RESTORATION OF $2\pi^0$ INCLUSIVE DISTRIBUTION FROM OBSERVED 2γ DATA

L. DIÓSI and B. LUKÁCS
Central Research Institute for Physics, Pf. 49, H-1525 Budapest 114, Hungary

Received 12 February 1988

Here a formula is derived to restore the $2\pi^0$ effective mass distribution from the observed 2γ one. The method is applied on bubble chamber events of moderate statistics, as a preliminary survey, and gives an exponential asymptotics in the relative rapidity.

1. Introduction

As it is well known, the two-particle rapidity distributions give a deeper insight into the dynamics of the strong interactions [1]. For charged secondaries a quite satisfactory amount of such data is available. However, the situation is far worse for neutral secondaries, as π^0, due to obvious difficulties in detection. Nevertheless, heavy-liquid bubble chamber experiments can offer at least inclusive many-particle distributions of substantial statistics [2]. So, the task is to restore the π^0 momentum distribution from the observed γ distribution. As several works [3–6] have shown, the $1\pi^0$ inclusive spectrum can be uniquely restored from the 1γ inclusive one. Here we show how to obtain a similar connection between two-particle inclusive spectra. Namely, we give the relationship between the two-particle effective mass distributions, because the relevant rapidity type variable of the $2\pi^0$ system is 2 arch $(M^2_{\pi\pi}/m^2_\pi)$, as demonstrated later.

2. Connection between two-particle inclusive γ and π^0 distributions

Consider the kinematics of the decay $\pi^0\to\gamma\gamma$. A single π^0 of four-momentum p yields the inclusive distribution $(2\pi)^{-1}\delta(4)\ (p-k_1-k_2)$, where k_1, k_2 are the four-momenta of the γ's. Hence one directly obtains the two-photon inclusive distribution as

$$f^{(2)}_\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)=\frac{1}{\pi}\int f^{(1)}_\pi(\boldsymbol{p})\delta^{(4)}(p-k_1-k_2)(\mathrm{d}\boldsymbol{p}/E)$$

$$+\frac{1}{\pi^2}\int\int f^{(2)}_\pi(\boldsymbol{p},\boldsymbol{p}')\delta(pk_1-\tfrac{1}{2}m^2_\pi)\delta(p'k_2-\tfrac{1}{2}m^2_\pi)(\mathrm{d}\boldsymbol{p}/E)(\mathrm{d}\boldsymbol{p}'/E')\,, \qquad (2.1)$$

where p, p' are the four-momenta of the pions; $f^{(1)}_\pi$, $f^{(2)}_\pi$ stand for the first- and second-order inclusive distributions of the pions while $f^{(2)}_\gamma$ denotes the two-γ inclusive distribution, and m_π is the π^0 rest mass.

The maximalist's goal would be a formula for $f^{(2)}_\pi(\boldsymbol{p},\boldsymbol{p}')$ by inverting eq. (2.1). However, here we will be contented with a more modest aim, namely to generate the $2\pi^0$ effective mass distribution from that of the 2γ pairs. Let us first introduce the normalized inclusive distributions ρ of the 2γ and 2π effective mass squares M^2:

$$\rho_\gamma(M^2_{\gamma\gamma})=\frac{1}{F^{(2)}_\gamma}\int f^{(2)}_\gamma(\boldsymbol{k}_1,\boldsymbol{k}_2)\delta(M^2_{\gamma\gamma}-(k_1+k_2)^2)(\mathrm{d}\boldsymbol{k}_1/k_1)(\mathrm{d}\boldsymbol{k}_2/k_2)\,, \qquad (2.2)$$

$$\rho_\pi(M^2_{\pi\pi}) = \frac{1}{F^{(2)}_\pi} \int f^{(2)}_\pi(\boldsymbol{p},\boldsymbol{p}')\delta(M^2_{\pi\pi}-(p+p')^2)(\mathrm{d}\boldsymbol{p}/E)(\mathrm{d}\boldsymbol{p}'/E') , \tag{2.3}$$

where $F^{(2)}_\alpha \equiv \langle n_\alpha(n_\alpha - 1)\rangle$ ($\alpha=\gamma$ or π) is the second factorial moment of the multiplicity distributions, respectively. Now substituting eq. (2.1) into (2.2) one obtains that the RHS contains the π^0 distribution only in the (2.3) combination:

$$\rho_\gamma(M^2_{\gamma\gamma}) = (\langle n_\gamma\rangle / F^{(2)}_\gamma)\delta(M^2_{\gamma\gamma}-m^2_\pi) + \frac{4F^{(2)}_\pi}{F^{(2)}_\gamma} \int_{4m^2_\pi}^{\infty} T(M^2_{\gamma\gamma}; M^2_{\pi\pi})\rho_\pi(M^2_{\pi\pi})\mathrm{d}M^2_{\pi\pi} , \tag{2.4}$$

where

$$T(M^2_{\gamma\gamma}; p, p') = T(M^2_{\gamma\gamma}, M^2_{\pi\pi})$$
$$= \frac{1}{(2\pi)^2} \int \delta(pk_1 - \tfrac{1}{2}m^2_\pi)\delta(p'k_2 - \tfrac{1}{2}m^2_\pi)\delta(M^2_{\gamma\gamma} - (k_1+k_2)^2)(\mathrm{d}\boldsymbol{k}_1/k_1)(\mathrm{d}\boldsymbol{k}_2/k_2) . \tag{2.5}$$

[Observe that T is a Lorenz scalar, therefore its p, p' dependence must occur only through the $2\pi^0$ effective mass square $M^2_{\pi\pi} \equiv (p+p')^2$.] Thus, by eqs. (2.4), (2.5), we have obtained a direct relation between the 2γ and the $2\pi^0$ effective mass inclusive distributions.

The kernel (2.5), after a straightforward and moderate computation, yields

$$\begin{aligned} T(M^2_{\gamma\gamma}; M^2_{\pi\pi})\mathrm{d}M^2_{\pi\pi} = \mathrm{d}Y_{\pi\pi} \quad & \times 2Y_{\pi\pi}, \quad z_{\gamma\gamma} \leqslant -Y_{\pi\pi} , \\ & \times Y_{\pi\pi} - z_{\gamma\gamma}, \quad -Y_{\pi\pi} \leqslant z_{\gamma\gamma} \leqslant Y_{\pi\pi} , \\ & \times 0, \quad Y_{\pi\pi} \leqslant z_{\gamma\gamma} , \end{aligned} \tag{2.6}$$

where, for convenience, we have introduced the shorthand notations $z_{\gamma\gamma}$ and $Y_{\pi\pi}$:

$$z_{\gamma\gamma} \equiv \ln(M^2_{\gamma\gamma}/m^2_\pi) \quad -\infty \leqslant z_{\gamma\gamma} \leqslant +\infty , \quad Y_{\pi\pi} \equiv 2\,\mathrm{arch}(M_{\pi\pi}/2m_\pi) \quad 0 \leqslant Y_{\pi\pi} \leqslant +\infty . \tag{2.7a,b}$$

Then, substituting the form (2.6) of the kernel into eq. (2.4), one obtains

$$F^{(2)}_\gamma \rho_\gamma(M^2_{\gamma\gamma}) = \langle n_\gamma\rangle \delta(M^2_{\gamma\gamma} - m^2_\pi)$$
$$+ 4F^{(2)}_\pi \int_{z_{\gamma\gamma}}^{\infty} \rho_\pi(M^2_{\pi\pi})(Y_{\pi\pi} - z_{\gamma\gamma})\mathrm{d}Y_{\pi\pi} + 4F^{(2)}_\pi \theta(-z_{\gamma\gamma}) \int_0^{|z_{\gamma\gamma}|} \rho_\pi(M^2_{\pi\pi})2Y_{\pi\pi}\mathrm{d}Y_{\pi\pi} , \tag{2.8}$$

where θ is the step-function. There are corresponding $M^2_{\gamma\gamma}$ values below and above the π^0-peak fulfilling the relation

$$\rho_\gamma(M^2_{\gamma\gamma}) + \rho_\gamma(m^4_\pi/M^2_{\gamma\gamma}) = \text{const.} \quad \text{for all } M^2_{\gamma\gamma} \neq m^2_\pi . \tag{2.9}$$

