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Particle Interferometry for Expanding Systems, $U_A(1)$ Restoration and Mass-Shifted Bosons

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Buda-Lund parameterization of BE corr. functions:
A simple, covariant form, for expanding systems
Notes on hydro parameterization
Correlation signals of partial $U_A(1)$ restoration
Correlations and Spectra
for Medium-Modified Quanta
Back-to-back correlations

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M. Gyulassy,
B. Lörstad,
D. Kharzeev,
A. Ster and
S. E. Vance

Two Gaussian parameterizations of BE Correlations:

Bertsch-Pratt : Not covariant formulation

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \lambda_* \exp\left[-R^2_{side}(\mathbf{K})Q^2_{side} - R^2_{out}(\mathbf{K})Q^2_{out} \right.$$
$$\left. -R^2_L(\mathbf{K})Q^2_L - 2R^2_{out,L}(\mathbf{K})Q_L Q_{out}\right] \quad (1)$$

where

$$\mathbf{K} = 0.5(\mathbf{k}_1 + \mathbf{k}_2) \quad (2)$$
$$\mathbf{\Delta k} = \mathbf{k}_1 - \mathbf{k}_2 \quad (3)$$
$$Q_L = k_{z,1} - k_{z,2} \quad (4)$$
$$Q_{out} = \mathbf{\Delta K} \cdot \mathbf{K}/|\mathbf{K}| \quad (5)$$
$$Q_{side} = |\mathbf{\Delta k} \times \mathbf{K}|/|\mathbf{K}| \quad (6)$$

Yano-Koonin-Podgoretskii : Covariant, but not the simplest

$$C(\mathbf{q}, \mathbf{K}) = 1 + \exp\left[-R^2_\perp(\mathbf{K})q^2_\perp - R^2_\parallel(\mathbf{K})(q^2_l - (q^0)^2) \right. \quad (7)$$
$$\left. - \left(R^2_0(\mathbf{K}) + R^2_\parallel(\mathbf{K})\right)(q \cdot U(\mathbf{K}))^2\right]$$

where the interpretation of $U(\mathbf{K})$ is the four-velocity of a fluid-element. YKP yields a diagonal Gaussian expression in the rest frame of the fluid-element.

Buda - Lund parameterization for HBT

Simplest possible covariant formulation:

- Buda - Lund :

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \exp\left(-R_{=}^2 Q_{=}^2 - R_{\parallel}^2 Q_{\parallel}^2 - R_{\perp}^2 Q_{\perp}^2\right) \tag{8}$$

$R_{=}$ reads as R-timelike,
(it measures the width of the proper-time distribution)
$R_{\parallel}$ reads as R-parallel,
(measures an invariant length parallel to the direction of the expansion)
$R_{\perp}$ reads as R-perpendicular.

Buda - Lund covariant form

$$C(\mathbf{\Delta k}, \mathbf{K}) = 1 + \exp\left(-R_{=}^2(\mathbf{K})Q_{=}^2(n) - R_{\parallel}^2(\mathbf{K})Q_{\parallel}^2(n) - R_{\perp}^2(\mathbf{K})Q_{\perp}^2\right) \quad (9)$$

Invariant time-like, longitudinal and transverse radii:

$$R_{=}^2 = R_{=}^2(\mathbf{K}) = \Delta\tau_*^2 \quad (10)$$
$$R_{\parallel}^2 = R_{\parallel}^2(\mathbf{K}) = \tau_s^2\Delta\eta_*^2 \quad (11)$$
$$R_{\perp}^2 = R_{\perp}^2(\mathbf{K}) = R_*^2 \quad (12)$$

Invariant time-like, longitudinal and transverse relative momenta:

$$Q_{=} = Q \cdot n = Q_0 \cosh[\eta_s] - Q_L \sinh[\eta_s] \quad (13)$$
$$Q_{\parallel} = \sqrt{Q^2 - (Q \cdot u)^2 - Q_{\perp}^2} = Q_0 \sinh[\eta_s] - Q_L \cosh[\eta_s] \quad (14)$$
$$Q_{\perp} = \sqrt{Q_{side}^2 + Q_{out}^2} = Q_t \quad (15)$$

I.e. $Q_{=}$ is the invariant relative momentum component parallel to the time-like normal-vector n, $Q_{\parallel}$ is the invariant relative momentum component in the longitudinal direction ("parallel to the beam"), and $Q_{\perp}$ is the remaining transversal component. Note that n is a normal-vector of the surface:

$$n = n(\overline{x}(\mathbf{K})) = (\cosh[\eta_s], 0, 0, \sinh[\eta_s]) \quad (16)$$
$$\eta_s = \eta_s(\mathbf{K}) = y + \frac{y_0 - y}{1 + \Delta\eta^2 m_t/T_0} \quad (17)$$

the last line is valid for the Buda-Lund hydro parameterization.

