Kinematics and dynamics of independent pion emission

Lajos Diósi*
Institute for Advanced Study, Wallotstrasse 19, D-14193 Berlin, Germany
and Research Institute for Particle and Nuclear Physics, H-1525 Budapest 114, POB 49, Hungary
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Multiparticle boson states, proposed recently for ''independently'' emitted pions in heavy-ion collisions, are reconsidered in standard second quantized formalism and shown to emerge from a simplistic chaotic current dynamics. Compact equations relate the density operator, the generating functional of multiparticle counts, and the correlator of the external current to each other. ''Bose-Einstein condensation'' is related to the external pulse. A quantum master equation is advocated for future Monte Carlo simulations.

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Multiparticle production in high-energy particle collisions is dominated by classical statistics. Bose-Einstein statistics of pions, nevertheless, proved to lead to quantum coherence effects which survive in the final multiparticle states [1]. The corresponding mechanism can be handled within standard quantum statistics. Yet, some recent works [2,3] continued, mainly for historic reasons as Weiner [4] points out, to use the ''traditional'' method of wave function with tedious explicit Bose-Einstein symmetrization. We will see how standard second quantization methods lead to the correct results in a shorter and clearer way. I discuss the dynamical conditions of Bose-Einstein condensation, and I outline a master equation suitable for Monte Carlo simulations.

For the concentrated study of the effects of Bose statistics, a simple scheme of ''independent'' emission has been proposed [3,5–7]. I recapitulate the features of these multiparticle states, sparing the burden of separate ''symmetrization.'' I then introduce the generating functional of the measured counts [8] directly in second quantized formalism. Introducing chaotic classical currents, advocated, e.g., by Ref. [1] and used, e.g., in [9,10], I construct the simplest quantum dynamics reproducing the corresponding multiparticle quantum states. The measured multidetector counts turn out to be identical to the corresponding spectral intensities of the effective current. I show that the existence of a Bose-Einstein condensate imposes explicit analytic constraints on the intensity and on the spectrum of the external effective current. Finally, I generalize the simple quantum dynamics and propose a quantum master equation suitable to the efficient Monte Carlo simulation of the multiparticle density matrix itself. The paper concludes with a summary.

When searching for a class of multiparticle density operators $\hat{\rho}$ representing independent bosons, consider first the Gibbs canonical state for noninteracting bosons at inverse temperature β. The bosons remain independent if, formally, we assign different instantaneous temperatures $\beta_{\mathbf{k}}$ to each mode $\mathbf{k}$, i.e., we assume $\hat{\rho}\sim\exp(-\Sigma_{\mathbf{k}}\beta_{\mathbf{k}}\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}})$. Moreover, the bosons remain independent if we assign different temperatures to a generic (maybe nonstationary) set of orthogonal modes instead of the momentum eigenstates. Hence, we arrive at the following class of ''independent multiboson states'' (IMS):

$$\hat{\rho}=\det(1-e^{-\beta})\exp\left(-\sum_{\mathbf{k},\mathbf{k}'}\beta_{\mathbf{k}'\mathbf{k}}\hat{a}_{\mathbf{k}'}^\dagger\hat{a}_{\mathbf{k}}\right), \tag{1}$$

where β is a positive matrix. Let us define the one-particle density matrix from the above state:

$$\rho_{\mathbf{k}'\mathbf{k}}=\frac{\langle\mathbf{k}'|\hat{\rho}|\mathbf{k}\rangle}{\sum_{\mathbf{k}}\langle\mathbf{k}|\hat{\rho}|\mathbf{k}\rangle}. \tag{2}$$

Using Eq. (1), we find the following matrix relation:

$$e^{-\beta}=\nu\rho\quad(\nu=\mathrm{tr}\,e^{-\beta}). \tag{3}$$

The IMS (1) can be rewritten in terms of the one-particle density matrix ρ and the parameter ν (whose physical interpretation remains a bit involved):

$$\hat{\rho}=\det(1-e^{-\nu\rho})\exp\left(\nu\sum_{\mathbf{k},\mathbf{k}'}\rho_{\mathbf{k}'\mathbf{k}}\hat{a}_{\mathbf{k}'}^\dagger\otimes\hat{a}_{\mathbf{k}}\right)\hat{\rho}_0. \tag{4}$$

This form might give an insight into the kinematics of the particle creation from the vacuum $\hat{\rho}_0$.

