Annalen der Physik. 7. Folge, Band 44, Heft 7, 1987, S. 488—492 J. A. Barth, Leipzig

# In Favor of a Newtonian Quantum Gravity

By L. Diósi and B. Lukács

Central Research Institute for Physics, Budapest, Hungary

Abstract. We list arguments for creating a unified theory of Newtonian Gravity and Quantum Mechanics. This nonrelativistic level has been historically bypassed, however even here one is confronted with conceptional problems anticipating some features of Relativistic Quantum Gravity. Bearing in mind Wigner's famous analysis on measurability in the relativistic case here a genuine uncertainty of the Newton potential is verified, leading to the breakdown of the Schrödinger equation when leaving microscopic regions.

## Für eine Newtonsche quantisierte Gravitation

Inhaltsübersicht. Wir geben eine Reihe von Argumenten für eine Vereinigung von Newtonscher Gravitationstheorie und Quantenmechanik. Dieser Schritt im Nichtrelativistischen wurde historisch umgangen. Man wird jedoch auch hier mit konzeptionellen Problemen konfrontiert, die Züge der relativistischen Quantengravitation tragen. Unter Beachtung von Wigners berühmter Analyse der Meßbarkeit im relativistischen Fall wird hier eine allgemeine Unbestimmtheit des Newtonschen Potentials verifiziert, die zu einem Zusammenbruch der Schrödingergleichung führt, wenn mikroskopische Regionen verlassen werden.

## 1. On the Consistency of Fundamental Laws of Physics

According to our present knowledge, there exist three fundamental physical laws of universal validity: the Gravity, the Relativity and the Quantization. Originally, each of these three principles was recognized for specific situations where both the other laws had negligible effects. Remember the Newtonian Gravity as the theory of planetary motion [1]; experiences with the propagation of light signals led to Special Relativity Theory [2], and Quantum Theory of Schrödinger and Heisenberg [3, 4] explained the atomic phenomena.

However, these brave theories firmly state the universality of themselves in arbitrary situations. Indeed, nature can produce less ideal phenomena where two or three of fundamental laws are essentially manifested. But, their original formulations were independent of each other thus one cannot expect they would be correct for all situations.

The simplest and most obvious formulation of the gravitational experiences was Newton's theory, with surprising precision. After the second fundamental law, the Relativity had become known by means of a simple Gedankenexperiment Einstein demonstrated that the Newtonian Gravity, Relativity and energy conservation are mutually inconsistent [5]. To avoid such an awkward situation one has to assume that gravity affects the geometry of the spacetime. Thus, a new theory had been born, General Relativity [6], unifying the old independent formulations of the Gravity and Relativity. A historical fact is that this unification was preceeded, stimulated and suggested by the recognition of the inconsistency between the two original laws.

The story of the unification of Quantization and Relativity shows quite similar motivations. It is well known how the formal inconsistency of the Schrödinger equation and of the Special Relativity has led Dirac to his famous new equation [7] which was the key to the relativistic Quantum Field Theory [8]. The QFT makes possible to explain typically, high energy particle reactions.

The full unification of our three fundamental laws has previously been merely a matter of principles. Recent investigations show however that the very Early Universe feels indeed the simultaneous manifestation of all these laws [9]. It seems to be straightforward that there are two ways for the full unification. The first one is the Wheeler-DeWitt theory [10, 11] which applies Schrödinger's Quantization to the General Relativity while the second one tries to apply the relativistic QFT to the General Relativity [13]. Unfortunately, such gravitational QFT's are never renormalizable.

Therefore the question of full unification still seems to be open. It is not pointless to revise the elementary steps and to muster the partial unifications. There are three possible couples: $(G, c)$, i.e. General Relativity; $(c, \hbar)$, i.e. relativistic QFT, and $(G, \hbar)$. This third one does not possess even a name; let us call it provisionally Newtonian Quantum Gravity (NQG). Moreover, it is not clear what specific phenomena are the typical applications of NQG. We realize that NQG has not been created because General Relativity was ready when full Quantum Theory became known; however this was rather a historical accident.

## 2. Some Mutual Limitations of Gravity and Quantum Physics

If one would try to elaborate NQG theory, the first stimulating problem is the likely inconsistency between the Newtonian Gravitation and the Schrödinger-Heisenberg Quantum theories. Here we give an order of magnitude estimation for a relevant inconsistency: we show that the Newtonian gravitational law (2.1) cannot be verified with arbitrary precision.

