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HYDRO MODEL PARAMETERS FROM HBT AND SPECTRUM OF NA22 π^- MESONS IN $(\pi^+/\mathrm{K}^+) + p$ AT 250 GeV/c

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Introduction

Hydro parameterization,

Applications to h+P (NA22), low and high energy heavy ion

Motivation: Cylindical Symmetry of boson source

+ the assumption of local thermalization

= prediction of spectra and correlations

Open problem:

Transplant these ideas to NA49

[1] T. Csörgő and B. Lörstad, Phys. Rev. C54 (1996) 1390

[2] T. Csörgő and B. Lörstad, in Proceedings of QM'95 Nucl. Phys. **A590** (1995)

[3] N. Agababyan, ... , T. Csörgő , ..., EHS/NA22 : hep-ex/9711009, Phys. Lett. B (1998) in print

ESTIMATION OF HYDRO MODEL PARAMETERS FROM SPECTRUM AND CORRELATIONS OF π^- MESONS IN $(\pi^+/\mathrm{K}^+)+p$ AT 250 GeV/c

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Introduction

Framework: hydrodynamical model for three-dimensionally expanding, cylindrically symmetric systems [1].

Fits to the invariant spectra of h^- mesons in the central rapidity region of $(\pi^+/\mathrm{K}^+)\mathrm{p}$ interactions at 250 GeV/c
and its combination with earlier results on two-particle correlations.

The first NA22 data on the pion inclusive spectra (with about half the statistics used in the present paper)

show their similarity in π^+ and K^+ induced reactions.

The present analysis, therefore,

is based on the combined $\pi^+\mathrm{p}$ and $\mathrm{K}^+\mathrm{p}$ data.

The parametrizations of invariant pion spectra

In refs. [1-2], the invariant spectrum of pions in rapidity y and transverse mass m_{t} is approximated by

$$
\begin{aligned}
f(y, m_{\mathrm{t}}) &= \frac{1}{N_{\mathrm{ev}}} \frac{\mathrm{d}N_\pi}{\mathrm{d}y \mathrm{d}m_{\mathrm{t}}^2} = \\
&= C {m_{\mathrm{t}}}^{\alpha} \cosh \eta_s \exp\left(\frac{\Delta\eta_*^2}{2}\right) \exp\left[-\frac{(y-y_0)^2}{2\Delta y^2}\right] \exp\left(-\frac{m_{\mathrm{t}}}{T_0}\right) \times \\
&\times \exp\left\{\frac{\langle u_{\mathrm{t}}\rangle^2 (m_{\mathrm{t}}^2 - m_\pi^2)}{2T_0[T_0 + (\langle u_{\mathrm{t}}\rangle^2 + \langle \frac{\Delta T}{T}\rangle) m_{\mathrm{t}}]}\right\}. \qquad (1)
\end{aligned}
$$

with

$$
\Delta y^2 = \Delta\eta^2 + \frac{T_0}{m_{\mathrm{t}}} \qquad (2)
$$

$$
\frac{1}{\Delta\eta_*^2} = \frac{1}{\Delta\eta^2} + \frac{m_{\mathrm{t}}}{T_0} \cosh \eta_{\mathrm{s}}, \qquad (3)
$$

$$
\eta_{\mathrm{s}} = \frac{y - y_0}{1 + \Delta\eta^2 \frac{m_{\mathrm{t}}}{T_0}}. \qquad (4)
$$

The width Δy of the rapidity distribution is determined by the width $\Delta\eta$ of the longitudinal space-time rapidity η distribution of the pion emitters and by the thermal smearing width $\sqrt{T_0/m_{\mathrm{t}}}$.

The parametrizations of invariant pion spectra II

T_0: the freeze-out temperature at the mean freeze-out time τ_f at the z axis, $T_0 = T_\mathrm{f}(r_\mathrm{T} = 0)$.

