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On the Motion of Solids in Modified Quantum Mechanics.

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Abstract. – In this paper we apply the unified dynamics of Ghirardi, Rimini and Weber to the translational and rotational motion of solids in three dimensions. We show that, in a certain approximation, the rotational equations can formally be reduced to the translational ones already known. We point out that the rotation of solids as well as their translation are practically of classical nature without any observable quantum effects.

1. Introduction.

Quantum mechanics teaches us that free microparticles cannot be arbitrarily localized; their position uncertainties usually tend to increase with time. It is, however, natural to expect that free macroscopic objects (*e.g.* solids) possess a certain *natural localization.* Without claiming completeness, we mention a few trends [1-6] from the last decades, each proposing original formalisms and/or mechanisms for the quantum localization of macro-objects.

Most recently, Ghirardi, Rimini and Weber (GRW) proposed a theory that utilized some of the techniques of the *continuous quantum measurement* theory of Barchielli *et al.* [5] and they have suggested *unified dynamics* for micro- and macro-objects [6]. In the following paragraphs, we describe the model for the spatial motion of solids, which we will use throughout this discussion. It is mostly the model of ref. [6] with a slight modification which we borrowed from ref. [5].

The usual unitary evolution of the quantum state ρ is modified as follows. An elementary *localization* [6] (or measurement [5]) *process* applies, with frequency λ, to each constituent of a given composite object:

$$\hat{\rho} \to T_{\{\tilde{q}\}}[\hat{\rho}] \equiv (\alpha/\pi)^{-3N/2} \exp\left[-1/2\,\alpha \sum_i^N (\hat{q}_i - \tilde{q}_i)^2\right] \hat{\rho} \exp\left[-1/2\,\alpha \sum_i^N (\hat{q}_i - \tilde{q}_i)^2\right], \qquad (1.1)$$

$\hat{\rho}$ is the density operator, $\hat{q}_i$ denotes the Cartesian coordinate operator of the i-th constituent ($i = 1, 2, \ldots, N$). We shall call $\tilde{q}_i$'s the *selected coordinates.* (In the terminology of Barchielli *et al.* [5] $\tilde{q}_i$'s are the measured coordinates.) The constant α rules the accuracy of the elementary localization. In the localization process (1.1) the coordinates are selected at

random according to the normalized probability distribution

$$p[\{\bar{\boldsymbol{q}}\}] = \mathrm{tr}\, T_{\{\bar{\boldsymbol{q}}\}}[\hat{\rho}]\,. \tag{1.2}$$

In the work of GRW [6] the following finite parameter values were proposed:

$$1/\sqrt{\alpha} \approx 10^{-5}\,\mathrm{cm}\,, \qquad \lambda \approx 10^{-16}\,\mathrm{s}^{-1}. \tag{1.3}$$

We follow, however, the original proposal of Barchielli *et al.* taking the *infinite-frequency zero-accuracy limit*

$$\alpha \to 0\,, \qquad \lambda \to \infty; \qquad \gamma \equiv \lambda\alpha = \mathrm{const} \approx 10^{-6}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}. \tag{1.4}$$

Originally, GRW proposed that the elementary localizations occurred at random instants for each constituent independently. In eq. (1.1) we can take the current elementary localizations at the same time, since, fortunately, the infinite frequency limit (1.4) is insensitive to this change.

In ref. [6] it has been shown that the loose elementary localizations of the constituents result in strong localization of the macro-object due to the large number N of constituents in it. Localization of solids has been discussed in a single spatial dimension. In this paper we are going to derive the GRW equations (in the limit (1.4)) for *solids performing translational and rotational motion in three dimensions*. We have to notice that the rotational motion will be discussed in a certain approximation suitable for most practical cases. Exact equations will be given elsewhere since they need more mathematical elaborations.

2. Localization process for solids.

In this section we are going to calculate the effect of localization (1.1), (1.2) on the centre-of-mass coordinate $\hat{\boldsymbol{Q}}$ and on the rotation angle $\hat{\boldsymbol{\theta}}$ of a given solid.