Therefore it is enough to restrict ourselves to $M^2_{\gamma\gamma} > m^2_\pi$. There, eq. (2.8) can be inverted by repeated differentiations as

$$\rho_\pi(M^2_{\pi\pi}) = \tfrac{1}{4}(F^{(2)}_\gamma / F^{(2)}_\pi)(\mathrm{d}/\mathrm{d}z_{\gamma\gamma})^2 \rho_\gamma(M^2_{\gamma\gamma})|_{z_{\gamma\gamma}=Y_{\pi\pi}} .$$

This is the cental result of the present paper. Eliminating the shorthand notations (2.7), this formula can be written into the final form

$$\rho_\pi(M^2_{\pi\pi}) = \tfrac{1}{4}(F^{(2)}_\gamma / F^{(2)}_\pi)(M^{-2}_{\gamma\gamma}\mathrm{d}/\mathrm{d}M^2_{\gamma\gamma})^2 \rho_\gamma(M^2_{\gamma\gamma})|_{M_{\gamma\gamma}=(M_{\pi\pi} \pm \sqrt{M^2_{\pi\pi} - 4m^2_\pi})/2} .$$

[The double sign in eg. (2.11) corresponds to the two halves of the $\rho_\gamma(M^2_{\gamma\gamma})$ distribution, separated by the π^0 peak, c.f. eq. (2.9).]

3. Application

Although eq. (2.11) is our final result we must note that this form is not too appropriate for direct use. Namely, it contains derivatives of a histogram which is an incorrectly defined numerical problem. For one-particle inclusive distributions this instability was circumvented [7] by special regularization methods [8], see also in ref. [6]. Let us postpone the problem for a moment.

The application of our formula on high statistics data will be the object of a subsequent paper. Here we want only to demonstrate how it works. Fig. 1 displays the $M_{\gamma\gamma}^2$ inclusive distribution taken from 2000 propane chamber events in 40 GeV π^-–p collisions [2]. While the symmetry (2.9) cannot be observed on the data, for this the responsibility can be relegated to an expected artefact, namely the excess of low energy photons originating partly from misinterpretation of low energy tertiaries, partly from unnoticed energy loss of e^+e^- pairs. the $M_{\gamma\gamma}^2 > m_\pi^2$ part of the measured distribution is more reliable, and contains the full information due to the symmetry (2.9).

Now, the upper part of the distribution should be put into eq. (2.11) in order to obtain the $2\pi^0$ effective mass distribution. However, as mentioned above, the required numerical differentiation is an awkward procedure. Nevertheless, as will be immediately shown, in the present case one can avoid this difficulty. Namely, fig. 2 displays the same distribution on a double logarithmic plot, and demonstrates that it is consistent with a power law:

$$\rho_\gamma(M_{\gamma\gamma}^2) = \text{const.}\cdot z_{\gamma\gamma}^{-\beta}, \quad \beta = 1.44 \pm 0.07, \quad z_{\gamma\gamma} > 0\,. \tag{3.1}$$

Then the differentiation in eqs. (2.10), (2.11) can be analytically performed, and the $2\pi^0$ effective mass distribution obtains the form

$$\rho_\pi(M_{\pi\pi}^2)\text{const.}\cdot\left(M_{\pi\pi} + \sqrt{M_{\pi\pi}^2 - 4m_\pi^2}\right)^{-2\beta}. \tag{3.2}$$

Now, consider the defining equation (2.7b) of $Y_{\pi\pi}$, and evaluate it in the limit $M_{\pi\pi} \gg m_\pi$. There, for negligible transversal momenta,

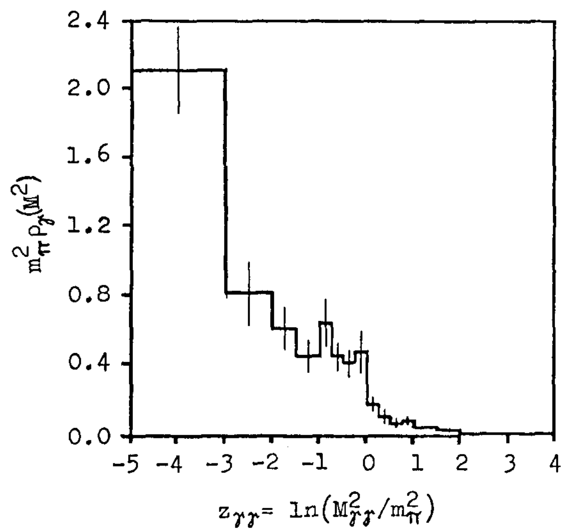

Fig. 1. Inclusive 2γ effective mass distribution for π^-–p at 40 GeV/c from the experiment of ref. [2]. For the details of the high-mass tail used in our evaluation, see fig. 2.

