Quest for Quantum Superpositions of a Mirror: High and Moderately Low Temperatures

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The Born-Markov master equation analysis of the vibrating mirror and photon experiment proposed by Marshall, Simon, Penrose, and Bouwmeester is completed by including the important issues of temperature and friction. We find that at the level of cooling available to date, visibility revivals are purely classical, and no quantum effect can be detected by the setup, no matter how strong the photon-mirror coupling is. Checking proposals of universal nonenvironmental decoherence is ruled out by dominating thermal decoherence; a conjectured coordinate-diffusion contribution to decoherence may become observable on reaching moderately low temperatures.

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The nature of the quantum-classical border along the mass scale is still poorly defined. There remain some 10 orders of magnitude unexplored between the heaviest molecules, for which the c.m. interference has been observed [1], and the lightest nanomechanical objects, for which no quantum behavior has been seen [2]. In trying to close the gap top down, the primary experimental task is to find firm evidence, never seen so far, that the spatial motion of a mass as large as a nanomechanical object does follow the Schrödinger equation, notwithstanding environmental interactions, or noise, which would quickly decohere the wave function. Only having succeeded in suppressing that effect so that interference of a heavy object is detected beyond any doubt, can we turn to checking the presence of spontaneous (also called universal or intrinsic) decoherence [3] on top of the environmental one.

An experimentally accessible system with potentialities to achieve the above goal is a photon in a high-quality resonating cavity, coupled by its radiation pressure to a nanomechanical oscillator, carrying one of the mirrors that close the cavity. After pioneering experiments [2] which did not detect any quantum effect on the mirror, as well as thoughtful theoretical analyses [4], a promising idea appeared for bridging the frequency gap and carrying out a genuine quantum test [5,6]. In that proposal, the vibrating mirror closes an optical cavity in arm A of a Michelson interferometer, arm B having another cavity with fixed mirrors. The vibrating mirror is expected to become entangled with a single photon traveling along both arms, the mirror being split into a kind of Schrödinger cat doublet. The interference of the photon is detected with the scope of extracting information about the quantum motion of the mirror. Since the vibrations of the mirror are much slower than the frequency of light, a shift of the interference pattern would be unobservable; the good chance is to record the *visibility* which is modulated by the motion of the mirror, creating revivals as the components of the superposition overlap again and again.

Highly worth doing as it is, this is a very hard experiment, for various reasons. One thing is that high-quality optical resonators are needed to keep the photon alive for several, or at least one, return of the mirror; a less familiar task is to preserve coherence of the vibrating mirror itself. The latter requires efficient cooling and a drastic reduction of various mechanisms of environment-induced decoherence, at least partly related to friction.

Drawing on previous analyses of the vibrating mirror and photon problem, the experimental proposal [5] has been analyzed by Adler *et al.* [7], bringing considerable new insight into the way decoherence influences the interferometric signal. The present Letter is meant to add the crucial features of finite temperature and friction, which have been but qualitatively described in Ref. [5]. The main point is that the requirement of sufficiently strong coupling to create entanglement enforces the use of a low-frequency vibrating mirror. Then, however, unless cooling performances are considerably improved, one remains in the high-temperature range, where no genuine quantum effect can be observed. We also confirm that testing theoretical proposals about universal nonenvironmental decoherence mechanisms has remained an extremely bold enterprise for some time to come. The only quantum effect accessible on moderate progress in cooling would be a refinement of the treatment of quantum friction, proposed in Ref. [8]; see below.

We start out in the framework posed by Adler *et al.*, calculating visibility of the photon interference as $\nu(t) = 2|\mathrm{Tr}_m \hat{\rho}_{\mathrm{OD}}(t)|$, where $\hat{\rho}_{\mathrm{OD}} = {}_A\langle 1|_B\langle 0|\hat{\rho}|0\rangle_A|1\rangle_B$ is the off-diagonal element of the full density matrix $\hat{\rho}$ of the mirror-photon system, where $|0\rangle_j$ and $|1\rangle_j$ are photon states with 0 or 1 photon, respectively, in arm $j = A, B$.

