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COMMENT

Relativistic formulation of quantum state diffusion?

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Abstract. The recently reported relativistic formulation of the well known non-relativistic quantum state diffusion is mistaken. It predicts, for instance, inconsistent measurement outcomes for the same system when seen by two different inertial observers.

1. Introduction

Breuer and Petruccione (BP) [1] have quite recently reported the construction of a relativistic generalization of the stochastic Ito–Schrödinger equations now widely used for open quantum systems. I have found that the model is incorrect. It predicts, for instance, inconsistent experimental results for slowly moving observers. The reader might ignore my own interpretation of BP's *concept* and read directly the *counterexample*.

2. The concept

In Dirac's electron theory, to each space-like hyperplane σ of the Minkowski-space a quantum state ψ is attributed. Such ψ is interpreted as the quantum state which is seen by the inertial observer residing on the hyperplane σ. Let, for concreteness, the hyperplanes be parametrized by their unit normal vectors n and by their distance a from the origin. If we vary the hypersurface, the state vector $\psi(\sigma) \equiv \psi(n, a)$ transforms unitarily:

$$\mathrm{d}\psi = -\mathrm{i}\mathrm{d}a H\psi - \mathrm{i}\mathrm{d}n^{\mu} K_{\mu}\psi \tag{1}$$

where the Hamiltonian H and the boost operator K_{μ} depend also on the hyperplane σ. Equation (1) can be split into two unitary equations:

$$\frac{\partial\psi}{\partial a} = -\mathrm{i}H\psi \tag{2}$$

$$(\delta^{\nu}_{\mu} - n_{\mu}n^{\nu})\frac{\partial\psi}{\partial n^{\nu}} = -\mathrm{i}K_{\mu}\psi. \tag{3}$$

This is standard Dirac theory so far‡. BP will retain the second unitary equation (3) while replacing the first equation (2) by the non-unitary Ito–Schrödinger equation of standard quantum state diffusion:

$$\frac{\partial\psi}{\partial a} = -\mathrm{i}H\psi + \text{nonlinear, stochastic terms.} \tag{4}$$

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‡ In standard Dirac theory, ψ as well as H and K_{μ} have well known expressions. BP have departed from the standard forms. Their wavefunction (11) (or (21)) is not Dirac's one!

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BP also present equations for the average state ρ, derived from equations (3), (4):

$$(\delta_\mu^\nu - n_\mu n^\nu)\frac{\partial\rho}{\partial n^\nu} = -\mathrm{i}[K_\mu, \rho] \tag{5}$$

$$\frac{\partial\rho}{\partial a} = -\mathrm{i}[H, \rho] + L\rho L^\dagger - \tfrac{1}{2}L^\dagger L\rho - \tfrac{1}{2}\rho L^\dagger L. \tag{6}$$

The first equation is unitary, the second is not. BP claim to prove that their equations (3), (4) as well as (5), (6) are compatible, preserve Lorentz-invariance, and that their translation invariance can also be pointed out in a specific sense†. Unfortunately, BP are unaware of further basic features of Dirac's theory. In particular, Dirac's theory assumes that various inertial observers have consistent experimental results. The BP equations fail to do so!

Assume, for instance, that two space-like hyperplanes σ_1 and σ_2 intersect at spacetime point x. Let A be a local scalar observable at x. Then in Dirac's theory the observable A transforms between σ_1 and σ_2 in such a way that its expectation value will not change:

$$\langle A\rangle_{\sigma_1} = \langle A\rangle_{\sigma_2} \tag{7}$$

where $\langle\cdots\rangle_\sigma$ stands for expectation values in quantum states ψ taken on the hyperplane σ. The claimed compatibility and relativistic invariance of the BP equations (3), (4) and (5), (6) are, in themselves, irrelevant since the physical consistency (7) of Dirac's theory is lost. How fatally it is lost the reader shall understand on a non-relativistic application.

3. The counterexample

We do not need BP's stochastic equations but the ensemble averaged ones (64), (65) (cf my equations (5), (6)), together with equation (63) which relates measured expectation values to the density operator (in the standard way, this time). Let us apply these equations to a typical non-relativistic situation. Consider a free non-relativistic electron in the authors' reference system $n_0 = (1, 0, 0, 0)$. We call the observer who rests in this reference system *R-observer*. Soon we need a moving *M-observer*, too, who rests in the reference system $n_v = (1, v/c, 0, 0)$ moving with a *small* non-relativistic velocity v with respect to the R-observer. The electron is non-relativisitic for both observers. Assume it remains *localized* along the right x-axis at $x \approx \ell$. Both observers will measure the same spin-observable:

$$A = |\uparrow\rangle\langle\downarrow| + \text{h.c.} \tag{8}$$

They will measure *in coincidence*! For instance, the M-observer switches on his apparatus at time $a = 0$, in coincidence with the R-observer's apparatus at (his/her) time $a_0 = \ell v/c^2$. We expect that the two measurements lead to the same result in the the non-relativistic limit $v/c \to 0$:

$$\mathrm{tr}\{A\rho(n_v, 0)\} = \mathrm{tr}\{A\rho(n_0, a_0)\} + \mathcal{O}(v/c). \tag{9}$$

Note that we can choose an arbitrary large distance ℓ so that $a_0 = \ell v/c^2$ remains relevant (e.g. constant) even for $v/c \to 0$.