New notation for the side and out components
(No mixing of Q_0 and $Q_o = Q_{out}$)

Symbolic notation in transverse direction:

(Thanks to O. Smirnova!)

$$Q_\perp^2 = Q_{side}^2 + Q_{out}^2 \tag{18}$$

$$Q_{side} \equiv Q_{..} \tag{19}$$

$$Q_{out} \equiv Q_{:} \tag{20}$$

Similarly, the radii can be denoted as

$$R_{side} = R_{..} \tag{21}$$

$$R_{out} = R_{:} \tag{22}$$

In a general Gaussian form, the Buda - Lund covariant BECF can be denoted as

$$C(Q,K) = 1 + \lambda_* \exp\left(-R_{=}^2 Q_{=}^2 - R_{\parallel}^2 Q_{\parallel}^2 - R_{..}^2 Q_{..}^2 - R_{:}^2 Q_{:}^2\right) \tag{23}$$

Note, that this equation is identical to eq. (44) of T. Cs. and B. Lörstad, PRC 54(1996) 1390. Only the notation has been modernized and the covariance emphasized.

Note that $n(\mathbf{K})$ normal-vector rests in the (t, r_z) plane for the Buda-Lund parameterization, while the $U(\mathbf{K})$ is in velocity-space.

Recapitulation of core/halo model

In ref. T. Cs. and B. Lörstad, PRC 54(1996) 1390, Gaussians appear as a particular approximations only. Exponentials, power laws, $\lambda \neq 1$ etc are possible also.

Core/halo separation:

$$S(x, K) \;=\; S_c(x, K) + S_h(x, K) \tag{24}$$

$S(x, K)$ is the emission function or covariant Wigner-function.

Core: resolvable part.

Halo: unresolvable part. E.g. pions from decays of long lived resonances, like η and η' mesons.

No partial coherence is assumed (heavy ion collisions (OK?))

Core-halo model (Marburg group + Budapest-Lund collab.)

$$N(\mathbf{k}) \;=\; E\frac{d^3n}{d\mathbf{k}} = N_c(\mathbf{k}) + N_h(\mathbf{k}) = \frac{1}{\sqrt{\lambda_*(\mathbf{k})}} N_c(\mathbf{k}) \tag{25}$$

$$C_2(k_1, k_2) \;=\; \frac{N_2(k_1, k_2)}{N_1(k_1) N_1(k_2)} = 1 + \lambda_*(\mathbf{k}) \frac{|\tilde{S}(\Delta k, K)|^2}{|\tilde{S}(0, K)|^2} \tag{26}$$

$\lambda_*(k)$ measures the core fraction at a given value of the momentum:

$$\lambda_*(k) \;=\; (N_c(k)/N_t(k))^{1/2} \tag{27}$$

Buda - Lund for non-Gaussian correlations

For systems expanding mainly in one direction (r_z)

$$S_c(x,K)d^4x = H_*(\tau)G_*(\eta)I(r_x,r_y)d\tau d\eta dr_x dr_y \tag{28}$$

$$\tau = \sqrt{t^2 - r_z^2} \tag{29}$$

$$\eta = 0.5\log\left[(t+r_z)/(t-r_z)\right] \tag{30}$$

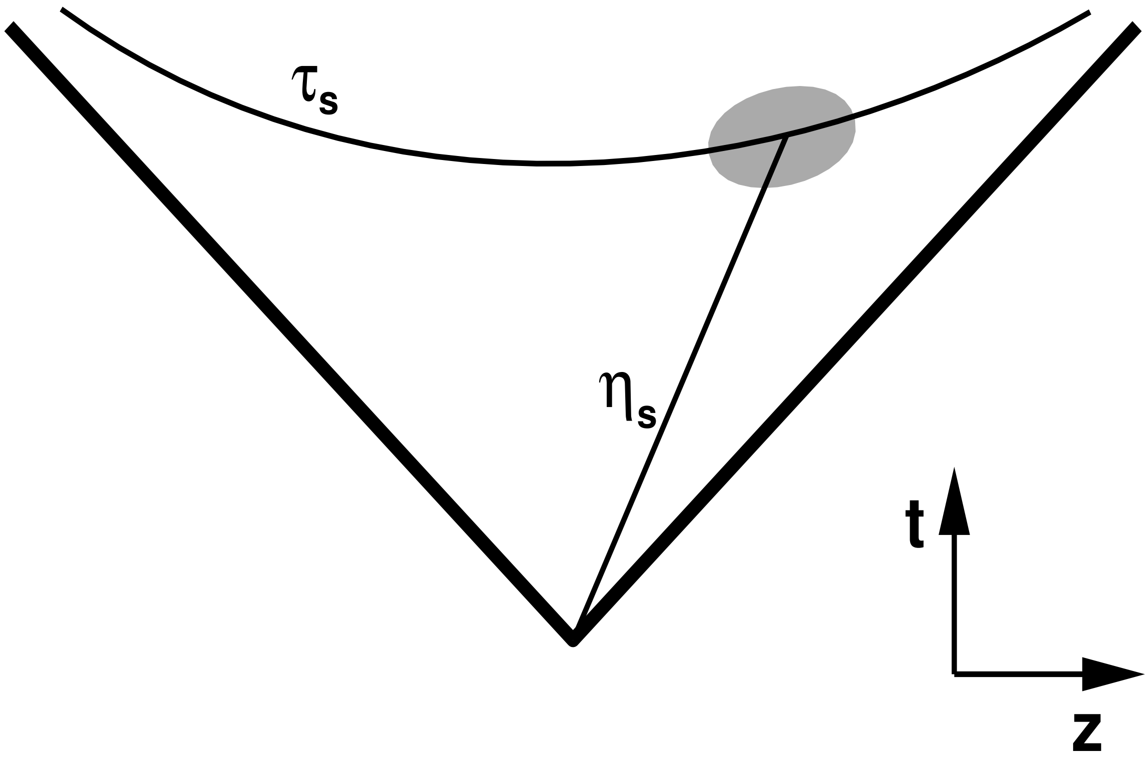
$H_*(\tau)$ = effective (K dependent) proper-time distribution
$G_*(\tau)$ = effective (K dependent) space-time rapidity distrib.
$I_*(r_x,r_y)$ = effective (K dependent) transverse distrib.

