PHYSICAL REVIEW A VOLUME 43, NUMBER 1 1 JANUARY 1991

Covariant formulation of multiple localized quantum measurements

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(Received 26 June 1990)

We have considered the dynamics of simple measuring devices coupled to quantized relativistic Heisenberg fields. We have defined localized observables and derived covariant equations for the joint probability distribution of the measured outcomes. The proposed formalism is based on standard field-theoretical terms. We have shown that our formalism yields the expected anticoincidence when two devices detect two wave packets of a single electron. Applications in the measurement theory of vacuum fluctuations have been suggested.

I. INTRODUCTION

The famous dichotomy of quantum dynamics on the one hand, and of quantum measurement on the other, has had a lasting effect on physicists' approach to locality and causality of quantum theory. Usually we are content with quantum dynamics because it has a relativistically invariant formulation in terms of local quantum field theories. The theory of quantum measurements has, however, remained in rather primitive form. It works with an instantaneous "collapse" of the wave function. This collapse is most believed to be unremovable from the theory and permanently obstructs the manifest covariance of measurement theory.

Nevertheless it may well be that the wave function is not the best frame in which to express the history of the dynamical evolution.[1] We should not use the wave function to describe the history; we need a covariant frame instead. In fact, we have had such a frame for a long time. In quantum field theory the Heisenberg operators of the fields carry all dynamical information in covariant form; moreover, they also respect causality, while the only role of the Heisenberg state (i.e., of the wave function) is to specify initial conditions. Thus in the Heisenberg operators we have the ingredients of a covariant formalism for quantum measurements.

In Sec. II we will define local observations of relativistic Heisenberg fields, with the help of von Neumann's measuring devices.[2] In Sec. III the device dynamics will be integrated out and then, as a result, the net (phenomenological) theory of multiple local quantum measurements will be presented in covariant form in Sec. IV. In Sec. V, the theory will be applied to two separate devices detecting two wave packets of a single electron. Section VI will discuss future applications to the measurement theory of free-field vacuum fluctuations. The conclusion and final remarks will be presented in Sec. VII.

II. HEISENBERG DYANMICS OF COUPLED FIELDS AND DEVICES

Consider a system whose dynamics is described by a given number of quantized local Heisenberg fields. Let us assume there are N localized observables $A_1, A_2, \ldots, A_N$ to be measured, each having the form

$$A_n = \int g_n(x)\phi_n(x)dx \ , \tag{1}$$

where each g_n is a normalized weight function with compact support. Sometimes we shall refer to this support as that of the localized observable (1). The ϕ_n's are local Hermitian bosonic fields, composite or primary ones, not necessarily differing from each other.

The simplest device capable of measuring A_n is described by the pair q_n, p_n of Heisenberg operators of canonical coordinate and momentum, respectively. The Hamiltonian H_n is assumed to vanish. The N devices are, of course, coupled to the fields, expressed by the Lagrangian density

$$\mathcal{L}(x) = -\sum_{n=1}^{N} g_n(x)\phi_n(x)q_n \ . \tag{2a}$$

The corresponding interaction Hamiltonian takes the form

$$H(t) = \sum_{n=1}^{N} \int g_n(t,\mathbf{x})\phi_n(t,\mathbf{x})d\mathbf{x}q_n \ . \tag{2b}$$

It should be admitted that Eq. (2b) assumes the introduction of time coördinates. Actually, any local time would be used for each device. However, the choice of time coordinates, as we shall see, does not affect the statistics of measurements, nor the covariance of our final results.

Having specified the coupled Heisenberg dynamics of the fields and of the devices, we also have to prescribe the initial conditions. As is known, they have to be expressed by the Heisenberg state vector. We assume the following form for the state of the fields and devices:

$$|\psi_{\text{in}}\rangle \otimes |D\rangle = |\psi_{\text{in}}\rangle \otimes |D_1\rangle \otimes |D_2\rangle \otimes \cdots \otimes |D_N\rangle \ , \tag{3}$$

where $|\psi_{\text{in}}\rangle$ is the field theory's Heisenberg state and the $|D\rangle$ stand for the Heisenberg states of each device, respectively.

