m}{\text{closed}} \underset{\beta_m}{\overset{\alpha_m}{\rightleftarrows}} \underset{m}{\text{open}}$$

$$\frac{dm(t)}{dt} = \alpha_m(V(t))(1-m(t)) - \beta_m(V(t))m(t) = \frac{m_\infty(V(t)) - m(t)}{\tau_m(V(t))}$$

$$m_\infty(V) = \frac{\alpha_m(V)}{\alpha_m(V) + \beta_m(V)}$$

$$\tau_m(V) = \frac{1}{\alpha_m(V) + \beta_m(V)}$$



P Dayan, LF Abbott: Theoretical Neuroscience, (MIT Press, Cambridge, 2001).

The Hodgkin-Huxley model / 3

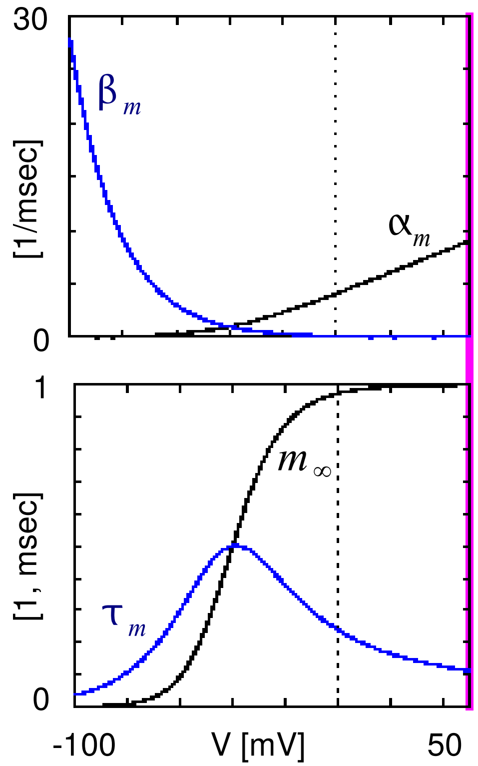
What is most important in the HH model: voltage dependent gating kinetics

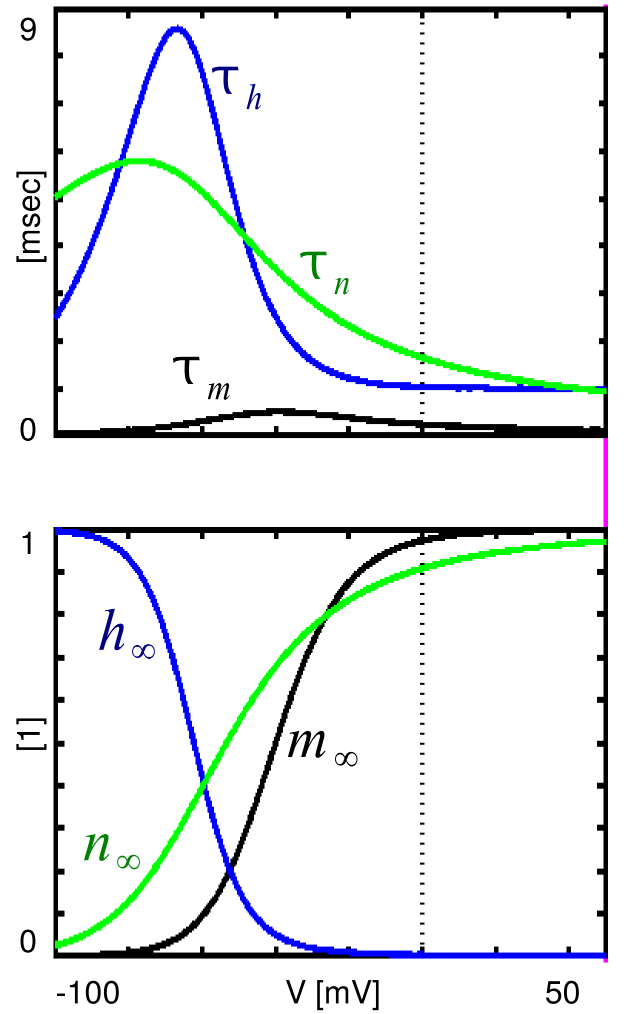
$$\underset{1-m}{\text{closed}} \; \underset{\beta_m}{\overset{\alpha_m}{\rightleftharpoons}} \; \underset{m}{\text{open}}$$

$$\frac{dm(t)}{dt} = \alpha_m(V(t))(1-m(t)) - \beta_m(V(t))\,m(t) = \frac{m_\infty(V(t)) - m(t)}{\tau_m(V(t))}$$