† That translation invariance would be broken by the author's dynamic equations we demonstrate easily. For instance, let the parameters of three space-like hyperplanes $\sigma_1, \sigma_2, \sigma_3$ be, in obvious notation, chosen as follows: $n_1 = n_2 = (1, 0, 0, 0)$, $n_3 = (2/\sqrt{3}, 0, 0, 1/\sqrt{3})$, $a_1 = 1$, $a_2 = 2$, $a_3 = 2/\sqrt{3}$. Let the two other reference frames R_1, R_2 have $(0, 0, 0, \sqrt{3} - 2)$ and $(0, 0, 0, 2\sqrt{3} - 2)$ as their shifted origins. Elementary calculations show that $a_1 = a_3$ in R_1 and $a_2 = a_3$ in R_2, while n_1, n_2, n_3 do not change. Consider the averaged states ρ_1, ρ_2, ρ_3, respectively, on the three hyperplanes. We apply equation (65) of BP in frame R_1 to relate ρ_1 and ρ_3: they turn out to be unitary equivalent. Similarly, we apply equation (65) in frame R_2 to ρ_2 and ρ_3: they, too, will be unitary equivalent. This implies that ρ_1 and ρ_2 must be unitary equivalent. But these latters are related non-unitarily by equation (64) (no matter which inertial frame is chosen).

For the electron's initial state in the frame of the R-observer we choose a superposition of the spin-up, spin-down states:

$$\rho(n_0, 0) = \tfrac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|). \tag{10}$$

(Observe that, initially, the R-observer would measure 1 for the expectation value of A.) We define the following Lindblad generator:

$$L(n_0, a) \equiv \frac{1}{\tau}|\uparrow\rangle\langle\uparrow|. \tag{11}$$

Let the reduction time τ, controlling the strength of the quantum state diffusion, satisfy the condition $\tau << a_0$. Then equation (64) turns the initial pure state (10) into the mixed one:

$$\rho(n_0, a_0) \approx \tfrac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|). \tag{12}$$

At time $a_0 = \ell v/c^2$, the quantum state has reduced to the mixture of $|\uparrow\rangle$ and $|\downarrow\rangle$. The expectation value of the Hermitian observable A becomes zero for the the R-observer's measurement.

Let's turn to the M-observer. According to the equation (65), he/she initially sees the state

$$\rho(n_v, 0) = \rho(n_0, 0) + \mathcal{O}(v/c) \tag{13}$$

which is identical to the rest observer's initial state upto terms of the order of v/c. So, the M-observer measures 1 up to terms $\mathcal{O}(v/c)$.

In summary, we can write

$$\langle A\rangle_{(n_v,0)} - \langle A\rangle_{(n_0,a_0)} = 1 + \mathcal{O}(v/c) \tag{14}$$

which, indeed, contradicts† the invariance condition (7) of Dirac's theory.

4. Conclusion

Abandoning the standard Dirac wavefunctions has not been the main reason for BP's failure. The fatal reason is that relativistic Wiener processes do not exist but trivial ones do, as is particularily shown in the context of continuous wavefunction reduction theories by, e.g. Pearle [2] and myself [3]. Relativistic models of continuous reduction theories should first relax the Markovian approximation, cf [4].

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References

[1] Breuer H-P and Petruccione F 1998 *J. Phys. A: Math. Gen.* **31** 33
[2] Pearle P 1990 *Sixty-Two Years of Uncertainty: Historical, Philosophical and Physics Inquires into the Foundation of Quantum Physics* ed A I Miller (New York: Plenum)
[3] Diósi L 1990 *Phys. Rev.* A **42** 5086
[4] Diósi L and Strunz W 197 *Phys. Lett.* **235** 569
Strunz W, Diósi L and Gisin N *Phys. Rev.* A to appear

† ℓ or $1/\tau$ (or both) diverge in the limit $v/c \to 0$. This does, however, not invalidate our counterexample. First, it makes sense to assign any finite values to ℓ and $1/\tau$. Second, the violation of equation (7) by equation (14) is obvious already at finite v/c, i.e. at finite ℓ and $1/\tau$.