QUANTUM MEASUREMENT AND GRAVITY FOR EACH OTHER

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INTRODUCTION

Typical quantum measuring devices are based on electromagnetic interactions and, when dealing with quantum measurements, most of us would not consider gravitation at all. Nevertheless, there have been old believers [1] of the role of gravitation in wavefunction collapse. I support this idea and I am going to present further arguments. My speculations may seem rather particular for some but this way has, in fact, led me to an elegant model of spontaneous wavefunction collapse.

MEASUREMENT PROBLEM

Quantum Mechanics consists of two equally important parts: the Schrödinger-equation and the von Neumann Measurement Theory. Remind that without the latter we would have nothing to compare with experiments. The Schrödinger-equation, in itself, is rather like a perfect computer program without output commands.

Many physicists deny the so-called Measurement Problem (MP) since the Quantum Mechanics as well as the von Neumann Measurement Theory are generally thought to be perfect. However, according to John Bell [2], they are not quite perfect, only for all practical purposes (FAPP in Bell's irony) are they perfect.

By quantummechanical MP people mean various things. Let me single out two general issues. i) The Schrödinger-equation allows unnatural macroscopic superpositions. ii) In von Neumann theory, measurement is of distinguished, not perfectly specified notion.

WHERE DOES MEASUREMENT PROBLEM CULMINATE?

To take MP more seriously we shall show a room in the nice building of Physics where Quantum Mechanics and, especially, von Neumann's Mesurement Theory still have not been confirmed as perfect FAPP.

I sketch the building of Physics by a triangle [3] with the three fundamental constants G, c and $\hbar$ at its corners, referring to the three fundamental theories: the Newtonian Gravity, the Special Relativity and the Schrödinger Quantum Mechanics, respec-

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tively. Dirac's Relativistic Quantum Mechanics must be put on the c-$\hbar$ side of the triangle, while the G-c side is for Einstein's General Relativity. What is the side G-$\hbar$ for? Interprete Newton's Gravity as the intrinsic macroscopic law on one hand; remind, on the other, that the Schrödinger-equation is the intrinsic law of microworld. What theory would then link intrinsic microscopic data and common macroscopic ones together?

That is just von Neumann's Measurement Theory, and we have chosen it for the G-$\hbar$ side of the Physics triangle. At least historically, it has been the first bridge from $\hbar$ to G. It is perfect FAPP, therefore the question is still open: where, in Physics triangle, is MP going to culminate?

MEASUREMENT PROBLEM CULMINATES IN QUANTUM COSMOLOGY

The central region of Physics triangle corresponds to a fully unified theory, parametrized by c, G and $\hbar$ together. Having such a Theory of Everything, one would be able to understand the Universe, especially its birth. The hot dense matter of the early Universe needs a relativistic unified theory of micro- and macrophysics.

The best known proposal for the Theory of Everything is the so-called Wheeler-DeWitt's Schrödinger-equation [4]:

$$H\Psi = 0 \tag{1}$$

where Ψ is the wavefunction of the Universe and H is its Hamiltonian.

For the Wheeler-DeWitt-equation (1) the MP has become acute indeed. i) Typical solutions are uninterpretable, there are no time, no space-time to define. ii) There is nothing (i.e. no apparatus, no observer, no von Neumann either) but the Eq.(1). From it, we have no classical output since Ψ is a huge superposition of Everything.

Obviously, we need the following two properties instead. Ad i): solutions must possess interpretable space-times (histories). Ad ii): there must be decoherence between alternative histories, in favor to assign them probabilities.

COARSE-GRAINING

Recently, Gell-Mann and Hartle (GMH) [5] have proposed coarse-graining to achieve the above goals. It is, however, not clear to me whether they assume that the Eq.(1) itself contains the wanted coarse-grained decohering structure or, on the contrary, GMH impose "alternative decohering histories" upon the Wheeler-DeWitt-equation (1) from outside, i.e. they admit a modified dynamics.

Let us summarize their proposal, perhaps not the most general version of it. Introduce a time-dependent complete set $\{P_\alpha(t); \alpha=1,2,\dots\}$ of orthogonal Hermitian projectors. Let us define a given coarse-grained history h by the sequence $h=(\alpha_1,\alpha_2,\dots,\alpha_N)$ and let the corresponding history-dependent coarse-garined quantum state be equal to

$$\Psi(h) = P_{\alpha_N}(t_N)\dots P_{\alpha_2}(t_2)P_{\alpha_1}(t_1)\,\Psi_0 \equiv$$
$$\equiv T\{\prod_n P_{\alpha_n}(t_n)\}\Psi_0\ , \tag{2}$$