In the simplest case, each device is prepared in a standing Gaussian state whose wave function in momentum representation reads

$$\langle p_n | D_n \rangle = (2\pi\sigma_n^2)^{-1/4} \exp(-p_n^2/4\sigma_n^2) ,\quad n=1,2,\ldots,N . \tag{4}$$

Consequently, the collective wave function of the N devices takes the following form:

$$\langle p_1,p_2,\ldots,p_N | D \rangle = C \exp\left[-\sum_{n=1}^{N} p_n^2/4\sigma_n^2 \right] \tag{5}$$

with the constant $C=\prod_{n=1}^{N}(2\pi\sigma_n^2)^{-1/4}$.

Now let us see how the Heisenberg dynamics works. The interaction (2) will, of course, modify the original field equations and thus the observables $A_n(1)$ as well. These changes are merely those unavoidable perturbations caused by any measuring device to the measured quantum system and always implicit in quantum measurements. It turns out that the quantities measured, in fact, are the Heisenberg operators (1) of the coupled dynamics of the fields and the devices.

The interaction Hamiltonian (2b) sets the dynamics of the devices in motion. Consider the Heisenberg equations of motion:

$$\dot{q}_n = i[H,q_n]=0 , \tag{6a}$$

$$\dot{p}_n = i[H,p_n] = \int g_n(t,\mathbf{x})\phi_n(t,\mathbf{x})d\mathbf{x} , \tag{6b}$$

for $n=1,2,\ldots,N$. Although the Heisenberg operators of the positions remain constants, the momentum operators vary during the period of the coupling. It is crucial to recognize that the excesses of the momenta are of covariant forms:

$$p_n^f = p_n^i + A_n, \quad n=1,2,\ldots,N \tag{7}$$

as is easily seen from Eqs. (1) and (6b). (p_n^i, p_n^f stand for the Heisenberg momenta "before" and "after" the coupling, respectively. Figure 1 shows the invariant space-time regions where the p_n^i's are still constants and, respectively, where the p_n^f's will not change anymore, i.e., are available for readout.)

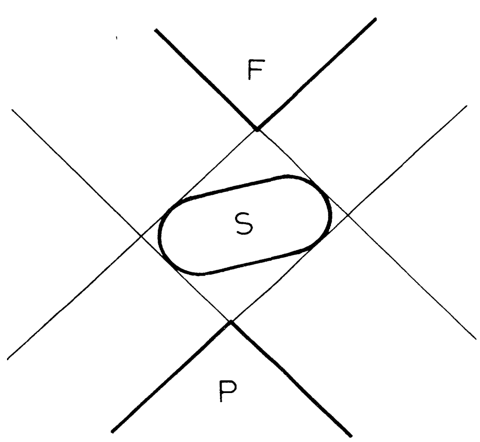

FIG. 1. Invariant past and future of localized observations. S denotes the support of a given localized observable. In the space-time region P, the measurement has not yet started ("before" measurement). In the space-time region F, the measurement is already over ("after" measurement) and the outcome is available everywhere in it.

Canonical momenta always commute with each other; thus, according to the principles of quantum mechanics, the final momentum operators (7) can be simultaneously measured, and the outcomes $\{\overline{p}_1^f,\overline{p}_2^f,\ldots,\overline{p}_N^f\}$ have a definite joint probability distribution $w(\overline{p}_1^f,\overline{p}_2^f,\ldots,\overline{p}_N^f)$. Although p_n^f is not equal to A_n in the strong sense [see Eq. (7)], in a weak sense it is. From Eq. (4), it follows that $\langle p_n^i \rangle = 0$ and $\Delta p_n^i = \sigma_n$, hence one can make the reservation that, up to a precision σ_n, the measured value $\overline{p}_n^f$ yields the measured value $\overline{A}_n$ of the local observable A_n.

