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Comments on Omnès' model for uniqueness of data

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Abstract

Standard methods of the theory of permanent state reduction are shown to offer an alternative realization of Omnès' project. Our proposal, as simple as Omnès' one, possesses a closed master equation for the ensemble density operator, assuring causality.

In his recent Letter [1], Omnès has outlined an appealing concept to generate unique data from quantum mechanics modified by a conjectured interaction between space(-time) and the dynamic system evolving in it. A concrete stochastic model has been presented. In our Comment we would like invoke recent ideas (see Ref. [2] and references therein) promoting a concept very much like Omnès' one. The corresponding theory is a realistic candidate to solve the data uniqueness problem [3]. It exploits the theory of permanent state reduction which has emerged from a great deal of parallel efforts (with milestones such as, e.g., Refs. [4-12]). These efforts have recently led to *standard* equations of permanent state reduction, i.e. the *quantum state diffusion* theory [13], extending earlier results [10] to arbitrary dimensions. All these well developed antecedents invite us to revise (also to correct, in some sense) the model [1] of Omnès.

Omnès starts with the *strong consistency condition.* It holds for the quantum state ρ of a macroscopic system if there exists a certain complete and orthogonal set of Hermitian projectors $\{E_\alpha\}$ such that

$$\rho=\sum_\alpha E_\alpha\rho E_\alpha=\sum_\alpha \pi_\alpha\rho_\alpha\,, \tag{1}$$

where $\pi_\alpha=\mathrm{tr}(E_\alpha\rho)$ and $\rho_\alpha=\pi_\alpha^{-1}E_\alpha\rho E_\alpha$. Initially, say at $t=0$, the strong consistency condition (1) may not be satisfied. As time goes on, *decoherence* can successively enforce the approximate validity of (1). In Omnès' model, a conjectured space interaction on the probability parameters π_a assures uniqueness. The π_α perform a specially chosen Brownian motion: one $\pi_\alpha(t)$ will end up becoming 1 with probability $\pi_\alpha(0)$; the other ones will be 0. In such a way the model yields the uniqueness of data concerning the properties $\{E_\alpha\}$.

One can (and has to, as we shall argue later) enrich Omnès' work by assuming a simple master equation for the density operator [7,5,10], assuring the approximate fulfillment of the consistency condition (1),

$$\dot\rho=\mathscr{L}_0\rho-\frac{1}{\tau}\rho+\frac{1}{\tau}\sum_\alpha E_\alpha\rho E_\alpha\,, \tag{2}$$

where $\mathscr{L}_0$ is the linear evolution superoperator of the system itself while the further terms on the r.h.s. come from the conjectured interaction with the space. These terms tend to make ρ block-diagonal on a time scale τ. It will really do it approximately, against the self-

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dynamics $\mathscr{L}_0\rho$ which might usually restore the damped off-diagonals.

Closely related to the master equation (2), let us introduce the following diffusion matrix [10],

$$W_{\alpha\beta}=\frac{1}{\tau}\pi_\alpha\pi_\beta\left(\delta_{\alpha\beta}-\pi_\alpha-\pi_\beta+\sum_\gamma\pi_\gamma^2\right). \quad (3)$$

Observe that the trace $w\equiv\sum_\alpha W_{\alpha\alpha}=\tau^{-1}(1-\sum_\alpha\pi_\alpha^2)$ vanishes *iff* all π_α are 0 but one equals 1. So, w is a good quantity to qualify the non-uniqueness of the data in question. Let us replace Omnès' diffusion matrix in his Eq. (4) [1] by (3),

$$\langle\dot\pi_\alpha(t)\dot\pi_\beta(t')\rangle=2W_{\alpha\beta}\delta(t-t'), \quad (4)$$

for all α, β. For times $t\gg\tau$, the above Brownian motion drives a given $\pi_\alpha(t)$ to 1 with probability $\pi_\alpha(0)$; the other ones tend to 0 (see the proof, e.g., in Ref. [5]). At this level, our model is equivalent to Omnès' one in offering data uniqueness.

