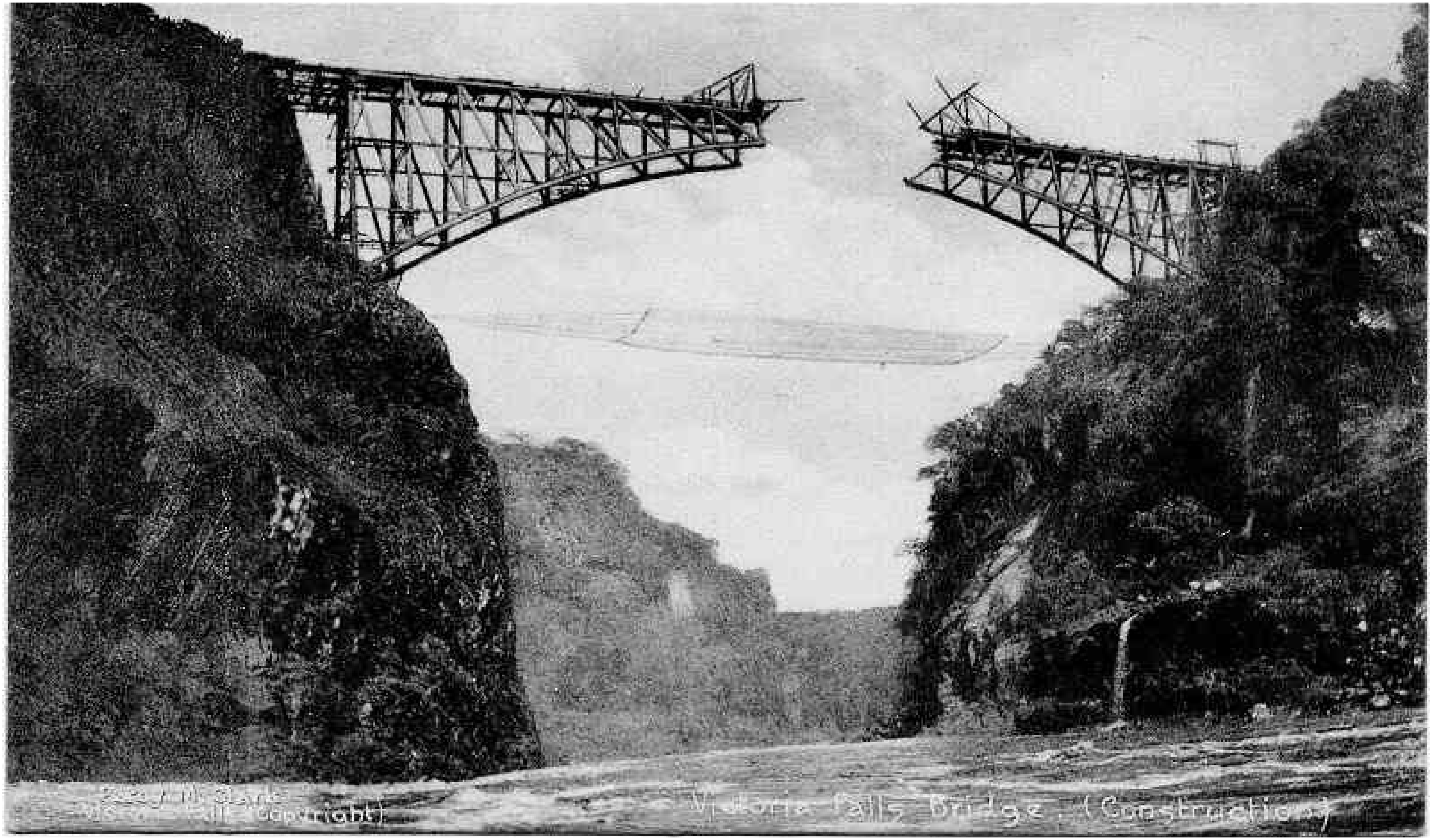
Computational Neuroscience



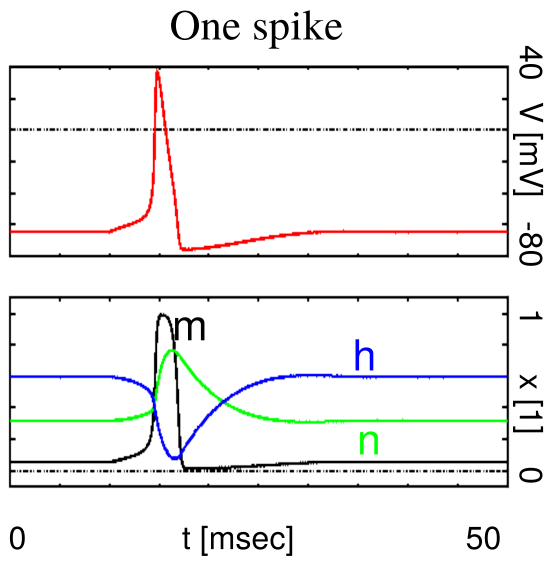
Structure – Dynamics – Implementation – Algorithm – Computation - Function

Simplified neuron models

Reminder: the HH-model

$$C_m \frac{dV}{dt} = \bar{g}_{Na}\left(E_{Na} - V(t)\right) m^3(t) h(t) + \bar{g}_K \left(E_K - V(t)\right) n^4(t) +$$

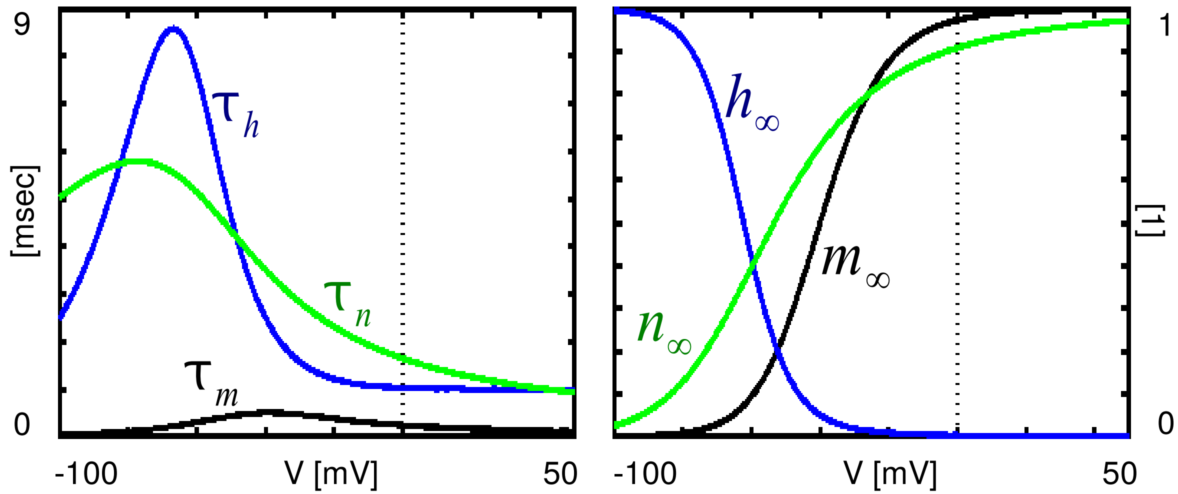
$$+ g_{leak}\left(E_{leak} - V(t)\right) + I_{external}(t)$$



$$\frac{dm}{dt} = \frac{m_\infty\left(V(t)\right) - m(t)}{\tau_m\left(V(t)\right)}$$

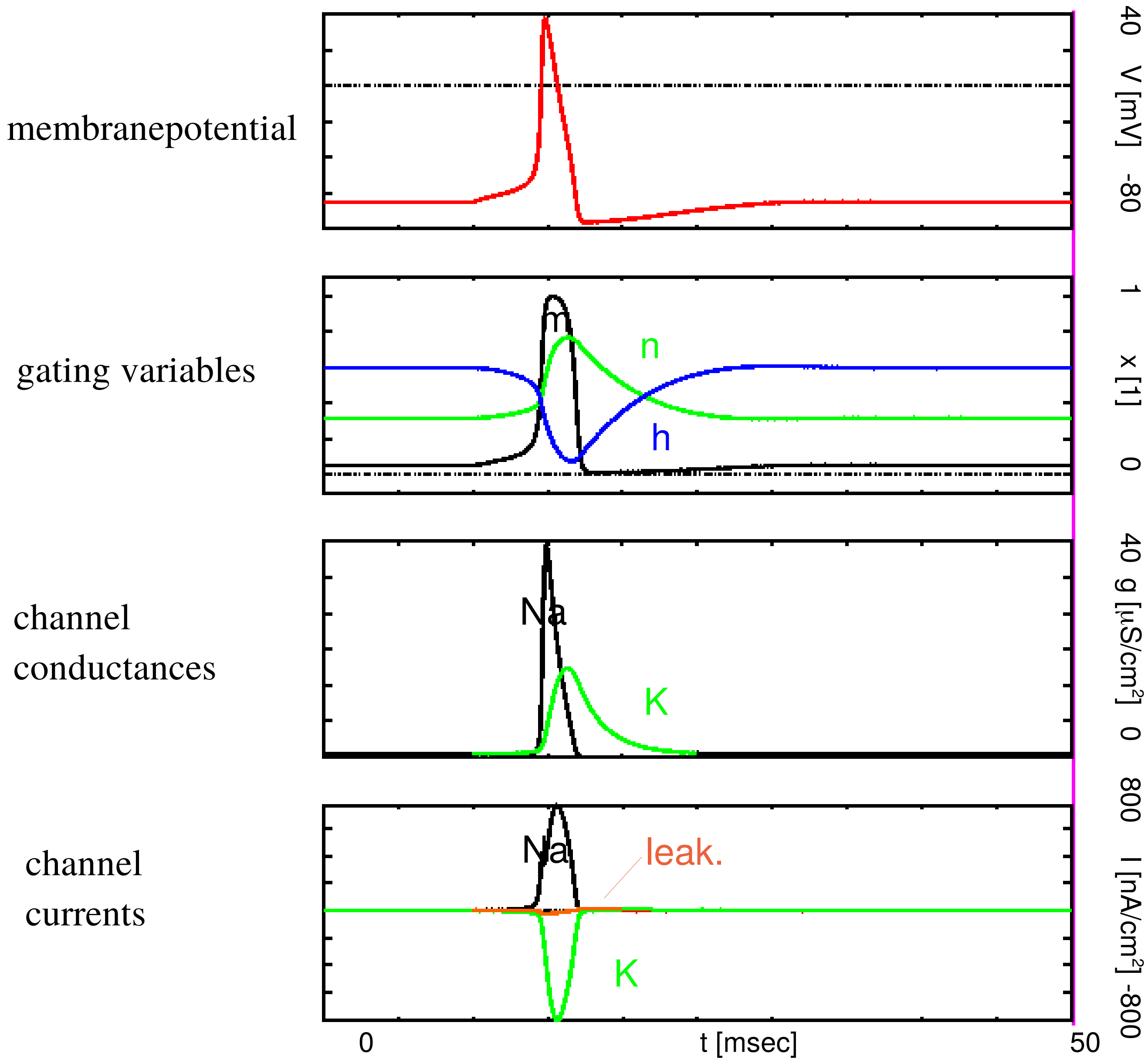
$$\frac{dh}{dt} = \frac{h_\infty\left(V(t)\right) - h(t)}{\tau_h\left(V(t)\right)}$$