First we need the geometrical connection between the coordinates $(\hat{\boldsymbol{Q}}, \hat{\boldsymbol{\theta}})$ of the solid and the constituent coordinates $\{\hat{\boldsymbol{q}}\}$. For an ideal classical solid, the Cartesian coordinate $\boldsymbol{q}_i$ of the i-th constituent can always be written as

$$\boldsymbol{q}_i = \boldsymbol{Q} + R_{\boldsymbol{\theta}}\,\boldsymbol{a}_i \qquad (i = 1, 2, \ldots, N)\,, \tag{2.1}$$

where

$$\boldsymbol{Q} = (1/N) \sum_{i=1}^{N} \boldsymbol{q}_i \tag{2.2}$$

is the centre-of-mass coordinate (for simplicity, assume identical constituents of unit mass). The 3×3 orthogonal matrix $R_{\boldsymbol{\theta}}$ stands for the rotation by angle $|\boldsymbol{\theta}|$ around the axis parallel to the vector $\boldsymbol{\theta}$. The unrotated reference (c.m.s.) coordinates $\{\boldsymbol{a}\}$ add up to zero: $\sum \boldsymbol{a}_i = 0$. We introduce the tensor of inertia I of the solid in the c.m.s.

$$I \equiv \sum [(\boldsymbol{q}_i - \boldsymbol{Q})^2\, \mathbf{1} - (\boldsymbol{q}_i - \boldsymbol{Q}) \circ (\boldsymbol{q}_i - \boldsymbol{Q})]\,, \tag{2.3}$$

where the symbol $\circ$ denotes tensor product. The tensor I can be expressed through the

angle $\boldsymbol{\theta}$ of rotation

$$I = I_{\boldsymbol{\theta}} = R_{\boldsymbol{\theta}} I_0 R_{-\boldsymbol{\theta}}, \tag{2.4}$$

where

$$I_0 = \sum (\boldsymbol{a}_i^2 \mathbf{1} - \boldsymbol{a}_i \circ \boldsymbol{a}_i) \tag{2.5}$$

is the tensor of inertia of the unrotated solid. The invariant determinant of tensor I will be denoted by $|I|$.

If θ is small (*i.e.* $|\boldsymbol{\theta}| \ll \pi$) then, by neglecting terms of the order of θ^2, eq. (2.1) takes the form

$$\boldsymbol{q}_i = \boldsymbol{Q} + \boldsymbol{a}_i + \boldsymbol{\theta} \times \boldsymbol{a}_i . \tag{2.6}$$

This approximation allows one to express the rotation angle by a symmetric expression of the constituent coordinates

$$\boldsymbol{\theta} = I_0^{-1} \sum \boldsymbol{a}_i \times \boldsymbol{q}_i . \tag{2.7}$$

After these preparations, let us turn to the quantum case. Equations (2.1)-(2.4) can obviously be applied to the corresponding operators $\hat{\boldsymbol{q}}_i$, $\hat{\boldsymbol{Q}}$ and $\hat{\boldsymbol{\theta}}$. Through the whole paper we *suppose that the quantum uncertainty* $\Delta\theta$ *of the rotation angle is small*: $\Delta\theta \ll \pi$. Furthermore, we perform the calculations of this section in a specially adapted coordinate system where the expectation value $\langle \hat{\boldsymbol{\theta}} \rangle$ vanishes; this choice can be taken without loss of generality. Then, approximations (2.6) and (2.7) remain valid for operators as well.

In analogy to eqs. (2.1) and (2.2), we introduce the centre-of-mass coordinate $\boldsymbol{Q}$ and the rotation angle $\boldsymbol{\theta}$ for the selected constituent positions $\{\boldsymbol{q}\}$ appearing in the localization process (1.1), (1.2)

$$\boldsymbol{Q} = (1/N) \sum \boldsymbol{q}_i , \tag{2.8}$$

$$\boldsymbol{\theta} = I_0^{-1} \sum \boldsymbol{a}_i \times \boldsymbol{q}_i , \tag{2.9}$$

which now means a certain «average» rotation fitted to the given set $\{\boldsymbol{q}\}$ of the selected constituent coordinates.

Having all necessary ingredients, *let us integrate the* r.h.s. *of eq.* (1.1) *over the selected coordinates* $\{\bar{\boldsymbol{q}}\}$, *while the coordinates* $(\bar{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})$ *of the solid* (cf. eqs. (2.8) and (2.9)) *are kept fixed*

$$\hat{\rho} \to T_{(\bar{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})}[\hat{\rho}] \equiv \int \delta(\bar{\boldsymbol{Q}} - N^{-1} \sum \bar{\boldsymbol{q}}_i)\, \delta(\bar{\boldsymbol{\theta}} - I_0^{-1} \sum \boldsymbol{a}_i \times \bar{\boldsymbol{q}}_i)\, T_{(\bar{\boldsymbol{q}})}[\hat{\rho}]\, \mathrm{d}^N \bar{\boldsymbol{q}} . \tag{2.10}$$

According to the GRW theory [6], $T_{(\bar{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})}$ represents the localization process for such case when no selection (measurement [5]) is made on the constituent coordinates $\{\hat{\boldsymbol{q}}\}$ but on the collective coordinates $(\hat{\boldsymbol{Q}}, \hat{\boldsymbol{\theta}})$ of the solid.

