NUCLEAR INSTRUMENTS AND METHODS 138 (1976) 241–244; © NORTH-HOLLAND PUBLISHING CO.

AN EXTENSION OF THE GENERATOR FUNCTIONAL TECHNIQUE; THE GENERAL METHOD OF THE CORRECTION OF DETECTION LOSSES IN HIGH ENERGY MEASUREMENTS

L. DIÓSI

High Energy Physics Department, Central Research Institute for Physics, Budapest, H-1525 Bp. 114 P.O.B. 49, Hungary

Received 20 July 1976

This paper deals with the correction of the cross sections of the high energy multiparticle production in the case when the detection of the secondary particles has a probability less than unity. Generalizing the usual generator functional technique we get a universal method. The possibility of getting the true distributions is proved under some conditions we have considered, and some examples are given.

Our results can be applied to bubble chamber measurements of neutral particles, to counter experiments, for inclusive reactions as well as for exclusive ones.

1. Introduction

This paper deals with the reconstruction of the true multiparticle production cross sections from the experimental ones when the detection of secondary particles has a probability less than unity[1]). Generalizing the familiar generator functional technique of multiparticle final states[2,3]), a universal method of reconstruction has been obtained.

In sect. 2 we recapitulate the phenomenology of multiparticle distributions and generator functionals, in sect. 3 the detection losses will be specified, in sect. 4 we shall present the reconstruction method. Sect. 5 deals with the applications, sect. 6 contains the conclusions.

2. Distribution functions of multiparticle final states[2])

Let us introduce the function $s^{(n)}$ denoting the exclusive distribution of n-particle final state and choose its norm as usual:

$$\int s^{(n)}(k_1, \ldots, k_n)\, \mathrm{d}k_1 \ldots \mathrm{d}k_n = n!\, p_n, \tag{1}$$

where p_n is the probability of the fixed n multiplicity and k_r is the momentum of the rth secondary particle.

Following Feynman[4]) let us define the inclusive distribution function of order j:

$$f^{(j)}(k_1, \ldots, k_j) = \sum_{n=j}^{\infty} \frac{1}{(n-j)!}\, s^{(n)}(k_1, \ldots, k_n)\, \mathrm{d}k_{j+1} \ldots \mathrm{d}k_n. \tag{2}$$

Both the exclusive and the inclusive distribution functions can be expressed by the derivatives of the very same functional $F[h(\cdot)]$:

$$s^{(n)}(k_1, \ldots, k_n) = \left.\frac{\delta^n F}{\delta h(k_1) \ldots \delta h(k_n)}\right|_{h\equiv 0}, \tag{3}$$

$$f^{(j)}(k_1, \ldots, k_j) = \left.\frac{\delta^j F}{\delta h(k_1) \ldots \delta h(k_j)}\right|_{h\equiv 1}. \tag{4}$$

From eqs. (3) and (4) the equivalence theorem can be deduced: The exclusive distributions can be expressed in terms of inclusive distributions[2]).

3. Measured distribution functions

If the detection efficiency of the secondary particles is less than 1, we measure a distribution different from the true one. Let us denote the measured distributions by $\bar{s}^{(n)}$, $\bar{f}^{(j)}$ and $\bar{p}_n$ consequently.

Let us assume that the detection probability ω of a certain secondary particle depends on the momentum of the particle and is independent of the other secondaries: $\omega = \omega(k)$. Now we can establish the following relation between the true and the measured exclusive distribution functions:

$$\bar{s}^{(n)}(k_1, \dots, k_n) = \sum_{m=n}^{\infty} \frac{1}{(m-n)!} \int s^{(m)}(k_1, \dots, k_m)\, \omega(k_1) \dots \omega(k_n)\, \tilde{\omega}(k_{n+1})\, \mathrm{d}k_{n+1} \dots \tilde{\omega}(k_m)\, \mathrm{d}k_m, \tag{5}$$

where we have used the notation $\tilde{\omega} = 1 - \omega$.

The question arising here is whether the true distributions $s^{(n)}$ could be obtained from the measurable $\bar{s}^{(n)}$ distributions or not. The answer is: in principle it can be done, the reconstruction method does exist.

4. Reconstructibility of the generator functional

We do not try to solve eq. (5) for $s^{(n)}$ but we are going to show: how to reconstruct the generator functional F defined by eq. (3) or eq. (4).

Using eq. (3), the generator functional can be obtained as:

$$F[h(\cdot)] = \sum_{n=0}^{\infty} \frac{1}{n!} \int s^{(n)}(k_1, \dots, k_n)\, h(k_1)\, \mathrm{d}k_1 \dots h(k_n)\, \mathrm{d}k_n. \tag{6}$$

Let us introduce the generator functional of the measurable distributions too:

$$\bar{F}[\bar{h}(\cdot)] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \bar{s}^{(n)}(k_1, \dots, k_n)\, \bar{h}(k_1)\, \mathrm{d}k_1 \dots \bar{h}(k_n)\, \mathrm{d}k_n. \tag{7}$$

If we substitute eq. (5) into eq. (7) we get:

$$\bar{F}[\bar{h}(\cdot)] = \sum_{m=0}^{\infty} \frac{1}{m!} \int s^{(m)}(k_1, \dots, k_m)\, [\omega(k_1)\, \bar{h}(k_1) + \tilde{\omega}(k_1)]\, \mathrm{d}k_1 \dots [\omega(k_m)\, \bar{h}(k_m) + \tilde{\omega}(k_m)]\, \mathrm{d}k_m. \tag{8}$$

Using again eq. (3) and putting it into the right-hand side of eq. (8), summing up the functional Taylor series the result is as follows:

$$\bar{F}[\bar{h}(\cdot)] = F[\omega(\cdot)\, \bar{h}(\cdot) + \tilde{\omega}(\cdot)]. \tag{9}$$

A further algebraic transformation of arguments leads to the fundamental reconstruction formula:

$$F[h(\cdot)] = \bar{F}\left[\frac{h(\cdot) + \omega(\cdot) - 1}{\omega(\cdot)}\right]. \tag{10}$$

This result makes possible the reconstruction of all original distributions from the measured data, at least in principle.