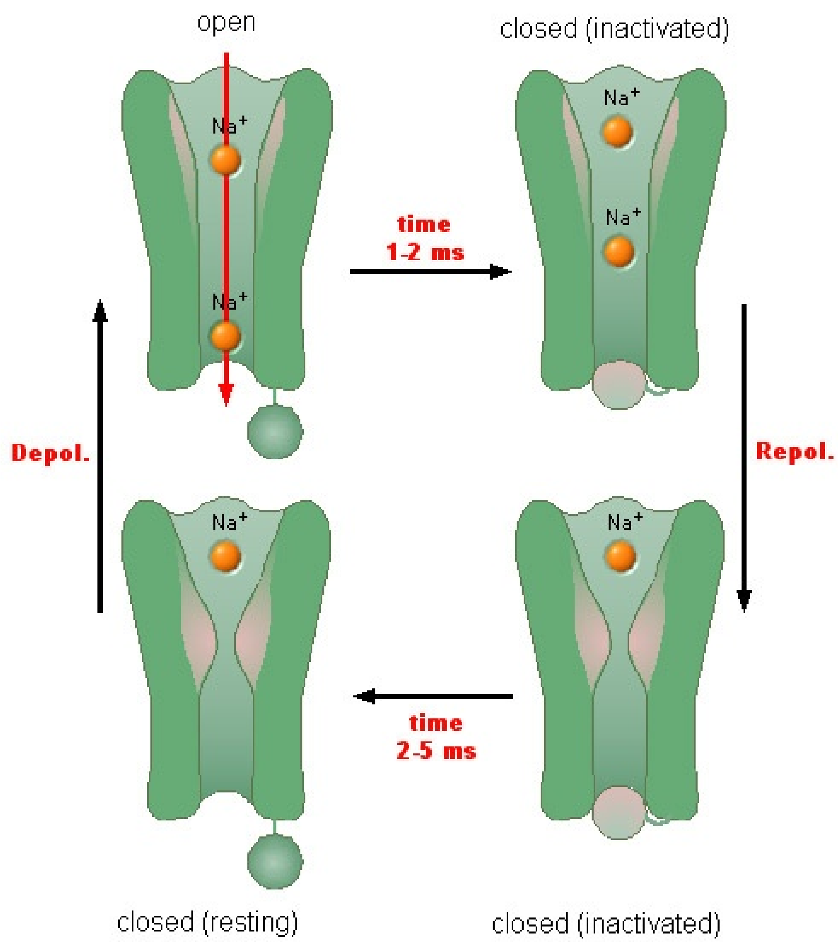
$$\frac{dn}{dt} = \frac{n_\infty\left(V(t)\right) - n(t)}{\tau_n\left(V(t)\right)}$$



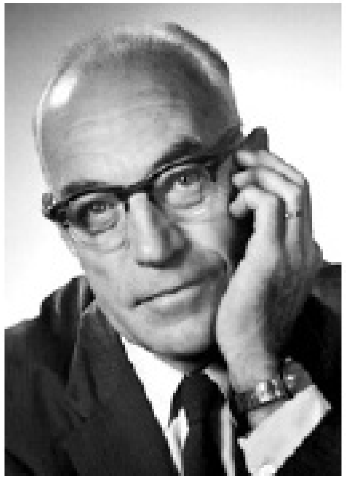
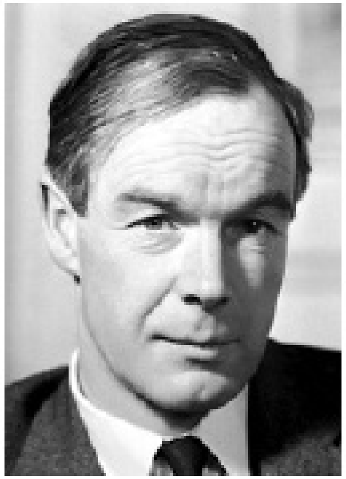
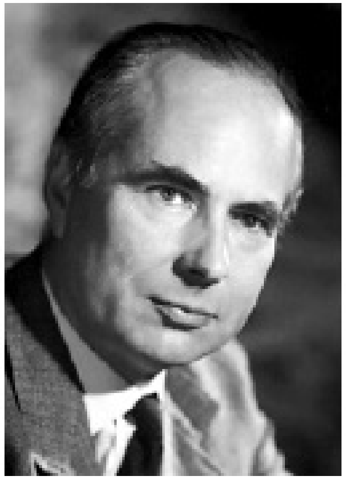
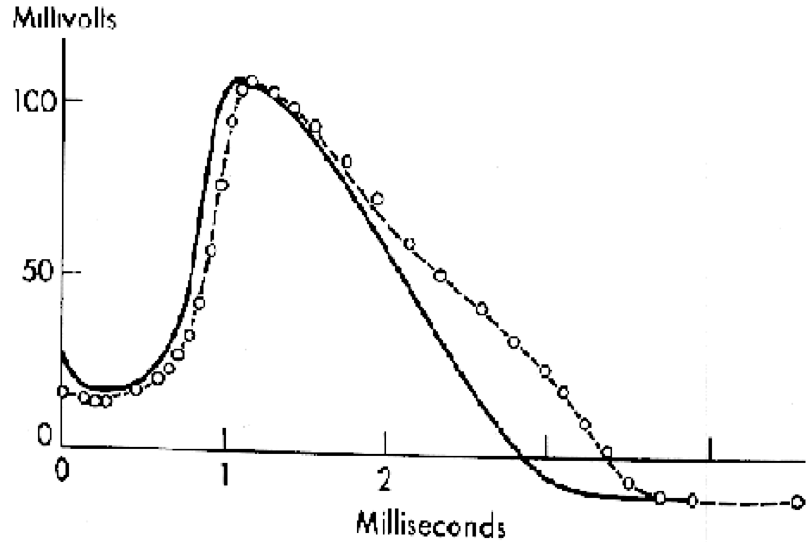
The Hodgkin-Huxley model / 4

HH model in work:

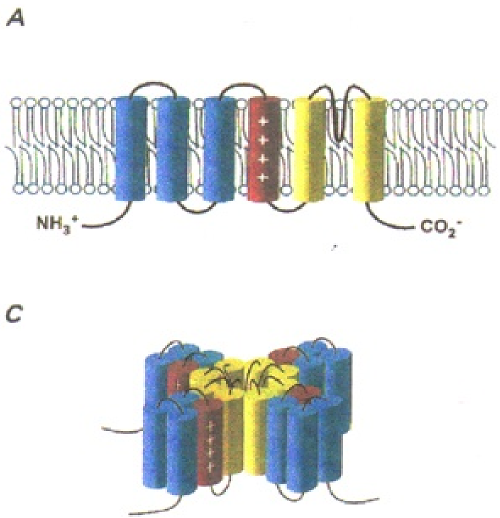


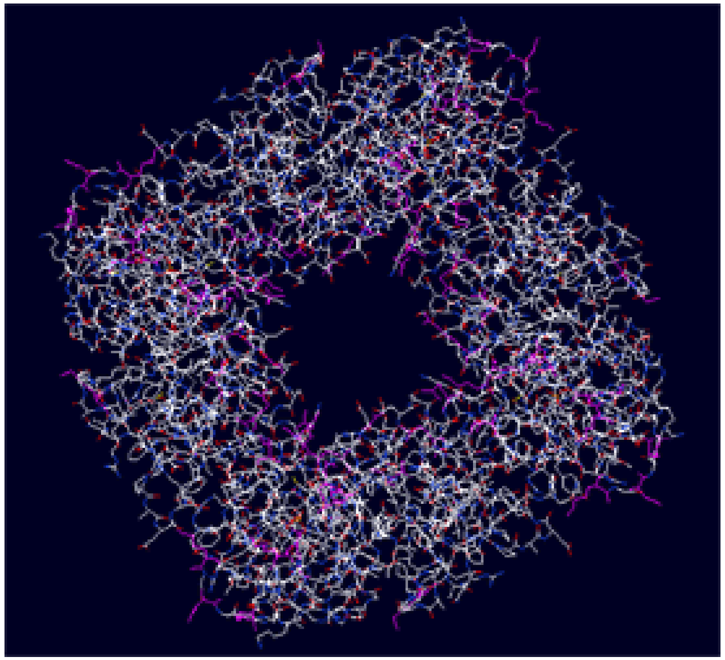
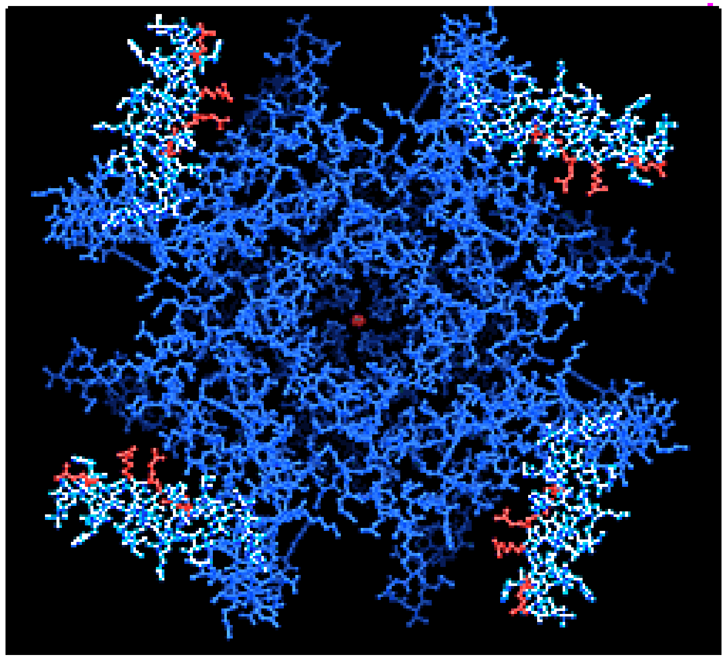