$$\exp(i\Delta E(t_1-t_2) - iQ_z(r_{z,1}-r_{z,2})) \simeq$$
$$\exp(iQ_=(\tau_1-\tau_2) - iQ_{||}\tau_s(\eta_1-\eta_2))$$

$$C(\Delta k, K) = 1 + \lambda_*(K)\frac{|\tilde{H}_*(Q_=)|^2}{|\tilde{H}_*(0)|^2}\frac{|\tilde{G}_*(Q_{||})|^2}{|\tilde{G}_*(0)|^2}\frac{|\tilde{I}_*(Q_{:},\, Q_{..})|^2}{|\tilde{I}_*(0;0)|^2} \tag{31}$$

This is the general Buda-Lund invariant decomposition of the BE correlation function; the resulting Fourier-transformed proper-time distributions can be of power-law, exponential, Gaussian or other types.

Particle production for exanding systems



Space-time picture particle emission for a given mean momentum of the pair.

Comments on the hydro model parameterization

The Buda - Lund hydro parameterization is recapitulated in the talks of Ster and Hakobyan. Main idea: to characterize the hydro fields (temperature profile, transversal flow, density distribution) with means and variances only. Similar models are studied by the Regensburg and the Kiev groups.

Recent activity on non-identical particle correlations:
Lednicky, Miskowiec et al.
pion and proton source effectively separate at forward rapidities.

A natural result in $Buda-Lund$ model and its various incarnations.

In the Buda - Lund parameterization, we have a K dependent normalvector, pointing towards the center of the particle production.

In the Longitudinal Comoving System (LCMS, T. Cs. & S. Pratt, 1990), we find

$$n = n(\overline{x}(\mathbf{K})) = (\cosh[\eta_s], 0, 0, \sinh[\eta_s]) \tag{32}$$

$$\eta_s^L = \eta_s^L(\mathbf{K}) = \frac{y_0 - y}{1 + \Delta\eta^2 m_t / T_0} \tag{33}$$

For non-identical particle correlations, e.g. π and p, the velocities of the particles must be similar.

If $v_\pi = v_p$, then we have $m_t^\pi << m_t^p$.

On the hydro model parameterization II

For pions and protons with the same velocity,
using $m_\pi = 140$ MeV, $m_p = 1$ GeV, $T = 140$ MeV and $\Delta\eta^2 = 2$ the Buda - Lund model yields:

$$\eta_s^p - \eta_s^\pi \simeq \frac{y - y_0}{3} \tag{34}$$

where y_0 is the mid-rapidity, y is the rapidity of the pion and the proton, and 3 is the numerically estimated coefficient.

The spatial separation between protons and pions is about $L \simeq 10$ fm/c (Miskowiec , Quark Matter'96) at $y - y_0 \simeq 2$. This can be used to estimate the mean freeze-out proper-time of particle production as$\tau_s = L/(\eta_s^p - \eta_s^\pi) \simeq 15$ fm/c.

Spectra and Correlations for Decaying Media (HBT for DCC)

Model Assumptions:

Model Hamiltonian

$$H = H_0 - \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x}-\mathbf{y}) \phi(\mathbf{y}). \tag{35}$$

where H_0 is the vacuum (or free) Hamiltonian,

$$H_0 = \int d^3\mathbf{x} \phi^2(\mathbf{x}) \left\{ \frac{1}{2} \partial_\mu \phi(\mathbf{x}) \partial^\mu \phi(\mathbf{x}) + \frac{1}{2} m_0^2 \phi^2(\mathbf{x}) \right\} \tag{36}$$

This Hamiltonian H corresponds to quasi-particles with a momentum-dependent medium-modified, effective mass.

