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Gravity-Related Wave Function Collapse

Is Superfluid He Exceptional?

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Abstract The gravity-related model of spontaneous wave function collapse, a long-time hypothesis, damps the massive Schrödinger Cat states in quantum theory. We extend the hypothesis and assume that spontaneous wave function collapses are responsible for the emergence of Newton interaction. Superfluid helium would then show significant and testable gravitational anomalies.

Keywords Wave function collapse · Newton gravity · Superfluid He

1 Introduction

Quantum and gravity are expected to interfere in relativistic cosmic phenomena only. Alternative speculations suggest that quantum and gravity meet in a new way already at nanoscales. At least this follows from the gravity-related spontaneous collapse (decay) model [1–7] of massive macroscopic quantum superpositions also called Schrödinger cat (Cat) states. The DP model (after the author and Penrose) is non-relativistic, the predicted collapse rate is proportional to the Newton constant G, and it becomes relevant at nanoscales already. Although spontaneous collapses are too tiny, efforts are being under way to detect them, cf., e.g., [8–14].

I outline a way to go beyond the DP model. A self-contained and elementary explanation and discussion of the DP model in Sects. 2–5 is followed by a hypothesis extending the DP model. Section 6 argues and conjectures that the Newton interaction is induced by the spontaneous collapses, it emerges stochastically at the same rate as the G-related collapse rate which is of the order of 1 ms for common condensed matter. However, for superfluid He the collapse time can be much longer! Our most challenging conjecture is that the emergence time of gravity depends on the quantum

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mechanical microstructure of the source. The Newton field of bulk superfluid He goes after the acceleration of the source at a substantial delay compared to common condensed matter (Sect. 7). Work [15] is closely related to Sects. 2–5, discusses the mass resolution and environmental decoherence issues in particular, and it conjectures the anomalous low collapse rate in liquid He.

2 Schrödinger Cat Problem

If we extend quantum mechanics from its original atom physical context for larger objects, we face the well-known Cat paradox. To put it concrete and tractable, we often consider mechanical Cat [1–8], e.g., a rigid massive ball in a superposition of two macroscopically different locations $\mathbf{x}$ and $\mathbf{x}'$:

$$|\mathrm{Cat}\rangle = |\mathbf{x}\rangle + |\mathbf{x}'\rangle. \tag{1}$$

This superposition is discontinued when a position measurement is applied to the ball, then it collapses randomly into $|\mathbf{x}\rangle$ or $|\mathbf{x}'\rangle$:

$$|\mathrm{Cat}\rangle \Longrightarrow \begin{cases} |\mathbf{x}\rangle \\ |\mathbf{x}'\rangle \end{cases} . \tag{2}$$

We applied standard quantum mechanics to the c.o.m. motion of the ball as if it were a single atom or molecule. The Cat paradox lies in that we never see a massive body superposed at two macroscopically different locations. The existence of the Cat state (1) is unlikely, its preparation seems impossible.

However, the Cat state (1) and collapse (2) imply more than a paradox, they imply an acute physical problem. While in collapses the basic conservation laws are statistically respected, a single branch after the collapse would violate them. Most embarrassing is the non-conservation of local mass density, i.e., violation of continuity. But non-conservation of total energy and momentum, the non-conservation of c.o.m. are also bad features. In our example, the c.o.m. of the Cat state is $(\mathbf{x} + \mathbf{x}')/2$ before the collapse, and it becomes either $\mathbf{x}$ or $\mathbf{x}'$ randomly after the collapse. Note that breakdown of conservation laws in separate branches of the collapse is acceptable in quantum measurement of a microscopic system because of its interaction with the measuring device. The same is not true for the Cat. The measuring device can be almost massless compared to the Cat, the device won't be able to compensate the shift of the Cat c.o.m. caused by the collapse.

The Cat paradox can be relaxed if one assumes that the collapse (2) happens spontaneously and universally, in the absence and independent of any measurement devices. Accordingly, one adds random spontaneous collapses to the standard quantum theory, resulting in non-linear and stochastic modification of the standard Schrödinger equation. Such dynamical collapse models [16] quickly damp the Cat states, ensuring that macroscopic variables have definite values, i.e., their quantum uncertainties remain suitably microscopical. But spontaneous universal collapse is no solution to the violation of conservation laws. Dynamical collapse theories themselves would steadily pump energy into the quantum system in question, this annoying side-effect has been known longtime ago, cf. [17] and references therein.