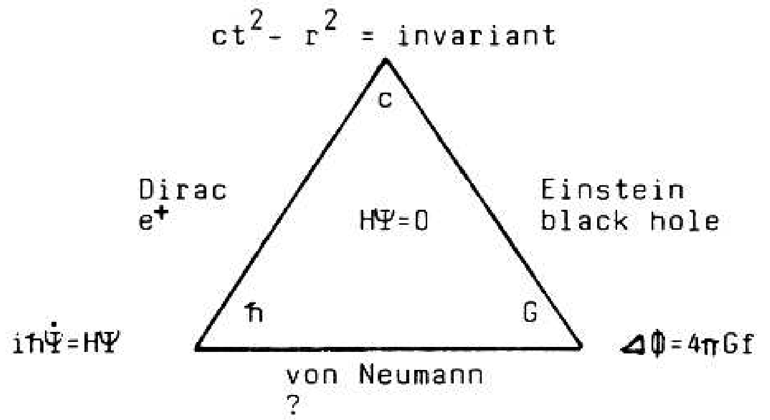

Fig.1. Scheme of Physics' Building. c=velocity of light, G=Newton's gravitational constant, ħ=Planck constant. The corners of triangle represent 3 fundamental theories, the sides correspond to partially unified theories while the middle symbolizes the fully unified theory.

where Ψ_0 is the original Heisenberg-state and T denotes time-ordering. To a given history h one assigns the probability

$$w(h) = \| \Psi(h) \|^2 \quad . \tag{3}$$

These probabilities are compatible with each other provided all pairs of considered histories decohere. GMH introduce decoherence functional D. Two histories h and h' are said to decohere if

$$D(h'|h) \equiv \Psi^+(h') \, \Psi(h) = 0 \quad . \tag{4}$$

Certainly, the construction (2-4) solves the MP, too. More precisely, it would solve if such construction existed. Let us translate the GMH proposal (2-4) into the common language of von Neumann Measurement Theory. Consider the time-dependent observable

$$Q(t) = \sum_{\alpha} \alpha P_{\alpha}(t) \tag{5}$$

composed of GMH's projectors. Observe that coarse-graining is mathematically equivalent to N subsequent measurements of $Q(t_1)$, $Q(t_2), \ldots, Q(t_N)$ à la von Neumann, while the history h is just the corresponding sequence of the measurement outcomes. Then the history-dependent state (2) turns out to be the resulting state after the N measurements, and the probability (3) of GMH's history can be recognized as the standard probability of the N subsequent wavefunction collapses.

We owe to translate the crucial decoherence criterion (4) as well. Let us consider the most simple case by taking N=2, and consider decoherence between histories $h=(\alpha_1,\alpha_2)$ and $h'=(\alpha'_1,\alpha'_2)$, respectively. According to GMH,

$$D(h'|h) = \Psi_0^+ P_{\alpha'_1} P_{\alpha'_2} P_{\alpha_2} P_{\alpha_1} \Psi_0 = 0 \quad , \text{ if } h' \neq h \quad . \tag{6}$$

(Our notation supresses time arguments t_1 and t_2.) If the decoherence conditions (6) fulfil then it is straightforward to prove the following statement: the expectation value of $Q_2 \equiv Q(t_2)$

is independent of whether $Q_1 \equiv Q(t_1)$ was earlier measured or not. Let us see the proof:

$$\begin{aligned} \langle Q_2 \rangle_{Q_1 \text{ measured}} &\equiv \sum_{\alpha_1} \Psi_0^+ P_{\alpha_1} Q_2 P_{\alpha_1} \Psi_0 = \\ &= \sum_{\alpha_1} \sum_{\alpha_2} \alpha_2 D(\alpha_1, \alpha_2 | \alpha_1, \alpha_2) = \\ &= \sum_{\alpha_1'} \sum_{\alpha_1} \sum_{\alpha_2} \alpha_2 D(\alpha_1', \alpha_2 | \alpha_1, \alpha_2) = \\ &= \sum_{\alpha_2} \alpha_2 D(\alpha_2 | \alpha_2) = \Psi_0^+ Q_2 \Psi_0 \equiv \langle Q_2 \rangle . \end{aligned} \quad (7)$$

In general, what we have to conclude is rather surprizing. GMH's coarse-graining is equivalent to performing von Neumann measurements, their decoherence criterion is equivalent to assuming a chain of nondisturbing von Neumann measurements.

If, in the present case, Q_1, Q_2,... were observables of that kind then we would certainly measure the history $h=(\alpha_1, \alpha_2, \dots, \alpha_N)$ without disturbing the original dynamics. This is an old dream of measurement theorists. If a nontrivial chain of nondisturbing measurements have had ever been constructed we would have eliminated the MP long ago.