The Hodgkin-Huxley model / 5

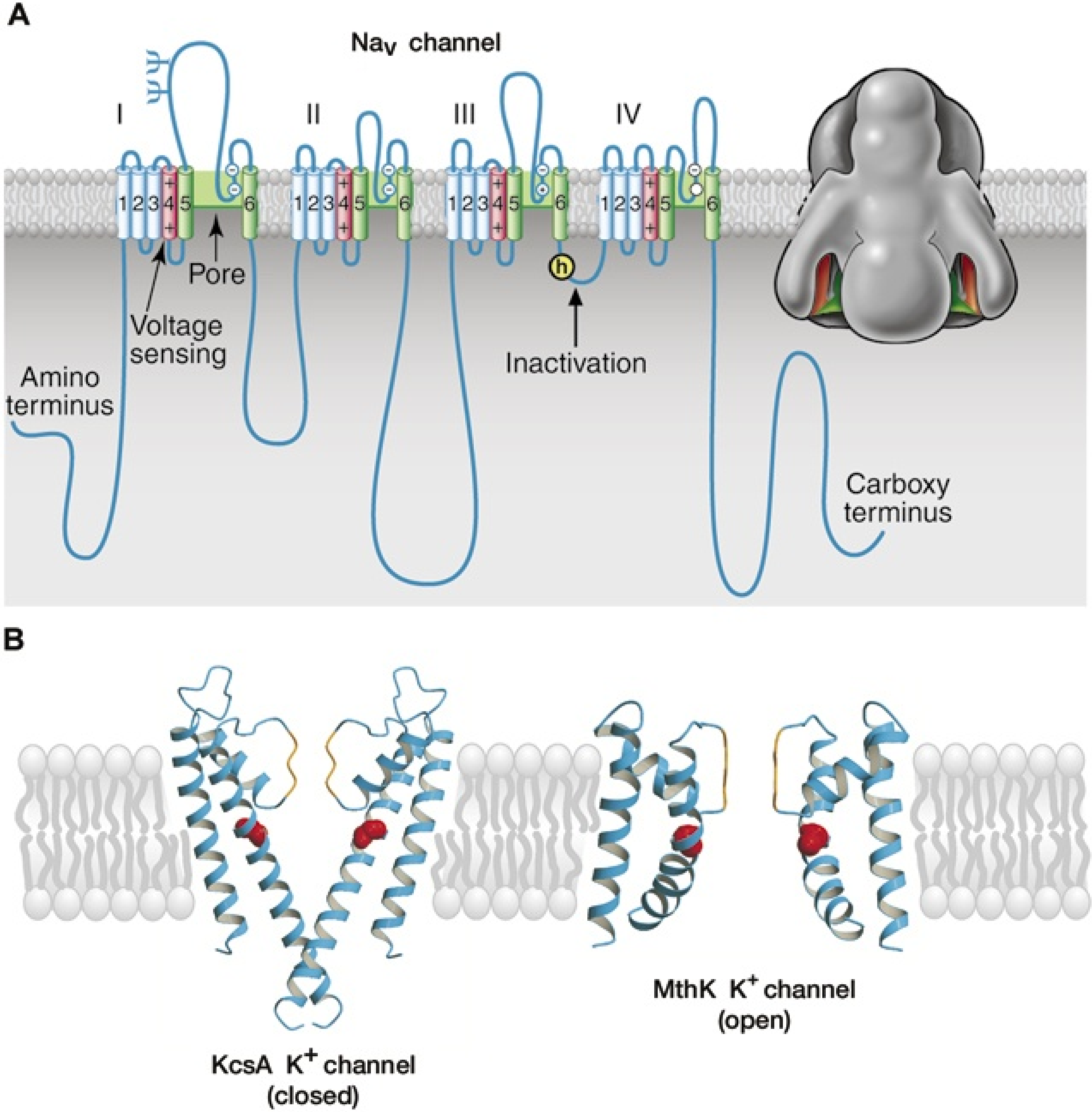





Sir John Carew Eccles, Alan Lloyd Hodgkin, Andrew Fielding Huxley: Awarded by Nobel-prize in medicine, 1963.



The Hodgkin-Huxley model / 6



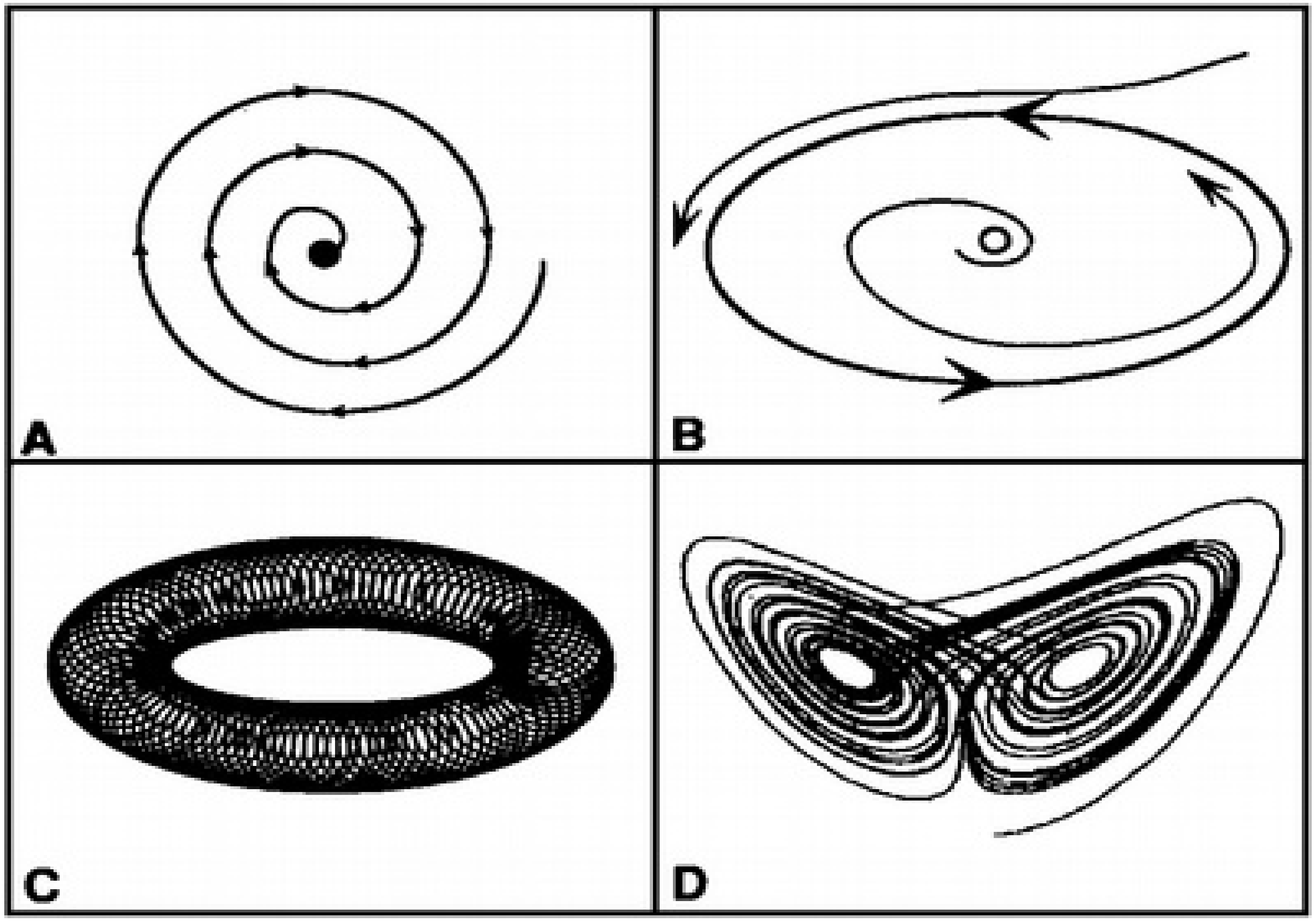
Types of attractors

A: Fixed point

B: Limit cycle

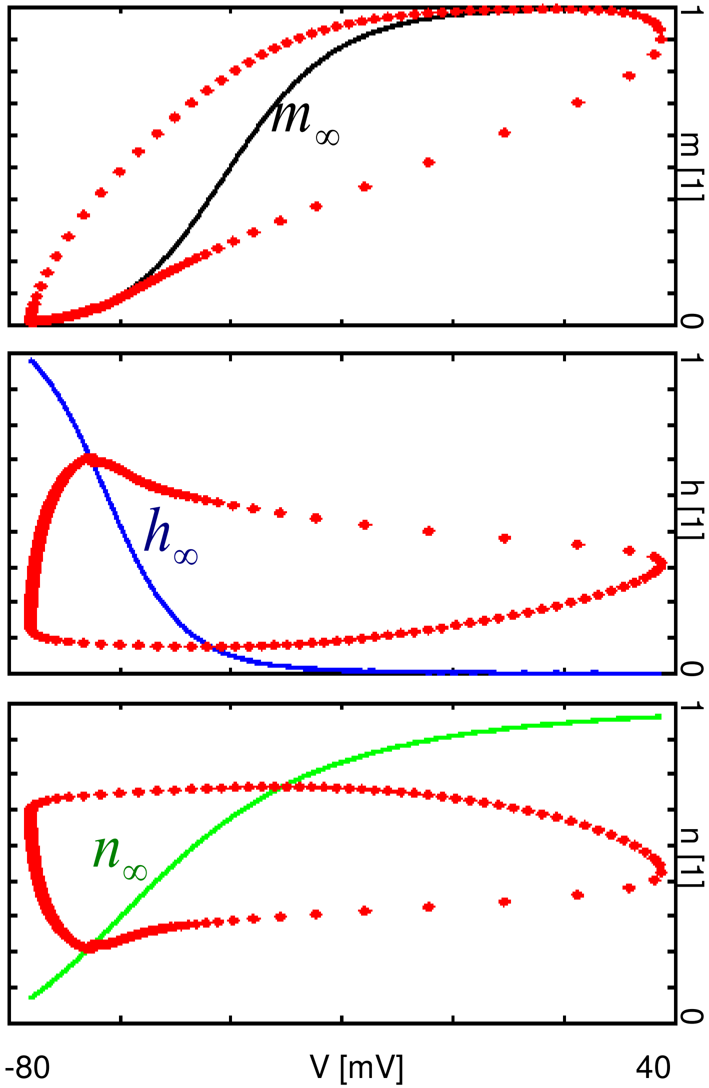
C: Quasi periodic: requires irrational ratio for at least two osscillations.

D: Chaotic: requires at least 3 dimensions.