We have to note that the IMS are nonstationary quantum states. Yet, the measured quantities $\hat{n}_{\mathbf{k}}=\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}}$ are not sensitive to the time evolution of the IMS $\hat{\rho}$. This will be formulated later in the paper.

We introduce generating functionals for the multiparticle final-state momentum distributions. A compact heuristic form of definition is the following:

$$G[u]=\mathrm{tr}\left(\hat{\rho}\prod_{\mathbf{k}}(u_{\mathbf{k}})^{\hat{n}_{\mathbf{k}}}\right), \tag{5}$$

where $u_{\mathbf{k}}$ are auxiliary variables and $\hat{n}_{\mathbf{k}}=\hat{a}_{\mathbf{k}}^\dagger\hat{a}_{\mathbf{k}}$. If we introduce the diagonal matrix u by $u_{\mathbf{k}'\mathbf{k}}=\delta_{\mathbf{k}'\mathbf{k}}u_{\mathbf{k}}$ then, using the IMS density operator (4), the generating functional takes the following form:

$$G[u]=\frac{\det(1-\nu\rho)}{\det(1-\nu u\rho)}. \tag{6}$$

The logarithmic generating functional $g=\ln G$ can be expressed through its Taylor expansion in a transparent way [5,7]:

*Electronic address: diosi@rmki.kfki.hu

$$g[u]=\sum_{r=1}^{\infty}\frac{\nu^r}{r}[\mathrm{tr}(u\rho)^r-\mathrm{tr}\,\rho^r]. \tag{7}$$

For $u_{\mathbf{k}}\equiv u$, it yields the (logarithmic) multiplicity generating function

$$g(u)=\sum_{r=1}^{\infty}\frac{\nu^r}{r}\mathrm{tr}\rho^r(u^r-1) \tag{8}$$

whose Taylor coefficients are the combinants (cf. [11]).

The derivatives of the generating functionals at $u=0$ yield the *exclusive* distribution/correlation functions. In experiments, we can easily measure the *inclusive* distributions instead, which are the derivatives at $u=1$ [8]. To make these derivations more convenient, let us substitute $\nu\rho$ in the generating functionals (6)–(8) by $\nu\rho=\alpha/(1+\alpha)$, where α will be the correlation matrix of currents mentioned later:

$$G[u]=\frac{1}{\det[1-(u-1)\alpha)]}, \tag{9}$$

$$g[u]=\sum_{r=1}^{\infty}\frac{1}{r}\mathrm{tr}(u\alpha-\alpha)^r, \tag{10}$$

$$g(u)=\sum_{r=1}^{\infty}\frac{1}{r}\mathrm{tr}\,\alpha^r(u-1)^r. \tag{11}$$

Comparing these expressions with Eqs. (6)–(8) we see that the inclusive distributions/correlations will depend on the current correlation matrix α exactly the same way as the exclusive distributions/correlations depend on (ν times) the one-particle density matrix ρ.

Multiparticle production cannot be derived from ''first principles.'' I can certainly not overcome the well-known difficulties. Instead, I present the simplest quantum dynamics which produces exactly the class (1) of IMS. I postulate the following effective Hamiltonian:

$$\hat{H}=\delta(t)\sum_{\mathbf{k}}(J_{\mathbf{k}}^{\star}\hat{a}_{\mathbf{k}}+J_{\mathbf{k}}\hat{a}_{\mathbf{k}}^{\dagger}), \tag{12}$$

where $J_{\mathbf{k}}$ denote the Fourier components of a certain effective external field J exciting the boson field. The ''current'' $J(x)=J(\mathbf{x})\delta(t)$ is nonzero in the collision area and we assume that the collision time can be taken infinitely short. Let us calculate the unitary effect of the Hamiltonian (5) on the vacuum:

$$|\psi_J\rangle=\exp\left(-i\sum_{\mathbf{k}}(J_{\mathbf{k}}^{\star}\hat{a}_{\mathbf{k}}+J_{\mathbf{k}}\hat{a}_{\mathbf{k}}^{\dagger})\right)|0\rangle$$
$$=\exp\left(-\frac{1}{2}\sum_{\mathbf{k}}|J_{\mathbf{k}}|^2-i\sum_{\mathbf{k}}J_{\mathbf{k}}\hat{a}_{\mathbf{k}}^{\dagger}\right)|0\rangle, \tag{13}$$

which is otherwise a product coherent state $\Pi_{\mathbf{k}}\otimes|-iJ_{\mathbf{k}}\rangle$.