The inhomogeneity related to the transverse expansion appears in the form of $\langle u_t \rangle$ and the transverse temperature inhomogeneity:

$$\left\langle \frac{\Delta T}{T} \right\rangle = \frac{T_0}{T_\mathrm{rms}} - 1, \tag{5}$$

where $T_\mathrm{rms} = T_\mathrm{f}(r_\mathrm{T} = r_\mathrm{T}(\mathrm{rms}))$ is the freeze-out temperature at the transverse rms radius $r_\mathrm{T}(\mathrm{rms})$ and at time τ_f.

The parameter α in (4) is related to the number d of dimensions in which the expanding system is inhomogeneous.

For one-dimensional inhomogeneity ($d = 1$, longitudinal expansion), $\alpha = 1 - 0.5d = 0.5$ (provided $\Delta\eta^2 \gg T_0/m_\mathrm{t}$).

The transverse inhomogeneity of the system leads to smaller values of α, typically to $\alpha = 0., ..., -0.5$.

The minimum value of $\alpha = -1$ is achieved at $d = 4$ for the special case of a three-dimensionally expanding system with temporal change of local temperature during the particle emission process.

Parameterization of Spectra 3.

The parameter y_0 in (4) is the midrapidity, and be can slightly differ from 0 due to different species of colliding particles.

C is an overall normalization coefficient.

The two-dimensional distribution (1) can be simplified for one-dimensional slices as:

1. At fixed $m_{\rm t}$, one has

$$f(y, m_{\rm t}) = C_m \exp\left[- \frac{(y - y_0)^2}{2\Delta y^2(m_t)} \right], \qquad (6)$$

where C_m is an $m_{\rm t}$-dependent constant. The width parameter Δy^2 extracted for different $m_{\rm t}$-slices is predicted to depend linearly on $1/m_{\rm t}$, with slope T_0 and intercept $\Delta\eta^2$ (cf. (2)):

$$\Delta^2 y = \Delta^2 \eta + \frac{T_0}{m_t} \qquad (7)$$

Note, that for static fireballs or spherically expanding shells (7) and (2) are satisfied with $\Delta\eta = 0$.

Parameterization of Spectra 4.

2. At fixed y, the m_t^2-distribution is approx.:

$$f(y, m_\mathrm{t}) = C_y m_\mathrm{t}^\alpha \exp\left(-\frac{m_\mathrm{t}}{T_\mathrm{eff}}\right) \tag{8}$$

where C_y is a y-dependent constant.

The y-dependent "effective temperature" $T_\mathrm{eff}(y)$ can be found as

$$T_\mathrm{eff}(y) = \frac{T_*}{1 + a(y - y_0)^2}, \tag{9}$$

where $T_* \neq T_0$ is the maximum of $T_\mathrm{eff}(y)$, achieved at $y = y_0$, and

$$a = \frac{T_0 T_*}{2m_\pi^2(\Delta\eta^2 + \frac{T_0}{m_\pi})^2} \tag{10}$$

with T_0 and $\Delta\eta^2$ as defined above.

These approximations predict a specific narrowing of the rapidity and transverse mass spectra with increasing m_t and y.

The data

The results of this paper are based on the analysis of 104145 events of $(\pi^+/\mathrm{K}^+)\mathrm{p}$ interactions containing at least one negative track with momentum resolution better than 4% (depending on the momentum, the average momentum resolution varies from 1% to to 2.5%). All negative particles are assumed to have the pion mass. The contamination from other particles is estimated to be $(7 \pm 3)\%$ [10]. Candidates for single diffraction dissociation with charge multiplicity $n \leq 6$ are excluded. For each event, a weight is introduced in order to normalize to the non-single diffractive topological cross sections [16].

The analysis is restricted to centrally produced π^--mesons with cms rapidity $|y| < 1.5$. The total number of pions used in the analysis is about 397 k.

The results

The rapidity distribution

The rapidity distributions fitted by (10) are given in Fig. 1 for 23 m_{t} slices and m_{t} values ranging from 0.16 to 0.63 GeV.

The fit quality is satisfactory for all slices ($0.7 < \chi^2/NDF < 1.5$).

The fitted Δy^2 values plotted in Fig. 2 demonstrate the widening of the y-distribution with increasing $1/m_{\mathrm{t}}$.

According to (6), $\Delta\eta^2 = 1.91 \pm 0.12$ and slope $T_0 = 159 \pm 38$MeV.