Let us evaluate the r.h.s. of eq. (2.10) in coordinate representation

$$\langle \boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_N | \, T_{(\bar{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})}[\hat{\rho}] \, | \, \boldsymbol{q}_1', \boldsymbol{q}_2', \ldots, \boldsymbol{q}_N' \rangle = \exp[-(1/4)\alpha \sum (\boldsymbol{q}_i - \boldsymbol{q}_i')^2] \times$$

$$\times (\alpha/\pi)^{3N/2} \int \delta(\bar{\boldsymbol{Q}} - N^{-1} \sum \bar{\boldsymbol{q}}_i)\, \delta(\bar{\boldsymbol{\theta}} - I_0^{-1} \sum \boldsymbol{a}_i \times \bar{\boldsymbol{q}}_i) \exp[-\alpha \sum [\bar{\boldsymbol{q}}_i - (1/2)(\boldsymbol{q}_i + \boldsymbol{q}_i')]^2 \, \mathrm{d}^N \bar{\boldsymbol{q}} \times$$

$$\times \langle \boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_N | \hat{\rho} | \boldsymbol{q}_1', \boldsymbol{q}_2', \ldots, \boldsymbol{q}_N' \rangle . \tag{2.11}$$

In analogy to eqs. (2.2) and (2.7), we introduce $(\boldsymbol{Q}', \boldsymbol{\theta}')$ as a function of the primed coordinates $\{\boldsymbol{q}'\}$. Then the sum $\sum (\boldsymbol{q}_i - \boldsymbol{q}_i')^2$ can be expressed in terms of $(\boldsymbol{Q}, \boldsymbol{\theta})$ and $(\boldsymbol{Q}', \boldsymbol{\theta}')$

$$\sum (\boldsymbol{q}_i - \boldsymbol{q}_i')^2 = N(\boldsymbol{Q} - \boldsymbol{Q}')^2 + (\boldsymbol{\theta} - \boldsymbol{\theta}')\, I_0 (\boldsymbol{\theta} - \boldsymbol{\theta}') \tag{2.12}$$

and, similarly, the 2nd line of eq. (2.11) will reduce to

$$(\alpha \sqrt{N}/\pi)^3 |I|^{1/2} \exp\left[- \alpha N [(1/2)(\boldsymbol{Q} + \boldsymbol{Q}') - \overline{\boldsymbol{Q}}]^2 - \right.$$

$$\left. - \alpha [(1/2)(\boldsymbol{\theta} + \boldsymbol{\theta}') - \bar{\boldsymbol{\theta}}]\, I_0 [(1/2)(\boldsymbol{\theta} + \boldsymbol{\theta}') - \bar{\boldsymbol{\theta}}] \right]. \tag{2.13}$$

Let us substitute identities (2.12) and (2.13) into the r.h.s. of eq. (2.11) and let us restore the operator formalism. The localization process (2.10) of solid will then have the following final form:

$$\hat{\rho} \to T_{(\overline{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})}[\hat{\rho}] = (\alpha \sqrt{N}/\pi)^3 |I|^{1/2} \times \exp\left[- (1/2)\, \alpha N (\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2 - (1/2) - \alpha (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})\, I_0 (\hat{\boldsymbol{\theta}}\, \bar{\boldsymbol{\theta}}) \right] \cdot$$

$$\cdot \hat{\rho} \exp\left[- (1/2)\, \alpha N (\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2 - (1/2)\, \alpha (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})\, I_0 (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}) \right]. \tag{2.14}$$

For the probability distribution $p(\overline{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})$ of the selected position and angle of the solid we have

$$p(\overline{\boldsymbol{Q}}, -\bar{\boldsymbol{\theta}}) = \operatorname{tr} T_{(\overline{\boldsymbol{Q}}, \bar{\boldsymbol{\theta}})}[\hat{\rho}] = (\alpha \sqrt{N}/\pi)^3 |I|^{1/2} \cdot$$

$$\cdot \operatorname{tr} \{ \hat{\rho} \exp\left[- \alpha N (\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2 - \alpha (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})\, I_0 (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}) \right] \}. \tag{2.15}$$

Equations (2.14) and (2.15) represent the main results of this section. By comparing them to eqs. (1.1) and (1.2) of the original localization for constituent positions $\{\hat{\boldsymbol{q}}\}$, it is easy to see that the *accuracy parameter α of the elementary localizations has now got a factor of N*. In a similar way, *the accuracy parameter of the orientation localization* is equal to $\alpha|I|$ which *is also proportional to the number N of the constituents* in the solid.