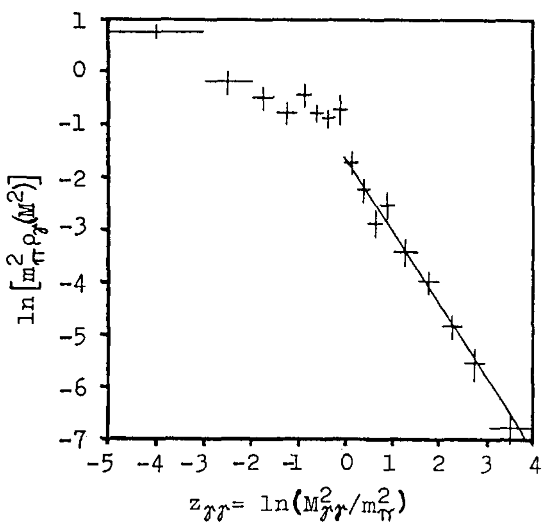

Fig. 2. As in fig. 1, but on doubly logarithmic scales for the better display of the tail. Observe the shoulder at the π^0 mass (instead of the theoretical δ-shaped peak). The tail is indeed conform with a decreasing exponential (best fitting: continuous line).

$$Y_{\pi\pi} \approx |y_1 - y_2| , \tag{3.3}$$

where y_i stands for the individual rapidities of the π^0's. Now observe that $f_\pi^{(2)}(Y_{\pi\pi}) = F_\pi^{(2)} \rho_\pi(M_{\pi\pi}^2) \mathrm{d}M_{\pi\pi}^2 / \mathrm{d}Y_{\pi\pi}$ is the inclusive distribution of the "relative rapidity" (3.3). Thus in the above limit eq. (3.2) yields the following asymptotical result for $f_\pi^{(2)}(|y_1 - y_2|)$:

$$f^{(2)}(|y_1 - y_2|) \sim \exp[-(\beta - 1)|y_1 - y_2|] , \tag{3.4}$$

with $\beta - 1 \approx \frac{1}{2}$ [cf. eq. (3.1)].

4. Conclusion

Here a rigorous relation has been constructed between the observable 2γ and theoretically relevant $2\pi^0$ inclusive distributions. A preliminary calculation has demonstrated the exponential asymptotic behaviour of the $2\pi^0$ relative rapidity distribution. Since experimental two-particle rapidity distributions are regarded as standard additional tests on production mechanisms [9], it seems worthwhile and promising to utilize this method for extending such tests to neutral secondaries. The needed $M_{\gamma\gamma}$ inclusive distributions of high statistics are already appearing in the literature (cf. e.g. refs. [10,11]).

Acknowledgement

The authors thank Professor T. Gèmesy, Professor Livia Jenik and Professor S. Krasznovszky for useful suggestions.

References

[1] C. Quigg, in: AIP Conf. Proc. on Particles and fields, eds. H.H. Bingham, M. Davies and G.R. Lynch (AIP, New York, 1973) p. 520.
[2] E. Abdurakhmanov et al., Phys. Lett. B 48 (1974) 277.
[3] R.G. Glasser, Phys. Rev. D 6 (1972) 1993.
[4] G.I. Kopylov, Phys. Lett. B 41 (1972) 371.
[5] R.N. Cahn, Phys. Rev. D 7 (1973) 247.
[6] J. Antos, Nucl. Instrum. Methods A 249 (1986) 241.
[7] A. Abdurakhimov et al., Nucl. Phys. B 83 (1974) 365.
[8] M. Dakowski, Yu.A. Lazarev, V.F. Turchin and L.S. Turovtseva, Nucl. Instrum. Methods 113 (1973) 195.
[9] P. Carruthers and I. Sarcevic, in: Hadronic multiparticle production (World Scientific, Singapore, 1988); see also Los Alamos report LA-UR-87-3716.
[10] A.A. Minaenko et al., Z. Phys. C 26 (1984) 27.
[11] M. Aguilar-Benitez et al., Z. Phys. C 34 (1987) 419.