Origin of the resting membrane potential

**Nernst-equation:**
relationship between potential difference and ion concentration in equilibrium for one sort of ion
\[
E = V_{IC} - V_{EC} = \frac{RT}{zF} \ln \left( \frac{[C]_{EC}}{[C]_{IC}} \right)
\]

**Goldman-Hodgkin-Katz-equation (GHK):**
resting membrane potential as a function of the ion concentrations and permeabilities
\[
V_{rest} = \frac{RT}{F} \ln \left( \frac{P_K [K^+]_{EC} + P_{Na} [Na^+]_{EC} + P_{Cl} [Cl^-]_{EC}}{P_K [K^+]_{IC} + P_{Na} [Na^+]_{IC} + P_{Cl} [Cl^-]_{EC}} \right)
\]

**Nernst-Planck equation:**
ionic flux (current) as a function of the electro-chemical potentials
Basics of the conductance-based models

\[ V \]

\[ C_m \frac{dV(t)}{dt} = -\frac{V(t)}{R_m} = -V(t)g_m \]

- Capacitive current
- Resistive or conductive current

EC fluid (matrix)

IC fluid (cytosol)

lipid core: capacitor

ionchannel: resistance (conductance)

Modellek az idegrendszer-kutatásban
2018, Neuroinformatika, Zalányi László
Model with parallel conductances

Equation for current equilibrium:

\[
C_m \frac{dV(t)}{dt} = g_{Cl}(E_{Cl} - V(t)) + g_{Na}(E_{Na} - V(t)) + g_{K}(E_{K} - V(t)) + I_{\text{insert}}(t)
\]

Nernst-potential

\{ driving force \}

\{ single ion current \}
The Hodgkin-Huxley model / 1

Equation for current equilibrium:

\[
C_m \frac{dV(t)}{dt} = g_{\text{leak}}(E_{\text{leak}} - V(t)) + g_{\text{Na}}(t)(E_{\text{Na}} - V(t)) + g_{\text{K}}(t)(E_{\text{K}} - V(t)) + I_{\text{insert}}(t)
\]

Leak current (mainly Cl\(^{-}\))

Equations for ionic currents:

\[
g_{\text{Na}}(t) = g_{\text{Na}}^{-} \cdot m^3(t) \cdot h(t)
\]

\[
g_{\text{K}}(t) = g_{\text{K}}^{-} \cdot n^4(t)
\]
What is most important in the HH model: voltage dependent gating kinetics

\[
\frac{dm}{dt} = \alpha_m(V(t)) \left[ 1 - m(t) \right] - \beta_m(V(t)) m(t) = \frac{m_\infty(V(t)) - m(t)}{\tau_m(V(t))}
\]

\[
m_\infty(V) = \frac{\alpha_m(V)}{\alpha_m(V) + \beta_m(V)}
\]

\[
\tau_m(V) = \frac{1}{\alpha_m(V) + \beta_m(V)}
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What is most important in the HH model: voltage dependent gating kinetics

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\]

\[
\tau_m(V) = \frac{1}{\alpha_m(V) + \beta_m(V)}
\]
The Hodgkin-Huxley model / 4

HH model in work:

membrane potential

- Graph showing membrane potential over time.

gating variables

- Graph showing gating variables (n, h) over time.

channel conductances

- Graph showing channel conductances (Na, K) over time.

channel currents

- Graph showing channel currents (Na, K) over time.

- Diagram illustrating the gating and conductance changes of Na+ and K+ channels.

- Time scales indicating 1-2 ms for opening and 2-5 ms for inactivation.

- States of the channels: open, closed (resting), closed (inactivated).
The Hodgkin-Huxley model / 5

The Hodgkin-Huxley model / 6

A

Na\textsubscript{v} channel

I
 II
 III
 IV

Pore
Voltage sensing

Inactivation

Amino terminus

Carboxy terminus

B

KcsA K\textsuperscript{+} channel
(closed)

MthK K\textsuperscript{+} channel
(open)