20 May 1999

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 454 (1999) 381–385

# KNO scaling in the neutral pion multiplicity distributions for $\pi^- p$ interactions at 40 and 250 GeV/c

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Received 15 February 1999; received in revised form 26 March 1999
Editor: L. Montanet

**Abstract**

We analysed the binomial multiplicity moments of the neutral pions, using an extension of the generating functional technique for detection losses. We applied this model-independent method to the individual $\gamma$ weights of 10000 events of $\pi^- p$ interactions at 40 GeV/c. We compared the obtained results to those of 250 GeV/c. We used the FRITIOF and a shifted KW distribution to describe the data. © 1999 Elsevier Science B.V. All rights reserved.

## 1. Introduction

Investigation of multiplicity distributions have so far been done mostly for charged particles. A comprehensive study of functional forms and fits for data have been lately reviewed in Warsaw [1]. Less information is available on production of $\pi^0$ meson. The $\pi^0$ decay product, gammas, may be observed in bubble chamber with low efficiency.

We shall analyze the moments of multiplicity distributions of $\gamma$-s for $\pi^- p$ and $\pi^- n$ interactions at 40 GeV. We will use the data from the Dubna 2m propane bubble chamber. The statistics includes about 10000 events for $\pi^- p$ and 3600 events for $\pi^- n$ interactions. We have 25% mean efficiency [2]. Every individual conversion weight of $\gamma$ is at our disposal from the data summary tape (DST).

Since the detection probability is lower than 100% the measured distribution is different from the true one. The problem is the following: how to take into account this difference in the analysis of the data.

## 2. General method of the correction of detector losses

The generating functional technique is both extremely general and useful. On the one hand it can be used to prove important theorems [3], on the other hand it permits the description of detection losses too [4]. In order to give some insight into this problem we will show a general model independent method by Diósi [4]. We shall recall some statements from these papers.

The true $n$-particle exclusive distribution $s^{(n)}$ with the proper normalization is the following:

$$\int s^{(n)}(k_1,\dots,k_n)\,dk_1,\dots dk_n = n!\,p_n \qquad (1)$$

where $p_n$ is the probability of fixed $n$ multiplicity and $k_n$ is the momentum of the $n$-th particle.


PII: S0370-2693(99)00388-3

With the aid of the detection probabilities $\omega = \omega(k)$-s we can describe the measured exclusive distribution

$$\bar{s}^{(n)}(k_1,\dots,k_n) = \sum_{m>n} \frac{1}{(m-n)!} \int s^{(m)}(k_1,\dots,k_m) \times \omega(k_1)\dots\omega(k_n) \times \tilde{\omega}(k_{n+1})\,dk_{n+1}\dots\tilde{\omega}(k_m)\,dk_m \quad (2)$$

where $\tilde{\omega} = 1 - \omega$.

By definition the generating functional:

$$F[h(.)] = \sum_{n=0} \frac{1}{n!} \int s^{(n)}(k_1,\dots,k_n) \times h(k_1)\,dk_1\dots h(k_n)\,dk_n \quad (3)$$

The exclusive distribution can be expressed by the derivatives of the generating functional:

$$s^{(n)}(k_1,\dots,k_n) = \left.\frac{\delta^n F}{\delta h(k_1)\dots\delta h(k_n)}\right|_{h=0} \quad (4)$$

The generating functional of the measurable distribution:

$$\bar{F}[\bar{h}(.)] = \sum_{n=0} \frac{1}{n!} \int \bar{s}^{(n)}(k_1,\dots,k_n) \times \bar{h}(k_1)\,dk_1\dots\bar{h}(k_n)\,dk_n \quad (5)$$

Using these eqs. the fundamental reconstruction formula can be obtained as:

$$\bar{F}[\bar{h}(.)] = F[\omega(.)\bar{h}(.) + 1 - \omega(.)] \quad (6)$$

$$F[h(.)] = \bar{F}[\omega^{-1}(.)h(.) + 1 - \omega^{-1}(.)] \quad (7)$$

We can check these formulae: if the argument $h(.)$ or $\bar{h}(.) = 1$ then $\bar{F}[1] = F[1]$ for arbitrary $\omega(.)$, and if $\omega(.) = 1$ then $F = \bar{F}$.

We should note that (6) and (7) is a generalization of Nifenecker's results for the generating function [5] if $h(.) \to z$; $\omega(.) \to$ constant, where the constant $\omega$ is the neutron detector efficiency.