As we shall see in the next section, for solids of nondegenerated form $\alpha|I|$ turns out to be much greater than unity. Therefore, eq. (2.15) together with the overall assumption $\Delta\theta \ll \pi$ of this section lead to $\|\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}\| \ll \pi$. This smallness of deviation between the two angles means that, in the same approximation, eqs. (2.14) and (2.15) become *covariant* against Galilean coordinate transformations if I_0 is replaced by the actual tensor $I_{\langle\hat{\theta}\rangle}$ of inertia (2.4). Consequently, the temporary assumption $\langle\hat{\boldsymbol{\theta}}\rangle = 0$ of this section is not necessary for the validity of the final formulae.

3. Motion of solids in modified quantum mechanics.

In the case of the ordinary dynamics the translational and the rotational motions of a free solid satisfy a separable set of equations. Let the quantum state $\hat{\rho}$ be of the form $\hat{\rho}_t \otimes \hat{\rho}_r$ where $\hat{\rho}_t$, $\hat{\rho}_r$ belong to the translational and, respectively, to the rotational degrees of freedom. Then $\hat{\rho}_t$ and $\hat{\rho}_r$ satisfy the following separate Schrödinger equations:

$$\dot{\hat{\rho}}_t = - (i/2\hbar N) [\hat{P}^2, \hat{\rho}_t], \tag{3.1}$$

$$\dot{\hat{\rho}}_r = - (i/2\hbar) [\hat{M} I_{\bar{\theta}}^{-1} \hat{M}, \hat{\rho}_r]. \tag{3.2}$$

Here $\hat{\boldsymbol{P}}$, $\hat{\boldsymbol{M}}$ denote the momenta canonically conjugated to the coordinates $\hat{\boldsymbol{Q}}$ and $\hat{\boldsymbol{\theta}}$, respectively.

The *localization processes* (2.14), (2.15) *can be factorized as well, into independent localizations of the position* $\hat{\boldsymbol{Q}}$ *and of the angle* $\hat{\boldsymbol{\theta}}$:

$$\hat{\rho_t} \to (\alpha N/\pi)^{3/2} \exp[-(1/2)\alpha N(\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2]\,\hat{\rho}_t \exp[-(1/2)\alpha N(\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2] \tag{3.3}$$

with probability $(\alpha N/\pi)^{3/2}\,\mathrm{tr}\{\hat{\rho}_t \exp[-\alpha N(\hat{\boldsymbol{Q}} - \overline{\boldsymbol{Q}})^2]\}$ and, furthermore,

$$\hat{\rho}_r \to (\alpha/\pi)^{3/2} |I|^{1/2} \exp[-(1/2)\alpha(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}) I_{\langle\hat{\theta}\rangle} (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})]\,\hat{\rho}_r \exp[-(1/2)\alpha(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}) I_{\langle\hat{\theta}\rangle} (\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})], \tag{3.4}$$

with probability $(\alpha/\pi)^{3/2}|I|^{1/2}\,\mathrm{tr}\{\hat{\rho}_r \exp[-\alpha(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}}) I_{\langle\hat{\theta}\rangle}(\hat{\boldsymbol{\theta}} - \bar{\boldsymbol{\theta}})]\}$.

The localization processes (3.3) and (3.4) repeatedly interrupt the unitary evolutions (3.1) and (3.2), respectively, at rate λ. Taking the infinite frequency limit (1.4), eqs. (3.1)-(3.4) represent the dynamics of the given solid.

The basic features of the translational equations (3.1) and (3.3) are relatively well known. On the one hand, if we do not record the selected coordinate $\overline{\boldsymbol{Q}}$, then the statistical operator obeys to the following master equation:

$$\dot{\hat{\rho}}_t = -(i/2\hbar N)[\hat{P}^2, \hat{\rho}_t] - (1/4)\gamma N[\hat{\boldsymbol{Q}}, [\hat{\boldsymbol{Q}}, \hat{\rho}_t]]. \tag{3.5}$$

From this equation the *suppression of the interference between far-away localized states* follows (cf. ref. [3-6] for details).