$$m_*^2(|\mathbf{k}|) = m_0^2 - \delta M^2(|\mathbf{k}|),$$

the vacuum mass is m_0. The mass-shift is limited to long wavelength collective modes:

$$\delta M^2(|\mathbf{k}| > \Lambda_s) = 0.$$

The Density Matrix

The hydrodynamic density matrix has the form

$$\rho = \prod_i \rho_i, \tag{37}$$

where the index i runs over all fluid cells. The locally thermalized density matrix in cell i is

$$\rho_i = \frac{\exp\left(-(H_i - \mu_i N_i)/T_i\right)}{\mathrm{Tr}\exp\left(-(H_i - \mu_i N_i)/T_i\right)} \tag{38}$$

In each cell, the field can be expanded with creation and annihilation operators as

$$\phi_i(x) \;=\; \sum_{\mathbf{k}} (a_{i,\mathbf{k}} \varphi_{i,\mathbf{k}}(x) + h.c.) \tag{39}$$

(Sinyukov and Makhlin, 1986-88)

The covariant back-to-back momenta are $k^-(x)$:

$$k^-(x) = 2u(x)(k \cdot u(x)) - k \tag{40}$$

The unperturbed energy of modes k and k^- are the same: $\omega = (ku(x)) = (k^- u(x))$, and in the $u_i^\mu = (1,0)$ frame $\mathbf{k}^- = -\mathbf{k}$.

Diagonalization of the Density Matrix

The thermal density matrix can be exactly diagonalized with the local multi-mode Bogoliubov transformation:

$$a^\dagger_{i,k} = \cosh r_{i,k}\, b^\dagger_{i,k} + \sinh r_{i,k}\, b_{i,k^-}, \quad (41)$$

$$a^\dagger_{i,k^-} = \cosh r_{i,k^-}\, b^\dagger_{i,k^-} + \sinh r_{i,k^-}\, b_{i,k}, \quad (42)$$

$$a_{i,k} = \sinh r_{i,k}\, b^\dagger_{i,k^-} + \cosh r_{i,k}\, b_{i,k}, \quad (43)$$

$$a_{i,k^-} = \sinh r_{i,k^-}\, b^\dagger_{i,k} + \cosh r_{i,k^-}\, b_{i,k^-}, \quad (44)$$

where

$$r_{i,k} = 0.5 \log\left(\frac{k \cdot u_i - \mu_i}{k^* \cdot u_i - \mu_i}\right), \quad (45)$$

where $k^* = (\Omega_i(\mathbf{k}), \mathbf{k})$ and $\Omega_i(\mathbf{k}) \ = \sqrt{m_0^2 - \delta M_i^2(|\mathbf{k}|) + \mathbf{k}^2}$, corresponding to the frequency of mode $\mathbf{k}$ propagating in the ith fluid cell.

Exact relationship between quanta in vacuum (a-quanta) and the quanta in medium (b-quanta). Since $k_{i,-} \cdot u_i = k \cdot u_i$, it follows from eq. (45) that

$$r_{i,k} = r_{i,k^-}. \quad (46)$$

Covariant generalization of of $r_{\mathbf{k}} = r_{-\mathbf{k}}$ appropriate for isotropic media.

Diagonalization II.

With the b-operators, the in-medium density matrix is diagonalized as

$$\rho_i = \frac{1}{Z_i} \exp\left[- \left(\sum_{\mathbf{k}} (\Omega_{i,|\mathbf{k}|_i} - \mu_i) b^\dagger_{i,\mathbf{k}} b_{i,\mathbf{k}} \right) \right]. \tag{47}$$

The important point here is that it is the b-quanta that are thermalized in medium. b-operators diagonalize the in-medium density matrix.

On the other hand, the b-quanta are not observed experimentally.

It is the a-quanta that are observed .

Therefore, in calculating final state observables, we have to evaluate the expectation value of $\mathcal{O}(a, a^\dagger)$ with the density matrix defined in the b-basis:

$$\langle \mathcal{O}(a, a^\dagger) \rangle = \mathrm{Tr}\, \rho_b \mathcal{O}(a, a^\dagger). \tag{48}$$

Recovering the standard results:

One can evaluate the expectation value of any scalar function over the fluid elements. One obtains that

$$\langle f(k_i, k_j, x_l)\rangle_l \;=\; \frac{g}{(2\pi)^3}\int d^3\Sigma_\mu(x_l) K^\mu_{i,j} \exp\left[i(k_i - k_j)x_l\right] f(K_{ij}, K_{i,j}, x_l),$$

where g is the degeneracy factor, $K_{12} = 0.5(k_1 + k_2)$ is the mean four-momentum, $q_{12} = k_1 - k_2$ is the relative four-momentum.

The remaining calculation is straightforward.