One can thus have the desired joint probability distribution of the outcomes of measuring the N local observables (1), in the following form:

$$w(\overline{A}_1,\overline{A}_2,\ldots,\overline{A}_N) = \langle \delta(\overline{A}_1 - p_1^f)\delta(\overline{A}_2 - p_2^f)\cdots\delta(\overline{A}_N - p_N^f)\rangle , \tag{8}$$

where quantum averaging $\langle\ \rangle$ stands for the expectation value in the Heisenberg state (3). This expression is invariant for the change of space-time coordinates. We recall the invariance of Eq. (7) that the invariance of Eq. (8) relies on.

III. ELIMINATION OF DEVICE VARIABLES

The simple form of the devices' Heisenberg states (4) and (5) allows us to perform a partial quantum averaging on the right-hand side (rhs) of Eq. (8). As a result, we shall obtain the simultaneous distribution of the measurement outcomes solely in terms of the field variables.

We solve the task in the interaction picture. The field operators satisfy their original field equations as if they were not coupled to the devices and, vice versa, the device variables q_n, p_n are constant operators again. The quantum state will, however, evolve according to the interaction Lagrangian (Hamiltonian) (2).

Fortunately, we only need the final asymptotic quantum state, since the momentum operators p_n^f on the rhs of Eq. (8) can be taken equally well at the asymptotic future. Hence, using the interaction Lagrangian (2a), we construct the S matrix, which transforms the initial state (3) into the final one:

$$S = T\exp\left[i\int \mathcal{L}(x)dx \right] = T\exp\left[-i\sum_{n=1}^{N} A_n q_n \right] . \tag{9}$$

The symbol T denotes time ordering of the local field operators. Time ordering is a covariant operation, since local field operators commute at spacelike separations.

Recall that we are in the interaction picture; hence all operators but the q_n's have changed their values as compared to the preceding section. They remain identical "before" the couplings are activated (see Fig. 1).

We rewrite the rhs of Eq. (8) in the following way: each Heisenberg operator p_n^f has to be replaced by p_n (which is always constant) while the state (3) is multiplied by the S matrix (9):

$$\langle S^\dagger \delta(\overline{A}_1 - p_1)\delta(\overline{A}_2 - p_2)\cdots\delta(\overline{A}_N - p_N)S\rangle \ . \tag{10}$$

Since the wave function (5) of the devices' initial state is known, we can evaluate the effect of the S matrix (9):

$$\langle p_1, p_2, \ldots, p_N|S|D's\rangle = CT\exp\left[-\sum_{n=1}^{N}\frac{(p_n - A_n)^2}{4\sigma_n^2}\right] . \tag{11}$$

By substituting this result into the expression (10), the device variables p_n $(n=1,2,\ldots,N)$ can be integrated out and we obtain Eq. (8) in terms of the field variables:

$$w(\overline{A}_1, \overline{A}_2, \ldots, \overline{A}_N) = C^2\left\langle \psi_{\rm in}\left|\tilde{T}\exp\left[-\sum_{n=1}^{N}(\overline{A}_n - A_n)^2/4\sigma_n^2\right]T\exp\left[-\sum_{n=1}^{N}(\overline{A}_n - A_n)^2/4\sigma_n^2\right]\right|\psi_{\rm in}\right\rangle , \tag{12}$$

where $\tilde{T}$ denotes anti-time-ordering.

IV. MULTIPLE LOCALIZED QUANTUM OBSERVATIONS IN SPACE TIME

It seems worthwhile to reconsider the various pictures that have so far occurred in the present paper. We defined the von Neumann measurement of localized observables in the Heisenberg picture of the coupled dynamics of fields plus devices (Sec. II). In Sec. III we used the interaction picture, and the coupling between the fields and devices played the role of the interaction. Since we have succeeded in eliminating the device variables, we are henceforth going to consider only the fields of the system in question, whose Heisenberg fields will then be identical to the interaction-picture fields of Sec. III.

