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Correlations and Spectra for Mass-Shifted Bosons (HBT for DCC)

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Theory of Correlations and Spectra
for Medium-Modified Quanta (DCC)
Discussing new concepts :
Back-to-back correlations
Lengths of In homogeneity

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Spectra and Correlations for Decaying Media (HBT for DCC)

Model Assumptions:

Our model Hamiltonian is

$$H = H_0 - \frac{1}{2} \int d^3\mathbf{x} d^3\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x}-\mathbf{y}) \phi(\mathbf{y}). \tag{1}$$

where H_0 is the vacuum (or free) Hamiltonian,

$$H_0 = \int d^3\mathbf{x} \phi^2(\mathbf{x}) \left\{ \frac{1}{2} \partial_\mu \phi(\mathbf{x}) \partial^\mu \phi(\mathbf{x}) + \frac{1}{2} m_0^2 \phi^2(\mathbf{x}) \right\} \tag{2}$$

This Hamiltonian H corresponds to quasi-particles that propagate with a momentum-dependent medium-modified, effective mass, that is related to the vacuum mass, m_0, via

$$m_*^2(|\mathbf{k}|) = m_0^2 - \delta M^2(|\mathbf{k}|),$$

The mass-shift is limited to long wavelength collective modes of the system:

$$\delta M^2(|\mathbf{k}| > \Lambda_s) = 0.$$

The Density Matrix

The hydrodynamic density matrix has the form

$$\rho = \prod_i \rho_i, \tag{3}$$

where the index i runs over all fluid cells. The locally thermalized density matrix in cell i is

$$\rho_i = \frac{\exp\left(-(H_i - \mu_i N_i)/T_i\right)}{\mathrm{Tr}\exp\left(-(H_i - \mu_i N_i)/T_i\right)} \tag{4}$$

In each cell, the field can be expanded with creation and annihilation operators as

$$\phi_i(x) \;=\; \sum_{\mathbf{k}} (a_{i,\mathbf{k}} \varphi_{i,\mathbf{k}}(x) + h.c.) \tag{5}$$

(Sinyukov and Makhlin, 1986-88)

We define covariantly the back-to-back momenta, denoted by $k^-(x)$. This is given by

$$k^-(x) = 2u(x)(k \cdot u(x)) - k \tag{6}$$

Note that the unperturbed energy of modes k and k^- are the same: $\omega = (ku(x)) = (k^- u(x))$, and in the $u_i^\mu = (1,0)$ frame $\mathbf{k}^- = -\mathbf{k}$.

Diagonalization of the Density Matrix

The thermal density matrix can be exactly diagonalized with the local multi-mode Bogoliubov transformation:

$$a^\dagger_{i,k} = \cosh r_{i,k}\, b^\dagger_{i,k} + \sinh r_{i,k}\, b_{i,k^-}, \tag{7}$$

$$a^\dagger_{i,k^-} = \cosh r_{i,k^-}\, b^\dagger_{i,k^-} + \sinh r_{i,k^-}\, b_{i,k}, \tag{8}$$

$$a_{i,k} = \sinh r_{i,k}\, b^\dagger_{i,k^-} + \cosh r_{i,k}\, b_{i,k}, \tag{9}$$

$$a_{i,k^-} = \sinh r_{i,k^-}\, b^\dagger_{i,k} + \cosh r_{i,k^-}\, b_{i,k^-}, \tag{10}$$

where

$$r_{i,k} = 0.5 \log\left(\frac{k \cdot u_i - \mu_i}{k^* \cdot u_i - \mu_i}\right), \tag{11}$$

where $k^* = (\Omega_i(\mathbf{k}), \mathbf{k})$ and $\Omega_i(\mathbf{k}) = \sqrt{m_0^2 - \delta M_i^2(|\mathbf{k}|) + \mathbf{k}^2}$, corresponding to the frequency of mode $\mathbf{k}$ propagating in the ith fluid cell.