In the gravity-related spontaneous collapse model (Sect. 3) a plausible way out appears: the energy–momentum balance might be restored by the gravitational field itself (Sect. 6).

3 Gravity-Related Spontaneous Collapse

When constructing the mechanism of spontaneous collapse in order to eliminate massive Cat states more general than (1), we assume that the spatial mass density $f(\mathbf{r})$ of our quantum system matters. The Cat state reads

$$|\mathrm{Cat}\rangle = |f\rangle + |f'\rangle, \tag{3}$$

where f and f' are 'macroscopically' different. This difference—we call it 'catness'—will be measured by a certain distance $\ell_G(f, f')$. The DP model [1–7] defines catness by the following combination of three different Newtonian interaction potentials:

$$\ell_G^2(f, f') = 2U(f, f') - U(f, f) - U(f', f') \tag{4}$$

where

$$U(f, f') = -G \int\int \frac{f(\mathbf{r})f'(\mathbf{s})}{|\mathbf{r}-\mathbf{s}|} d\mathbf{r}d\mathbf{s}. \tag{5}$$

As we said, we assume that Nature makes $|\mathrm{Cat}\rangle$ decay spontaneously:

$$|\mathrm{Cat}\rangle \Longrightarrow \begin{cases} |f\rangle \\ |f'\rangle \end{cases} . \tag{6}$$

Small catness ℓ_G should allow for slow decay, large catness should spark fast decay. Hence the DP model postulates the Cat characteristic life-time in the form $\tau_G = \hbar/\ell_G^2(f, f')$ which, with (4), yields the following collapse rate:

$$\frac{1}{\tau_G} = \frac{2U(f, f') - U(f, f) - U(f', f')}{\hbar}. \tag{7}$$

This expression guarantees immediate decay (fast rate) for macroscopic cats (Cats) and no decay (extreme slow rate) for atomic 'cats'.

The DP model diverges for point-like constituents. If, e.g., we consider a single point-like object of mass M at location $\mathbf{x}$ (and $\mathbf{x}'$), it yields singular mass distributions:

$$\begin{aligned} f(\mathbf{r}) &= M\delta(\mathbf{r}-\mathbf{x}), \\ f'(\mathbf{r}) &= M\delta(\mathbf{r}-\mathbf{x}'). \end{aligned} \tag{8}$$

The self-interaction terms $U(f, f)$ and $U(f', f')$ become $-\infty$, leading to a divergent collapse rate (7).

4 Resolution of Mass Density $f(\mathbf{r})$ Matters

To treat the divergence of the DP model, we introduce a short length cutoff to limit the spatial resolution of the mass density f, i.e., to coarse-grain f. Two extreme cutoffs have been considered: (i) f is resolved microscopically down to the nuclear size $\sim 10^{-12}$ cm or (ii) f is coarse-grained over the atomic scales $\sim 10^{-8}$ cm. As we show below, the predicted collapse rates are extremely different: for the microscopic resolution (10^{-12} cm) collapse may take milliseconds, for the macroscopic resolution (10^{-8} cm) it may take hours. To see all this, we return to our mechanical Cat (1) and apply the DP model (Sect. 3) to it.

We consider the c.o.m. motion of a rigid spherical object of macroscopic mass M and radius R. Let us first assume that the spatial cutoff is the bigger one (10^{-8} cm), then we coarse-grain f (and f') over the atomic structure. E.g., a macroscopically homogeneous ball yields

$$\begin{aligned} f(\mathbf{r}) &= \rho\theta(|\mathbf{r}-\mathbf{x}| \leq R), \\ f'(\mathbf{r}) &= \rho\theta(|\mathbf{r}-\mathbf{x}'| \leq R), \end{aligned} \tag{9}$$

where $\rho = M/(4\pi R^3/3)$ is the constant mass density, θ is the step-function. The central quantity is the collapse rate (7) of the c.o.m. wave function. It becomes the function of the c.o.m. distance $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}'$, and we calculate it in the first non-vanishing order [1–3]:

$$\frac{1}{\tau_G} = \text{const} \times \frac{M\omega_G^2}{\hbar}(\Delta\mathbf{x})^2. \tag{10}$$

The R-dependence has been absorbed into the parameter $\omega_G = \sqrt{4\pi G\rho/3}$ which we call the frequency of the Newton oscillator, cf. Appendix. This frequency is purely classical, its order of magnitude is $\omega_G \sim 10^{-3}$/s in typical condensed matter where ρ is a few times 1 g/cm^3. The above collapse rate expression is valid if $|\Delta\mathbf{x}| \ll R$.