The m_{t} distribution

The m_{t}-distributions fitted by (11), are given in Fig. 3 for 11 y-slices for y-values between -1.5 to +1.5. For all slices $\chi^2/NDF < 1.4$.

$T_{\mathrm{eff}}(y)$ (Fig. 4) decreases with increasing $|y|$ approximately as predicted, with $T_* = 160 \pm 1$ MeV, $a = 0.083 \pm 0.007$ and $y_0 = -0.065 \pm 0.039$.

Note, that fit of $T_{\mathrm{eff}}(y)$ by a Lorentzian has low confidence level: $\chi^2/NDF = 29/11$.

An asymmetry is found in the T_{eff} distribution with respect to $y = 0$. Except for the last point, T_{eff} is higher in the meson than in the proton hemisphere.

Figure 1

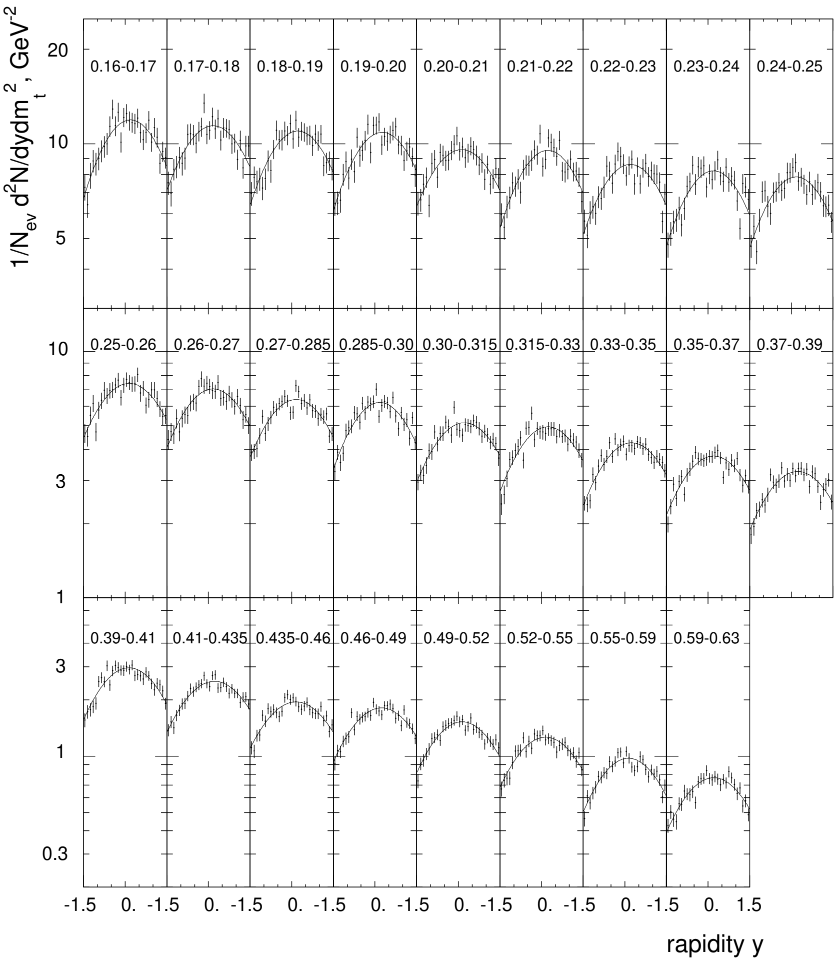


Fig. 1

The rapidity distributions of centrally produced pions ($|y| < 1.5$) for different m_t-slices given. The curves are the fit results obtained according to parameterization (10).