If the mass-shift is vanishing, the well-known results are re-obtained in terms of the function $G(1,2)$,

$$G(1,2) = \langle b_1^\dagger b_2\rangle \tag{49}$$

the single-particle and the two-particle momentum distribution can be written as

$$N_1(1) \;=\; E_1 \frac{d^3N}{d\mathbf{k}_1} = G(1,1), \tag{50}$$

$$N_2(1,2) \;=\; E_1E_2 \frac{d^6N}{d\mathbf{k}_1 d\mathbf{k}_2} = G(1,1)G(2,2) + G(1,2)G(2,1). \tag{51}$$

Hence, in the absense of squeezing, the well known two-particle inclusive correlation function is given by

$$C(1,2) = \frac{N_2(1,2)}{N_1(1)N_1(2)} = 1 + \frac{|G(1,2)|^2}{G(1,1)G(2,2)} \tag{52}$$

Results for multi-mode squeezing

In the presence of local squeezing as given by (45), we obtainr:

$$N_1(1) = G_c(1,1) \tag{53}$$
$$N_2(1,2) = G_c(1,1)G_c(2,2) + |G_c(1,2)|^2 + |G_s(1,2)|^2 + \Delta G^2(1,2) \tag{54}$$

which yields a new expression for the two-particle correlation function:

$$C_2(1,2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{\Delta G^2(1,2)}{G_c(1,1)G_c(2,2)}$$

where we have introduced three auxiliary quantities, that generalize the amplitude $G(i,j)$ of the non-squeezed case (49) to the case where the hadrons are mass-shifted.

We obtain these as

$$G_c(1,2) = \langle |c_{1,2}|^2 n_{1,2} + |s_{-1,-2}|^2 n_{-1,-2} + |s_{-1,-2}|^2 \rangle, \tag{55}$$
$$G_s(1,2) = \langle c^*_{1,-2} s_{-2,1} n_{1,-2} + c^*_{2,-1} s_{-1,2} n_{-1,2} + c^*_{2,-1} s_{-1,2} \rangle \tag{56}$$
$$\Delta^2 G(1,2) = \langle |c_{2,-1}|^2 \rangle \langle |s_{-1,2}|^2 \rangle - |\langle c_{2,-1} s^*_{-1,2} 2 \rangle|^2 \tag{57}$$

The last term describes the fluctuations of the squeezing on the freeze-out hypersurface, and this term is always non-negative due to the Schwartz inequality.

Auxiliary quantities:

We have introduced for notational convenience the following auxiliary quantities:

$$c_{a,b} = \cosh(r(a,b,x)) \quad , \quad s_{a,b} = \sinh(r(a,b,x)) \quad , \tag{58}$$

where $a, b = \{\pm 1, \pm 2\}$ refer to the momentum dependence with local mass modifications.

The squeezing parameter is defined as

$$r(a,b,x) = 0.5 \log \left(\frac{K^*_{a,b} \cdot u(x) - \mu(x)}{K_{a,b} \cdot u(x) - \mu(x)} \right) \tag{59}$$

and as the back-to-back momentum pairs are defined locally, and $K_{a,b}u(x) = K_{b,a}u(x)$ and $K^*_{a,b}u(x) = K^*_{b,a}u(x)$ for any combination of $a, b = \{\pm 1, \pm 2\}$, we have the symmetry

$$r(a,b,x) = r(b,a,x) \tag{60}$$

$$c_{a,b} = c_{b,a} \tag{61}$$

$$s_{a,b} = s_{b,a} \tag{62}$$

For local equilibrium, $n_{a,b}(x)$ is given by the Bose-Einstein distribution

$$n_{a,b} = \frac{1}{\exp\left[\frac{K^*_{a,b}(x) \cdot u(x) - \mu(x)}{T(x)}\right] - 1} \tag{63}$$

where $a, b = \pm 1, \pm 2$ and $K^*_{i,j}(x)$ is medium modified.

Infinite Volume Homogenous Limit

We find that the single inclusive distribution for a two-mode squeezed state in the limit of infinite homogenous systems is given by

$$N_1(\mathbf{k}) = \langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle = |c_{\mathbf{k}}|^2 \, n_{\mathbf{k}} + |s_{-\mathbf{k}}|^2 n_{-\mathbf{k}} + |s_{-\mathbf{k}}|^2, \qquad (64)$$

where $c_{\mathbf{k}} = \cosh r_{\mathbf{k}}$, $s_{\mathbf{k}} = \sinh r_{\mathbf{k}}$.

If we assume uniformity, $n_{\mathbf{k}} = n_{-\mathbf{k}}$ and $r_{\mathbf{k}} = r_{-\mathbf{k}}$, the spectrum of eq. (64) will become the same as the spectrum obtained for one-mode squeezed states.

In this homogenous limit, we get $C_2(\mathbf{k}, \pm\mathbf{k})$ as follows:

$$C_2(\mathbf{k}, \mathbf{k}) = 2, \qquad (65)$$

$$C_2(\mathbf{k}, -\mathbf{k}) = 1 + \frac{|c^*_{\mathbf{k}} s_{\mathbf{k}} n_{\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}} n_{-\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}}|^2}{N_1(\mathbf{k})\, N_1(-\mathbf{k})}. \qquad (66)$$