These final states $|\psi_J\rangle$ are pure states whereas the IMS are mixed ones. Obviously, no unitary dynamics can create mixed states from pure ones. Therefore, I consider unitary dynamics in *random* external fields: I assume a Gaussian distribution for the stochastic fluctuations of the current J. Let the mean values $M[J_{\mathbf{k}}]$ be always zero. Also we assume that $M[J_{\mathbf{k}'}J_{\mathbf{k}}]\equiv 0$, which is equivalent to a random phase for all $J_{\mathbf{k}}$. We denote the only nonvanishing correlations by the non-negative Hermitian matrix α:

$$M[J_{\mathbf{k}'}J_{\mathbf{k}}^{\star}]=\alpha_{\mathbf{k}'\mathbf{k}}. \tag{14}$$

After these preparations, we can define the density operator $\hat{\rho}$ of the final state as the stochastic mean value of the pure coherent states (13):

$$\hat{\rho}=M[|\psi_J\rangle\langle\psi_j|]. \tag{15}$$

Substituting Eq. (13) and taking the stochastic mean over J of the Gaussian correlation (14) we are led directly to form (4) of the IMS density operators. The one-particle density matrix ρ and the parameter ν are related to the correlation matrix α of the current by easily invertible matrix relations:

$$\nu\rho=\frac{\alpha}{1+\alpha},\quad \alpha=\frac{\nu\rho}{1-\nu\rho}. \tag{16}$$

Measuring the one-particle density matrix we could, up to the validity of the model, calculate the structure of the external current. Although such a measurement is (so far) not completely possible we will see later that the inclusive correlation function gives the modulus of α directly. It is also seen from Eqs. (16) that a Gaussian shape, like [7]

$$\rho_{\mathbf{k}'\mathbf{k}}\sim\exp\left(-\frac{1}{2\Delta^2}\mathbf{k}_+^2-\frac{1}{2}R^2\mathbf{k}_-^2\right),$$
$$\mathbf{k}_+=\frac{\mathbf{k}+\mathbf{k}'}{2},\quad \mathbf{k}_-=\mathbf{k}-\mathbf{k}', \tag{17}$$

for the one-particle density matrix is not compatible with a Gaussian-shaped current correlation matrix $\alpha_{\mathbf{k}'\mathbf{k}}$ and *vice versa*.

The final-state distributions in IMS can be directly related to the currents J. The generating functional (5) can conveniently be re-expressed as an averaged functional over the fluctuating external current J:

$$G[u]=M\left[\exp\left(\sum_{\mathbf{k}}(u_{\mathbf{k}}-1)|J_{\mathbf{k}}|^2\right)\right], \tag{18}$$

which is of course equivalent to Eqs. (6) or (9).

The above equation has numerous useful consequences. The multiplicity distribution can be written in this form:

$$p_r=M\left[\left(\sum|J|^2\right)^r\exp\left(-\sum|J|^2\right)\right], \tag{19}$$

while the factorial moments take the same form but without the exponential factor $\exp(-\Sigma|J|^2)$, i.e.,

$$F_r=M\left[\left(\sum|J|^2\right)^r\right]. \tag{20}$$

This phenomenon also characterizes the differences between the expressions of the exclusive and the inclusive distribution functions, respectively,

$$f(1,2,\ldots,r)=M[|J_1|^2,|J_2|^2,\ldots,|J_r|^2 \times \begin{cases} \exp(-\Sigma|J|^2)] & \text{(exclusive)} \\ ] & \text{(inclusive)} \end{cases} \tag{21}$$

as well as of the correlation functions. In particular, the inclusive correlation functions take the following form:

$$C(1,2,\ldots,r)=M[|J_1|^2|J_2|^2\ldots|J_r|^2]_c, \tag{22}$$

the exclusive ones would contain the ominous exponential factor, too. The notation $M[\ldots]_c$ means that in the ''expectation value'' only the ''connected grafs'' are to be taken into the account. In the case of Eq. (22) it yields $(r-1)!$ ''cycles,'' i.e., the ''cycle'' $\alpha_{12}\alpha_{23}\ldots\alpha_{r1}$ and its variants for permutations of $2,\ldots,r$ [5].