This gives exact relationship between quanta in vacuum (a-quanta) and the quanta in medium (b-quanta). Since $k_{i,-} \cdot u_i = k \cdot u_i$, it follows from eq. (11) that

$$r_{i,k} = r_{i,k^-}. \tag{12}$$

This is the covariant generalization of of $r_{\mathbf{k}} = r_{-\mathbf{k}}$ appropriate for isotropic media.

Diagonalization II.

With the b-operators, the in-medium density matrix is diagonalized as

$$\rho_i = \frac{1}{Z_i} \exp\left[- \left(\sum_{\mathbf{k}} (\Omega_{i,|\mathbf{k}|_i} - \mu_i) b^\dagger_{i,\mathbf{k}} b_{i,\mathbf{k}} \right) \right]. \tag{13}$$

The important point here is that it is the b-quanta that are thermalized in medium. b-operators diagonalize the in-medium density matrix.

On the other hand, the b-quanta are not observed experimentally.

It is the a-quanta that are observed.

Therefore, in calculating final state observables, we have to evaluate the expectation value of $\mathcal{O}(a, a^\dagger)$ with the density matrix defined in the b-basis:

$$\langle \mathcal{O}(a, a^\dagger) \rangle = \mathrm{Tr}\, \rho_b \mathcal{O}(a, a^\dagger). \tag{14}$$

Recovering the standard results:

One can evaluate the expectation value of any scalar function over the fluid elements. One obtains that

$$\langle f(k_i, k_j, x_l)\rangle_l \;=\; \frac{g}{(2\pi)^3} \int d^3\Sigma_\mu(x_l) K^\mu_{i,j} \exp\left[i(k_i - k_j)x_l\right] f(K_{ij}, K_{i,j}, x_l),$$

where g is the degeneracy factor, $K_{12} = 0.5(k_1 + k_2)$ is the mean four-momentum, $q_{12} = k_1 - k_2$ is the relative four-momentum.

The remaining calculation is straightforward.

If the mass-shift is vanishing, the well-known results are re-obtained in terms of the function $G(1,2)$,

$$G(1,2) = \langle b_1^\dagger b_2 \rangle \tag{15}$$

the single-particle and the two-particle momentum distribution can be written as

$$N_1(1) \;=\; E_1 \frac{d^3 N}{d\mathbf{k}_1} = G(1,1), \tag{16}$$

$$N_2(1,2) \;=\; E_1 E_2 \frac{d^6 N}{d\mathbf{k}_1 d\mathbf{k}_2} = G(1,1)G(2,2) + G(1,2)G(2,1). \tag{17}$$

Hence, in the absense of squeezing, the well known two-particle inclusive correlation function is given by

$$C(1,2) = \frac{N_2(1,2)}{N_1(1)N_1(2)} = 1 + \frac{|G(1,2)|^2}{G(1,1)G(2,2)} \tag{18}$$

Results for multi-mode squeezing

In the presence of local squeezing as given by (11), we obtainr:

$$N_1(1) = G_c(1,1) \tag{19}$$

$$N_2(1,2) = G_c(1,1)G_c(2,2) + |G_c(1,2)|^2 + |G_s(1,2)|^2 + \Delta G^2(1,2) \tag{20}$$

which yields a new expression for the two-particle correlation function:

$$C_2(1,2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{\Delta G^2(1,2)}{G_c(1,1)G_c(2,2)}$$

where we have introduced three auxiliary quantities, that generalize the amplitude $G(i,j)$ of the non-squeezed case (15) to the case where the hadrons are mass-shifted.

We obtain these as

$$G_c(1,2) = \langle |c_{1,2}|^2 n_{1,2} + |s_{-1,-2}|^2 n_{-1,-2} + |s_{-1,-2}|^2 \rangle, \tag{21}$$

$$G_s(1,2) = \langle c^*_{1,-2} s_{-2,1} n_{1,-2} + c^*_{2,-1} s_{-1,2} n_{-1,2} + c^*_{2,-1} s_{-1,2} \rangle \tag{22}$$

$$\Delta^2 G(1,2) = \langle |c_{2,-1}|^2 \rangle \langle |s_{-1,2}|^2 \rangle - |\langle c_{2,-1} s^*_{-1,2} 2 \rangle|^2 \tag{23}$$

The last term describes the fluctuations of the squeezing on the freeze-out hypersurface, and this term is always non-negative due to the Schwartz inequality.