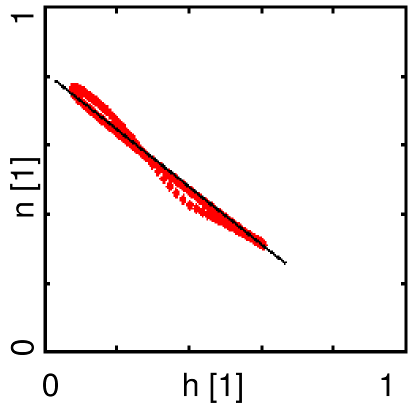


The dimension reduction of the HH-model

One spike in the state space (three projections)



FAST ACTIVATING VARIABLES *(V)*



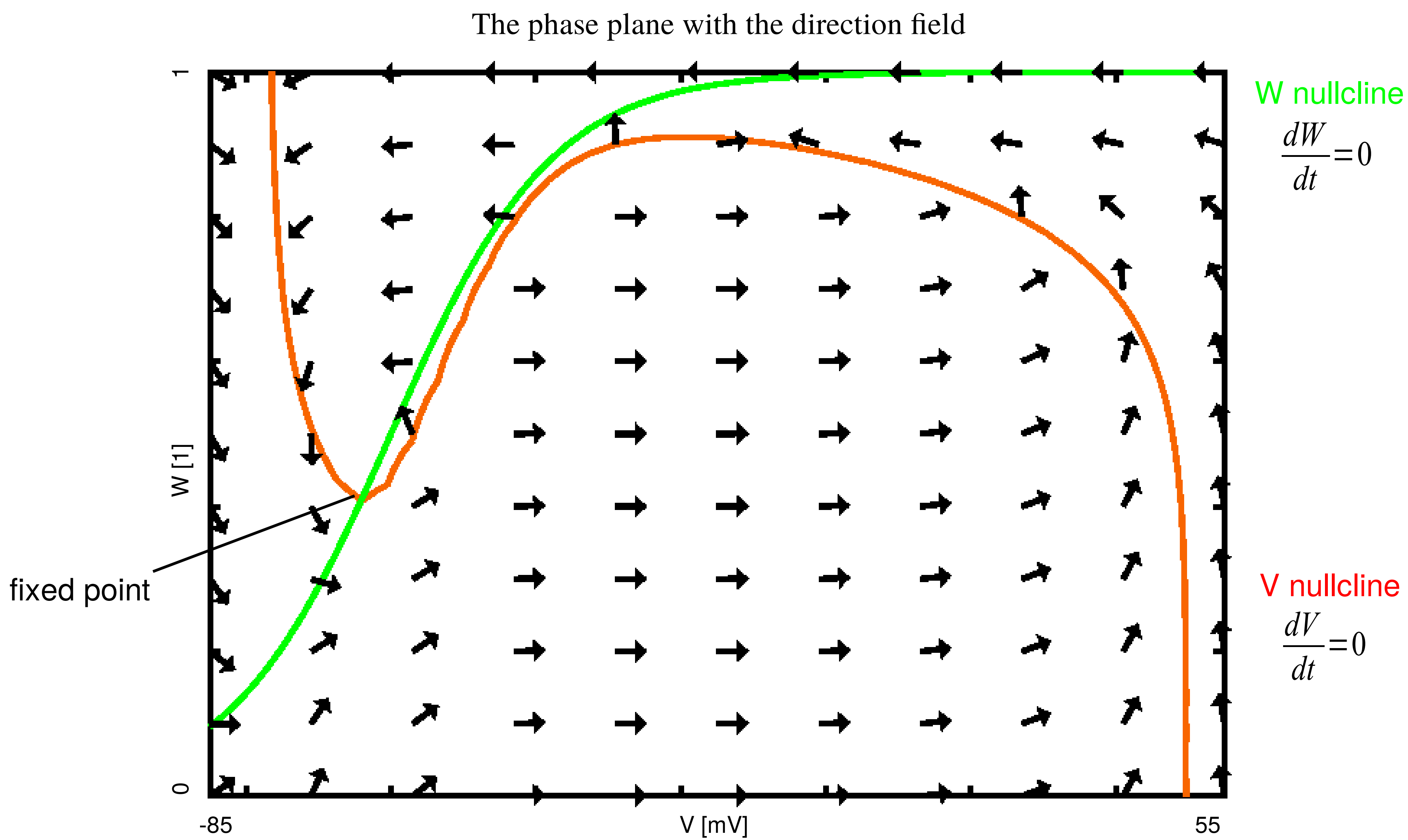
$$C_m \frac{dV}{dt} = \bar{g}_{Na}\left(E_{Na} - V(t)\right) m_\infty^3(t)\left(1 - W(t)\right) + $$
$$+ \bar{g}_K\left(E_K - V(t)\right)\left(\frac{W(t)}{s}\right)^4 +$$
$$+ g_{sziv}\left(E_{leak} - V(t)\right) + I_{external}(t)$$

$$\frac{dW}{dt} = \frac{W_\infty\left(V(t)\right) - W(t)}{\tau_W\left(V(t)\right)}$$

SLOW, INACTIVATING VARIABLES

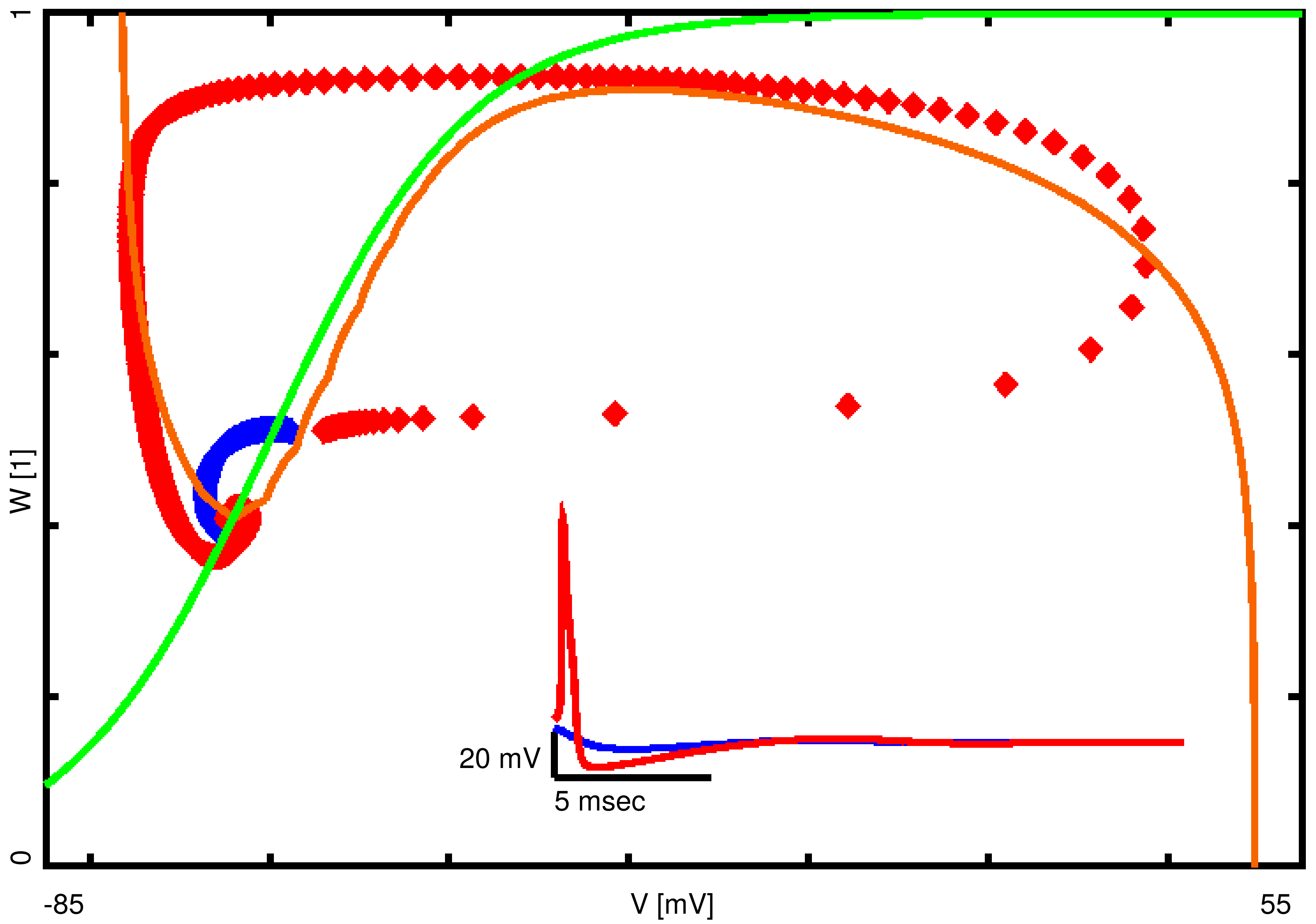
(W)

Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 1

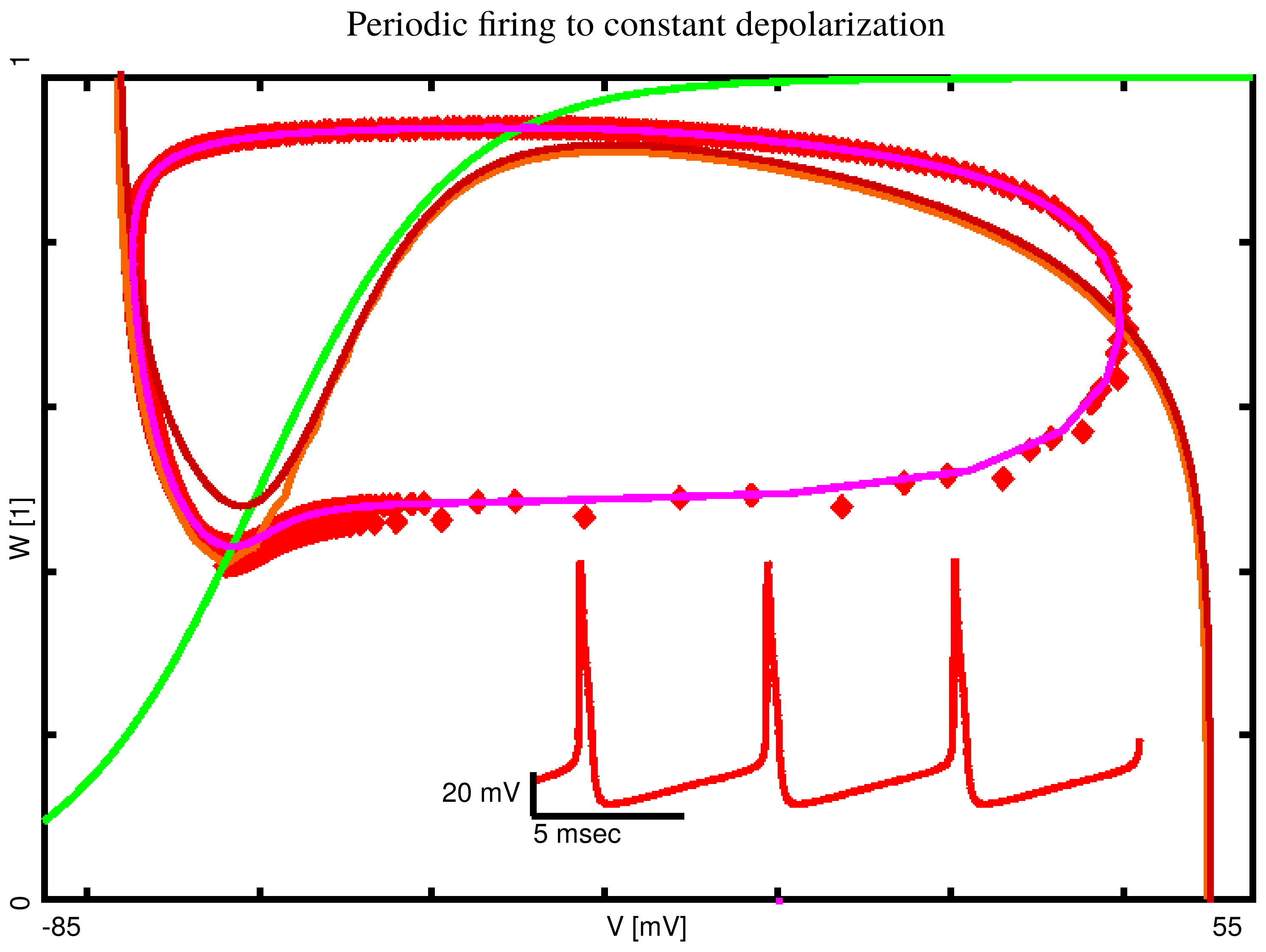


Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 2

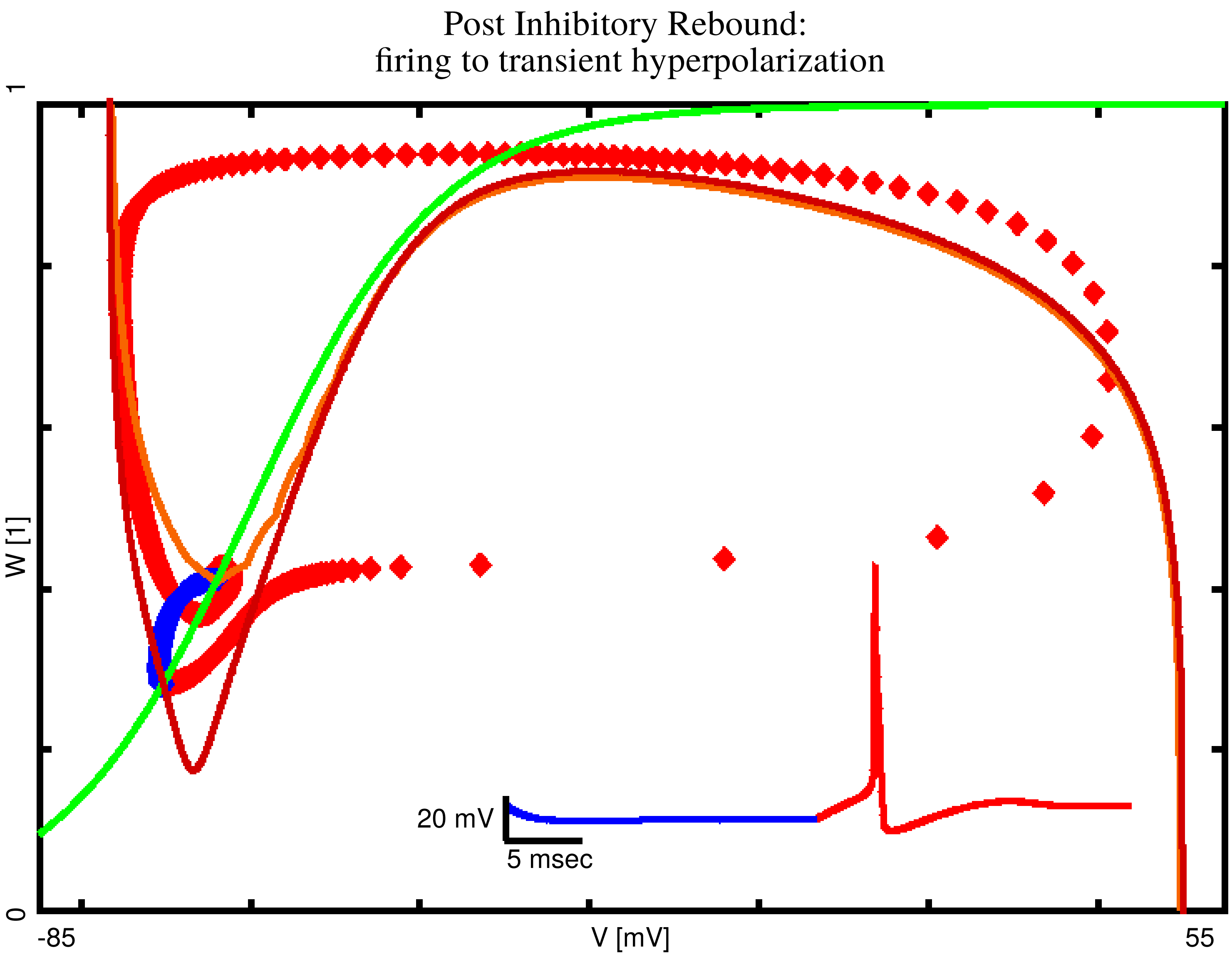
Excitability: firing is a threshold phenomenon (all-or-none)



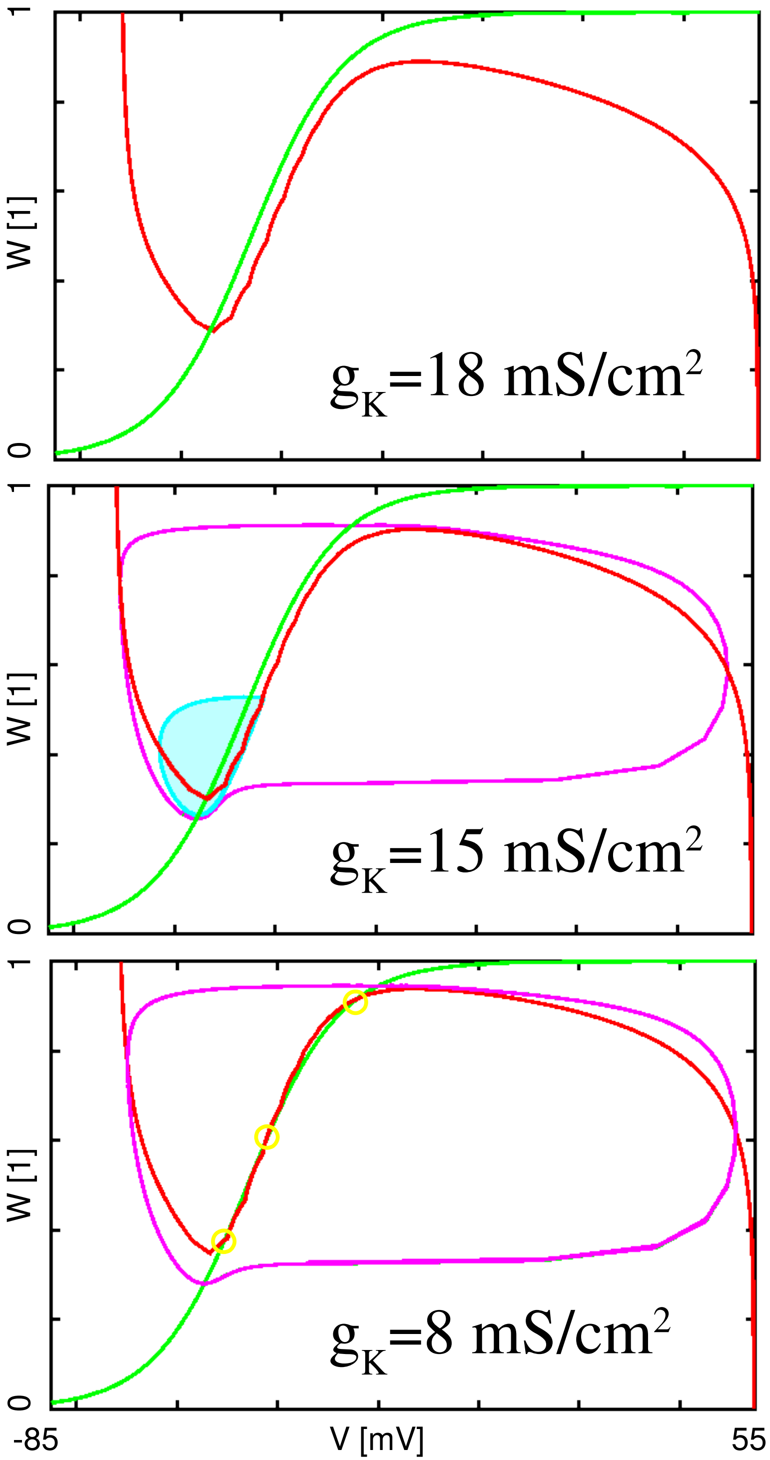
Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 3