On the other hand, let us make a full selection of the states on the basis of the coordinates $\overline{\boldsymbol{Q}}$. Then it has been conjectured by GRW [6] that there exists a certain *stationary regime* for the selected states. In this regime the wave function becomes spherical symmetric Gaussian with the following spreads [6]:

$$\begin{cases} \Delta Q = \sqrt{3/N}\,(\hbar/2\gamma m)^{1/4} \approx 10^{-11}\,\mathrm{cm}, \\ \Delta P = \sqrt{3N}\,\hbar(\gamma m/2\hbar)^{1/4} \approx 10^{-16}\,\mathrm{gcms}^{-1}. \end{cases} \tag{3.6}$$

Here the dimension of the constituent mass m has been restored; the $\sqrt{3}$ factor is due to the presence of the 3 independent spatial directions. The numeric values correspond to $Nm \approx 1$ g. The expectation values $\langle\hat{\boldsymbol{Q}}\rangle$ and $\langle\hat{\boldsymbol{P}}\rangle$ move along classical trajectories apart from a certain stochastic spread around them. This *anomalous Brownian motion* of the centre-of-mass is, however, completely unobservable due to its smallness [6].

As far as the rotational motion of solids in the GRW theory is concerned, we shall restrict ourselves to the case of a rigid sphere of radius R, with uniformly distributed constituents. Then the tensor of inertia (2.3) is proportional to the unit tensor: $I = (2/5)NR^2\,\mathbf{1}$. From here, the letter I will denote the scalar factor $(2/5)NR^2$.

Observe that eqs. (3.2) and (3.4) of the rotation can be obtained from eqs. (3.1) and (3.3), respectively, by the substitutions $\boldsymbol{Q} \to \boldsymbol{\theta}$, $\boldsymbol{P} \to \boldsymbol{M}$, $N \to I$. All that is known about the translation of solids in GRW theory can thus be transferred unto the rotation.

So, for instance, the counterpart of the master equation (3.5) will ensure the *suppression of the interference between different orientations* of the solid:

$$\dot{\hat{\rho}}_r = -(i/2\hbar I)[\hat{\boldsymbol{M}}^2, \hat{\rho}_r] - (1/4)\gamma I[\hat{\boldsymbol{\theta}}, [\hat{\boldsymbol{\theta}}, \hat{\rho}_r]]. \tag{3.7}$$

From the second term on the r.h.s. we see that the characteristic time of damping the

interference between two different orientations $\boldsymbol{\theta}$, $\boldsymbol{\theta}'$ is equal to $((1/4)\gamma I\|\boldsymbol{\theta}-\boldsymbol{\theta}'\|^2)^{-1}\approx$ $\approx(\gamma NR^2\|\boldsymbol{\theta}-\boldsymbol{\theta}'\|^2)^{-1}\approx 10^{-16}\,\mathrm{s}\|\boldsymbol{\theta}-\boldsymbol{\theta}'\|^{-2}$. This very fast damping assures that the generic quantum states of a solid satisfy our basic approximation $\Delta\theta\ll\pi$ taken in sect. 2.

In the *stationary regime* the rotational wave function of the solid ($Nm\approx 1$ g, $R\approx 1$ cm) will be a Gaussian function of the rotation angle $\boldsymbol{\theta}$. We obtain the quantum uncertainties of the orientation $\hat{\boldsymbol{\theta}}$ and of the angular momentum $\hat{\boldsymbol{M}}$ from eqs. (3.6) with the substitutions $\boldsymbol{Q}\to\boldsymbol{\theta}$, $\boldsymbol{P}\to\boldsymbol{M}$, $N\to I=(2/5)NR^2$ yielding

$$\begin{cases}\Delta\theta=\sqrt{5/2}\,\Delta Q/R\approx 10^{-11},\\ \Delta M=\sqrt{2/5}\,R\Delta P\approx 10^{-16}\,\mathrm{gcm^2s^{-1}}.\end{cases}\tag{3.8}$$

Similarly to the case of translation, the expectation values of the rotational canonical coordinates $\langle\hat{\boldsymbol{\theta}}\rangle$, $\langle\hat{\boldsymbol{M}}\rangle$ approximately satisfy the classical equations of the symmetric rotator. The true trajectories are perturbed by a practically unobservable anomalous Brownian motion.

Needless to say that the quantum uncertainties of the orientation $\Delta\theta$ and of the angular momentum ΔM are both unobservable either.

4. Conclusion.

In this paper we have shown that the theory of spontaneous wave function localization proposed in ref. [6] leads to a certain natural localization of the position and orientation of a given free (macroscopic) solid. Technically, the problem of rotation of solids in GRW theory has been reduced to the known equations of the translation of solids. One sees that, similarly to the translation of solids, the rotation can also be interpreted in classical terms, since the GRW theory allows no room for observable quantum effects in the spatial motion of macroscopic bodies (at least not for solids).

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