Auxiliary quantities:

We have introduced for notational convenience the following auxiliary quantities:

$$c_{a,b} = \cosh(r(a,b,x)) \quad , \quad s_{a,b} = \sinh(r(a,b,x)) \quad , \tag{24}$$

where $a, b = \{\pm 1, \pm 2\}$ refer to the momentum dependence with local mass modifications.

The squeezing parameter is defined as

$$r(a,b,x) = 0.5 \log\left(\frac{K^*_{a,b} \cdot u(x) - \mu(x)}{K_{a,b} \cdot u(x) - \mu(x)}\right) \tag{25}$$

and as the back-to-back momentum pairs are defined locally, and $K_{a,b}u(x) = K_{b,a}u(x)$ and $K^*_{a,b}u(x) = K^*_{b,a}u(x)$ for any combination of $a, b = \{\pm 1, \pm 2\}$, we have the symmetry

$$r(a,b,x) = r(b,a,x) \tag{26}$$

$$c_{a,b} = c_{b,a} \tag{27}$$

$$s_{a,b} = s_{b,a} \tag{28}$$

For local equilibrium, $n_{a,b}(x)$ is given by the Bose-Einstein distribution

$$n_{a,b} = \frac{1}{\exp\left[\frac{K^*_{a,b}(x) \cdot u(x) - \mu(x)}{T(x)}\right] - 1} \tag{29}$$

where $a, b = \pm 1, \pm 2$ and $K^*_{i,j}(x)$ is medium modified.

Infinite Volume Homogenous Limit

We find that the single inclusive distribution for a two-mode squeezed state in the limit of infinite homogenous systems is given by

$$N_1(\mathbf{k}) \;=\; \langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle = |c_{\mathbf{k}}|^2\, n_{\mathbf{k}} + |s_{-\mathbf{k}}|^2 n_{-\mathbf{k}} + |s_{-\mathbf{k}}|^2, \qquad (30)$$

where $c_{\mathbf{k}} = \cosh r_{\mathbf{k}}$, $s_{\mathbf{k}} = \sinh r_{\mathbf{k}}$.

If we assume uniformity, $n_{\mathbf{k}} = n_{-\mathbf{k}}$ and $r_{\mathbf{k}} = r_{-\mathbf{k}}$, the spectrum of eq. (30) will become the same as the spectrum obtained for one-mode squeezed states.

In this homogenous limit, we get $C_2(\mathbf{k}, \pm\mathbf{k})$ as follows:

$$C_2(\mathbf{k}, \mathbf{k}) \;=\; 2, \qquad (31)$$

$$C_2(\mathbf{k}, -\mathbf{k}) \;=\; 1 + \frac{|c^*_{\mathbf{k}} s_{\mathbf{k}} n_{\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}} n_{-\mathbf{k}} + c^*_{-\mathbf{k}} s_{-\mathbf{k}}|^2}{N_1(\mathbf{k})\, N_1(-\mathbf{k})}. \qquad (32)$$