Alternatively, we consider the DP model with the 'nuclear' cutoff 10^{-12} cm. We therefore take f such that the total mass M is localized in the nuclei of size $\sim 10^{-12}$ cm and of density $\rho^{\text{nucl}} \sim 10^{12} \times \rho$. To recalculate the collapse rate (7) for very small displacements $|\Delta\mathbf{x}|$, note that we can ignore pair-wise contributions of the nuclei. Each nucleus contributes separately like a single ball of density ρ^{nucl}. Hence their total contribution amounts to the expression (10) with ω_G^{nucl} instead of ω_G:

$$\frac{1}{\tau_G} = \text{const} \times \frac{M(\omega_G^{\text{nucl}})^2}{\hbar}(\Delta\mathbf{x})^2, \tag{11}$$

where $\omega_G^{\text{nucl}} = \sqrt{4\pi G\rho^{\text{nucl}}/3}$ is the frequency of the Newton oscillator in nuclear matter. Its order of magnitude is $\omega_G^{\text{nucl}} \sim 10^3$/s, cf. Appendix. The rate expression (11) is valid if $|\Delta\mathbf{x}| \ll 10^{-12}$ cm.

With the 'nuclear' resolution, Cat life-time τ_G has become cca 10^{12} times shorter! Without this huge enhancement [18] of the spontaneous collapses, the experimental test of the DP model would be too requesting, as recognized in [12–14].

5 Equilibrium Rate of Spontaneous Collapse

We are going to investigate a possible universal feature of the standard quantum dynamics modified by the DP spontaneous collapses. We expect that the unitary and collapse mechanisms, respectively, reach a certain balanced coexistence. For the ideal case of free Cat motion an exact proof is known, cf. [3]. Here we present the underlying idea leading to the correct qualitative results [2]. For the free c.o.m. motion of the Cat, the standard kinetic term in the Schrödinger equation tends to spread the wave function, competing with the DP spontaneous collapses which tend to shrink the wave function. These counteracting tendencies reach the balance (equilibrium) when the corresponding two rates are equal:

$$\frac{\hbar}{M(\Delta\mathbf{x})^2} \sim \frac{M\omega_G^2(\Delta\mathbf{x})^2}{\hbar}, \tag{12}$$

where the l.h.s. is the rate of kinetic changes, the r.h.s. is the rate (10) of collapses. Now, calculate the geometric mean of the l.h.s. and the r.h.s., you get ω_G, hence both l.h.s. and r.h.s. must be of the order of ω_G!

For us, the important conclusion is the following. The estimated equilibrium collapse rate of the free Cat is ω_G which is of the order of 10^{-3}/s:

$$\frac{1}{\tau_G^{\mathrm{eq}}} \sim \omega_G \sim \frac{1}{\mathrm{hour}}. \tag{13}$$

The equilibrium collapse rate is independent of the mass M and size R of the Cat. The equilibrium width of localization comes out as $|\Delta\mathbf{x}| \sim \sqrt{\hbar/M\omega_G}$—a tiny scale for massive Cats. The equilibrium collapse rate ω_G is fully classical, independent of $\hbar$. However, the obtained rate, i.e., one collapse per hour, seems too low to be relevant under natural circumstances.

The situation turns around if we assume the DP model with the microscopic mass density resolution: we expect enhanced equilibrium collapse rate. Indeed, ω_G in the balance condition (12) must be replaced by ω_G^{nucl}, meaning that the equilibrium collapse rate is cca 10^6 times higher than before:

$$\frac{1}{\tau_G^{\mathrm{eq}}} \sim \omega_G^{\mathrm{nucl}} \sim \frac{1}{\mathrm{ms}}. \tag{14}$$

This is a remarkably high rate of spontaneous collapse. Although it was obtained for the free rigid ball Cat, it is plausible to expect that a similar universal equilibrium collapse rate exist in long wavelength hydrodynamic modes of condensed (or just bulk) matter under natural circumstances.