Armed with this technique we invert Eq. (2):

$$s^{(n)}(k_1,\dots,k_n) = w(k_1)\dots w(k_n) \sum_{i=0} \frac{(-1)^i}{i!} \times \int \bar{s}^{(n+i)}(k_1,\dots,k_{n+i}) \times \tilde{w}(k_{n+1})\,dk_{n+1}\dots\tilde{w}(k_{n+i})\,dk_{n+i} \quad (8)$$

Where:

$$w = \frac{1}{\omega}; \quad \tilde{w} = w - 1.$$

Taking a simple case, if $\omega =$ constant then from Eq. (2)

$$\bar{p}_{\bar{n}} = \sum_{n \ge \bar{n}} p_n \binom{n}{\bar{n}} \omega^{\bar{n}} (1-\omega)^{n-\bar{n}} \quad (9)$$

where $\bar{n}$ is the measured multiplicity, and with

$$\tilde{w} = \frac{1}{\omega} - 1 = -\left(1 - \frac{1}{\omega}\right)$$

we can calculate from (8)

$$p_n = \frac{1}{n!} \sum_{i=0} \left(\frac{1}{\omega}\right)^n \left(1 - \frac{1}{\omega}\right)^i \frac{1}{i!} (n+i)!\,\bar{p}_{n+i}$$

Substituting $n + i \to \bar{n}$ we obtain the so called Diven formula [6] for

$$p_n = \sum_{\bar{n} \ge n} \bar{p}_{\bar{n}} \binom{\bar{n}}{n} \left(\frac{1}{\omega}\right)^n \left(1 - \frac{1}{\omega}\right)^{\bar{n}-n} \quad (10)$$

If $\omega$ is small we can arrive at a solution containing large oscillating and sometimes even negative components of $p_n$[7]. On the other hand it was demonstrated that the moments ($\langle n \rangle$ and $D_n^2$) of the same $p_n$ prove to be very stable in the case of multiplicities of fission neutrons [7]. We show these results in Table 1.

The same conclusion can be drawn from analytical calculation for Poisson distribution [4].

In the general case $\omega = \omega(k)$ we can prove [4] that the true binomial moment

$$B_j = \frac{1}{j!} \int w_1 \cdots w_j \bar{f}_j(w_1,\dots,w_j)\,dw_1 \cdots dw_j \quad (11)$$

where $\bar{f}_j$ is the measured inclusive distribution.

Table 1
True moments of distributions for different experiments [7]

| No. exp. | $\omega$% | No. events | $\langle n \rangle$ | $D_n^2$ |
|---|---|---|---|---|
| 1 | 48.3 | 7169 | $2.690 \pm 0.036$ | $1.388 \pm 0.076$ |
| 2 | 48.2 | 65015 | $2.690 \pm 0.015$ | $1.290 \pm 0.025$ |
| 3 | 44.4 | 6928 | $2.690 \pm 0.038$ | $1.212 \pm 0.084$ |
| 4 | 39.9 | 20359 | $2.690 \pm 0.025$ | $1.173 \pm 0.057$ |
| 5 | 23.7 | 4039 | $2.690 \pm 0.071$ | $1.230 \pm 0.272$ |
| 6 | 22.0 | 4039 | $2.690 \pm 0.075$ | $1.587 \pm 0.311$ |

## 3. Gamma moments from the data summary tape

$\omega$ is the probability of $e^+e^-$ pair creation of a secondary $\gamma$:

$$\omega = 1 - \exp\left(\frac{L_x}{L}\right) = \frac{1}{w} \tag{12}$$

where $L_x$ is the potential length, $L = L(k)$ is the radiation length and $w$ denotes the conversion weight. The general prescription for the true binomial moments [4]

$$B_k = \left\langle \tilde{B}_k \right\rangle_{\mathrm{DST}} \tag{13}$$

where $\tilde{B}$ is the following for every event:

$$\widetilde{B_k} = \begin{cases} 0 & \text{if} \quad \bar{n} < k \\ \sum_{\alpha} w_{i_1} \cdot w_{i_2} \cdots w_{i_k} & \end{cases} \tag{14}$$

where $\alpha = \binom{\bar{n}}{k}$ and $\bar{n}$ is the detected number of gammas in an event and the summation goes for all the $\alpha$ different set of indices. E.g. $\bar{n} = 4$, $k = 2$, $\alpha = 6$

$$\tilde{B}_2 = w_1 w_2 + w_1 w_3 + w_1 w_4 + w_2 w_3 + w_2 w_4 + w_3 w_4$$

In addition to $B_k$ we calculated the errors and the correlations of $B_k$ from the DST:

$$(\Delta B_k)^2 = \left\langle \left( \tilde{B}_k - B_k \right)^2 \right\rangle_{\mathrm{DST}} \tag{15}$$

$$\Delta B_k \, \Delta B_l = \left\langle \left( \tilde{B}_k - B_k \right) \left( \tilde{B}_l - B_l \right) \right\rangle_{\mathrm{DST}} \tag{16}$$