$$m_\infty(V) = \frac{\alpha_m(V)}{\alpha_m(V)+\beta_m(V)}$$

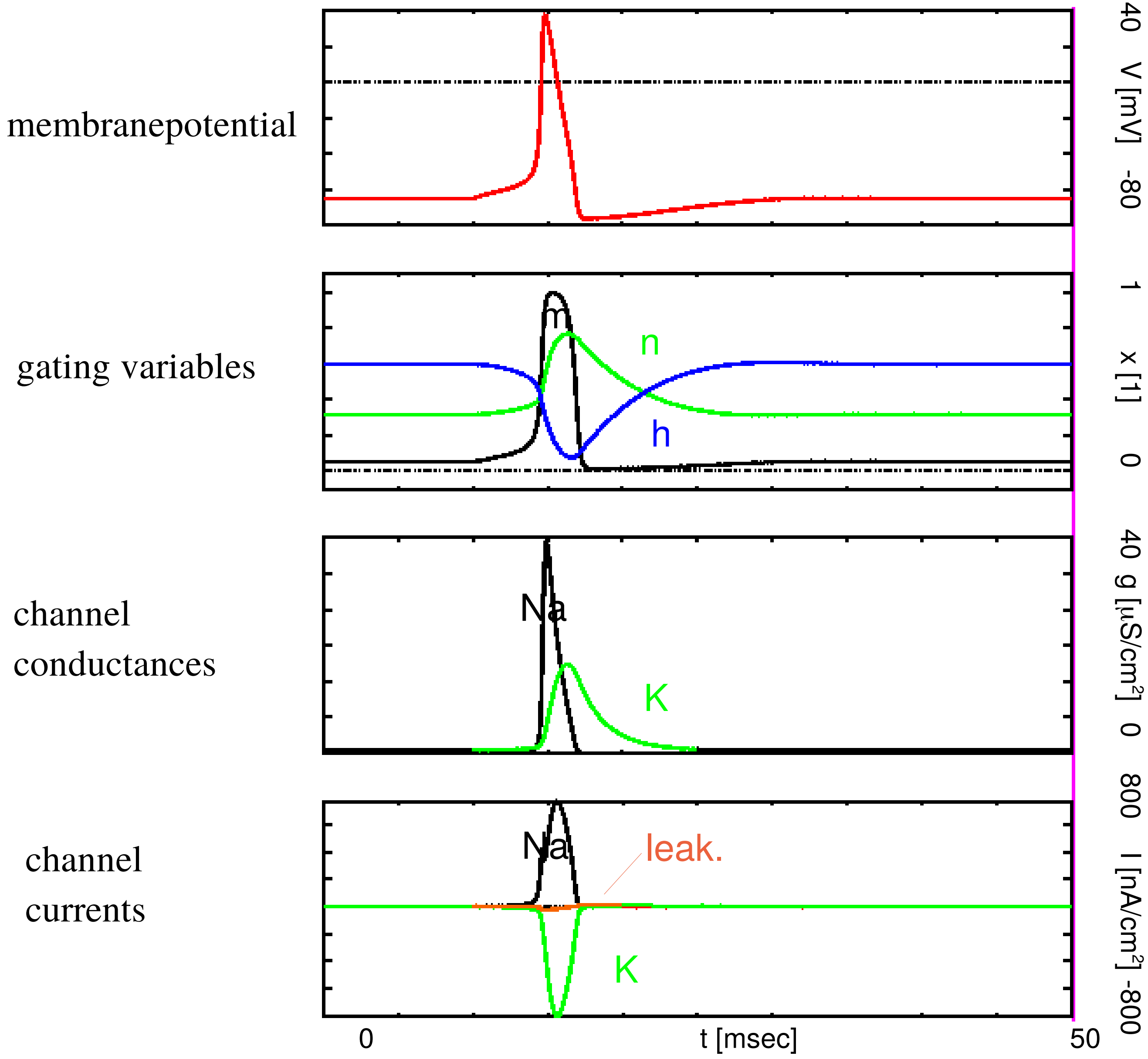
$$\tau_m(V) = \frac{1}{\alpha_m(V)+\beta_m(V)}$$