Figure 2

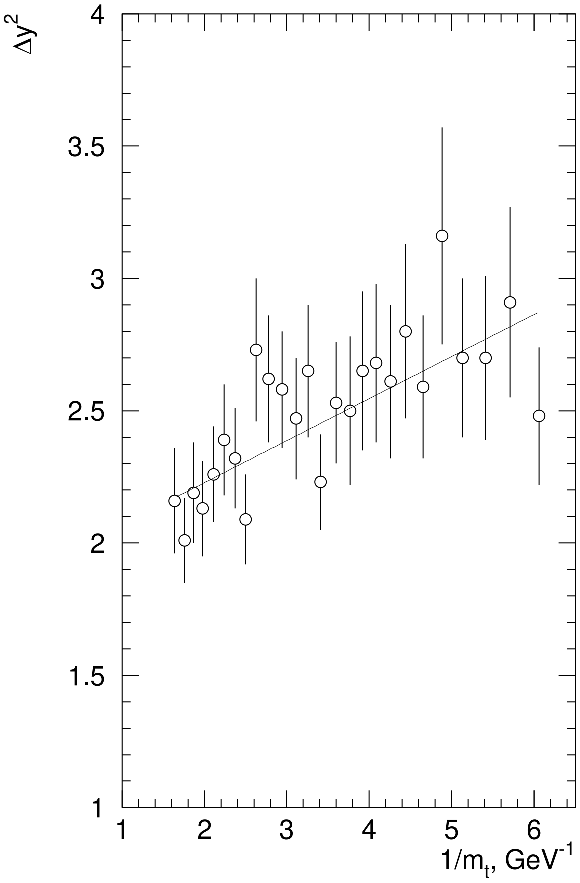


Fig. 2

The $(1/m_t)$-dependence of $(\Delta y)^2$ for inclusive π^- meson rapidity distributions at $|y| < 1.5$. The straight line is the fit result according to parametrization (2).

Figure 3

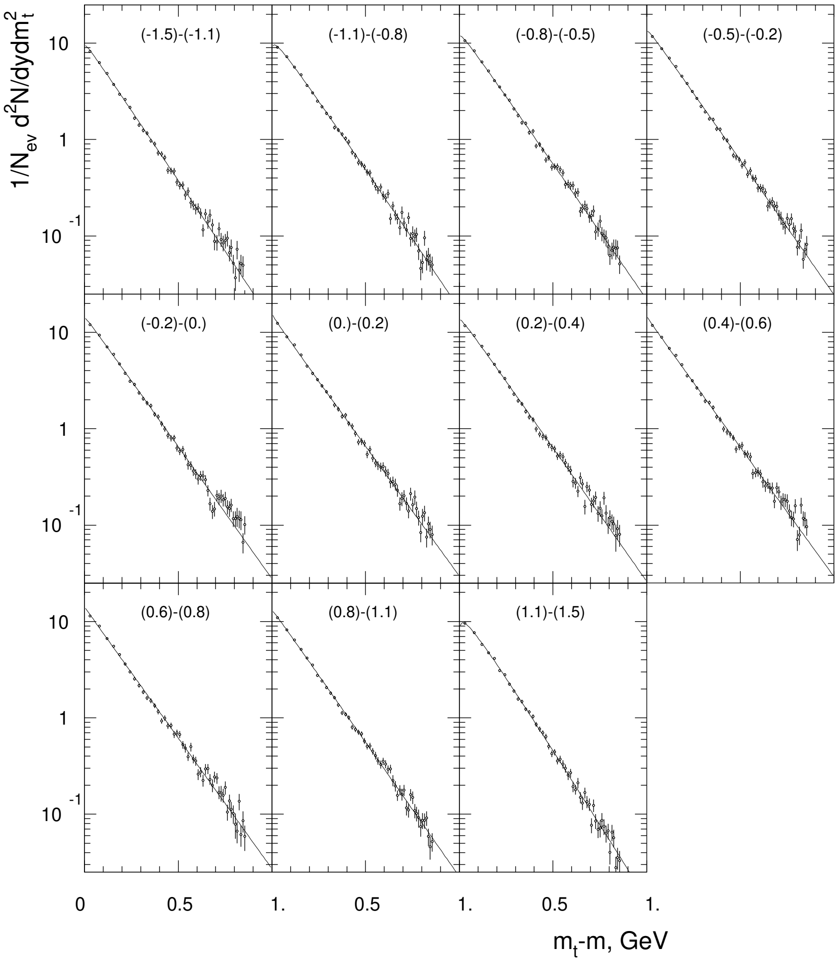


Fig. 3

The $m_{\rm t}$-distribution of centrally produced pions for different y-slices, as indicated. The solid lines are the fit results obtained according to parameterization (11).