For $\hat{\rho}_{\mathrm{OD}}$, Adler *et al.* [7] derive a master equation, with a position decoherence term of strength D_{pp}. To include friction of constant γ in the treatment, we follow the usual theoretical pathway using the high-temperature Markovian master equation [9] of quantum friction. Following Ref. [8], we also include a correction term, negligible for high temperatures but relevant for moderately low ones, taking the form of a momentum decoherence term of strength D_{qq}; we shall tune it toward its theoretical minimum value $\gamma^2/16D_{pp}$ assuring to preserve positivity of the density matrix [8]. This correction, as we shall see later, may turn out to govern the only quantum effect detectable at moderately low temperatures.

With all those extra terms, using units with $\hbar = 1$, the master equation reads

$$\frac{\partial \hat{\rho}_{\rm OD}}{\partial t} = -i\hat{H}^A\hat{\rho}_{\rm OD} + i\hat{\rho}_{\rm OD}\hat{H}^B - D_{pp}[\hat{x},[\hat{x},\hat{\rho}_{\rm OD}]] - i\frac{\gamma}{2}[\hat{x},\{\hat{p},\hat{\rho}_{\rm OD}\}] - D_{qq}[\hat{p},[\hat{p},\hat{\rho}_{\rm OD}]], \quad (1)$$

with

$$\hat{H}^B = \frac{M\omega_m^2}{2}\hat{x}^2 + \frac{1}{2M}\hat{p}^2, \qquad \hat{H}^A = \hat{H}^B - \omega_c\frac{\hat{x}}{L}, \quad (2)$$

where ω_m is the frequency of the vibrating mirror [4], L the cavity length, ω_c the frequency of the photon, and M the mass of the mirror.

Besides using the width $\sigma = 1/\sqrt{2M\omega_m}$ of the ground-state wave packet of the mirror and the oscillator quality factor $Q_m^{-1} = \gamma/\omega_m$, the following dimensionless parameters will be of central importance for the discussion below: the photon-mirror coupling constant $\kappa = (\omega_c/\omega_m)(\sigma/L)$ [4], the decoherence strength $\Lambda = (\sigma^2/\omega_m)D_{pp}$, and the combination $\chi = D_{qq}/(\omega_m\sigma^2)$. Using these notations, and temporarily introducing units of time with $\omega_m = 1$, the final form of the master equation becomes

$$\frac{\partial \hat{\rho}_{\rm OD}}{\partial t} = -i\sigma^2[\hat{p}^2,\hat{\rho}_{\rm OD}] - \frac{i}{4}\sigma^{-2}[\hat{x}^2,\hat{\rho}_{\rm OD}] + i\kappa\sigma^{-1}\hat{x}\hat{\rho}_{\rm OD} - \Lambda\sigma^{-2}[\hat{x},[\hat{x},\hat{\rho}_{\rm OD}]] - \frac{i}{2}Q_m^{-1}[\hat{x},\{\hat{p},\hat{\rho}_{\rm OD}\}] - \chi\sigma^2[\hat{p},[\hat{p},\hat{\rho}_{\rm OD}]]. \quad (3)$$

The above equation can be solved analytically, e.g., via the trace expression [10]:

$$\tilde{\rho}_{\rm OD}(k,\Delta) = {\rm Tr}_m[\hat{\rho}_{\rm OD}\exp i(\sigma^{-1}k\hat{x} + \sigma\Delta\hat{p})], \quad (4)$$

where k, Δ are dimensionless Fourier variables [11]. Using this representation, the master Eq. (3) results in the following equation of motion:

$$\frac{\partial\tilde{\rho}_{\rm OD}(k,\Delta)}{\partial t} = 2k\frac{\partial}{\partial\Delta}\tilde{\rho}_{\rm OD} - \frac{1}{2}\Delta\frac{\partial}{\partial k}\tilde{\rho}_{\rm OD} - \Lambda\Delta^2\tilde{\rho}_{\rm OD} + \kappa\left(\frac{\partial}{\partial k} + i\frac{\Delta}{2}\right)\tilde{\rho}_{\rm OD} - Q_m^{-1}\Delta\frac{\partial}{\partial\Delta}\tilde{\rho}_{\rm OD} - \chi k^2\tilde{\rho}_{\rm OD}. \quad (5)$$

We aim at finding a temperature-averaged solution. Following the tradition [4], we first solve the equations for an arbitrary pure coherent state $|\alpha_0\rangle$ of the mirror, for which Eq. (4) takes the Gaussian form

$$\tilde{\rho}_{\rm OD}(k,\Delta) = \tfrac{1}{2}e^{-[c_1k^2 + c_2k\Delta + c_3\Delta^2 + ic_4k + ic_5\Delta + c_6]}, \quad (6)$$

with the initial values

$$c_1(0) = \tfrac{1}{2}, \quad c_2(0) = 0, \quad c_3(0) = \tfrac{1}{8}, \quad c_4(0) = -2{\rm Re}[\alpha_0], \quad c_5(0) = -{\rm Im}[\alpha_0], \quad c_6(0) = 0. \quad (7)$$