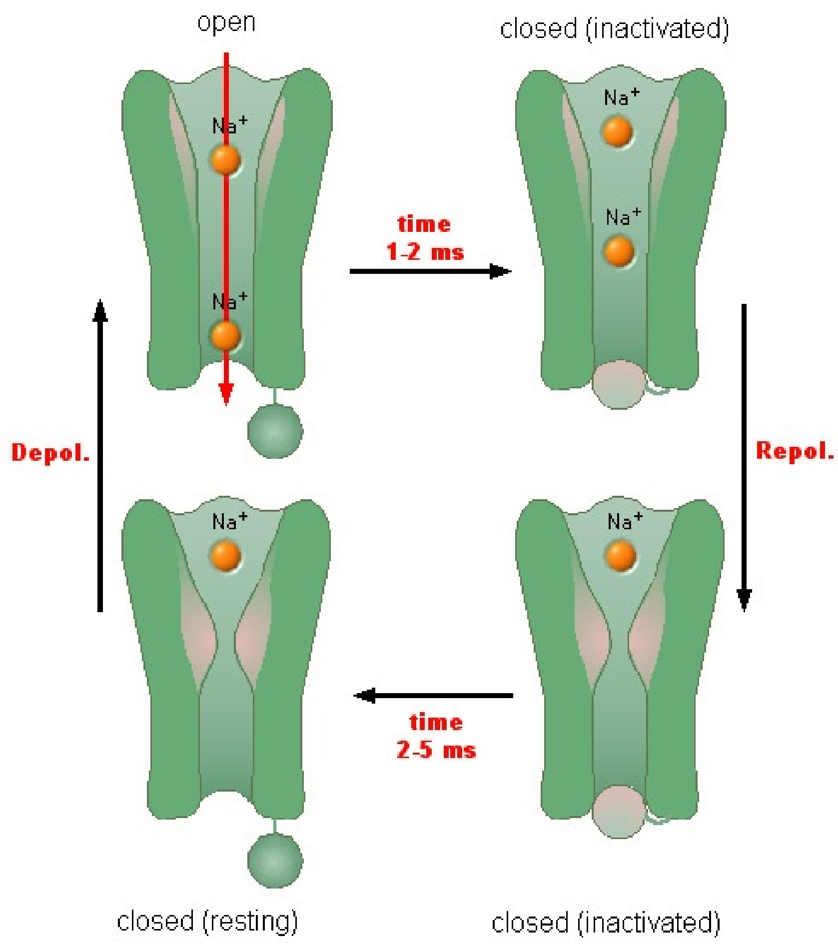




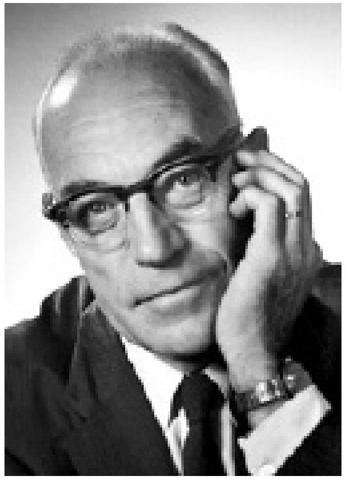
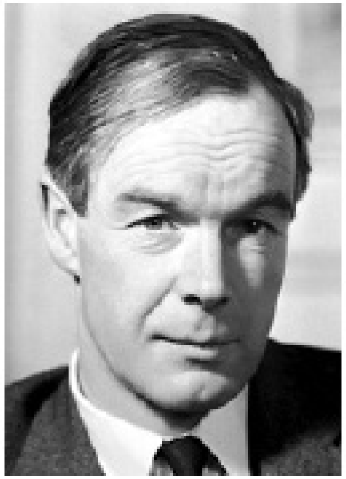
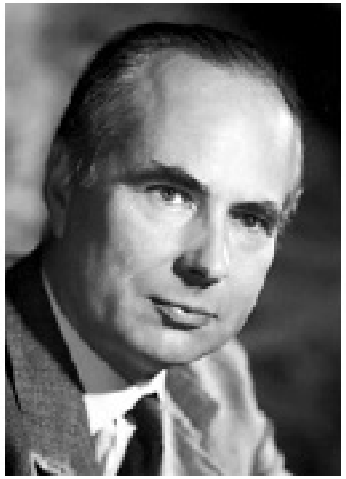
The Hodgkin-Huxley model / 4

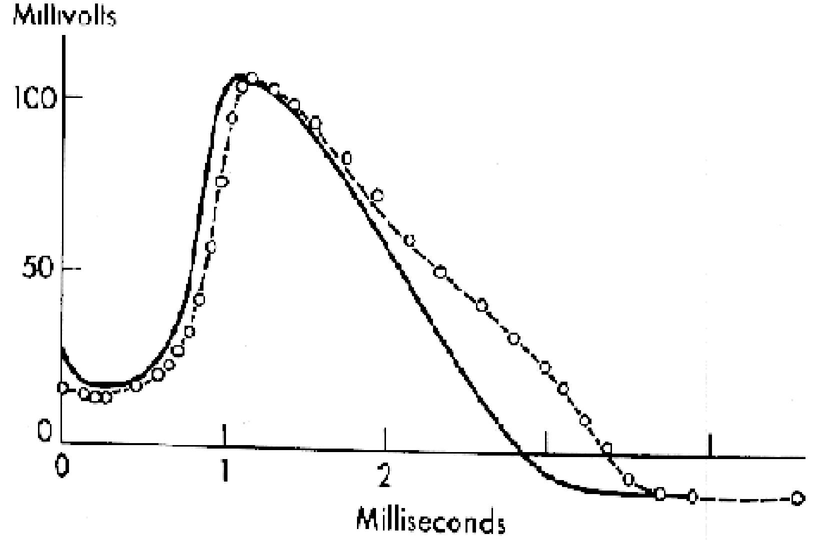
HH model in work:



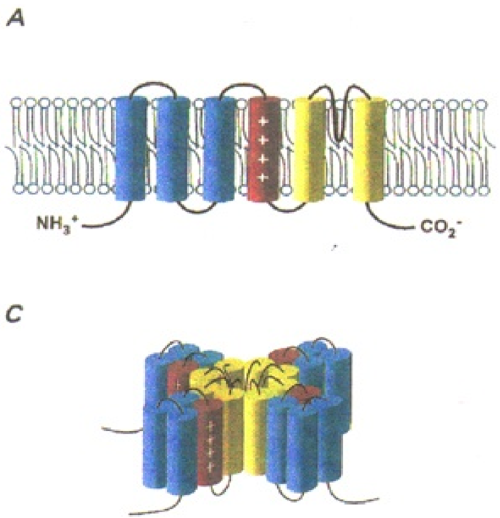


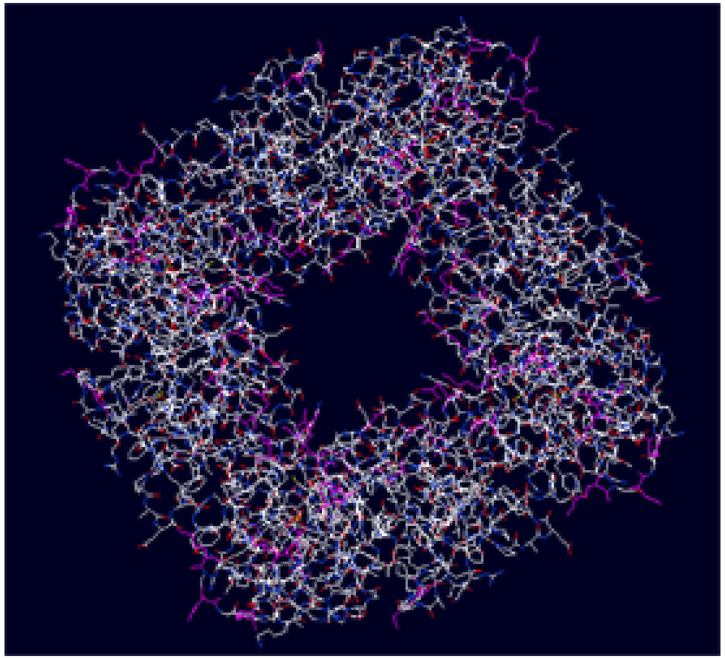
The Hodgkin-Huxley model / 5

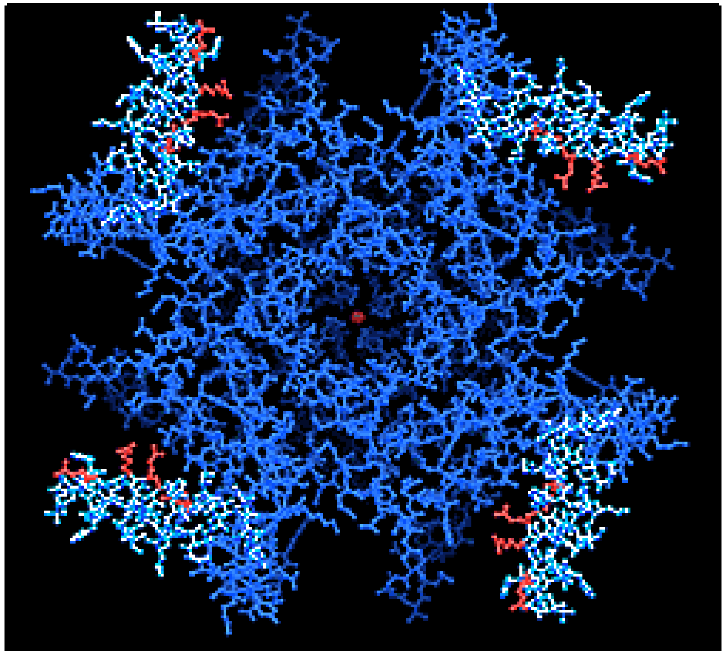






Sir John Carew Eccles, Alan Lloyd Hodgkin, Andrew Fielding Huxley: Awarded by Nobel-prize in medicine, 1963.







The Hodgkin-Huxley model / 6