One can easily summarize the main result of this section as follows. The counts $n_{\mathbf{k}}$, measured simultaneously in a collision event, are *statistically identical* to the corresponding spectral intensities $|J_{\mathbf{k}}|^2$. Like their distributions, their corresponding moments are identical as well:

$$\langle n_1 n_2\ldots n_r\rangle=M[|J_1|^2|J_2|^2\ldots|J_r|^2]. \tag{23}$$

The IMS class of density operators (1) has a particular asymptotics. The ''inverse temperature'' matrix β must be positive. If it were degenerate the state (1) would not exist at all. A degenerate β can formally be interpreted as if the mode of the zero eigenvalue became infinitely hot. This mode is, in fact, becoming more and more populated but the infinite population is unattainable. Nonetheless, an IMS with almost degenerate β would really be a Bose-Einstein condensate since this only requires a big finite number of bosons in a single quantum state. Speculations that the point of degeneracy, i.e., the point $\nu=1/\|\rho\|$, is the point of condensation (like in thermal Bose systems) cannot be verified for the IMS.

Let us first recapitulate the kinematics of an IMS condensate. The condensate mode does not interfere with the other modes so we can discuss it separately. We assume that our IMS is dominated by the condensate mode. The one-particle density matrix has the form $\rho_{\mathbf{k}'\mathbf{k}}=\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}}^{\dagger}$ where $\varphi_{\mathbf{k}}$ is the condensate mode's wave function. If we introduce the condensate absorption operator $\hat{a}_c=\Sigma_{\mathbf{k}}\varphi_{\mathbf{k}}\hat{a}_{\mathbf{k}}$ then, using Eqs. (1)–(4), the condensate IMS can be written as a thermal equilibrium state at temperature $T=-1/\ln\nu$:

$$\hat{\rho}_c=(1-e^{-1/T})\exp\left(-\frac{\hat{a}_c^{\dagger}\hat{a}_c}{T}\right). \tag{24}$$

This state assumes a Hamiltonian $\hat{a}_c^{\dagger}\hat{a}_c$ which is not the real case, the condensate is not even stationary in general. Yet, the form (24) is completely proper to calculate characteristics of the state by a thermal analogy. For instance, Eq. (5) yields directly the generating functional in the form

$$G[u]=\frac{1-e^{-1/T}}{1-\exp(-1/T)\sum_{\mathbf{k}} u_{\mathbf{k}}|\varphi_{\mathbf{k}}|^2}, \tag{25}$$

with the canonical thermal multiplicity distribution

$$p_n=(1-e^{-1/T})e^{-n/T} \tag{26}$$

of the mean multiplicity

$$\langle n\rangle=\frac{1}{e^{n/T}-1}. \tag{27}$$

Let us observe that approaching the ''condensation point'' corresponds to $T\to\infty$ and the population of the ''Bose condensate'' increases to the infinity while it is remaining thermally distributed all the time.

Now I turn to the dynamic conditions for the fluctuating external current J. In the special case of the condensate IMS, the second relation in Eq. (16) becomes simply $\alpha_{\mathbf{k}'\mathbf{k}}=\langle n\rangle\rho_{\mathbf{k}'\mathbf{k}}=\langle n\rangle\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}}^{\star}$. Recall the definition (14) of α as the current's correlation matrix, which yields the following relation:

$$M[J_{\mathbf{k}'}J_{\mathbf{k}}^{\star}]=\langle n\rangle\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}}^{\star}. \tag{28}$$