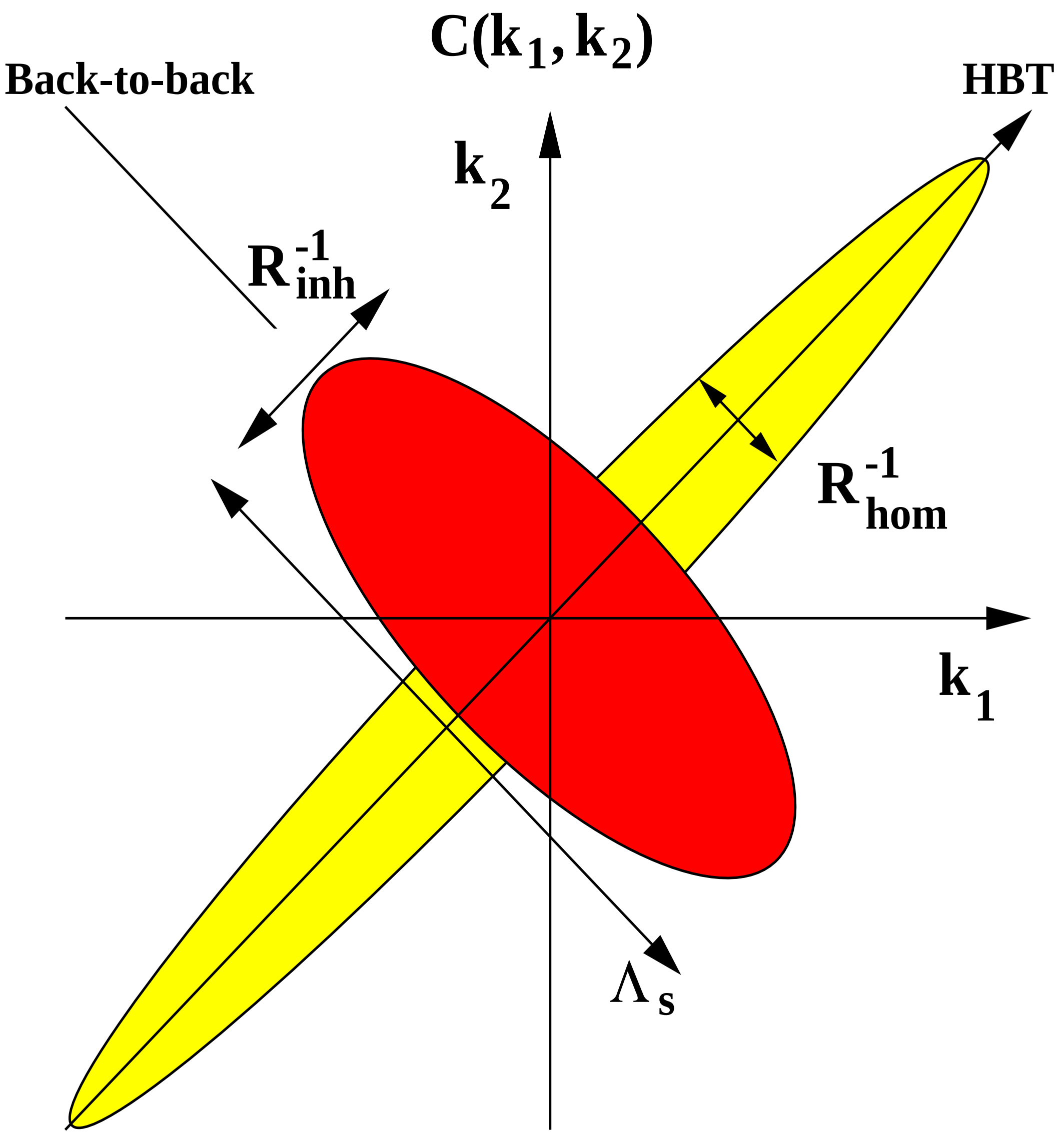
In all other cases, the two-particle correlation function is 1.

This implies two important issues,

(i) the intercept of the correlation function $C_2(\mathbf{k}, \mathbf{k})$ remains the canonical value of 2, even if quanta in medium are different from those in the vacuum,