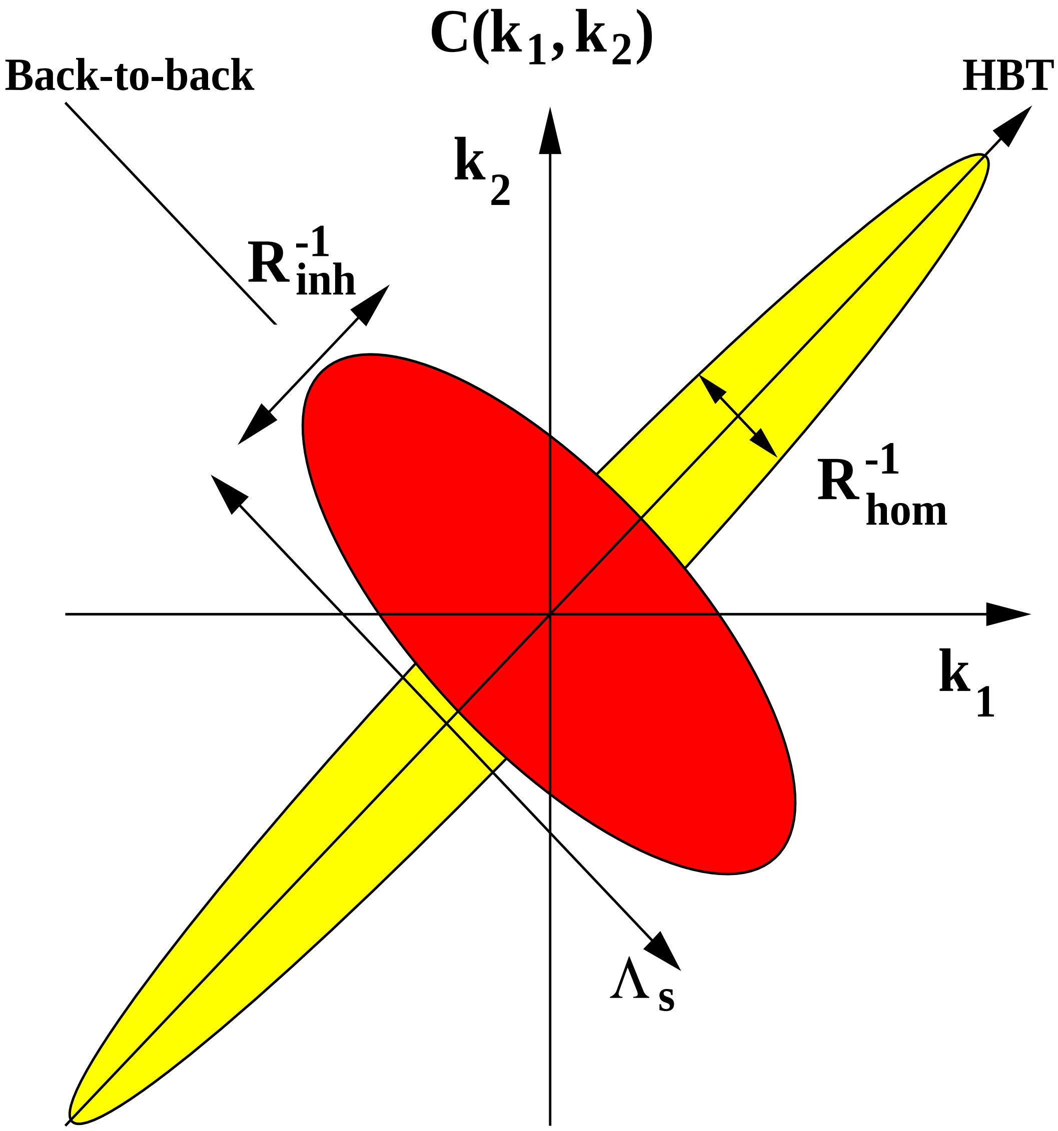
In all other cases, the two-particle correlation function is 1.

This implies two important issues,

(i) the intercept of the correlation function $C_2(\mathbf{k}, \mathbf{k})$ remains the canonical value of 2, even if quanta in medium are different from those in the vacuum,