We make a tactical decision and define the DP model with the nuclear size 10^{-12} cm cutoff. Hence the effect of DP collapses are enhanced and likely to be relevant even under natural circumstances. The decoherence caused by spontaneous collapses is the main prediction of the DP model (as well as of all dynamical collapse models [16]). Unfortunately, the direct observation of the decoherence in equilibrium Cat is rather hopeless. Therefore, as argued in [18] (cf. also in [19,20]) the DP model should be refined, developed, extended. The upgraded model should predict more characteristic phenomena than the spontaneous collapses.

6 If G-Related Collapse is the Cause of Gravity?

Although the Newton interaction and/or the Einstein equation played a role in the arguments [1–7] leading to the DP model, the parameter G in the resulting DP model controls the spontaneous collapses instead of the Newton interaction. The G-related spontaneous collapses damp Cat states but there is no gravitational dynamics (Newton acceleration). We can introduce it by hand, through the Newtonian pair-potential. But we choose the alternative option of an emergent Newton interaction.

The main motivation comes from what we claimed to be the Cat problem in Sect. 1. The standard collapse of a massive Cat strongly violates conservation laws. This is invariably so with spontaneous collapses. In the equilibrium situation of Sect. 5, the Cat state is being continuously damped by the spontaneous collapses which are still violating the conservation laws—on a tinier scale this time. One would think that these stochastic defects of conservation might be theoretically corrected if we construct a stochastic external field to compensate them. Obviously, external fields cannot restore all conservation laws. The non-conservation of the local density could only be compensated topologically, not dynamically. But momentum and energy non-conservation can, in principle, be cured dynamically. We have recently presented a combination of analytic and heuristic arguments to show that a random external field to compensate the statistical non-conservation of the Cat momentum will on average contribute to what the Cat's own Newton field should be [21]. Until a more rigorous, convincing proof becomes available we explore a different perspective. In a sense, we go one step back, we don't apply analytical tools (like the Ito-calculus in [21]). We conjecture heuristically instead.

We have argued previously why the G-related collapses might induce the phenomenon of Newton field and interaction. We don't intend to concretize the mechanism of the emergence, rather we formulate the following plausible feature of it. *If the G-related collapses induce Newton gravity then, independently of the detailed mechanism, the emergence rate/time of Newton gravity is related (proportional) to the wave function collapse rate/time of the sources.* We outline how this hypothesis would lead to a particular anomaly w.r.t. standard Newton theory.

7 Testing Gravity's Laziness: Is He Exceptional?

Recall Sect. 6 where we defined the DP model with the low (nuclear size 10^{-12} cm) cutoff, leading to the equilibrium collapse time $\tau_G^{\mathrm{eq}} \sim 1$ ms. In Sect. 6 we assumed that this time-scale should be the characteristic emergence time of Newton gravity.

Diósi [22] has tried to reconciliate the existence of a 1 ms emergence time with state-of-the-art Newtonian gravity. The available experimental, both astronomic and laboratory, evidences have poor time-resolution, perhaps not better than 1 ms. How immediate is the creation of the Newton field of accelerating mass sources? Apparently, we cannot exclude a delay if it is $\sim$1 ms or less. This poor state of art may be advanced if we re-analyze previous evidences (or perform new Cavendish experiments) with accelerating (revolving) sources. That is a decent task for classical Newton gravity research. If there is a delay, it may depend on the geometry (i.e., on the wavelength) of the sources. However, it may be quite hard to propose a concrete non-relativistic model where the Newton interaction is delayed. Diósi [22] offers a fenomenological equation to start with. We don't open this chapter now.