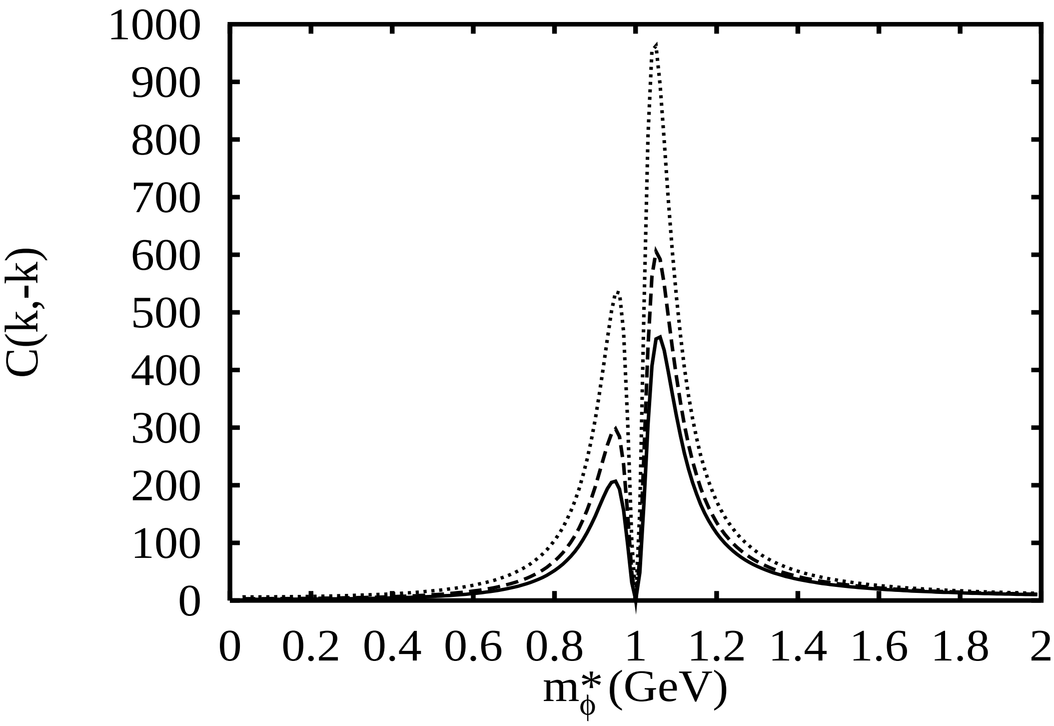
(ii) back-to-back correlation ($C_2(\mathbf{k}, -\mathbf{k}) \neq 1$) is generated by hadron modification.

Back-to-Back Correlations



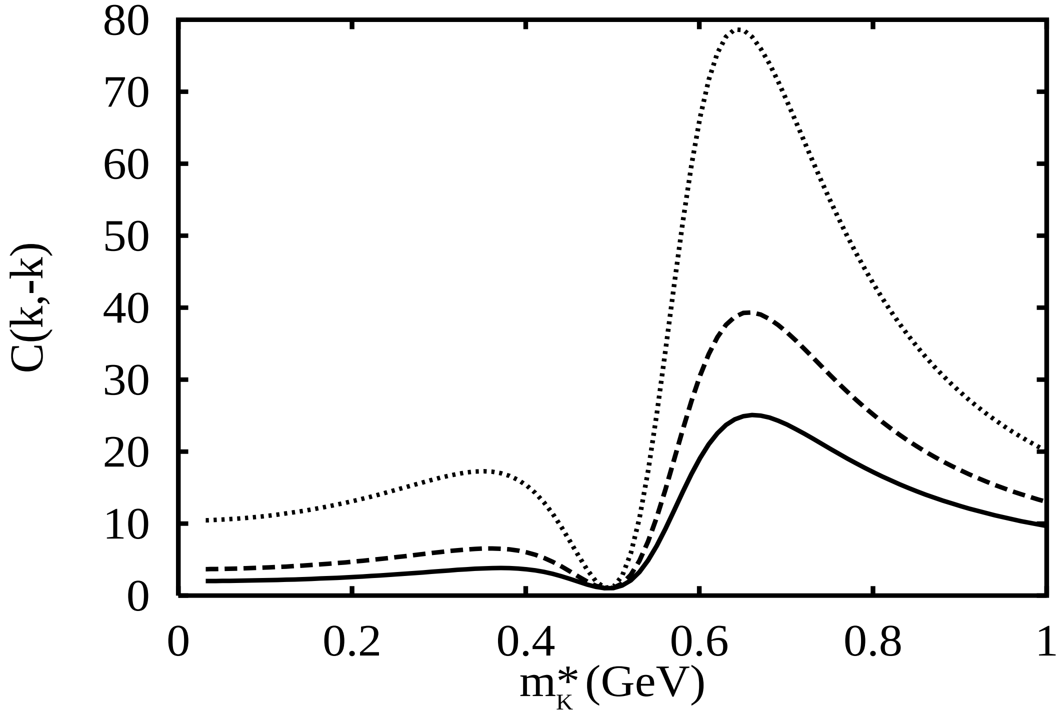
Back-to-back correlations appear due to mass-shifts in medium.