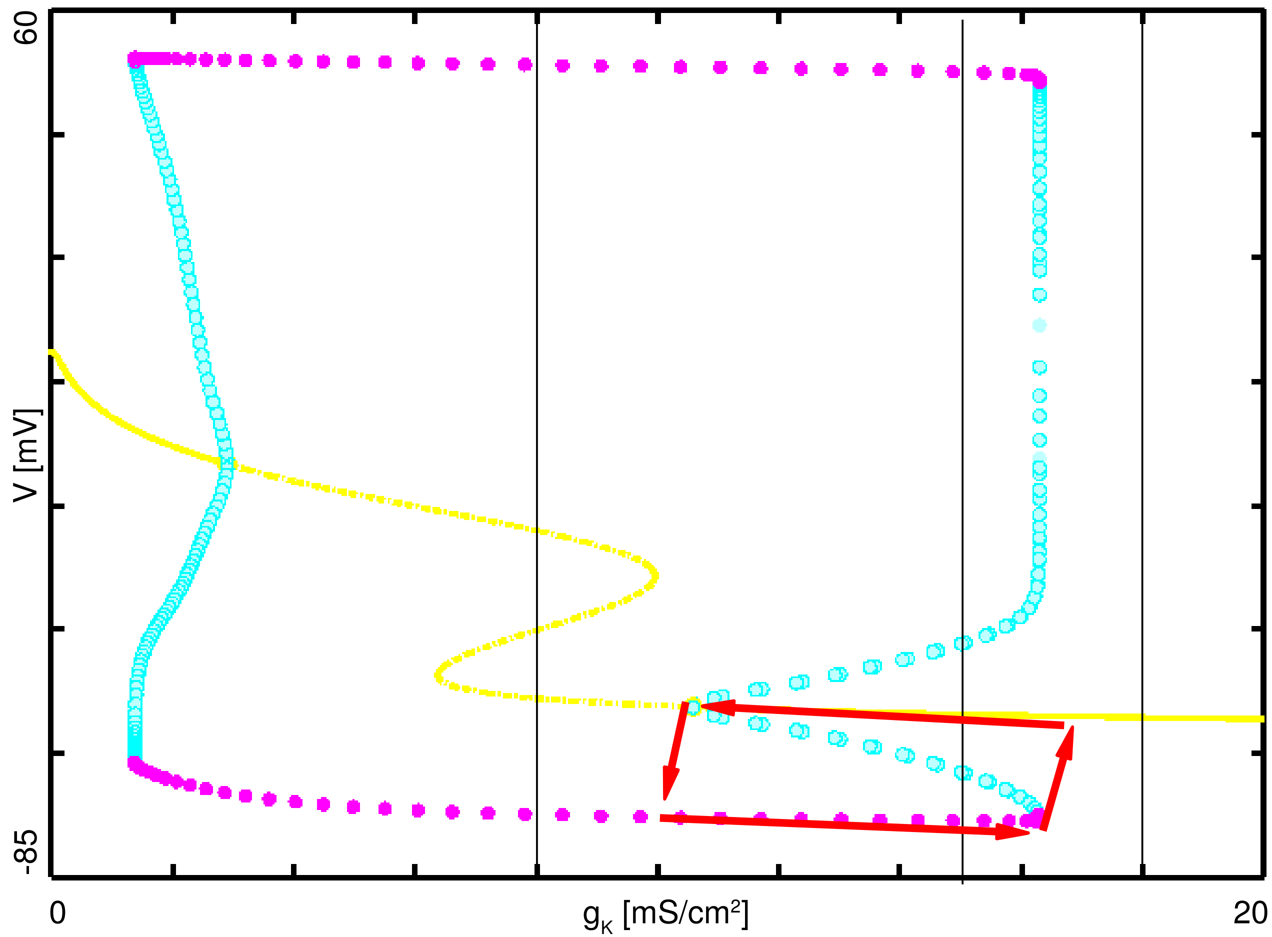


Phase Plane Analysis: the FitzHugh- (Nagumo-, Rinzel-) model / 4



Phase Plane Analysis: Bursting / 1

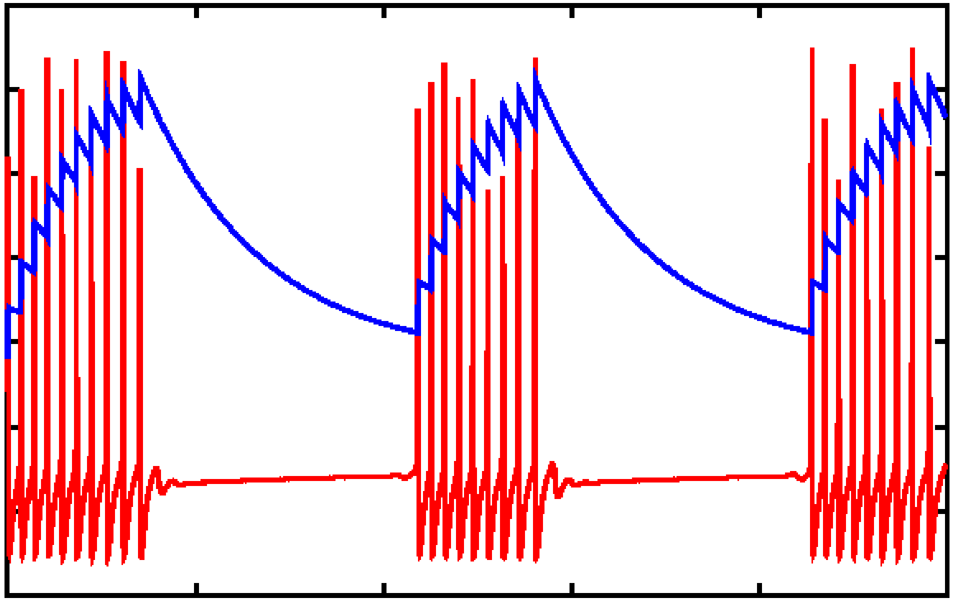




$$\frac{d(g_K)}{dt}=Z(V)(V-V_{rest})-\frac{g_K-g_K^{rest}}{\tau_{g_K}}$$

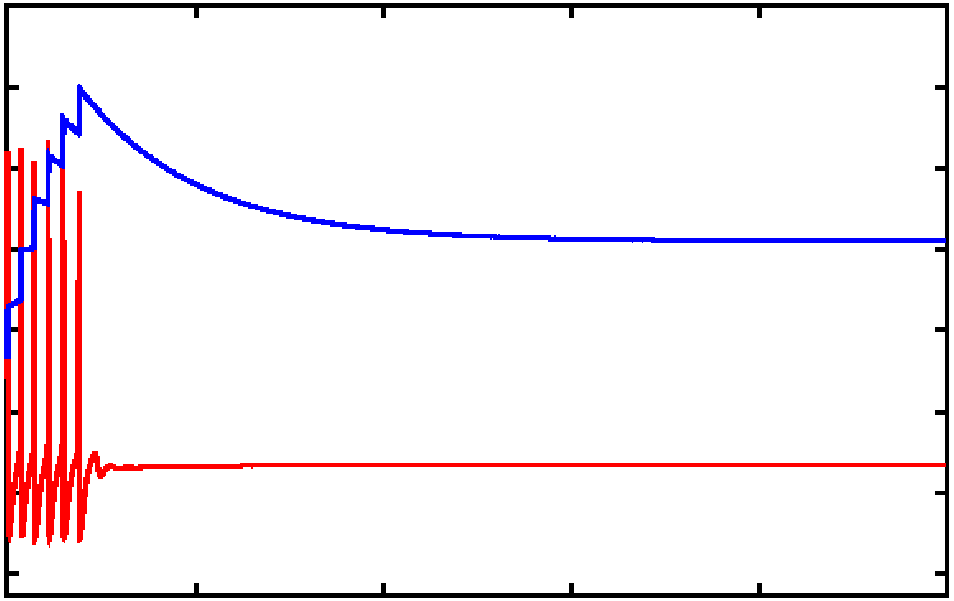
Phase Plane Analysis: Bursting / 2

Endogeneous Burster



$g_K^{rest} = 8\,mS/cm^2$

Conditional Burster



$g_K^{rest} = 12\,mS/cm^2$

Types of bifurcations

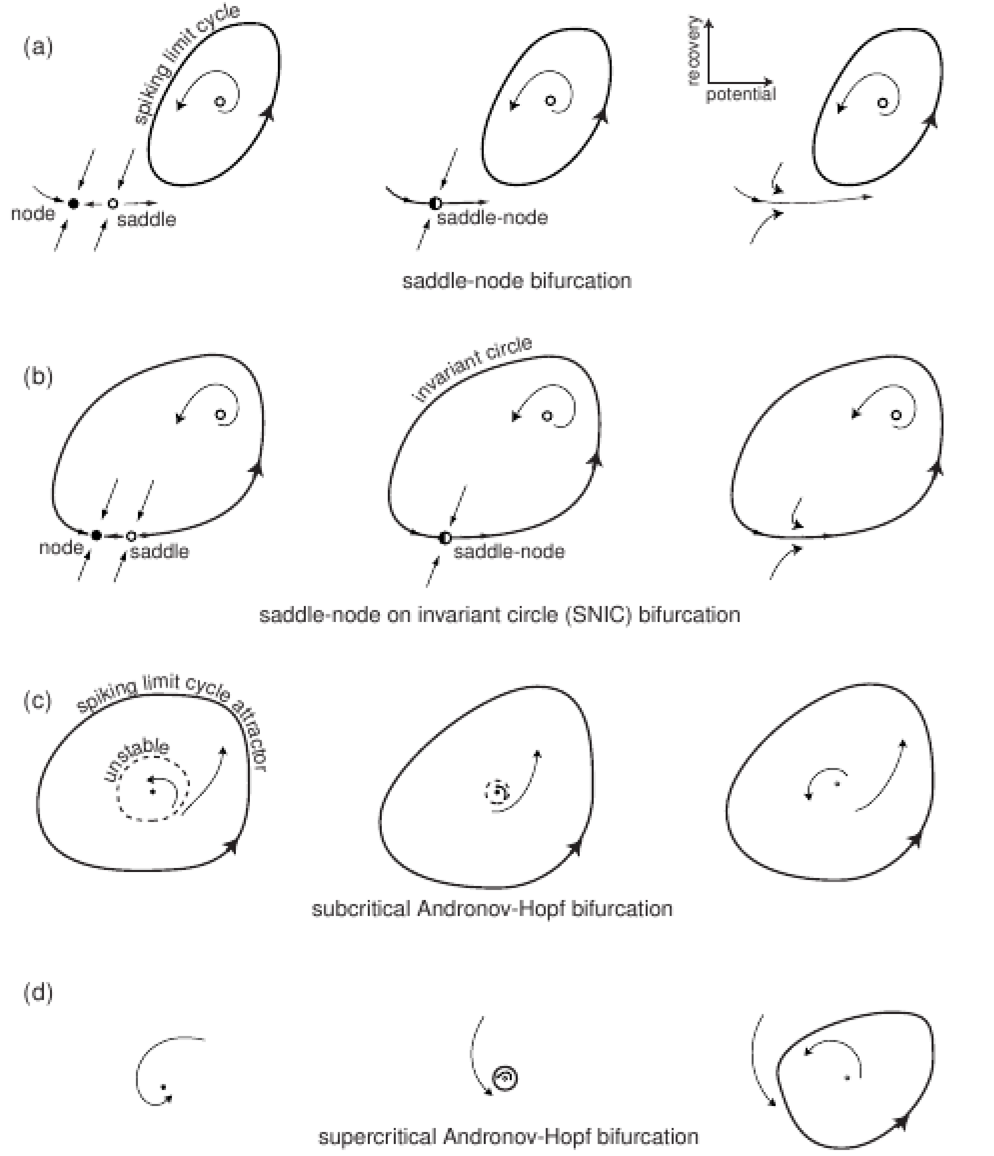


Figure 1.12: Four generic (codimension-1) bifurcations of an equilibrium state leading to the transition from resting to periodic spiking behavior in neurons.

coexistence of resting and spiking states

subthreshold oscillations	YES (bistable)	NO (monostable)
NO (integrator)	saddle-node	saddle-node on invariant circle
YES (resonator)	subcritical Andronov-Hopf	supercritical Andronov-Hopf

Figure 1.13: Classification of neurons into monostable/bistable integrators/resonators according to the bifurcation of the resting state in Fig.1.12.

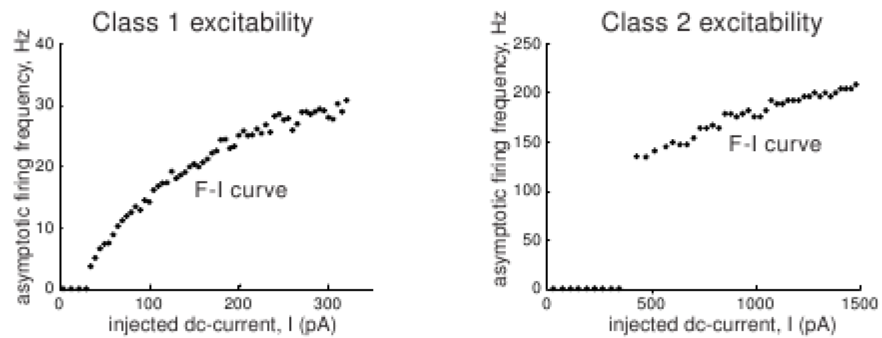


Figure 1.14: Frequency-current (F-I) curves of cortical pyramidal neuron and brainstem mesV neuron from Fig.7.3. These are the same neurons used in the ramp experiment in Fig.1.11.

Class 1 ~ saddle-node on invariant circle

Class 2 ~ saddle-node or Andronov-Hopf

Types of bifurcations 2.