Regarding that $M[J_{\mathbf{k}'}J_{\mathbf{k}}]$ should vanish by assumption. the Gaussian fluctuations satisfying the above relation must take the form

$$J_{\mathbf{k}}=z\sqrt{\langle n\rangle}\varphi_k \tag{29}$$

for all $\mathbf{k}$, where z is a random complex number of the standard Gauss distribution $(1/\pi)\exp(-|z|^2)d^2z$. Taking the stochastic mean of the modulus square of both sides we obtain

$$|J_{\mathbf{k}}|^2=\langle n\rangle|\varphi_{\mathbf{k}}|^2, \tag{30}$$

which also leads to

$$\langle n\rangle=\sum_{\mathbf{k}}|J_{\mathbf{k}}|^2. \tag{31}$$

Equations (29)–(31) show the simple way the pulse of the effective current J determines the condensate wave function and the mean population. Actually, the mean multiplicity is identical to the overall intensity of the current pulse (31). The pulse's normalized spectral density is equal to the modulus square of the condensate wave function (30). Equation (29) seems, however, to be very restrictive since it imposes the same random phase and weight simultaneously for all current amplitudes $J_{\mathbf{k}}$.

I outline a possible generalization of the simple dynamics proposed earlier. Let us replace the Hamiltonian (12) by

$$\hat{H}(t)=g(t)\sum_{\mathbf{k}}[J_{\mathbf{k}}^{\star}(t)\hat{a}_{\mathbf{k}}+J_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}}^{\dagger}], \tag{32}$$

where $g(t)$ is a normalized function of characteristic width Δt, controlling the intensity of particle creation. The time-dependent currents $J_{\mathbf{k}}(t)$ are random functions of zero mean; let their correlator be of the nonstationary white-noise type:

$$g(t)M[J_{\mathbf{k}'}(t')J_{\mathbf{k}}^{\star}(t)]=\delta(t'-t)\alpha_{\mathbf{k}'\mathbf{k}}. \tag{33}$$

In the limit $\Delta t \rightarrow 0$, the random dynamics represented by Eqs. (32),(33) reduces to the simplistic dynamics (12),(14) . The Hamiltonian (32) with the white-noise currents (33) yield the following master equation [12] for the noise-averaged density operator (in the interaction picture):

$$\frac{d\hat{\rho}}{dt}=g\sum_{\mathbf{k},\mathbf{k}'}\alpha_{\mathbf{k}'\mathbf{k}}\left(\hat{a}^{\dagger}_{\mathbf{k}'}\hat{\rho}\hat{a}_{\mathbf{k}}-\frac{1}{2}\{\hat{a}_{\mathbf{k}}\hat{a}^{\dagger}_{\mathbf{k}'},\hat{\rho}\}\right). \qquad (34)$$

One solves the master equation with the vacuum initial state. If $1/\Delta t$ is much greater than the typical pion energy then, via the relationships (16), the final state will tend to the IMS (1). For larger Δt the simple relations (16) do not hold. Though analytic calculations are still possible one can turn to very powerful Monte Carlo (MC) methods [13] developed for Markovian master equations. These MC algorithms will yield the density operators of the multiparticle final states without ''struggling through'' [14] the usual Wigner-function formalism.

The aim of this paper was partly pedagogical. To avoid Bose symmetrization ''by hand,'' I used standard quantum-mechanical considerations to construct and to analyze the ''independent multiboson states.'' I showed how these states emerge from a simplistic version of chaotic current models and I derived the relationship between the IMS states and the correlator of the currents. I briefly recapitulated the generating functional representation of multiparticle counts. Beyond methodological matters, I found that the Bose-Einstein condensate would be thermally populated and the condensation point corresponds to the infinite hot state. I restricted my analysis to the IMS and the simplest chaotic current mechanisms with trivial time dependences. The simple choice allows for transparent relationships between the current and the final multiparticle state. (Other works, like, e.g., Ref. [10], incorporate a more realistic space-time evolution of the multiparticle source.) There is, nonetheless, a particular advantage of any underlying dynamics whether realistic or not. Usually it allows economic simulation methods for the physical quantities of interest. To this end, I proposed a quantum master equation known to be suitable for efficient Monte Carlo simulations.

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