Thus we obtained significant results for the first three binomial moments. Assuming that all gammas come from neutral pions:

$$p_{2n}^{(\gamma)} = p_n^{(\pi^0)}, \quad p_{2n+1}^{(\gamma)} = 0$$

we can calculate arbitrary $\pi^0$ moments using the proper generating function:

$$G^{(\gamma)}(z) = \sum p_n^{(\gamma)} z^n$$

and

$$G^{(\pi^0)}(z) = \sum p_n^{(\pi^0)} z^n = G^{(\gamma)}(z^{1/2})$$

In this way we calculated the average multiplicity in accordance with earlier published data [2] and

$$\frac{\langle n^{\pi^+} \rangle + \langle n^{\pi^-} \rangle}{2} = \frac{2.18 + 2.81}{2} = 2.5$$

which is equal to $\langle n^{(\pi^0)} \rangle = 2.49 \pm 0.04$ for $\pi^- p$ interactions at 40GeV. The behaviour of $\pi^- n$ data on $c_2 = 1.69 \pm 0.11$ and $c_3 = 3.61 \pm 0.43$ are similar to $c_2 = 1.64 \pm 0.07$ and $c_3 = 3.38 \pm 0.32$ found for $\pi^- p$ data.

We can compare our results with the 5m hydrogen bubble chamber data on $\pi^- p$ at 250 GeV [8]. The statistics is larger (20000 events) but $\langle \omega \rangle = 14\%$ is smaller, the experimental details have been described in [8].

At 250 GeV $c_2 = 1.55 \pm 0.12$ and $c_3 = 3.04 \pm 0.51$.

Within the errors the KNO moments $c_2$ and $c_3$ do not show a violation of KNO scaling [9] between 40 and 250 GeV.

## 4. FRITIOF and shifted KW distribution for $\psi^{(\pi^0)}$

We have generated 14500 FRITIOF events at every sample. The FRITIOF reproduces the mean multiplicities, the second scaled moments $c_2$ and $c_3$, which can be seen in Table 2.

We should note that a shifted KW distribution has successfully described [10] the KNO moments and the distributions for the single hemisphere data of DELPHI and OPAL collaborations. In order to predict the third scaled moments $c_3$–s we use a shifted KW distribution with parameter $m = 2$, which has proved to be successful for charged particles in inelastic pp collisions [11,12]. We carry out a shift with $+1$, since the KW distribution is equal to zero in $n = 0$, and we use the so called stick approximation. It means that we use the continuous (denoted by $^*$) KW distribution

$$P_n^* = \frac{m}{\langle n \rangle_* \, \Gamma(A)} F^A z^{mA-1} \exp[-F z^m]$$

where

$$F = \frac{\Gamma^m\left(A + \frac{1}{m}\right)}{\Gamma^m(A)}, \quad z = \frac{n}{\langle n \rangle_*}, \quad m = 2$$

Taking the sum of $P_n^*$ for $n = 0,1,2,\ldots$ we can define the remaining $\epsilon$ in the Euler-MacLaurin formula:

$$\sum_n P_n^*(A, \langle n \rangle_*) = 1 + \epsilon$$

Table 2
Calculated parameters of shifted KW and FRITIOF distributions for $c_3$

| Experiment | $1+\epsilon$ | $A$ | $\langle n \rangle_*$ | $c_3$ [pred] | $c_3$ [exp] | $c_3$ [FRITI] |
|---|---|---|---|---|---|---|
| $\pi^- n$ 40 GeV | 0.955 | 0.71 | 3.16 | 3.57 | $3.61 \pm 0.43$ | $2.91 \pm 0.11$ |
| $\pi^- p$ 40 GeV | 0.965 | 0.75 | 3.37 | 3.38 | $3.38 \pm 0.32$ | $2.85 \pm 0.10$ |
| $\pi^- p$ 250 GeV | 0.978 | 0.77 | 4.43 | 2.98 | $3.04 \pm 0.51$ | $2.53 \pm 0.09$ |

We can form the discrete probabilities

$$P_n = \frac{P^*_{n+1}(A, \langle n \rangle_*)}{1+\epsilon}$$

fulfilling the requirements:

$$\langle n \rangle = \sum n P_n, \quad c_2 = \frac{\sum n^2 P_n}{\langle n \rangle^2}$$

We display the calculated parameters $\epsilon$, $\langle n \rangle_*$ and $A$ in Table 2.

