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Laser linewidth hazard in optomechanical cooling

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I discuss the robustness of pumped cavity dynamics against phase diffusion of the laser and conclude that optomechanical cooling has extreme sensitivity compared to laser cooling of atoms. Certain proposals for ground-state optomechanical cooling by a single cavity would require an unrealistically sharp laser linewidth or, equivalently, a very low level of phase noise. A systematic way to cancel classical excess phase noise is the interferometric twin-cavity pumping, initiated for optically trapped macromirrors of future gravitational-wave detectors.

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Very recently, numerous works [1–7] have predicted or suggested that laser cooling can bring a nanomechanical oscillator (nanomirror) close to its quantum ground state. It is hard to miss the conceptual similarities between optomechanical cooling and the standard (e.g., Doppler) laser cooling of atoms. This time the cooled object is the spatial motion of the mirror instead of the atom, and the refrigerator is an optical cavity oscillator instead of the atom's internal two-level system. In both cases, the refrigerator has high optical excitation frequencies, the object has thermally excited modes of low (radio) frequencies; therefore object-refrigerator coupling is practically missing. What really turns the atomic two-level system or the cavity into a refrigerator is the external laser field. Typical limitations of atom cooling are determined by the spontaneous decay rate κ of the atom; hence laser imperfections (linewidth Γ_l, basically) do not influence the mechanism as long as

$$\Gamma_l \ll \kappa. \tag{1}$$

I will conjecture that for optomechanical cooling the condition becomes

$$n\Gamma_l \ll \kappa, \tag{2}$$

where κ is the decay rate of the cavity field. This condition puts a fatally stronger limit on Γ_l because of the large factor n, the steady-state excitation number of the pumped cavity. Violating this condition will not invalidate optomechnical cooling in general. Ground-state cooling, however, becomes more problematic than was thought before.

Let us follow the standard theory and Langevin equation formalism, shared by most of the cited works, to introduce the time-dependent phase ϕ of the laser field into the equation of the cavity mode absorption operator:

$$\dot{a} = -(\kappa + i\Delta)a + Ee^{-i\phi} + \sqrt{2\kappa}a_{in} + \cdots, \tag{3}$$

where $\Delta > 0$ is the detuning of the cavity mode, and E is proportional to the pump field [1,5,7,13]. The third term on the right-hand side denotes the quantum noise coming from the vacuum environment at $T=0$ [8]:

$$\langle a_{\rm in}(t)a_{\rm in}^\dagger(s)\rangle = \delta(t-s), \tag{4}$$

and the ellipsis stands for the coupling to the position of the mirror. The diffusion of the phase ϕ of the laser light is determined by the white-noise correlation

$$\langle\dot{\phi}(t)\dot{\phi}(s)\rangle = 2\Gamma_l\delta(t-s). \tag{5}$$

This standard ansatz corresponds to a flat power spectrum $S(\omega)=2\Gamma_l$ of frequency fluctuations. The assumption will be refined later. We perform two subsequent canonical transformations $a \to ae^{-i\phi}$ and $a \to a+\alpha$ where $\alpha = E/(\kappa+\Gamma_l+i\Delta)$ is the large mean amplitude and a becomes a small perturbation around it. We obtain

$$\dot{a} = -(\kappa + \Gamma_l + i\Delta)a + i\alpha\dot{\phi} + \sqrt{2\kappa}a_{in} + \cdots. \tag{6}$$

Note that we have approximated the term $i(\alpha+a)\dot{\phi}$ by $i\alpha\dot{\phi}$.

With the choice of small detuning $\Delta > 0$, our refrigerator becomes equivalent to a central oscillator of low frequency Δ, which can have strong, even resonant, coupling to the mirror's mechanical oscillation. One would think that we obtained a low-frequency refrigerator operating at $T=0$ almost for free. In reality, however, the main resource of cooling is the perfect periodic driving field. The relevant imperfection is the finite linewidth Γ_l of the laser. Indeed, we must assure in Eq. (6) that the contribution of the phase noise (5) remain much less than the contribution of the quantum noise (4), which means $|\alpha|^2\Gamma_l \ll \kappa$. This is just our condition (2), since $|\alpha|^2 = n$ for large α. If the condition is not satisfied, the phase noise will impose an effective nonzero temperature on the cavity oscillator and it cannot act as a refrigerator to $T=0$ any longer. Let us ignore the structural difference between the noises $a_{\rm in}$ and $\dot{\phi}$, and imagine that the contribution of the large phase noise (5) is equivalent to the contribution of the quantum noise at a certain high (effective) temperature T:

$$\langle a_{\rm in}(t)a_{\rm in}^\dagger(s)\rangle = \frac{k_B T}{\hbar\Delta}\delta(t-s). \tag{7}$$

