Comment on "Uniqueness of the Equation for Quantum State Vector Collapse"

A recent Letter [1] investigated Markovian stochastic Schrödinger equations (SSEs) under the assumption of no-faster-than-light signaling [2]. I found that Theorem 1, claiming that the evolution of the density matrix ρ must be completely positive (CP), is incorrect. Theorem 2 constructs the most general diffusive SSE for the wave function ψ, which looks different from the simpler results in Ref. [3]. I prove that the difference is redundant.

If Theorem 1 were true, no Markovian SSE would exist for the non-CP qubit master equation [4]

$$\frac{d\rho}{dt} = \sum_{k=1}^{3} c_k(\sigma_k\rho\sigma_k - \rho), \qquad c_1 = c_2 = -c_3 = 1. \quad (1)$$

I consider the following SSE (cf. Ref. [5] for a jump process):

$$d\psi = -\frac{1}{2}\sum_{k=1}^{3} c_k(\sigma_k - n_k)^2\psi dt + \sqrt{2}n_z\psi_\perp dW \quad (2)$$

where $n_k = \langle\psi|\sigma_k|\psi\rangle$ and $\psi_\perp$ is orthogonal to ψ, and we can express it by $\psi_\perp = (1-n_z^2)^{-1/2}(n_y\sigma_x - n_x\sigma_y)\psi$. The SSE (2) yields the master equation (1) for $\rho = \mathbb{E}|\psi\rangle\langle\psi|$. The proof goes like this. From Eq. (2) we get

$$\frac{d\rho}{dt} = -\frac{1}{2}\mathbb{E}\sum_{k=1}^{3} c_k\{(\sigma_k - n_k)^2, |\psi\rangle\langle\psi|\} + 2\mathbb{E}n_z^2|\psi_\perp\rangle\langle\psi_\perp|. \quad (3)$$

One can confirm the identity

$$2n_z^2|\psi_\perp\rangle\langle\psi_\perp| = \sum_{k=1}^{3} c_k(\sigma_k - n_k)|\psi\rangle\langle\psi|(\sigma_k - n_k), \quad (4)$$

which when inserted into Eq. (3), leads to the linear master equation (1). Hence, Theorem 1 cannot be correct. The proof fails clearly if the number n of independent Lindblad operators L_k is bigger than the dimension d [6].

For CP master equations, the Letter's Theorem 2 is correct. The authors mention that Ref. [3] had answered the same question, but the Letter does not compare the results. I remedy the omission. An additional gauge transformation $\psi \to \exp(-id\chi)\psi$ with phase $d\chi = \mathrm{Im}\sum_k\langle\psi|L_k^{(\psi)}|\psi\rangle(\ell_k^{(\psi)}dt + dW_k)$ brings the Letter's SSE (4) to the form

$$d\psi = \Big[-iHdt + \sum_{k=1}^{N}\sum_{j=1}^{n} u_{kj}^{(\psi)}(L_j - \langle L_j\rangle)dW_k - \frac{1}{2}\sum_{k=1}^{n}(L_k^\dagger L_k - 2\langle L_k\rangle^\star L_k + |\langle L_k\rangle|^2)dt\Big]\psi \quad (5)$$

where $\langle L_k\rangle = \langle\psi|L_k|\psi\rangle$. The matrix u has gone from the drift part. The resulting SSE coincides exactly with Eq. (8.1) in Ref. [3], implying the following relationship between the noises of Ref. [3] and the Letter, respectively:

$$d\xi_j^* = \sum_{k=1}^{N} dW_k u_{kj}, \qquad j = 1,2,\ldots,n \le N. \quad (6)$$

In Ref. [3], all physically different SSEs are uniquely parametrized by the $n\times n$ complex symmetric correlation matrices $s_{jl} = (\mathbb{E}d\xi_j d\xi_l)/dt$ (to avoid confusion, here we use s for u of Eq. (4.1) in Ref. [3]). Now Eq. (6) establishes the correspondence between the u and s,

$$s_{jl}^* = \sum_{k=1}^{N} u_{kj}u_{kl}, \qquad j,l = 1,2,\ldots,n \le N. \quad (7)$$

As I said, the matrix s_{jl}, only constrained by $||s||$, cf. Eq. (4.3) in Ref. [3], is in one-to-one correspondence with the physically different SSEs at a given CP evolution of ρ. The matrix u_{kj} is not; its part $N \ge j > n$ is redundant. Now Eq. (7) shows a further redundancy: both u and Ou, with any $N\times N$ orthogonal matrix O, yield the same SSE.

Reference [3] derived the SSEs under a CP master equation from standard quantum monitoring. The SSE (2) is the first diffusive SSE considered ever that underlies a non-CP master equation; its physical relevance, if any, needs further studies.

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[6] The proof is false for any number $n > 1$; it would be good to know which further natural conditions might render the theorem true (private communication from the authors of Ref. [1]).