The corresponding visibility is $2{\rm Tr}_m\hat{\rho}_{\rm OD}(t,\alpha_0) = e^{-c_6(t)}$, to be evaluated using Eq. (5) which preserves the Gaussian structure (6) in time, with coefficients evolving according to the following simple linear equations:

$$\dot{c}_1 = 2c_2 + \chi, \qquad \dot{c}_2 = 4c_3 - c_1 - Q_m^{-1}c_2, \qquad \dot{c}_3 = -\tfrac{1}{2}c_2 - 2Q_m^{-1}c_3 + \Lambda, \qquad \dot{c}_4 = 2c_5 - 2i\kappa c_1, \qquad \dot{c}_5 = -\tfrac{1}{2}c_4 - \kappa(ic_2 + \tfrac{1}{2}) - Q_m^{-1}c_5, \qquad \dot{c}_6 = i\kappa c_4. \quad (8)$$

The solution depends on α_0 in the form $c_6(t) = \kappa^2 f_1(t) - i\kappa\{{\rm Re}[\alpha_0]f_2(t) - {\rm Im}[\alpha_0]f_3(t)\}$; then we do the thermal averaging of $e^{-c_6(t)}$ over α_0 [12] to obtain

$$\nu(t) = 2\left|\int P_T(\alpha_0){\rm Tr}_m\hat{\rho}_{\rm OD}(t;\alpha_0)d^2\alpha_0\right| = |e^{-\kappa^2[f_1(t) + (\bar{n}/4)(f_2^2(t) + f_3^2(t))]}|, \quad (9)$$

where $P_T(\alpha_0) = e^{-|\alpha_0|^2/\bar{n}}/(\pi\bar{n})$ is the P function of the initial thermal equilibrium state of the mirror, with $\bar{n} = [\exp(\hbar\omega_m/k_BT) - 1]^{-1}$. The functions f_1, f_2, f_3 are obtained in analytical form; however, the resulting formulas are not transparent enough to be displayed in full generality. Simpler results are obtained for the relevant case of a high-quality mechanical oscillator: $Q_m^{-1} \ll 1$. Then, while evaluating the complex frequencies in full accuracy, in the amplitudes one has to keep only the leading-order corrections in Q_m^{-1}. With that simplification, returning to physical units of time and introducing $\tilde{\omega}_m = \sqrt{\omega_m^2 - (\gamma/2)^2}$, which is the frequency of the damped classical oscillator, we arrive at our final result:

$$\nu(t) = \exp\{-(\bar{n} + 1/2)\kappa^2[1 + e^{-\gamma t} - 2e^{-(\gamma/2)t}\cos(\tilde{\omega}_m t)]\} \times \exp\left(-6\kappa^2\Lambda\left\{\tilde{\omega}_m t\left[\frac{1 - e^{-\gamma t}}{3\gamma t}\left(1 + \frac{\chi}{4\Lambda}\right) + \frac{2}{3}\right] - \frac{4}{3}e^{-(\gamma/2)t}\sin(\tilde{\omega}_m t) + \frac{1}{6}e^{-\gamma t}\sin(2\tilde{\omega}_m t)\left(1 - \frac{\chi}{4\Lambda}\right)\right\}\right). \quad (10)$$

In the first of the two factors above it is easy to recognize the visibility revival effect as originally proposed by Marshall *et al.* [5], modified by the temperature averaging already discussed by Bose *et al.* [4], as well as the mechanical effect of friction. The second factor describes decoherence effects, in accordance with the result of Adler *et al.* [7], now including the coordinate-diffusion correction χ [8] (see below), also modified by friction.