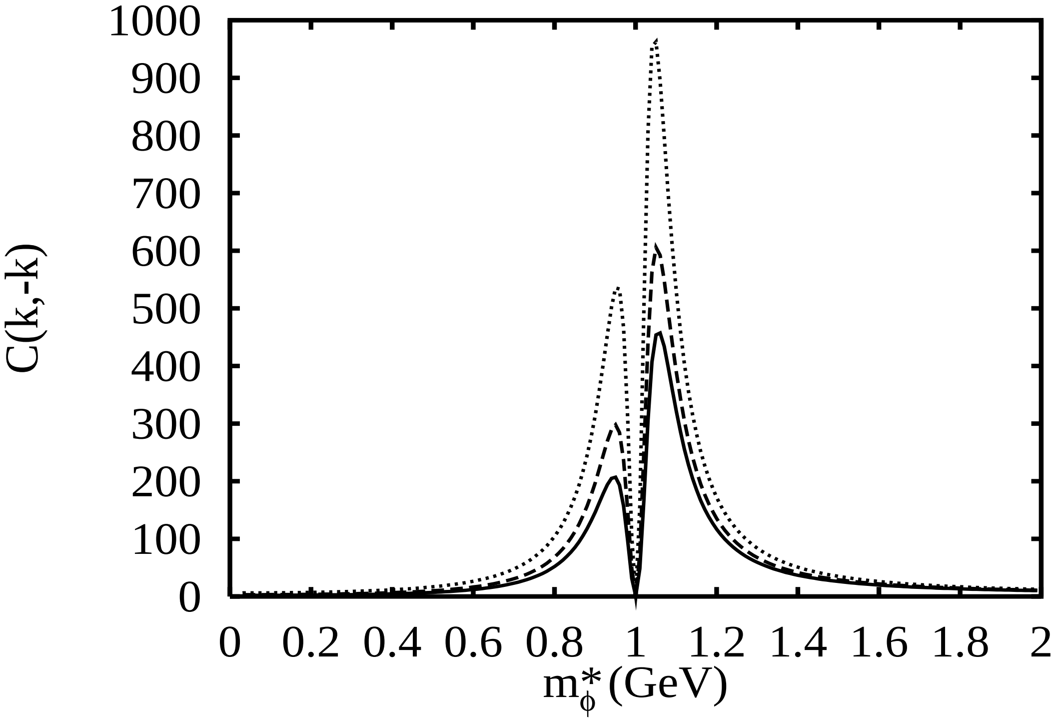
(ii) back-to-back correlation $(C_2(\mathbf{k}, -\mathbf{k}) \neq 1)$ is generated by hadron modification.

Back-to-Back Correlations



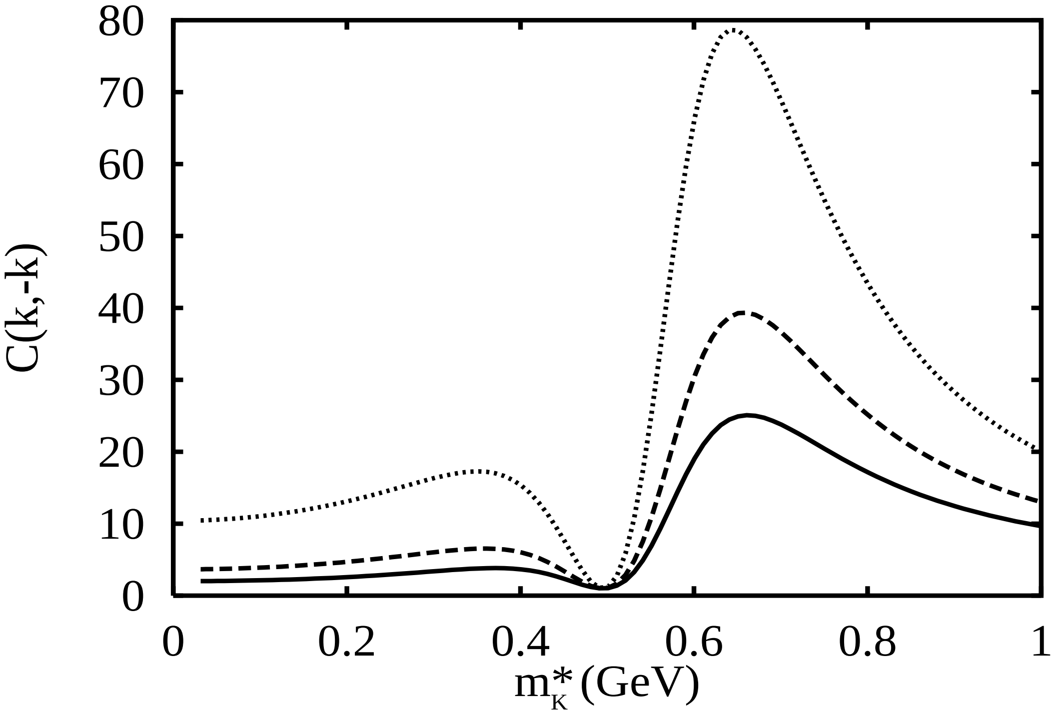
Back-to-back correlations appear due to mass-shifts in medium.