Using these parameters we can predict $c_3$, which are in good agreement with the true experimental data. With the shifted $P^*_{n+1}$ we create the continuous KNO function

$$\psi(\tilde{z}) = \langle n \rangle_* P^*_{n+1} = (\langle n \rangle + 1)(1+\epsilon)^2 P_n(\tilde{z}, A)$$

where

$$\langle n \rangle_* = (\langle n \rangle + 1)(1+\epsilon), \quad \frac{n+1}{\langle n \rangle_*} = \hat{z}$$

A KNO function with parameters $A = 0.743$ and $m = 2$ represents the calculated points In Fig. 1. KNO scaling is seen as a function of $\hat{z}$ at two energies.

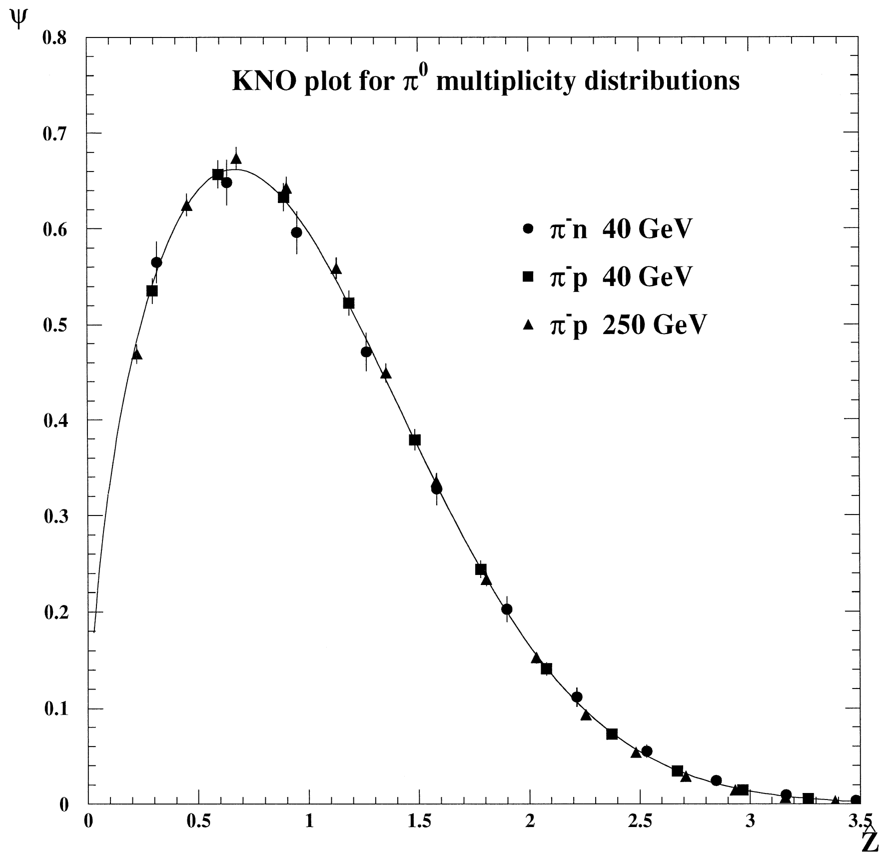


Fig. 1.

## 5. Conclusions

1. The use of generating functional technique provides an elegant and concise derivation of formulae relating the true distribution function to the measured ones. With the aid of this general method we have immediately obtained Nifeneckers [5] results on the generator functions and Diven [6] formula for the true distribution of fission neutron multiplicity, as a special case: $\omega =$ constant.
2. Since the mean detection efficiency for $\gamma$ is 25% in propane (at 40 GeV) the reconstruction of the true multiplicity distribution from the measured one is not efficient, but still the first three binomial moments of the true distribution can be obtained.
3. We have compared our results at 40 GeV with the published results at 250 GeV. It was found that the KNO moments $c_2$ and $c_3$ of $\pi^0$ are consistent with KNO scaling within their large (15 percent) errors up to third moment, in a model-independent way.
4. The experimental KNO moments are in agreement with the FRITIOF simulation and with a shifted KW distribution containing the first two binomial moments as an input. It is remarkable that this KW distribution can predict the measured $c_3$. All the points of $\pi^0$ multiplicity distributions calculated as a shifted KW show a clear scaling curve between 40 and 250 GeV.

## Acknowledgements

This work was supported by the Hungarian Academy of Sciences (Grant No. OTKA T 016377).

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