Then the following estimation can be made for the temperature of the effective cavity mode, caused by the phase noise:

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$$k_B T \sim \hbar\Delta \frac{n\Gamma_l}{\kappa}. \tag{8}$$

Finally, let us consider the concrete magnitudes of the parameters considered, e.g., in Ref. [5]. Accordingly, we take $\kappa \sim \Delta \sim 10$ MHz; the 50 mW laser power at 1064 nm wavelength yields $E \sim 10^{13}$ Hz, and we are led to $n = |\alpha|^2 \sim 10^{10} - 10^{11}$. This huge number would, via condition (2), impose a requirement of Γ_l less than $10^{-4} - 10^{-3}$ Hz. This range is far from being available now. The recent experimental work [9] estimates the deteriorating influence of phase noise in the alternative regime $\kappa \ll \Delta$ and for a stiffer oscillator. The lowest achievable excitation scales with $\sqrt{T\Gamma_l}$. Ground-state cooling of a 40 MHz oscillator from a cryogenic temperature T will still require 10^5 times smaller noise intensity than the value ($\sim$400 kHz) observed in the experiment.

As anticipated above, we refine the standard ansatz (5). Since the detuning Δ is used in resonance with a high-quality oscillator, it is only the frequency noise spectrum $S(\omega)$ in a narrow band around Δ that matters [9]. In reality, the strength $S(\Delta)$ can be, or can be made, much different from the linewidth Γ_l. Our calculations and considerations can invariably be retained when we just replace $2\Gamma_l$ by $S(\Delta)$. Obviously, the formulated demands should concern $S(\Delta)$ and its vicinity rather than the whole spectrum $S(\omega)$, rather than the linewidth Γ_l. The reduction of phase noise in a narrow band above 1 MHz might be a less difficult task than the reduction of the total spectrum and linewidth.

I have restricted my calculations and arguments for the behavior of the cavity oscillator (refrigerator). In mind, I had the back-action (self-cooling) method, while the active feedback control (cold damping) method may turn out less vulnerable to the laser instabilities. Clearly, the coupled linearized quantum Langevin equations must be extended and solved exactly for the steady state in the presence of the phase noise term. It is likely that the full "cost" of the ground-state optomechanical refrigerator will contain the cost of extreme laser stability.

Nonetheless, an idea that emerged in gravitational-wave interferometry might neutralize the laser instability for nanomirror cooling as well. Consider two identical cavities pumped by the same laser at the same phase. Then we have two cavity amplitudes a and b of identical behavior, including the identity $\alpha = \beta$ of their respective steady-state mean amplitudes. By introducing the modes $(a-b)/\sqrt{2} \to a$ and $(a+b)/\sqrt{2} \to b$, the "differential" mode satisfies

$$\dot{a} = -(\kappa + \Gamma_l + i\Delta)a + i\dot{\phi}a + \sqrt{2\kappa}a_{in} + \cdots. \tag{9}$$

Note that the large noise term $i\alpha\dot{\phi}$ has canceled; we have to retain the small one $i\dot{\phi}a$. This mode is a $T=0$ refrigerator, indeed. Its performance is limited only by the constraint (1), instead of Eq. (2). The coupling of the mirror motion to this mode is straightforward if, e.g., we use a shared movable end mirror, silvered on both sides, between the two cavities. Such setups have been suggested and analyzed for gravitational-wave interferometer macromirrors to cancel the influence of laser instabilities [10] and to project quantum mechanical tests [11,12]. The double-cavity concept itself exists for nanomirrors as well, so far unrelated to the laser noise issue [6], and with independent pumping [13]. To implement interferometric twin cavities in ground-state cooling of nanomirrors seems a reasonable, if not unavoidable, next step.

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