The gravity theory contains a field of acceleration $\boldsymbol{g}(\boldsymbol{x}, t)$, originated from the potential $\Phi(\boldsymbol{x}, t)$, which is determined via the Poisson equation.

$$\Delta\Phi(\boldsymbol{x}, t) = 4\pi\, G\varrho(\boldsymbol{x}, t), \quad \boldsymbol{g}(\boldsymbol{x}, t) = -\nabla\Phi(\boldsymbol{x}, t), \tag{2.1}$$

where $\varrho$ is the mass density. Now, in realistic measurements only a time and volume average can be measured, e.g.

$$\tilde{\boldsymbol{g}}(\boldsymbol{x}, t) = \frac{1}{VT} \int \boldsymbol{g}(\boldsymbol{x}', t')\, d^3x'\, dt'. \tag{2.2}$$

$$|t' - t| < T/2$$

$$|\boldsymbol{x}' - \boldsymbol{x}| < R$$

$V = 4\pi R^3/3$ and $T$ are defined by the specific apparatus. Now, the gravity theory uses a sharp $\boldsymbol{g}$ field; can it be measured with unlimited precision?

For simplicity, take a spherical object with volume $V$ and mass $M$ affected by the $g$ field, and a detector registrating the change of momentum of the particle; this is the measuring apparatus for $\boldsymbol{g}$. The free parameters of the apparatus are $V$, $T$ and $M$. Obviously, the wave packet of the mass point has to be confined practically in the volume $V$ between $t - T/2$ and $t + T/2$. Therefore it would not be a good strategy to start with a very sharp packet, when the spreading would be rapid. Starting with an original width $\sim R$, the maximal time allowed for the measurement is

$$t_{\max} \sim MR^2/\hbar. \tag{2.3}$$

First we assume that $T < t_{\max}$. If the particle is moving, it may leave the volume, therefore the optimum is a particle at rest at the beginning (or at the middle) of the measurement period. It collects a momentum

$$P \simeq M\tilde{g}T\,. \tag{2.4}$$

On the other hand, due to quantum effects, its momentum is not defined better than

$$\delta P \sim \hbar / R. \tag{2.5}$$

Hence the sensitivity of the measurement is limited by

$$\sigma(\tilde{g}) \sim \frac{\hbar}{MRT}\,. \tag{2.6}$$

It is pointless to increase $R$ or $T$ here; since the error of the averaged field would decrease, contrary to the local and instantaneous value, used in Newtonian theory.

However, $M$ can be increased until a new indefiniteness does not take over. Namely, the probe itself disturbes the $g$ field as

$$\boldsymbol{g}^M(\boldsymbol{x}, t) = -\nabla \frac{GM}{|\boldsymbol{x} - \boldsymbol{x}_M(t)|}, \tag{2.7}$$

see eq. (2.1). This should be taken into account, but $\boldsymbol{x}_M$ is not better known than $\delta x_M \sim R$ due to quantum mechanics, then an uncertainty

$$\delta\tilde{g} \sim \frac{GM}{R^2} \tag{2.8}$$

remains. Thus the final sensitivity limit is at the minimum of the right hand sides of eqs. (2.6) and (2.8):

$$M^{\text{optimal}} \sim \sqrt{\frac{\hbar R}{GT}}\,, \quad \sigma(\tilde{g})^{\text{optimal}} \sim \sqrt{\frac{\hbar G}{VT}}\,. \tag{2.9}$$

This is an absolute limitation for the simple apparatus. In order to see it, two assumptions are to be discussed. First, we have supposed that $t_{\max} > T$; if not, $T$ can be divided into $N$ subintervals shorter than $t_{\max}$. The error of one such measurement will larger than in eq. (2.9) by a factor $\sqrt{T/t_{\max}} \sim \sqrt{N}$, which will then be just conpensated when averaging the $N$ independent measurements. The second, tacit assumption was that the particle does not leave the volume under the influence of $\boldsymbol{g}$ during a time interval $T$. However, this is a limitation for $R$ and $T$, not for $M$, anyway, in more sophisticated apparatuses the effect can be compensated.