Inference about decoherence mechanisms can be extracted from the height of the *first* revival at $t_1 = 2\pi/\tilde{\omega}_m$: for times as short as that, damping through mechanical friction can be fully neglected, and (10) simplifies to

$$\nu(t_1) = \exp\{-\pi\kappa^2(12\Lambda + \chi)\} \tag{11}$$

(see Fig. 1). We postpone the discussion of χ, which is negligible at present-day temperatures (see below), and write tentatively $\Lambda = \Lambda_T + \Lambda_{\text{nonenv}}$. Concerning the first, dominant term, in thermal environment classical friction is always accompanied by classical momentum diffusion of strength $D_{pp}^T = Mk_BT\gamma$. This mechanism survives for quantum friction as well, causing the familiar thermal position decoherence

$$\Lambda_T = (k_BT/2\hbar\omega_m)Q_m^{-1}, \tag{12}$$

where we have restored the true physical scale of $\hbar$. Substituting the Marshall *et al.* figures [5] about present-day possibilities, $\omega_m = 3 \times 10^3\ \text{s}^{-1}$, $T = 2 \times 10^{-3}$ K, and $Q_m = 10^5$, we obtain $\Lambda_T \approx 0.5$. That is the background against which nonenvironmental decoherence mechanisms expected from the models Ghirardi-Rimini-Weber, "quantum mechanics with universal position localization," or "continuous spontaneous localization" (CSL) [3] should be tested, according to the suggestion of Marshall *et al.* The estimates range from $\Lambda_{\text{CSL}} \approx 0.2 \times 10^{-8}$ to much smaller figures for the gravitation-related universal collapse model [13]. This indicates that, for the thermal background, many orders of magnitude should be gained in friction and cooling before nonenvironmental decoherence proposals might show up in the experiment.

We must not forget our basic task, to see if the proposal [5] can yield evidence at least for the quantum behavior of

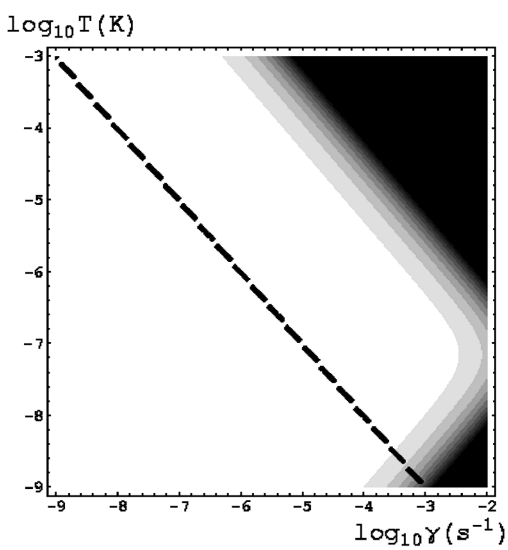

FIG. 1. Contours of equal visibility (white for good, black for poor), as a function of temperature T and friction γ, based on Eqs. (11), (12), and (14). Upper-right corner, present-state possibilities; dashed line, where CLS decoherence becomes observable. The turnback of the contours in the lower-right corner corresponds to the coordinate-diffusion contribution [8], with $\kappa = 1$ and $\lambda = 1$.

the vibrating mirror. At temperatures of mK and vibration frequencies of kHz envisaged for the mirror, we have $\bar{n} \sim 10^5$, which means the mirror is well in its high-temperature regime; accordingly, quantum effects are expected to be masked altogether, which is confirmed by the parameter combinations appearing in Eq. (10). Indeed, the parameter χ can be ignored at high temperatures, and the other two relevant combinations,

$$\kappa^2\Lambda_T = \frac{k_BT}{4M\omega_m^2L^2}\frac{\omega_c^2}{\omega_m^2}Q_m^{-1}, \qquad \kappa^2\bar{n} = \frac{k_BT\omega_c^2}{2M\omega_m^4L^2}, \tag{13}$$

turn out to be fully classical, containing no factor of $\hbar$. The first of them is the visibility extinction coefficient (cf. the similar result by Bose *et al.* [4]); the second is the parameter that controls thermal narrowing of the duration of visibility recurrences. Since $\kappa^2\bar{n} \gg 1$ at high temperatures, the duration of visibility revivals will be much shorter than the vibration period. That temperature-related narrowing effect has been already mentioned by Marshall *et al.* [5] as a challenge to the stability of the experimental setup.

The existence of visibility revivals in no way contradicts to the full classicality manifest in our results. Indeed, mirror-photon entanglement and cyclically returning disentanglements *coincide* with classical mirror-light correlation and returning decorrelations (classical radiation being scaled to one-photon strength), as the mirror repeatedly passes through its initial position, *independently of initial conditions*. That robust periodicity is specific to harmonic oscillator dynamics. All that can be followed in detail through the appropriate equations [14].