5. Applications

5.1. Exclusive distributions

Using eqs. (3), (7) and the Taylor expansion of eq. (10) around zero function, the formula giving $s^{(n)}$ in terms of the functions $\bar{s}^{(n)}$ can be obtained:

$$s^{(n)}(k_1, \dots, k_n) = w(k_1) \dots w(k_n) \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} \int \bar{s}^{(n+i)}(k_1, \dots, k_{n+i})\, \tilde{w}(k_{n+1})\, \mathrm{d}k_{n+1} \dots \tilde{w}(k_{n+i})\, \mathrm{d}k_{n+i}, \tag{11}$$

where the statistical weight functions w and $\tilde{w}$ have been introduced; see ref. 1:

$$w = 1/\omega, \qquad \tilde{w} = w - 1. \tag{12}$$

Eq. (11) is of practical value only if ω is close to unity. In such a case the first term will be dominating in the right-hand side of eq. (11) and the further terms are monotonously decreasing with i and can be taken as corrections. For example the measured two-particle exclusive distribution can be corrected as

$$s^{(2)}(k_1, k_2) = w(k_1)\, w(k_2) \left\{ \bar{s}^{(2)}(k_1, k_2) - \int \bar{s}^{(3)}(k_1, k_2, k_3)\, \tilde{w}(k_3)\, \mathrm{d}k_3 + \frac{1}{2} \int s^{(4)}\, \tilde{w}(k_3)\, \mathrm{d}k_3\, \tilde{w}(k_4)\, \mathrm{d}k_4 \pm \dots \right\}, \tag{13}$$

where the usual approximation for the distribution is $\bar{s}^{(2)}$.

5.2. Inclusive distributions

Using eqs. (4) and (10), the relation between the true and the measured inclusive distributions can be obtained as:

$$f^{(j)}(k_1, \dots, k_j) = \bar{f}^{(j)}(k_1, \dots, k_j)\, w(k_1) \dots w(k_j). \tag{14}$$

We remark that the reconstruction of the inclusive distributions is much simpler than that of the exclusive distributions.

5.3. Multiplicity distribution and moments

Using the normalization of $s^{(n)}$, having integrated eq. (11) we obtain the multiplicity distribution as:

$$p_n = \frac{1}{n!} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} \int \bar{s}^{(n+i)}(k_1, \dots, k_{n+i})\, w(k_1) \dots w(k_n)\, \tilde{w}(k_{n+1}) \dots \tilde{w}(k_{n+i})\, \mathrm{d}k_1 \dots \mathrm{d}k_{n+i}. \tag{15}$$

This formula can be applied practically only if ω is close to unity. If it is not fulfilled, we have to confine ourselves to reconstruct the moments of the distribution p_n. If we integrate eq. (2), the following result can be obtained:

$$\int f^{(j)}(k_1, \dots, k_j)\, \mathrm{d}k_1 \dots \mathrm{d}k_j = \sum_{n=j}^{\infty} p_n \frac{n!}{(n-j)!} \equiv F_j, \tag{16}$$

where F_j is the so-called factorial moment of order j. Substituting eq. (14) into eq. (16) we can obtain the simple reconstruction formula of F_j which remains useful for application for moderate j even if ω is small:

$$F_j = \int \bar{f}^{(j)}(k_1, \dots, k_j)\, w(k_1)\, \mathrm{d}k_1 \dots w(k_j)\, \mathrm{d}k_j. \tag{17}$$

The details of the multiplicity estimation method can be found in refs. 5 and 6.

6. Conclusions

We have considered the most frequent type of losses occurring in the detection of secondary particles in high energy physics experiments, and the following results have been obtained:

1) It has been shown that the generator functional technique is adequate to handle the detection losses.

2) In the limit of infinitely large statistics the reconstructibility of an arbitrary distribution from the measured data of the secondaries has been proved.

3) Applicability of the formulae:
   (a) the reconstruction of the inclusive distributions is relatively simple;
   (b) exclusive distributions can also be reconstructed when the detection losses are small;
   (c) if the above-mentioned conditions are not fulfilled, the reconstruction of the multiplicity distribution is not too efficient, but the binomial moments still can be obtained.

I would like to express my sincere gratitude for the continuous help of Dr. S. Krasznovszky and the bubble chamber group. I would like to thank Dr. B. Lukács for many discussions.

References

[1]) Budapest–Dubna–Hanoi, Dubna prep. P1-6928 (1973).
[2]) Z. Koba, H. B. Nielsen and P. Olesen, NBI-HE-71-7 (1971).
[3]) I. Montvay, Thesis (Budapest, 1972).
[4]) R. P. Feynman, Phys. Rev. Lett. **23** (1969) 1415.
[5]) L. Diósi, Thesis (Eötvös Lóránd University Budapest, 1975).
[6]) L. Diósi, to be published.