Figure 4

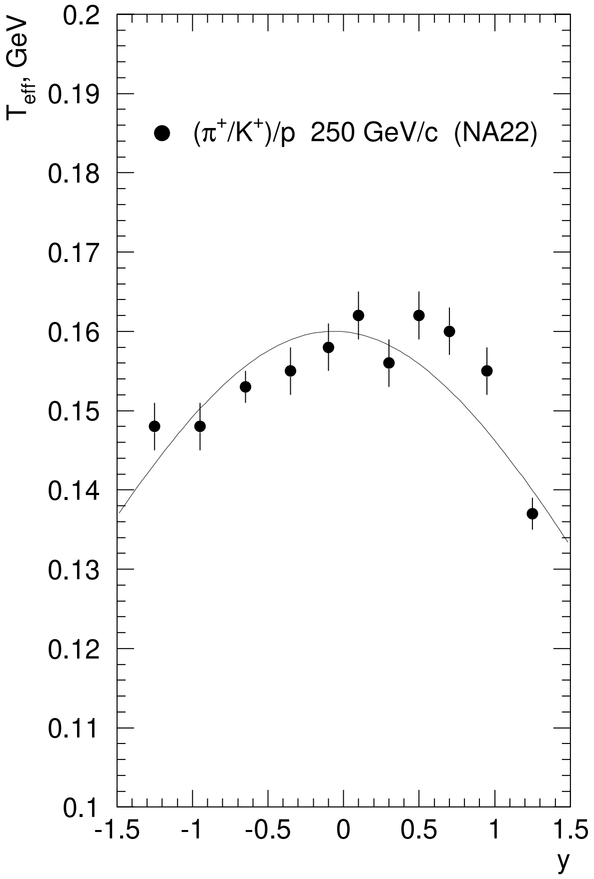
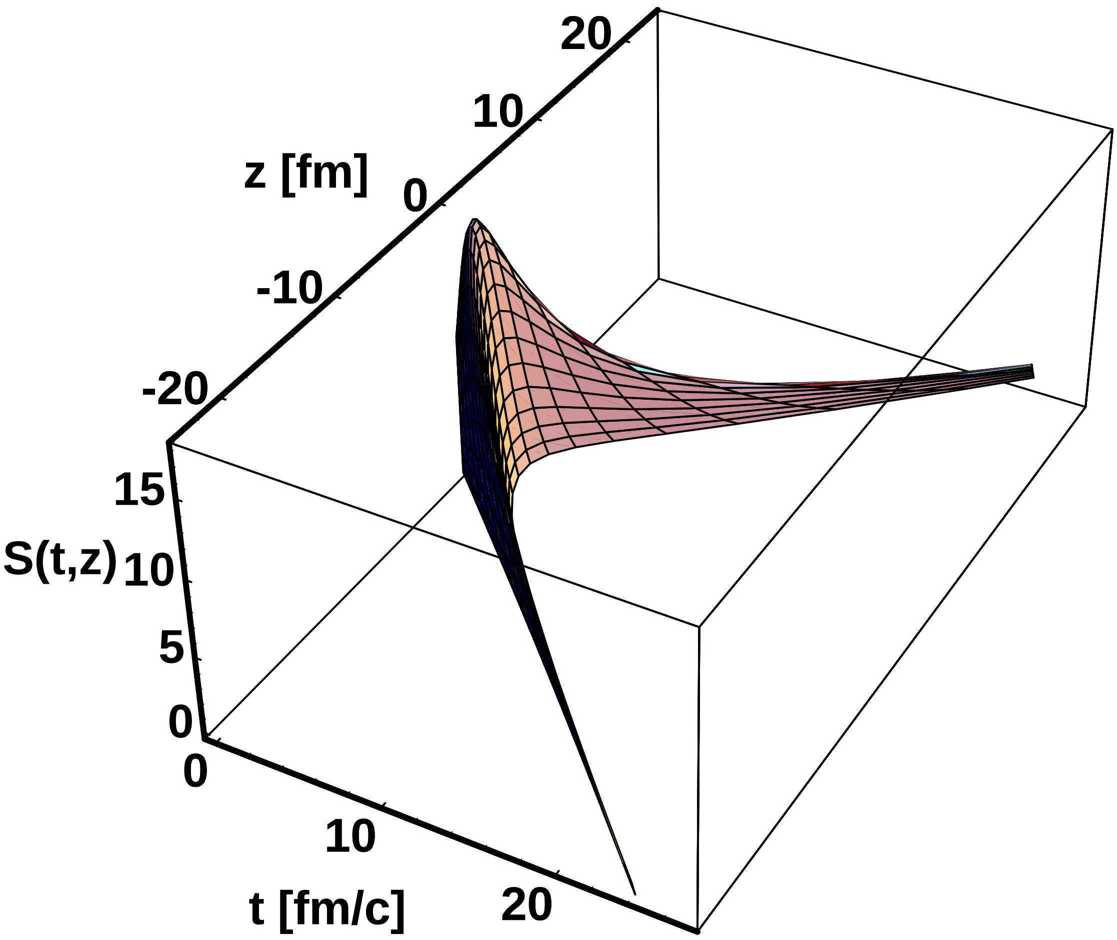