In order to detect the quantum behavior of the mirror, we must cool it to medium temperatures where $\bar{n}$ is a smaller number, say, 5 or 10. This could be done with GHz oscillators; see, e.g., [15]. Our frequency cannot be that high though: a hard, high-frequency oscillator resists the push the photon exerts on the mirror, as expressed in the low value of the coupling parameter κ. If $\kappa < 1$, the push is not strong enough to split the mirror into a well-separated superposition cat doublet, and no entanglement is created. This strong-coupling requirement forces the photon-mirror system into a vicious circle: to obtain a quantum effect, one must use a relatively soft (low-frequency) vibrating mirror, which is hard to cool down close to its ground state; therefore it is hard to leave the high-temperature range.

If we still succeed in pushing temperature down to $k_BT/\hbar\omega_m \approx \mathcal{O}(10)$, which for the soft oscillator envisaged by Marshall *et al.* corresponds to the range of a few μK, the quantum correction proportional to D_{qq} [8] enters the dynamics (1) and may become accessible to measurement, as discussed already by Jacobs *et al.* [4]. Let us tune D_{qq} toward its theoretical minimum; i.e., we assume $D_{qq} = \lambda\gamma^2/16D_{pp}^T$, where $\lambda \gtrsim 1$ is a small number to be extracted from experiment. Evaluating the factor $1 + \chi/4\Lambda_T$ in Eq. (10), we get the following medium-temperature quantum correction to the classical visibility extinction coefficient (13):

$$\frac{k_B T}{4M\omega_m^2 L^2}\frac{\omega_c^2}{\omega_m^2}Q_m^{-1}[1+\lambda(\hbar\omega_m/4k_B T)^2], \qquad (14)$$

which may reach measurability at moderately low temperatures. Approaching even that moderately low-temperature range is a bold enterprise with the photon-mirror combination. Refinement of the theoretical treatment for low temperatures is also desirable.

In summary, by including friction and temperature averaging in the theoretical framework set by Adler *et al.* [7], we gave an overall theoretical analysis of the experimental setup proposed by Marshall *et al.* [5]. We find that although photon visibility revivals are expected to be detected in the proposed setup, at the cooling level currently available they do not allow one to conclude that a macroscopic body might exhibit genuine quantum behavior. In agreement with the conclusion of Adler *et al.* [7], we also confirm that detection of any of the hypothetical nonenvironmental decoherence mechanisms is a remote scope, being orders of magnitude weaker than present-day thermal background decoherence. Nevertheless, on reaching moderately low temperatures, there it the chance to detect a different quantum effect: a coordinate-diffusion-related contribution to decoherence. Anyway, unprecedented progress in cooling a soft mirror is the only obvious way towards seeing both robust quantum effects and eventual violation of standard quantum mechanics, which is an aim of extreme importance.

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[11] Usually, one transforms the density matrix (in our case, its off-diagonal element) into the corresponding Wigner function $\rho_{\mathrm{OD}}(x,p)$. Then the master equation (3) becomes a generalized Fokker-Planck partial differential equation for $\rho_{\mathrm{OD}}(x,p)$, which we can integrate in Fourier picture; see, e.g., L. Diósi and C. Kiefer, J. Phys. A **35**, 2675 (2002); P.J. Dodd, Phys. Rev. A **69**, 052106 (2004). The present procedure takes a shortcut to the doubly Fourier-transformed Wigner function $\tilde{\rho}_{\mathrm{OD}}(k,\Delta)$.

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[13] Both Diósi and Penrose [3] would suggest the same decoherence rate $D_{pp}^{GR}(x-x')^2$ for the spatial superposition of the mirror at locations x, x', with $D_{pp}^{GR}\sim(G/\hbar)m^2/R^3$ where G is Newton's constant and R is the size of the bulky mirror. The dimensionless decoherence rate $\Lambda_{GR}=(\sigma^2/\omega_m)D_{pp}^{GR}$ would then be $\sim 10^{-14}\ \mathrm{s}^{-1}$, i.e., much less than Λ_{CSL}.

[14] At high temperature, the Wigner function $\rho_{\mathrm{OD}}(x,p)$ and the corresponding complex classical phase space density turn out to satisfy exactly the same Fokker-Planck–type equation, making quantum and classical visibilities coincide, apart from the medium-temperature quantum correction proportional to D_{qq}.

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