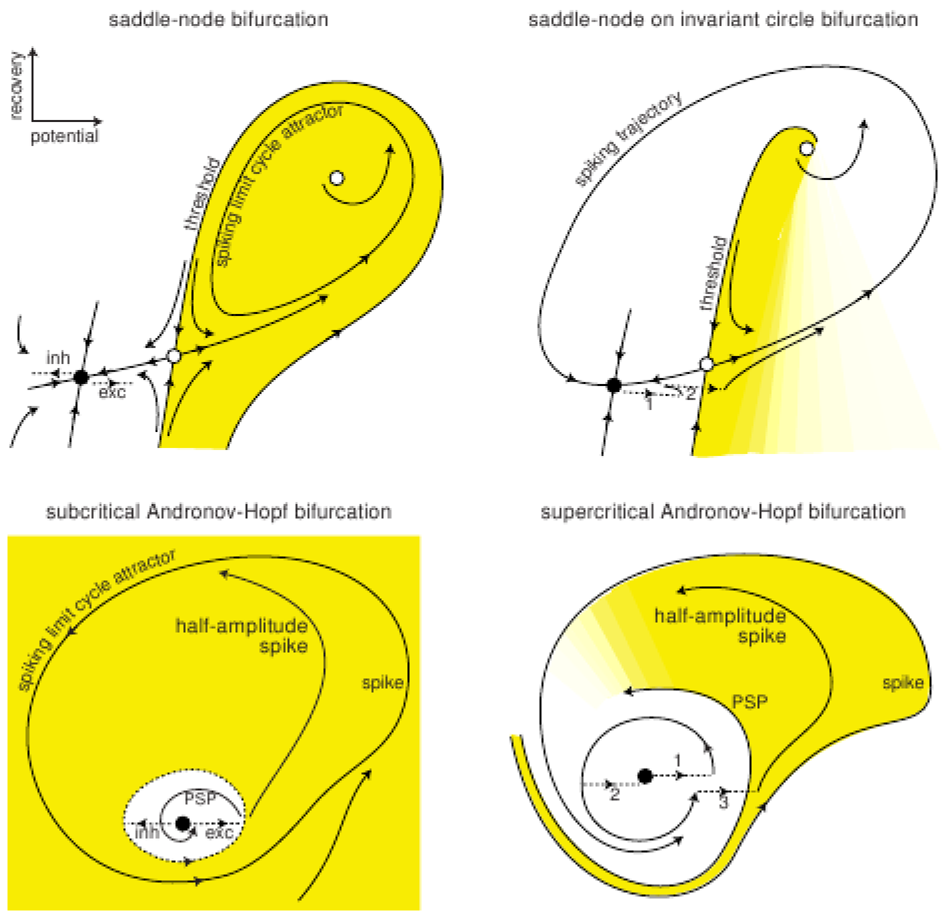


Figure 1.15: The geometry of phase portraits of excitable systems near four bifurcations can explain many neurocomputational properties (see section 1.2.4 for details).

subthreshold oscillations	coexistence of resting and spiking states: YES (bistable)	NO (monostable)
NO (integrator)	saddle-node	saddle-node on invariant circle
YES (resonator)	subcritical Andronov-Hopf	supercritical Andronov-Hopf

Figure 1.13: Classification of neurons into monostable/bistable integrators/resonators according to the bifurcation of the resting state in Fig.1.12.

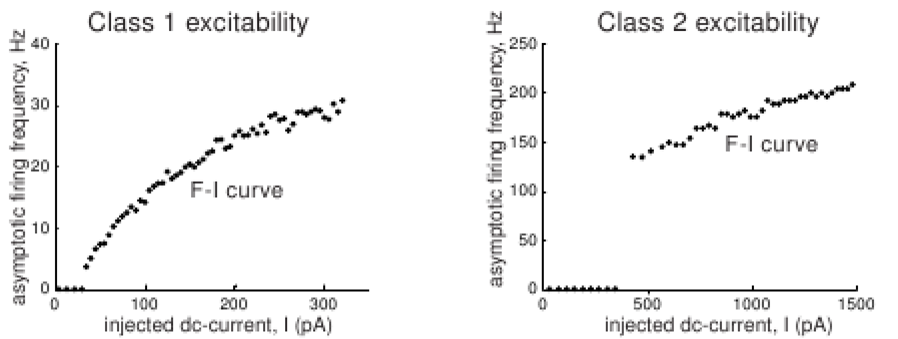
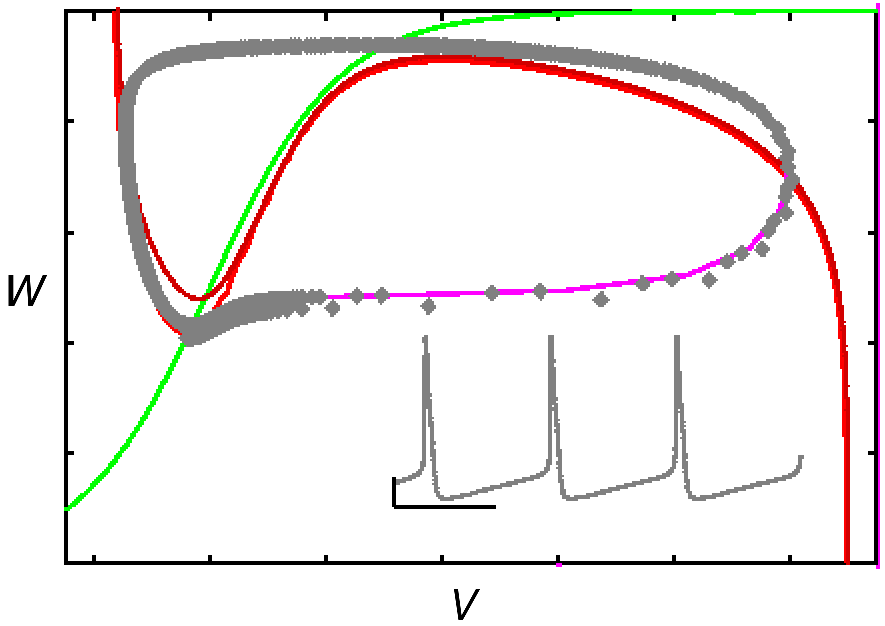
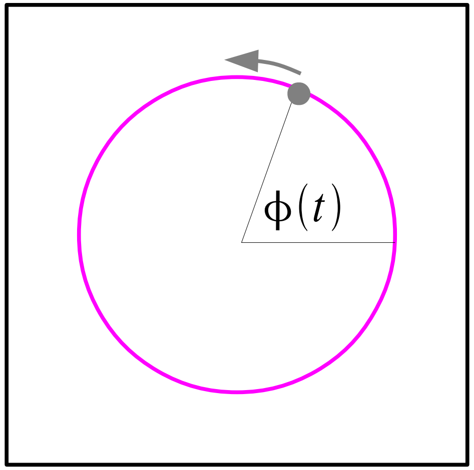


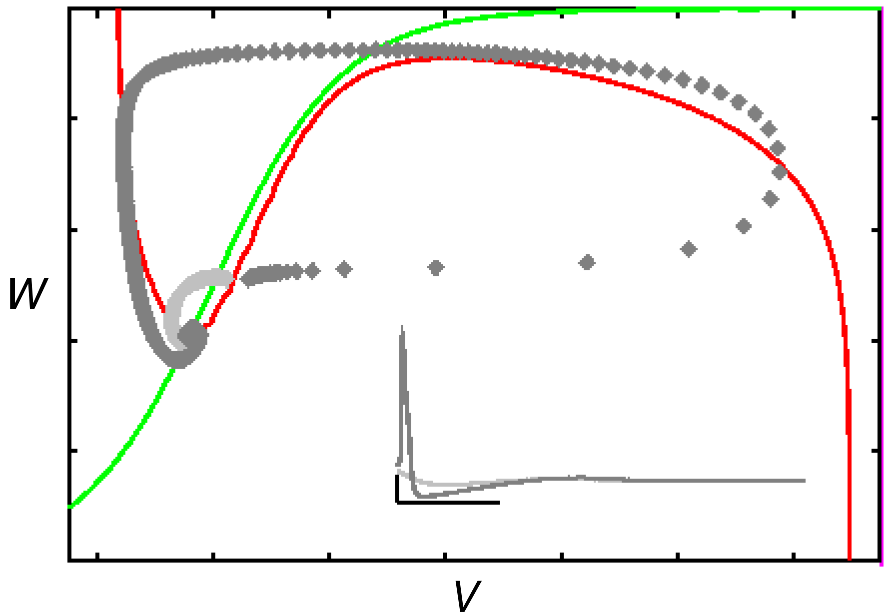
Figure 1.14: Frequency-current (F-I) curves of cortical pyramidal neuron and brainstem mesV neuron from Fig.7.3. These are the same neurons used in the ramp experiment in Fig.1.11.