Rather we anticipate a phenomenon which should, in the postulated framework, have experimental significance. Let us recall that the 'short' equilibrium collapse (and emergence) time $\tau_G^{\mathrm{eq}} \sim 1\,\mathrm{ms}$ comes from the fine-grained granular subatomic mass distribution of the sources. This is so with typical condensed matter sources. Superfluid He is the only non-relativistic exception where separate nuclei have no identity, no localization. A 'ball' of superfluid He, with given c.o.m., is rather described by a smooth mass density (9) than the granular one. Hence, in superfluid He, DP-collapses may happen very slowly, the Newton field would emerge with a very long delay. For ideal homogeneous Cat the equilibrium collapse (and Newton field emergence) time would be 1 h (13). Superfluid's mass density $f(\mathbf{r})$, when the c.o.m. is fixed, looks perhaps half-way between nuclear granularity and homogeneity, standard condensed matter physics must yield the concrete answer. Perhaps, the DP equilibrium collapse (and emergence) rate happens to be somewhere half-way between 1 ms and 1 h, say, 1 s. That would imply that using a liquid He source in a Cavendish experiment, the pendulum would not react for about a second if we accelerate (e.g. remove) the source.

8 Summary

We pointed out that the Cat quantum state represents an acute theoretical problem: violation of elementary conservation laws, yielding, e.g., non-conservation of the momentum. We have, in simple terms, recapitulated the basics of the DP model of gravity-related spontaneous wave function collapses. The DP model would successfully damp the Cat states whereas it cannot treat the violation of conservation laws. On the other hand, DP collapses show a universal feature when they reach a balance (equilibrium) with the counteracting unitary dynamics. The collapse time-scale turns out to be the fully classical quantity $1/\sqrt{4\pi G\rho^{\mathrm{nucl}}/3} \sim 1\,\mathrm{ms}$ for condensed matter. This circumstance makes us formulate a conjecture beyond the DP model. We argue that the equilibrium stochastic DP collapses of massive objects are accompanied by the stochastic emergence of the Newton field around these objects, with the tendency to restore the momentum balance. Since DP collapses are now claimed to be responsible for the emergence of the Newton interaction, the emergence time of gravity should be proportional to the collapse time 1 ms of the condensed matter source. It follows from our underlying arguments that gravity's emergence time may depend on the quantum mechanical structure. Superfluid He is exceptional, having a very long equilibrium collapse time (like a second, maybe). We predict that a Cavendish experiment with superfluid He source would detect the time-delay of the field if, e.g., we quickly remove the source.

The author has been aspiring after a consistent realization of *gravity from collapse* by some suitable mathematical extension of the DP collapse model. These aspirations which have brought limited successes so far, have surfaced the preliminary conjectures presented above. They are extremely speculative, more than the DP model itself. Yet, they can be falsified or proved in straightforward experiments.

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Appendix: Newton Oscillator

Take a homogeneous ball of mass density ρ, bore a narrow diagonal hole through it, and gently place a probe somewhere into the hole (Fig. 1). The probe oscillates harmonically at frequency

$$\omega_G = \sqrt{4\pi G\rho/3},$$

where G is the Newton constant. It is remarkable that the frequency is the function of density ρ only, it does not depend separately on the size and the mass of the ball. In typical condensed matter the density ρ is a few times $1\,\mathrm{g/cm}^3$, the frequency of the Newton oscillator is $\omega_G \sim 10^{-3}/\mathrm{s}$, the period is as long as cca 1 h.

Formally, we can consider the Newton oscillator inside a homogeneous ball of nuclear density $\rho^{\mathrm{nucl}} \sim 10^{12}\,\mathrm{g/cm}^3$. The oscillator frequency

$$\omega_G^{\mathrm{nucl}} = \sqrt{4\pi G\rho^{\mathrm{nucl}}/3}$$

becomes of the order of $10^3/\mathrm{s}$, the period is as small as cca 1 ms.

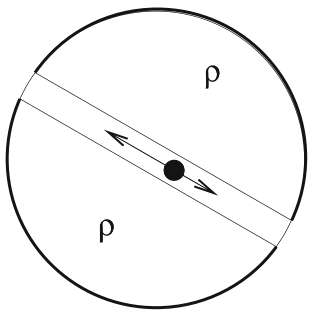

Fig. 1 Schematic view of a homogeneous ball of density ρ, with an infinite narrow diagonal hole where the probe is oscillating at frequency $\omega_G = \sqrt{4\pi G\rho/3}$ under the directional force of the Newton field of the ball

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