Comment on "Nonclassical States: An Observable Criterion"

In a recent Letter [1], Vogel has adopted a simple definition of classicality. The state of a quantized harmonic oscillator is said to have a classical counterpart if its P function is equivalent to a probability measure. The author claims that classicality, defined in such a way, will be violated "if and only if there exist values of k and φ for which the condition

$$|\tilde{G}(k,\varphi)| > 1$$

is fulfilled" where, according to the Letter, $\tilde{G}(k,\varphi)$ is the characteristic function of the "noise-subtracted quadrature distribution." This claim cannot be correct since I can easily construct a counterexample. Let us, for instance, start from the standard chaotic state [2] of mean population 1, diagonal in the occupation number representation:

$$\hat{\rho} = \frac{1}{2}\sum_{n=0}^{\infty} 2^{-n}|n\rangle\langle n|\,.$$

Its P function is $(1/\pi)\exp(-|\alpha|^2)$. Now we measure the occupation number and discard the vacuum states. The above classical state will thus transform into the truncated chaotic state:

$$\hat{\rho}' = \sum_{n=1}^{\infty} 2^{-n}|n\rangle\langle n|\,.$$

This state is nonclassical since its P function,

$$P(\alpha) = \frac{2}{\pi}\,e^{-|\alpha|^2} - \delta(\alpha),$$

yields a negative measure at $\alpha = 0$. The characteristic function of the "noise-subtracted" distribution of quadratures takes this form:

$$\tilde{G}(k,\varphi) = 2e^{-k^2} - 1$$

whose modulus will *never* exceed 1, contrary to Vogel's claim. Furthermore, the characteristic function $G(k,\varphi) \equiv \exp(-k^2/2)\tilde{G}(k,\varphi)$ as a function of k does *not* decay "more slowly than the characteristic function of the ground state." Also this is in disagreement with the Letter's central statement on nonclassical states.

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