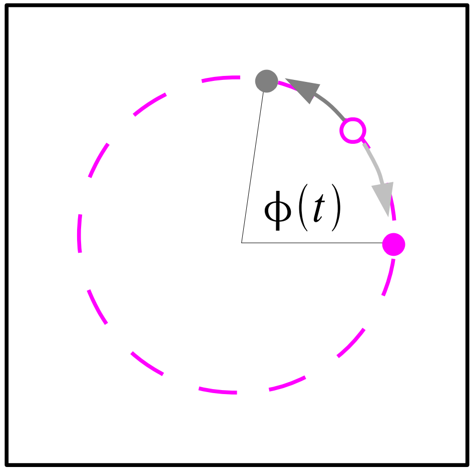
The phase-model





$$\frac{d\phi}{dt} = \omega\left(I(t)\right) + h\left(\phi(t), \phi(t)'\right)$$

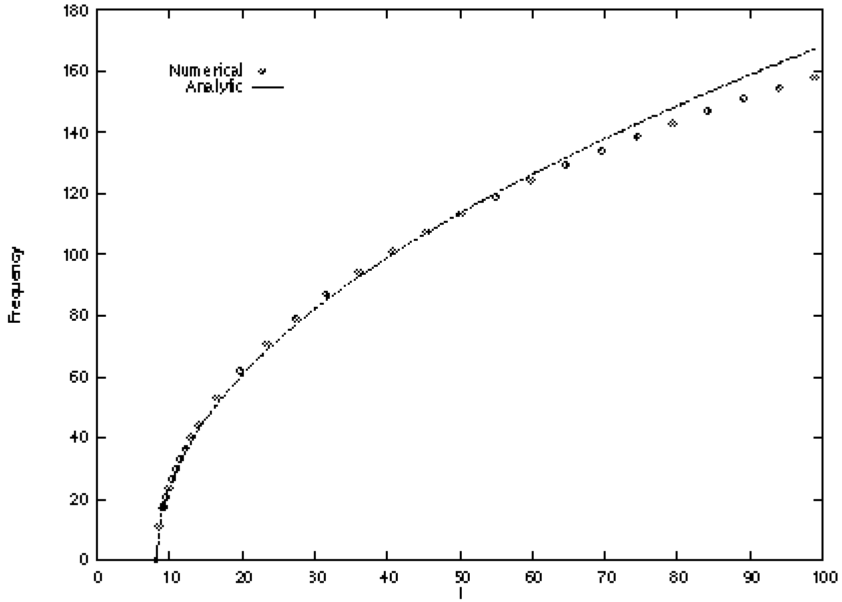




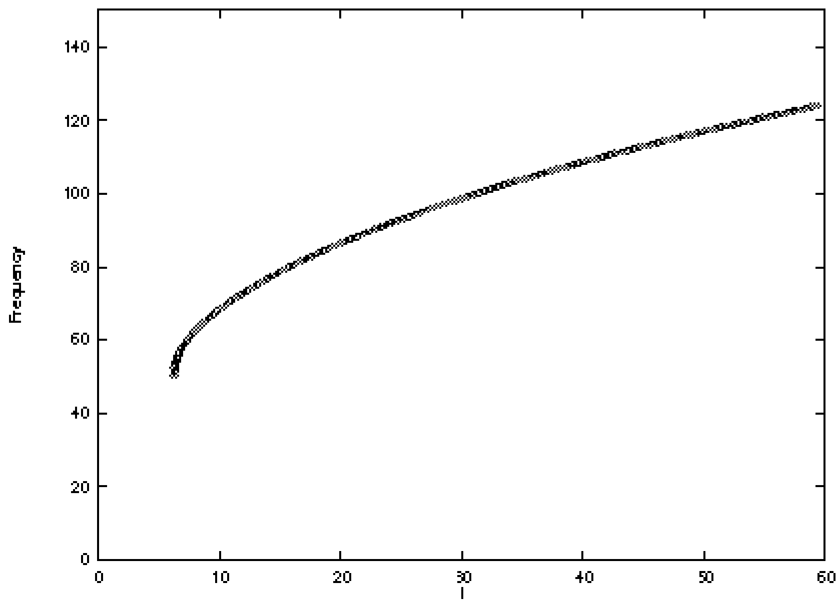
$$\frac{d\phi}{dt} = \omega\left(I(t), \phi(t)\right) + h\left(\phi(t), \phi(t)'\right)$$

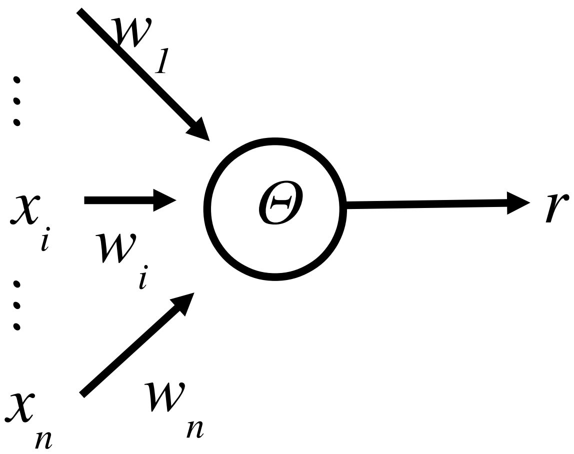
The rate-model

Type I

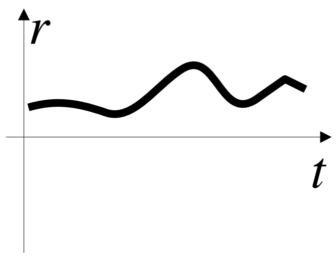


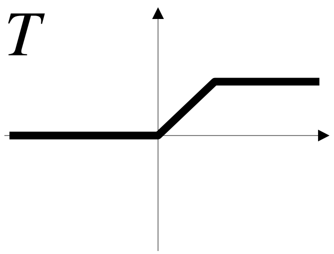
Type 2 (HH)



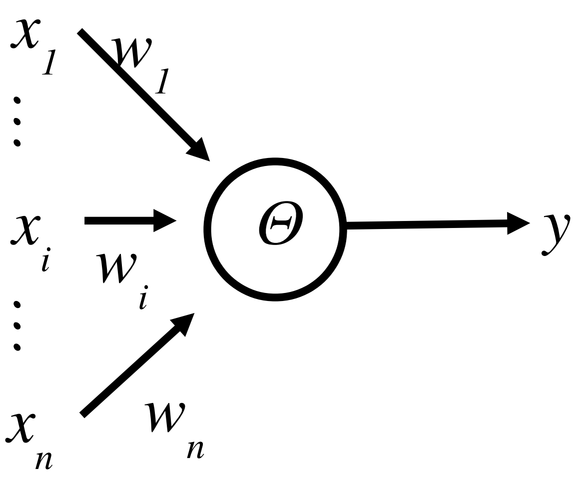


$$r(t) = T\left(\sum_{i=1}^{n} w_i \cdot x_i(t) - \Theta\right)$$

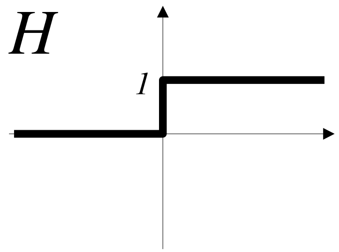




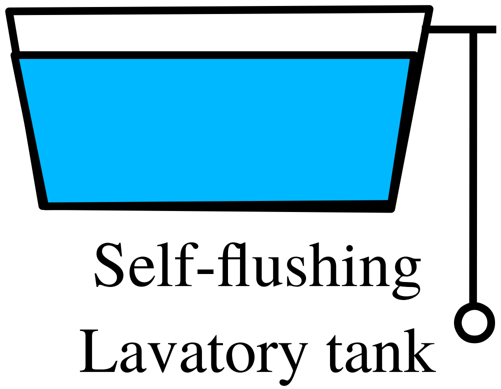
The McCulloch-Pitts model



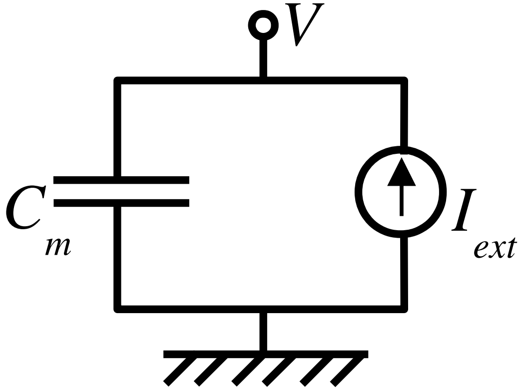
$$y(t)=H\left(\sum_{i=1}^{n} w_i \cdot x_i(t)-\Theta\right)$$



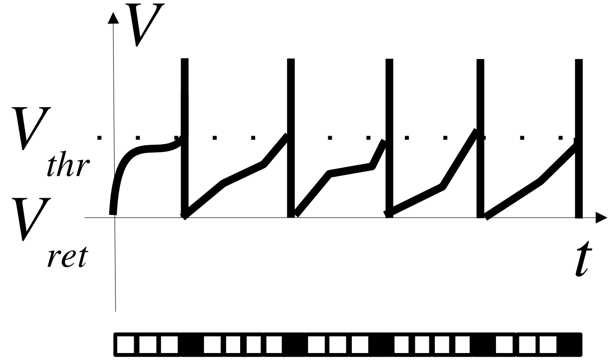
Integrate & Fire neuron



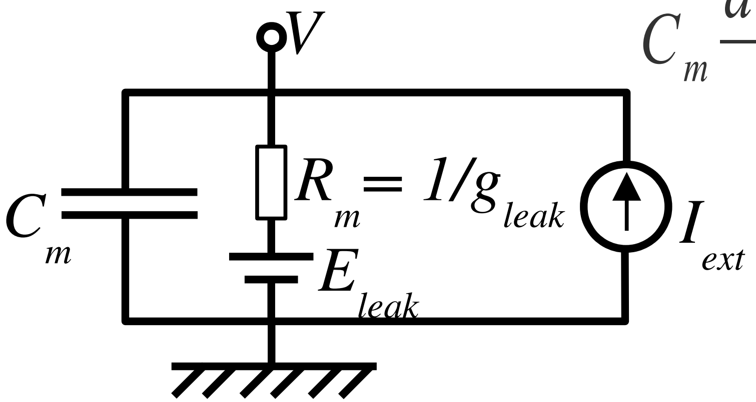
ha $V(t) > V_{thr} \longrightarrow$ ***spike*** $\longrightarrow V(t) := V_{ret}$



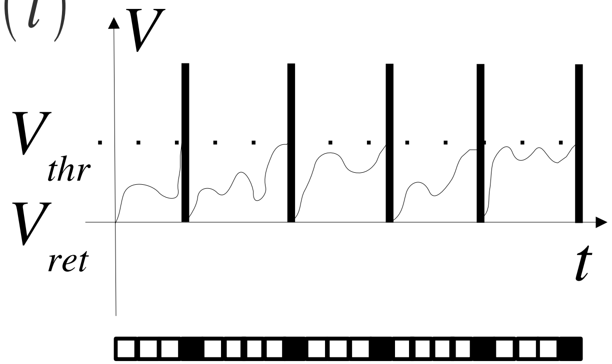
$$C_m \frac{dV(t)}{dt} = I_{ext}(t)$$



Leaky Integrator



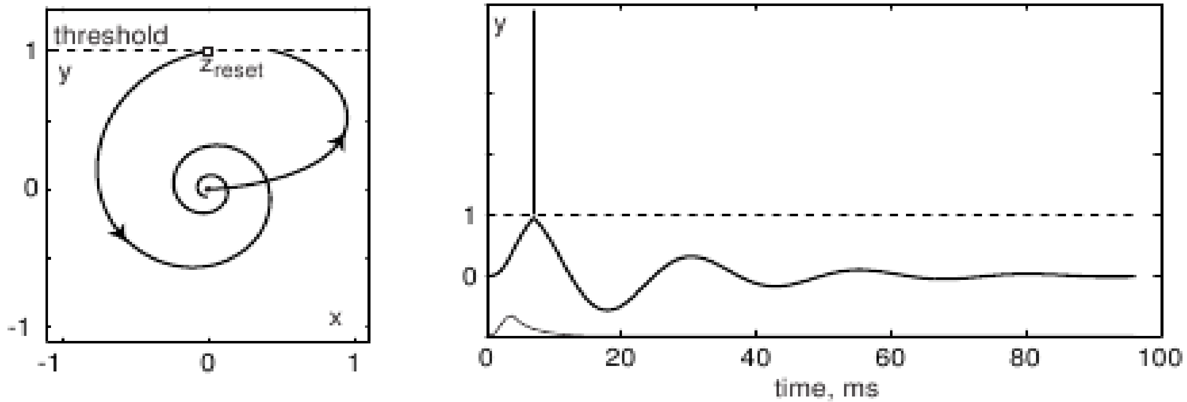
$$C_m \frac{dV(t)}{dt} = \frac{E_{leak} - V(t)}{R_m} + I_{ext}(t)$$



Resonate & Fire neuron

if $V(t) > V_{thr}$ ⟶ ***spike*** ⟶ $V(t) := V_{ret}$

$$C_m \frac{dZ(t)}{dt} = (b + i\omega) Z + I_{ext}(t)$$



Re(Z) is a potential-, *Im(Z)* is a current-like variable