Now let us summarize the net results of the preceding section, expressed by Eq. (12). Given a relativistic field theory by its Heisenberg fields and by its Heisenberg state $|\psi_{\rm in}\rangle$, we assume N localized observables

$$A_n = \int g_n(x)\phi_n(x)dx, \quad n=1,2,\ldots,N \tag{13}$$

of compact supports [c.f. Eq. (1)], with measuring accuracies σ_n, respectively. The random outcomes of the measurements are denoted by $\overline{A}_1, \overline{A}_2, \ldots, \overline{A}_N$. The measurements will perturb the system. The perturbation is taken into account by the nonunitary evolution of the state vector. The unnormalized final state vector has the following form:

$$\psi(\text{final}; \overline{A}_1, \overline{A}_2, \ldots, \overline{A}_N) = T\exp\left[-\sum_{n=1}^{N}\frac{(\overline{A}_n - A_n)^2}{4\sigma_n^2}\right]|\psi_{\rm in}\rangle . \tag{14}$$

The simultaneous probability distribution of the measurement outcomes is proportional to the squared norm of this final-state vector [cf. Eq. (12)]:

$$w(\overline{A}_1, \overline{A}_2, \ldots, \overline{A}_N) = C^2\|\psi(\text{final}; \overline{A}_1, \overline{A}_2, \ldots, \overline{A}_N)\|^2 , \tag{15}$$

where $C=\prod_{n=1}^{N}(2\pi\sigma_n^2)^{-1/4}$.

Each measurement outcome $\overline{A}_n$ can be communicated through the future cone of the localized observable A_n (see Fig. 1). Hence they are allowed to influence the Heisenberg fields and also the other devices. For example, A_m may depend on $\overline{A}_n$ if the support of A_m is located "after" the A_n's support.

Thus we have formulated the central result of our work. Equations (13)–(15) can also be recovered as a special case of continuous measurement of fields, in the recent phenomenological theory of Ref. 3. In Sec. III we have shown that this phenomenology follows from the von Neumann measurement of the Heisenberg fields, introduced in Sec. II. (We emphasize that, in contrast, e.g., to Ref. 3 or 4, the present work is not intended to develop a general covariant theory of state reduction.)

One can see that the time ordering T on the rhs of Eq. (14) is responsible for the causality of all perturbations that the measurements could cause each other. For example, assume the simplest case when the support of the localized observable A_{n+1} is entirely in the future cone of the observable A_n, for each $n<N$. Then the rhs of Eq. (14) can be rewritten in factorized form:

$$T\exp[-(\overline{A}_N - A_N)^2/4\sigma_N^2]T\exp[-(\overline{A}_{N-1} - A_{N-1})^2/4\sigma_{N-1}^2]\cdots\times\cdots T\exp[-(\overline{A}_1 - A_1)^2/4\sigma_1^2]|\psi_{\rm in}\rangle . \tag{16}$$

If the supports of the two observables, say of A_k and of A_{k+1}, respectively, become spacelike separated from each other, then the k'th and the $(k+1)$'th factors of the above product become commutative. When the two supports in question overlap, their T factors will lose their factorizability and their contribution on the rhs of Eq. (14) will keep the unfactorized form

$$T\exp[-(\overline{A}_k - A_k)^2/4\sigma_k^2 - (\overline{A}_{k+1} - A_{k+1})^2/4\sigma_{k+1}^2] . \tag{17}$$

Finally we note that if in case of interacting fields the interaction picture is to be used then Eqs. (13-15) can remain unchanged while Eq. (14) should be replaced by

$$\psi(\text{final};\overline{A}_1,\overline{A}_2,\ldots,\overline{A}_N)=T\exp\left[i\int\mathcal{L}_{\text{int}}dx-\sum_{n=1}^{N}(\overline{A}_n-A_n)^2/4\sigma_n^2\right]|\psi_{\text{in}}\rangle\ , \tag{18}$$

where $\mathcal{L}_{\text{int}}(x)$ is the interaction Lagrangian density of the fields.