Fig. 4

$T_{\rm eff}$ as a function of y fitted according to parametrization (8).

Figure 5



The reconstructed $S(t,z)$ emission function in arbitrary units, as a function of time t and longitudinal coordinate z. The best fit parameters of $\Delta\eta = 1.36$, $y_0 = 0.082$, $\Delta\tau = 1.3$ fm$/c$ and $\tau_{\rm f} = 1.4$ fm$/c$ are used to obtain this plot.

The transverse direction

The results of the full, (m_t, y) dependent fit are presented in Table 1.

The fitted value of of $\alpha = 0.26 \pm 0.02$ corresponds to an effective number $d_{\rm eff} \approx 1.5$ of dimensions in which the expanding system is inhomogeneous.

This can be at least partly attributed to a transverse inhomogeneity.

The mean transverse four-velocity $\langle u_{\rm t} \rangle = 0.20 \pm 0.07$ is rather small.

A rather large temperature inhomogeneity, $\langle \frac{\Delta T}{T} \rangle = 0.71 \pm 0.14$ is found.

The freeze-out temperature decreases from $T_0 = 140 \pm 3$ MeV at the central axis of the hydrodynamical tube to $T_{\rm rms} = 82 \pm 7$ MeV at the r.m.s.

α	$\Delta\eta$	T_0 (GeV)	y_0	$\langle u_{\rm t} \rangle$	$\langle \frac{\Delta T}{T} \rangle$	χ^2/NDF
0.26 $\pm$ 0.02	1.36 $\pm$ 0.02	0.140 $\pm$ 0.003	0.082 $\pm$ 0.006	0.20 $\pm$ 0.07	0.71 $\pm$ 0.14	642/683

Fit results according to parametrization (4) for $|y| < 1.5$.

Combination with two-particle correlations

$$C_2(\mathbf{p}_1, \mathbf{p}_2) \sim 1 + \exp[-R_{\mathrm{L}}^2 q_{\mathrm{L}}^2 - R_{\mathrm{o}}^2 q_{\mathrm{out}}^2 - R_{\mathrm{s}}^2 q_{\mathrm{side}}^2], \tag{11}$$

In the LCMS, where $Y = 0$, the effective radii can be approximately expressed [1] approximately as

$$R_{\mathrm{L}}^2 = \tau_{\mathrm{f}}^2 \Delta\eta_*^2 \tag{12}$$

$$R_{\mathrm{o}}^2 = R_*^2 + \beta_{\mathrm{t}}^2 \Delta\tau_*^2 \tag{13}$$

$$R_{\mathrm{s}}^2 = R_*^2 \tag{14}$$

with

$$\frac{1}{\Delta\eta_*^2} = \frac{1}{\Delta\eta^2} + \frac{M_{\mathrm{t}}}{T_0} \tag{15}$$

$$R_*^2 = \frac{R_{\mathrm{G}}^2}{1 + \frac{M_{\mathrm{t}}}{T_0}(\langle u_{\mathrm{t}}\rangle^2 + \langle\frac{\Delta T}{T}\rangle)}, \tag{16}$$

where parameters $\Delta\eta^2, T_0, \langle u_{\mathrm{t}}\rangle$ and $\langle\frac{\Delta T}{T}\rangle$ are estimated from the invariant spectra;