In spite of the extremely simple apparatus in the Gedankenexperiment, we conjecture that limitation (2.9) for $\sigma(\tilde{g})$ is universal; let us first accept it and later we will name the fact suggesting this. If the limitation is objective, it would have to be reflected somehow in the full formalism of the unification of Gravity and Quantum physics; here we can approximate it by an indeterministic contribution to $\boldsymbol{g}(\boldsymbol{x}, t)$, independent of the classical field created by massive objects:

$$\boldsymbol{g}(\boldsymbol{x}, t) = \boldsymbol{g}_{cl}(\boldsymbol{x}, t) + \boldsymbol{g}_{st}(\boldsymbol{x}, t), \tag{2.10}$$

where $\boldsymbol{g}_{cl}$ is the solution of eq. (2.1), while $\boldsymbol{g}_{st}$ is a stochastic variable of vanishing mean. To recover eq. (2.9) one has to demand for the average of type (2.2) that

$$\langle(\tilde{g}_{st})^2\rangle \sim \frac{\hbar G}{VT}, \tag{2.11}$$

where the symbol $\langle\rangle$ stands for stochastic mean.

Now observe that the squared dispersion of the averaged $\tilde{g}_{st}$ is inversely proportional to the cell $VT$ of averaging. This shows that $g_{st}$ values are independent if $(\boldsymbol{x}, t)$ and $(\boldsymbol{x}', t')$

do not coincide; therefore

$$\langle g_{st}(\boldsymbol{x}, t)\, g_{st}(\boldsymbol{x}', t')\rangle \sim I\hbar G\, \delta(\boldsymbol{x} - \boldsymbol{x}')\, \delta(t - t'). \tag{2.12}$$

If one uses the gravitational potential $\Phi$ (2.1), $\Phi$ becomes stochastic with moments

$$\langle \Phi(\boldsymbol{x}, t)\rangle = \Phi_{cl}(\boldsymbol{x}, t),$$

$$\langle \Phi(\boldsymbol{x}, t)\, \Phi(\boldsymbol{x}', t')\rangle - \langle \Phi(\boldsymbol{x}, t)\rangle \langle \Phi(\boldsymbol{x}', t')\rangle \sim \frac{\hbar G}{|\boldsymbol{x} - \boldsymbol{x}'|}\, \delta(t - t'), \tag{2.13}$$

where $\Phi_{cl}$ satisfies the Poisson equation (2.1).

Note that the laws determining the dispersion of $\Phi$ do not contain any parameter but $\hbar$ and $G$; this is the fact suggesting that the obtained limitation may indeed be universal.

## 3. Gravitational Bounds on Quantum Mechanics

If we were able to elaborate on the intimate unification of Newtonian Gravity and Quantum Theory, eqs. (2.13) would be a fundamental consequence of the NQG, analogous to the uncertainty principle in Quantum Mechanics. NQG will obviously describe ordinary microobjects on one hand, and, gravitating macroobjects on the other. We show that eqs. (2.13) yield sufficient information to estimate a bound where usual Quantum Mechanics breaks down.

Consider a point-like particle of mass $M$ with a wave packet of characteristic width $R$. If the particle behaves microscopically the Schrödinger equation must be a decent approximation:

$$i\hbar \frac{\partial}{\partial t} \psi(\boldsymbol{x}, t) = -\frac{\hbar^2}{2M} \Delta\, \psi(\boldsymbol{x}, t) + M\Phi(\boldsymbol{x}, t)\, \psi(\boldsymbol{x}, t). \tag{3.1}$$

However, the gravitational potential $\Phi$ has become stochastic, c.f. eqs. (2.13). We restrict ourselves to a free particle case when $\langle \Phi\rangle = \Phi_{cl} \equiv 0$.

The familiar deterministic Schrödinger equation is recovered if the stochastic term is negligible with respect to the kinetic energy $\sim \hbar^2/MR^2$. The characteristic cell size of the change of $\psi$ is $R^3T$ where $T \sim MR^2/\hbar$, see also eq. (2.3). Thus, the order of the averaged $\tilde{\Phi}$ in this cell can be obtained from eqs. (2.13) as

$$\tilde{\Phi} \sim \sqrt{\frac{\hbar G}{TR}} \sim \sqrt{\frac{\hbar^2 G}{MR^3}}\,. \tag{3.2}$$

By comparing the kinetic and potential terms of eq. (3.1) one gets that the microbehaviour breaks down at

$$\hbar^2/MR^2 \sim M\tilde{\Phi}, \tag{3.3}$$

so the particle is genuine microscopic if

$$M^3R \ll \hbar^2/G \sim 10^{-47}\ \mathrm{cmg}^3. \tag{3.4}$$

Obviously well above this bound a genuine macrobehaviour is expected; in the transition region specific phenomena may exist, which, nevertheless, cannot be predicted without a well formulated NQG.

