Erratum: Non-Markovian Continuous Quantum Measurement of Retarded Observables [Phys. Rev. Lett. 100, 080401 (2008)]

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Some corrections to my Letter [1] follow directly from the criticism by Wiseman and Gambetta [2]; further corrections restore the validity of my Letter. Contrary to my suggestion there, the given continuous measurement schemes cannot yield *pure state* trajectories but *mixed-state* ones [2]. Yet, it is possible to retain my claim that the NMSSE (5) describes true time-continuous measurement—with *delay* and *retrodiction*. The necessary corrections are the following.

A sentence completes the abstract: "However, the generic non-Markovian trajectories are mixed-state trajectories." To the text below Eq. (17), one appends this: "Contrary to our assumption, the state (17) is not pure even if it started from a pure $\hat{\rho}_0$; the continuous readout of x_t cannot provide sufficient information for a pure state $\psi_t[x]$, as shown by Wiseman and Gambetta [2]." Before the summary, the following part is inserted.

"*Delayed continuous readout.*—Unfortunately, the chosen readout schedule alters the reduced dynamics (2) because the detector modes (18) are not retarded, and hence the coupling between the system and the detector mode z_τ continues after time τ; cf. Ref. [2]. It ceases, nonetheless, at $\tau + T$ provided $T > 0$ is much larger than the reservoir correlation time so that $\alpha(T) = 0$ be already a good approximation. We can thus keep the reduced dynamics (2) invariant if we apply continuous readout with a finite *delay* T. We read out each pointer of *label* τ (i.e., z_τ) at *time* $\tau + T$. The conditional state (20) of the system at time $t > T$ must be replaced by

$$\hat{\rho}_t[z;\mathrm{delay}=T] = \frac{1}{p_t[z;\mathrm{delay}=T]}\int \hat{\rho}_t[z;z]\prod_{\tau\notin[0,t-T]} dz_\tau.$$

It turns out that $p_t[z;\mathrm{delay}=T] = p_{t-T}[z]$; i.e., the statistics of the delayed continuous readouts obtained until time t is identical to the statistics of zero-delay (and all-in-one [2]) readouts until time $t - T$. The delayed-readout state obviously coincides with the following average of the zero-delay-readout states (20):

$$\hat{\rho}_t[z;\mathrm{delay}=T] = \frac{1}{p_{t-T}[z]}\int \hat{\rho}_t[z]p_t[z]\prod_{\tau\in[t-T,t]} dz_\tau = \frac{1}{p_{t-T}[z]}\int \psi_t[z]\psi_t^\dagger[z]p_t[z]\prod_{\tau\in[t-T,t]} dz_\tau.$$

The second equality follows from the insertion of $\hat{\rho}_t[z] = \psi_t[z]\psi_t^\dagger[z]$ where, according to Eq. (23), the pure state $\psi_t[z]$ must be the normalized solution $\psi_t[z] = \Psi_t[z]/\|\Psi_t[z]\|$ of the NMSE (5). As we see, $\psi_t[z]$ does not directly describe a continuously measured quantum trajectory because the values z_τ for $\tau \in [t - T, t]$ would belong to the all-in-one measurement at time t. Still, the above partial average of the pure states $\psi_t[z]$ over those fictitious z_τ does fully describe our (delayed) non-Markovian continuous measurement. The normalized solutions $\psi_t[z]$ of the NMSSE (5) do correspond to retrodicted pure states of the system; the proof and physical interpretation will be given elsewhere."

Reference [2] must be added to the bibliography of [1].

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[1] L. Diósi, Phys. Rev. Lett. **100**, 080401 (2008).
[2] H. M. Wiseman and J. Gambetta, Phys. Rev. Lett. **101**, 140401 (2008).