V. MEASUREMENTS ON THE STATE OF TWO WAVE PACKETS

We are going to illustrate how the machinery of Eqs. (13)–(15) works in the case when $|\psi_{\text{in}}\rangle$ is a one-electron state,

$$|\psi_{\text{in}}\rangle=\sqrt{1/2}|1\rangle+\sqrt{1/2}|2\rangle\ , \tag{19}$$

such that state $|1\rangle$ corresponds to the electron going along a certain world tube "1" (world tube is a smeared world line) while state $|2\rangle$ represents it moving along another well-separated world tube "2."

$$A_n=\lambda_n\int g_n(x)j(x)dx\quad(n=1,2)\ , \tag{20}$$

where $j(x)$ is the electromagnetic current-vector operator, and λ_1 and λ_2 are coupling constants, tuning the sensitivity of the devices. Of course, the weight functions g_1,g_2 also become Lorentz vectors. The supports of A_1,A_2 are disjoint and overlap the world tubes "1" and "2," respectively. We shall say that the nth device has detected the electron whenever the absolute value $|\overline{A}_n|$ of the outcome is much greater than the accuracy σ_n of the measurement, which we shall set by $\sigma_1=\sigma_2=\sigma=1$, for simplicity's sake. Of course, the two devices are not expected to detect simultaneously if they (i.e., the supports of A_1,A_2) are spacelike separated. Let us see whether our theory will respect this claim.

If the supports of A_1 and A_2 are spacelike separated, then, invoking Eqs. (19) and (20), we obtain Eq. (14) in the following form:

$$\psi(\text{final};\overline{A}_1,\overline{A}_2)=\sqrt{\tfrac{1}{2}}T\exp[-(\overline{A}_1-A_1)^2/4]\times T\exp[-(\overline{A}_2-A_2)^2/4](|1\rangle+|2\rangle)\ . \tag{21}$$

Note that the two T factors commute due to the spacelike separation of the supports of A_1 and A_2.

To make our example even simpler, we may reduce the problem to the level of one-electron states, by ignoring all "radiative corrections." In this approximation, $j(x)|\psi_{\text{in}}\rangle$ vanishes everywhere but at the world tubes of the electron propagation. In accordance with this and with Eqs. (20), we have the following trivial equations:

$$T\exp[-(\overline{A}_2-A_2)^2/4]|1\rangle=\exp(-\overline{A}_2^2/4)|1\rangle\ ,$$
$$T\exp[-(\overline{A}_1-A_1)^2/4]|2\rangle=\exp(-\overline{A}_1^2/4)|2\rangle\ . \tag{22}$$

These relations simplify reducing the rhs of Eq. (21) as follows:

$$\psi(\text{final};\overline{A}_1,\overline{A}_2)=\sqrt{\tfrac{1}{2}}\exp(-\overline{A}_2^2/4)T\exp[-(\overline{A}_1-A_1)^2/4]|1\rangle+\sqrt{\tfrac{1}{2}}\exp(-\overline{A}_1^2/4)T\exp[-(\overline{A}_2-A_2)^2/4]|2\rangle\ . \tag{23}$$

By invoking Eq. (15), we obtain the following joint probability distribution of the outcomes $\overline{A}_1,\overline{A}_2$ in the two measurements:

$$w(\overline{A}_1,\overline{A}_2)=\tfrac{1}{2}(2\pi)^{-1}\exp(-\overline{A}_2^2/2)\|T\exp[-(\overline{A}_1-A_1)^2/4]|1\rangle\|^2+\tfrac{1}{2}(2\pi)^{-1}\exp(-\overline{A}_1^2/2)\|T\exp[-(\overline{A}_2-A_2)^2/4]|2\rangle\|^2\ , \tag{24}$$

where we have applied Eqs. (22) again, and used the commutativity between the first T factor and the second anti-T factor (and vice versa), along with the obvious orthogonality relation $\langle 1|2\rangle=0$.