R_{G} is related to the transverse geometrical rms radius of the source as $R_{\mathrm{G}}(\mathrm{rms}) = \sqrt{2} R_{\mathrm{G}}$; τ_{f} is the mean freeze-out (hadronization) time; $\Delta\tau_*$ is related to the duration time $\Delta\tau$ of pion emission and to the temporal inhomogeneity of the local temperature;

an approximate relation $\Delta\tau \geq \Delta\tau_*$ holds if $d \approx 1.5$; the variable β_{t} is the transverse velocity of the pion pair.

Bose-Einstein + Spectra II.

HBT + Spectra allows for estimating $(\tau_{\rm f}, \Delta\tau_*, R_{\rm G})$.

The interferometric radii $R_{\rm L}, R_{\rm o}, R_{\rm s}$ were extracted by NA22 in

N.M. Agababyan et al. (NA22 Coll.); Z. Phys. C71 (1996) 409 .

This dependence on $M_{\rm t}$ matches well the predicted hydro behaviour.

Using (11) and (14) with $T_0 = 140 \pm 3$ MeV and $\Delta\eta^2 = 1.85 \pm 0.04$ (Table 1), one finds $\tau_{\rm f} = 1.4 \pm 0.1$ fm/c.

The transverse-plane radii $R_{\rm o}$ and $R_{\rm s}$ are measured for the whole $M_{\rm t}$ range : $R_{\rm o} = 0.91 \pm 0.08$ fm and $R_{\rm s} = 0.54 \pm 0.07$ fm. One obtains (at $\beta_{\rm t} = 0.484c$) : $\Delta\tau_* = 1.3 \pm 0.3$ fm/c.

The mean duration time of pion emission can be estimated as $\Delta\tau \geq \Delta\tau_* = 1.3 \pm 0.3$ fm/c.

The geometrical rms transverse radius is $R_{\rm G}(\mathrm{rms}) = \sqrt{2} R_{\rm G} = 1.2 \pm 0.2$ fm, turns out to be larger than the proton rms transverse radius.

The space-time distribution of π emission

The momentum-integrated emission function along the z-axis, i.e., at $\mathbf{r}_\mathrm{t} = (r_x, r_y) = (0,0)$ is given by

$$S(t,z) \propto \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)\exp\left(-\frac{(\eta-y_0)^2}{2\Delta\eta^2}\right). \qquad (17)$$

It relates the parameters fitted to our data with particle production in space-time. Note that the coordinates (t,z), can be expressed as $(\tau\cosh(\eta), \tau\sinh(\eta))$.

We find a structure looking like a boomerang (or string), i.e., particle production takes place close to the regions of $z = t$ and $z = -t$, with gradually decreasing probability for ever larger values of space-time rapidity.

Although the mean proper-time for particle production is $\tau_\mathrm{f} = 1.4$ fm$/c$, and the dispersion of particle production in space-time rapidity is rather small, $\Delta\eta = 1.35$ fm, we see a long tail of particle emission on both sides of the light-cone, a total of 40 fm maximal longitudinal extension in z and a maximum of about 20 fm$/c$ of particle production in time t.

Highlights:

Hydro model parameterization describes NA22 data

Reconstruction of a large and expanding source

for the first time in hadron-proton reactions. Contrast: Previous results on a source size $R_{inv} = 1$ fm.

Analitical predictions confirmed

New ways of extracting the freeze-out times for expanding sources.

Our prediction from 1994-95 was that a simultaneous analysis of spectra and correlations will be necessary to reconstruct the space-time picture. Model-valued analysis is presented (surprisingly) not for NA49, but for NA22 !

Similar results for NA44 are in preparation with A. Ster .

Open problem: How to implement the same analysis for NA49 ?