What else can our alternative model offer? Most importantly, a closed evolution equation, modified by the conjectured interaction with space, can be constructed for the system's density operator. There are separate paths $\rho(t)$ for each realization of the π_α corresponding to a given pattern of interaction with space. The corresponding paths $\rho(t)$ are random (Brownian) paths embedded in the space of density operators. To specify such a ρ-valued Brownian motion, let us define the diffusion super-matrix and the drift, too, as follows,

$$\langle\dot\rho\otimes\dot\rho\rangle=\frac{1}{\tau}\sum_\alpha[(E_\alpha-\pi_\alpha)\rho\otimes\rho(E_\alpha-\pi_\alpha)$$
$$+\rho(E_\alpha-\pi_\alpha)\otimes(E_\alpha-\pi_\alpha)\rho], \quad (5)$$

$$\langle\dot\rho\rangle=\mathscr{L}_0\langle\rho\rangle-\frac{1}{\tau}\langle\rho\rangle+\frac{1}{\tau}\sum_\alpha E_\alpha\langle\rho\rangle E_\alpha. \quad (6)$$

The above equations each need a comment. The diffusion equation (5) leads directly to the diffusion equation (4) of the probability parameters, via the relations $\pi_\alpha=\mathrm{tr}(E_\alpha\rho)$. The drift term is, as it should be, identical to the r.h.s. of the master equation (2), apart from the notational difference. (In Eqs. (1) and (2), ρ denotes the ensemble state; in the subsequent part, however, the same symbol ρ is to denote the state of a sub-ensemble of a particular interaction pattern, and $\langle\rho\rangle$ must have been introduced for the ensemble state.)

What we have presented so far is an alternative concrete realization of Omnès' concept of data uniqueness from modified quantum mechanics. Due to the achievements of previous parallel researches, perhaps our model goes beyond Omnès' one. In Omnès' model no closed evolution (master) equation exists for the ensemble density operator. This would lead to acausal effects [8,9]. Obviously, only models *without* the master equation allow complete reduction within finite time. Models *with* master equation have asymptotic reduction, not a high price for causality.

The concept of Ref. [1] has a further delicate requirement: only the probabilities π_α of the *collective spatial properties* E_α are to be modified (in favor of the uniqueness of the latter); the internal quantum degrees of freedom must behave completely unchanged. This criterium has been perfectly met in Ref. [2], with a suitable cutoff [14]. The mechanism, however, differs from that of Omnès' model. Let us outline it, changing the original self-consistent presentation and adopting again the terminology and the setting of Ref. [1].

The collective spatial property α is identified with the mass density distribution f of the macroscopic system. That f is not countable needs extra considerations, of course. The space interaction is derived from the Newtonian limit of very tiny stochastic fluctuations of the space–time metric, calculated heuristically (conjectured, after all) [15]. Then the analogue of the master equation (2), is derived routinely [16]. From the master equation, the analogues of diffusion Eqs. (4)-(6) follow automatically, according to the quantum state diffusion theory. As a result of diffusion, the probability parameters π_f of the large scale mass distribution f of the macroscopic system become unique in the very sense of Omnès' concept. At the same time, the *tiny* space fluctuations we started with will not have any observable effect on the microscopic quantum degrees of freedom. The order of magnitude of time necessary for generating a unique position of a macro- or mesoscopic object turns out to be $\hbar/E_{\mathrm{grav}}$ where E_{grav} is the Newtonian gravitational self-energy of the extended object [2]. This yields periods as short as, e.g., 10^{-9} s for ordinary density objects of size $R\approx10^{-2}$ cm (a plausible value for the pointer's thickness). For elementary particles and atomic systems, however, similar effects would require astronomical times and this circumstance reassures quantum non-uniqueness for the microscopic properties.

Finally, we risk a filological remark [17]. For recent years, two independent schools of successful researches have been approaching the same robust problem in quantum theory: schools of *decoherent history* and of *quantum state diffusion*, respectively. Omnès' Letter presents a particular example to put the two together. Our Comment tried, above all, to show that the overlap of the two is more fertile than thought so far. A conceptual comparison and unification of both is to be published [18].

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