## 4. Discussion

We have listed arguments in favour of elaborating a unified non-relativistic theory of Gravity and Quantization (NQG), historically bypassed. Starting with Wigner's original ideas [14], in non-relativistic situations, an absolute lower bound is obtained for the uncertainty of measuring the gravitational potential therefore it must possess a certain smeared nature.

This uncertainty may influence the Schrödinger equation at $M^3R \sim \hbar^2/G$, i.e. for substantial masses. This means that ordinary elementary particles would show the familiar microbehaviour even at an astronomical scale of wave function. There the gravitation is irrelevant [15]. Note that the bound itself is familiar from other theories, too, as e.g. semiclassical gravitation [17] and smeared space-time model [18].

Surprising similarities may be seen between our "non-measurability" statement of sect. 2 and Hawking's unpredictability derived from quantum gravity fluctations [12]. This unpredictability appears, however, on Planck scale. Absolute limitations on this scale are genuine consequences of all three fundamental theories [16]. Our opinion is that the accuracy limit shown in sect. 2 reflects certain important features of the full Relativistic Quantum Gravity theory and they survive in its non-relativistic limit.

Acknowledgement. Authors would like to thank Dr. A. Frenkel for discussions. This work was triggered and inspired by F. Károlyházy's oeuvre.

## References

[1] NEWTON, I.: Philosophiae Naturalis Principia Mathematica. London: Streater 1687.
[2] EINSTEIN, A.: Ann. Phys. **17** (1905) 891.
[3] SCHRÖDINGER, E.: Ann. Phys. **79** (1926) 361, 489.
[4] HEISENBERG, W.: Z. Phys. **33** (1925) 879.
[5] EINSTEIN, A.: Ann. Phys. **35** (1911) 898.
[6] EINSTEIN, A.: Ann. Phys. **49** (1916) 898.
[7] DIRAC, P. A. M.: Proc. Roy. Soc. **A 117** (1926) 610; **A 118** (1926) 341.
[8] BJORKEN, J. D.; DRELL, S. D.: Relativistic Quantum Fields. New York: McGraw-Hill 1965.
[9] LANGACKER, P.: Phys. Rep. **720** (1981) 185.
[10] WHEELER, J. A.: In: Battelle Recontres. (Ed. C. DEWITT and J. A. WHEELER). New York: Benjamin 1968.
[11] DEWITT, B. S.: Phys. Rev. **160** (1967) 1113.
[12] HAWKING, S. W.: Commun. Math. Phys. **87** (1982) 395.
[13] VAN NIEUWENHUIZEN, P.: Phys. Rep. **68** (1981) 189.
[14] WIGNER, E. P.: Rev. Mod. Phys. **29** (1957) 255; c. f. also BOHR, N.; ROSENFELD, L.: Kgl. Danske Videnskab. Selskab. Mat.-Fys. Medd. **12** No. 8 (1933).
TREDER, H.-J.: In: Astrofisica e Cosmologia-Gravitazione-Quanti e Relativita. Firenze: Giunti Barbera 1979.
UNRUH, W. G.: In: Quantum Theory of Gravity. (Ed. S. M. CHRISTENSEN). Bristol: Adam Hilger Ltd. 1984.
[15] v. BORZESZKOWSKI, H.-H.; TREDER, H.-J.: Found. Phys. **12** (1982) 413.
[16] v. BORZESZKOWSKI, H.-H.; TREDER, H.-J.: Ann. Phys. **40** (1983) 287.
[17] DIÓSI, L.: Phys. Lett. **105 A** (1984) 199.
[18] KAROLYHÁZY, F.; FRENKEL, A.; LUKÁCS, B.: In: Physics as Natural Philosophy. (Ed. A. SHIMONY and H. FESCHBACH). Cambridge, Mass.: MIT Press 1982.

Bei der Redaktion eingegangen am 11. November 1985.

Anschr. d. Verf.: Dr. L. DIÓSI, Dr. B. LUKÁCS
Hungarian Academy of Sciences
Central Research Institute for Physics
H-1525 Budapest 114
P.O.B. 49