Comment on "Stable Quantum Computation of Unstable Classical Chaos"

In a recent Letter [1], Georgeot and Shepelyansky discussed a certain coherent quantum simulation of the classical Arnold map. They claim that this "classical chaotic system can be simulated on a quantum computer with exponential efficiency compared to classical algorithms." This Comment will question their statement. I argue that, as long as the classical evolution is concerned, the classical algorithm can be made exactly equivalent with the quantum one.

Following the Letter, consider the discretized classical Arnold map. Start a classical trajectory from i, j and let i', j' denote the new coordinates after many iterations including the time-reversal on halfway. The full protocol of the Letter's quantum simulation goes obviously like this. First, the quantum amplitudes a_{ij} are introduced to represent the initial classical phase-space density $|a_{ij}|^2$. Second, the quantum algorithm is performed on the amplitudes, using a polynomial number of simple quantum gates. In the ideal case of perfect gates, the resulting unitary transformation reads

$$a_{ij} \longrightarrow a_{i'j'}. \tag{1}$$

Third, quantum measurements are performed regarding those quantum observables which do possess interpretation for the classical Arnold system as well.

The point is the latter restriction. All information on the final state of the classical Arnold system has been encoded into the squared moduli $|a_{i'j'}|^2$. The complete classical information can thus be obtained by measuring the projectors,

$$|x_k\rangle|y_l\rangle\langle x_k|\langle y_l|, \tag{2}$$

simultaneously for each $k, l = 1, 2, \ldots, N$. I emphasize that no interpretation exists within the framework of the classical Arnold system for observables which are not diagonal in the basis $\{|x_k\rangle|y_l\rangle; k, l = 1, 2, \ldots, N\}$. Any simulation of nondiagonal observables would be redundant for the classical evolution. Now, I am going to show that the measured statistics of the classically relevant quantum observables (2) can equivalently be simulated by a classical algorithm whose logic steps are just identical with that of the quantum algorithm.

I propose a trivial classical protocol. First, we generate a random pair i, j with probability represented by the given initial phase-space density. Second, the classical algorithm is performed on i, j, using the sequence of simple classical gates each being the classical equivalent of the quantum gates in the Letter's quantum algorithm. With perfect gates this leads to i', j'. Third, we read out (trivially) the contribution of the result to the phase-space density in the final state. Obviously, this contribution will be of the same statistics which we would have obtained in quantum measurements of observables (2) after quantum computation (1). No one could distinguish between the data taken from the quantum or from the classical computers, respectively.

This equivalence remains valid if the coarse-grained or the Fourier-transformed version of set (2) is analyzed. Furthermore, the equivalence survives if logical gates are not perfect. Assume you have a classical computer to perform the map $i, j \rightarrow i', j'$, using simple reversible gates. And imagine that, at your alternative wish, you can run the same gates coherently. This is how the Letter's quantum algorithm can be related to the classical one. I have already proven that the coherent and incoherent runs give the same statistics for the classical Arnold map, provided the gates are perfect. If they are not, we can still assume that the bit-error rates are independent of whether we run the gates coherently or not [2]. Hence, the quantum and classical computations will be equivalent for nonideal gates, also. (The gates' phase errors do not influence the results of the quantum protocol.)

For the classical chaotic evolution, the claimed advantage of the Letter's quantum algorithm is illusory. It has disappeared when we have concretized the statistical analysis, left undetailed by the authors, of the final quantum state.

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[1] B. Georgeot and D. L. Shepelyansky, Phys. Rev. Lett. **86**, 5393 (2001).
[2] The Letter assumes different errors for quantum and classical gates.