The two terms on the rhs of Eq. (24) describe two peaks, each of the norm $\frac{1}{2}$. The first peak is centered around $\overline{A}_1=\langle 1|A_1|1\rangle, \overline{A}_2=0$ while the other peak is at $\overline{A}_1=0, \overline{A}_2=\langle 2|A_2|2\rangle$. The first peak enhances the first term on the rhs of Eq. (23), while the second peak enhances the second term. These peaks separate well if the quantum expectation values $\langle 1|A_1|1\rangle,\langle 2|A_2|2\rangle$ are much greater than the measurement accuracies, i.e., if the devices' sensitivities and accuracies are high enough.

Hence Eqs. (23) and (24) provide the following statistics for the outcome of the measurements. In most cases only one of the two devices will detect the electron, each with a probability about $\frac{1}{2}$. If the first device detects (i.e., $|\overline{A}_1|\gg 1$), then the final state of the electron is nearly equal to

$$\text{const}\times T\exp[-(\overline{A}_1-A_1)^2/4]|1\rangle\ . \tag{25a}$$

If the second device detects, the final state is much closer to

$$\text{const}\times T\exp[-(\overline{A}_2-A_2)^2/4]|2\rangle\ , \tag{25b}$$

which is orthogonal to the previous state (25a).

There may also occur detections in coincidence. Their rate depends on the sensitivities and on the accuracies of the devices: the more reliable the measurements are, the lower the rate of false coincidences is.

VI. MULTIPLE MEASUREMENTS OF FREE FIELDS

The theory of multiple local measurements, given by Eqs. (13)–(15) in Sec. IV, is calculable rather well when the observed fields ϕ_n of Eq. (13) are free bosonic fields, not necessarily different ones, and the initial quantum state $|\psi_{\rm in}\rangle$ is coherent.

In particular, when $|\psi_{\rm in}\rangle$ is the vacuum state, we guess that Eq. (15) yields a Gaussian form for the outcomes of the measurements:

$$w(\overline{A}_1,\overline{A}_2,\ldots,\overline{A}_N) = \text{const}\times\exp\left[-\tfrac{1}{2}\sum_{\substack{m=1\\n=1}}^{N} C_{mn}\overline{A}_m\overline{A}_n\right], \quad (26)$$

where the coefficients C_{mn} are calculable in a finite number of steps.

Thus we have a relativistic formalism to investigate vacuum fluctuations as experienced by several local observers located arbitrarily in space time. A delicate task would be, at the same time, to test Bell-type inequalities between various joint distributions (26). Another possible application would be to consider devices moving relative to each other.

VII. CONCLUSIONS AND FURTHER REMARKS

The local quantum field theories on the one hand, and the dynamics of simple measuring devices on the other hand, coupled together, have led to the covariant formalism [Eqs. (13)–(15)] of multiple local observations. The observables are localized expressions of the Heisenberg fields. The formalism needs no quantum states (wave functions) but the initial Heisenberg-state $|\psi_{\rm in}\rangle$.

It is worthwhile to note that the introduction of the final state (14) could have been completely ignored. Any kind of physical information is available by using Eq. (12); the interpretation [(14) and (15)] is a matter of convenience. The final state (14) would be relevant in a similar covariant formalism of the weak measurement theory[5] on preselected and postselected ensembles.

The field-theoretical technique that our formalism is based on could pave the way for standard relativistic calculations in the theory of measurements. The proposed language of localized observations may anticipate a language of local "beables,"[6–8] still spoken in outmoded quantum theorists' jargon.

ACKNOWLEDGMENTS

I am deeply indebted to Professor GianCarlo Ghirardi for his hospitality at the Department of Theoretical Physics of the University of Trieste and I also acknowledge the financial support from the Istituto Nazionale di Fisica Nucleare (INFN). I thank G. C. Ghirardi, Renata Grassi, and F. Strocchi for stimulating discussions. The author would also like to thank Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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