Results for ϕ mesons



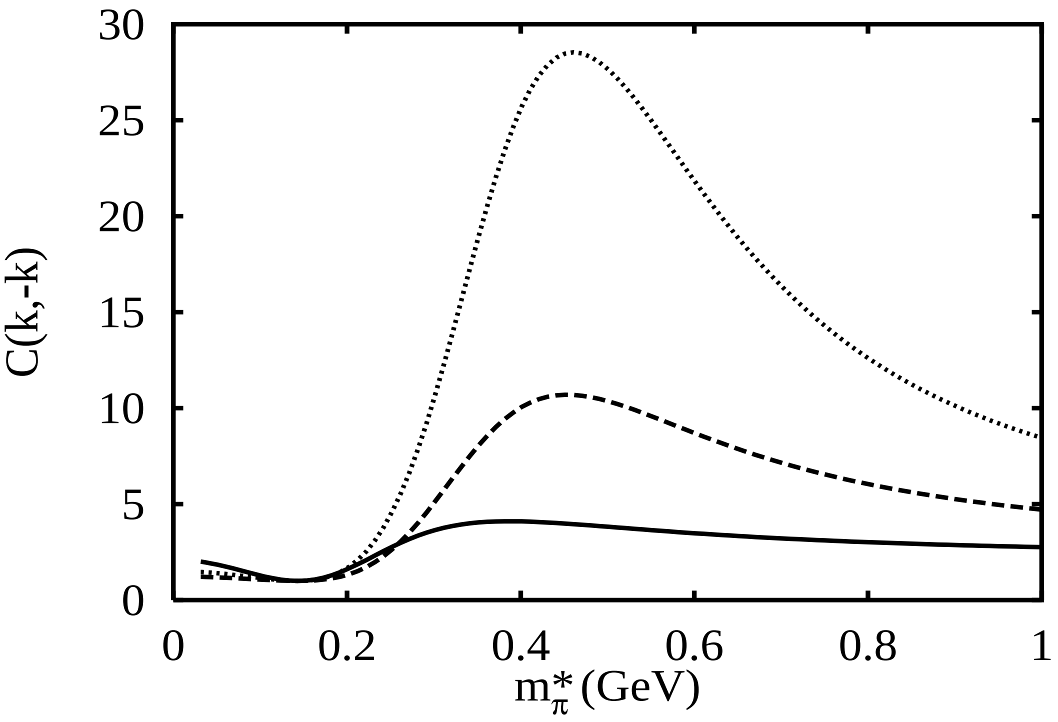
The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m^*_ϕ for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Results for K mesons



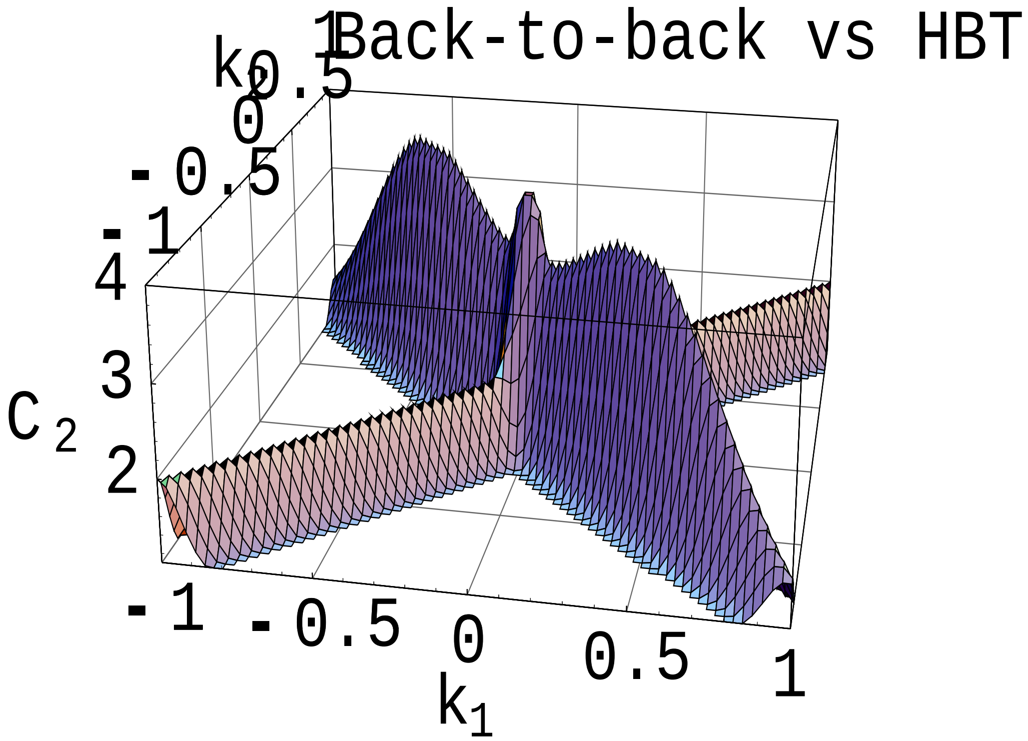
The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m_K^* for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Results for π mesons



The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m_π^* for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Finite Size Effects



Flying Eagle of back-to-back correlations sitting on the standard HBT correlation function.

Summary

Buda-Lund covariant parameterization: $Q_=$, $Q_\|$, $Q_{..}$, $Q_{:}$. (This decomposition of the relative momentum can be done for any parameterization, not only Gaussian).

For Gaussians: $R_=$, $R_\|$, $R_{..}$, $R_{:}$.

Buda-Lund hydro model $\rightarrow$ effective separation of the proton and pion source.

New signal of partial $U_A(1)$ restoration: A low-pt hole in the intercept parameter $\lambda_*(m_t)$.

The theory of particle correlations and spectra for bosons with in-medium mass-shifts is formulated for finite systems.

It predicts huge back-to-back correlations of ϕ and K mesons, for small values of the mass-shifts (PHENIX and STAR at RHIC).