Comment on “Quantum Backreaction on ‘Classical’ Variables”

In his recent Letter, Anderson [1] proposed a canonical formalism to couple quantum and (quasi)classical dynamic variables. Although the proposal may promise good physics (cf. Ref. [2]) its mathematical realization seems questionable. It seems the author takes lightly the fact that his quasiclassical bracket [Eq. (2) of [1]] is not antisymmetric. In fact, the lack of antisymmetry leads, in due course, to unacceptable consequences for time evolution of dynamic variables.

Consider the equation of motion, Eq. (4) of the Letter [3]. It will violate the Leibniz rule of differentiation as well as hermiticity of the dynamic variable $A$. In Anderson’s first example, the Hamiltonian is $\frac{1}{2} kp^2$ and yields $\dot{q} = kp$ and $\dot{x} = \frac{1}{2} p^2$ for the time derivatives of the canonical coordinates. From them, applying the Leibniz rule first, we can calculate the (initial) time derivative of the dynamic variable $A$ and obtain $A = \dot{x} q + x \dot{q} + \dot{q} x + q \dot{x} = \frac{1}{2} p^2 q + \frac{1}{2} q p^2 + 2 x k p$. If we calculated $A$ directly from the equation of motion (4) we would obtain a different expression $A = q p^2 + 2 x k p$. It is hardly an acceptable result since it is not Hermitian and the Leibniz rule fails obviously to hold.

Similar effects will occur quite generally. Consider, e.g., a quantum particle and another (quasi)classical one, interacting via translation invariant potential $V(q - x)$. The Letter’s Eq. (4) preserves the total momentum $p + k$ but it leads to an anti-Hermitian time derivative $-i \Delta V(q - x)$ when applied to the square $(p + k)^2$ of the total momentum. Anderson himself notices that, e.g., the energy of a conservative system might not be conserved in his theory. These controversies would not arise at all had we chosen antisymmetric bracket of Aleksandrov [3] and of Boucher and Traschen [4]:

\[
[A, B]_{q-c} = [A, B] + \frac{i}{2} \{A, B\} - \frac{i}{2} \{B, A\}
\]

instead of the Letter’s choice (2). I admit that I have failed to see enough reason of Anderson’s departures from the above bracket, especially since the antisymmetric bracket can even be derived from quantum mechanics in proper (quasi)classical approximation as shown by Aleksandrov [3]. This should be a maximum justification in favor of the antisymmetric bracket even if the Letter’s algebraic construction happened to result in a consistent theory.

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