Results for ϕ mesons



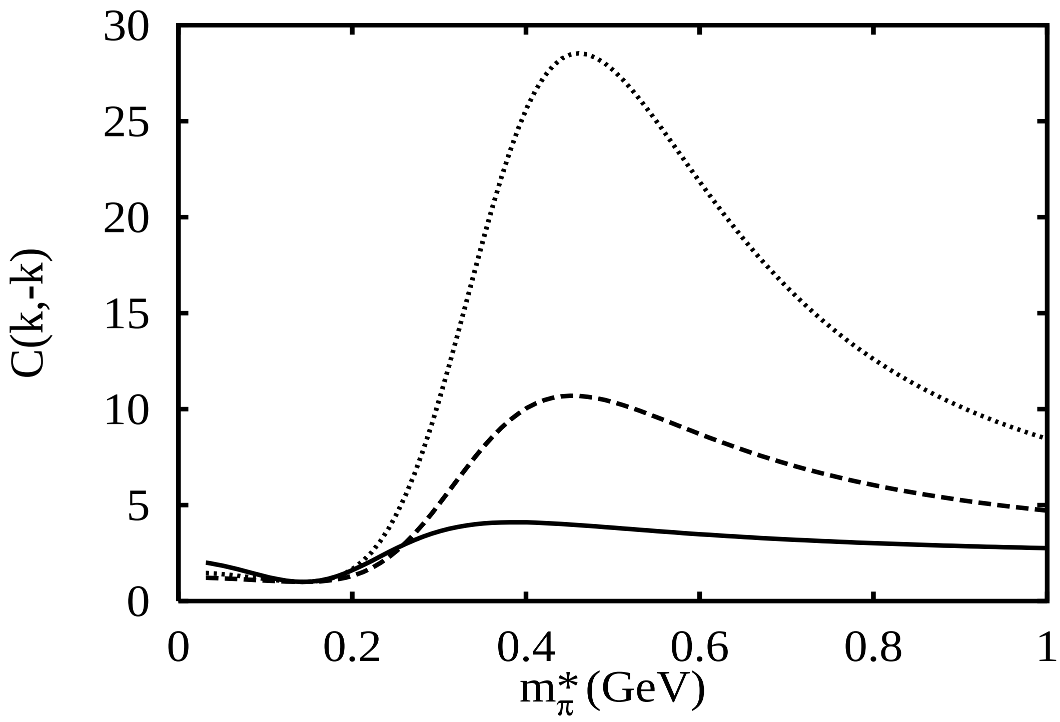
The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m^*_ϕ for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Results for K mesons



The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m^*_K for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Results for π mesons



The strength of the back-to-back correlations is shown at $\mathbf{K} = 0$ as a function of m_π^* for $T = 140$ MeV. Solid, dashed and dotted lines stand for $\mathbf{k_1} = 0$, 300 and 500 MeV, respectively.

Summary

The theory of particle correlations and spectra for bosons with in-medium mass-shifts is formulated for finite systems.

It predicts huge back-to-back correlations of ϕ and K mesons, for small values of the mass-shifts (PHENIX and STAR at RHIC).

The scales of the back-to-back correlations are given by the lengths of inhomogeneity . E.g. $\tau_0|y_1 - y_2|$.

A new tool to determine the total scales of expanding systems.

More work is needed to work out various expanding model scenarios.