FROM COARSE-GRAINING TO CONTINUOUS MEASUREMENT

The concept of nondisturbing measurements is too restrictive to solve the MP and, consequently, we should weaken its requirements. In particular, we can specify fuzzy measurements instead of von Neumann's ones. As a reward, the concept of continuous measurement [6] (i.e. of permanent coarse-graining) can be introduced.

If a certain quantized variable $Q(t)$ is to be coarse-grained I propose to assume a formal continuous measurement for it. The c-number function $\bar{Q}(t)$ of measurement outcomes will represent the measured history, according to GMH's philosophy. (I shall write $\bar{Q}$ for histories, instead of h.) The history-dependent state is then equal to

$$\Psi(\bar{Q}) = T \exp[-\gamma/2 \int (Q-\bar{Q})^2 dt] \, \Psi_0 \quad (8)$$

where γ characterizes the strength of continuous measurement, i.e. the strength of the continuous coarse-graining.

You may observe the expression (8) is a smoothed time-continuous version of GMH's one (2). For completeness, let us write down the counterpart of GMH's Eq.(3) for the probability of a given history:

$$w(\bar{Q}) \equiv D(\bar{Q}|\bar{Q}) = \| \Psi(\bar{Q}) \|^2 \quad . \quad (9)$$

The Eqs.(8) and (9) of continuous measurement theory can be cast into stochastic differential equations [7] which offer very flexible mathematical tools for explicit calculations.

The properly weakened version of GMH's decoherence criterion (4) will be the following:

$$D(\bar{Q}'|\bar{Q}) \equiv \Psi^+(\bar{Q}') \, \Psi(\bar{Q}) \approx 0 \;, \quad \text{if } \bar{Q}' \text{ and } \bar{Q} \text{ much differ.} \quad (10)$$

The fulfillment of such criterion follows from the very nature of continuous measurement. Exact results are obtained, e.g., for a free particle subjected to continuous position measurement [8].

The concept of continuous measurement has a few formal results in Quantum Cosmology [9]. Of course, one expect more applications in the future. If we, however, go back, for a while, to the $\hbar$-G side of the Physics triangle, I can present definite results.

CANDIDATE FOR $\hbar$-G THEORY OF SPONTANEOUS COLLAPSE

I am going to outline a certain theory parametrized by $\hbar$ and G (but not by c), which may replace von Neumann's Measurement Theory. It is, at the same time, the nonrelativistic caricature of GMH's Quantum Cosmology.

The coarse-grained (i.e. continuously measured) history variable is, by assumption, the Newtonian gravitational field strength

$$Q(r,t) \equiv G\ \nabla\int f(r',t)\frac{d^2r'}{|r-r'|} \quad , \tag{11}$$

where f is the mass distribution operator of the given system.

From an earlier Landau-Peierls-Bohr-Rosenfeld-type Gedankenexperiment we know that Q of Eq.(11) possesses the following intrinsic coherent fluctuations [3]:

$$\langle \delta Q(r',t')\delta Q(r,t)\rangle \ = \kappa\hbar G\delta^3(r'-r)\delta(t'-t) \tag{12}$$

where κ is order of unity. We are not allowed to assume higher accuracy for the continuous measurement of Q (11) than its inherent uncertainty (12). The maximum possible strength parameter is then the following:

$$\gamma = \kappa/\hbar G \quad . \tag{13}$$

The continuous measurement theory (8),(9) together with the Eqs.(11) and (13) give us the <u>unified $\hbar$-G-theory</u> which is free from MP: i) unnatural macroscopic superpositions are destroyed and ii) there is no distinguished concept for measurements since von Neumann Measurement Theory is automatically recovered in the proper limit.

Ref.10 contains a detailed presentation and discussion of the above $\hbar$-G-theory. It has been shown that, very similarly to the well-known GRW theory [11], the $\hbar$-G-theory offers an elegant unification of micro- and macrodynamics. For its consistency, however, one needs a spatial cutoff; one can, e.g., borrow it from the GRW theory [12], see also Ref.13.

Afterall, I still believe that a certain nonrelativistic theory, parametrized by $\hbar$ and G, must be considered. It would solve MP and, what is more important, it would be the autonomic theory of some, still unknown, new quality of physical phenomena. Remember Dirac's $\hbar$-c-theory discovered antimatter and Einstein's G-c-theory predicted space-time singularities. The genuine $\hbar$-G--theory must yield something unexpected, too.

SUMMARY

I hope I have succeeded to convince you about the relevance and the stimulating power of the Measurement Problem in Physics. Recent considerations in Quantum Cosmology have justified the longstanding discontent of measurement theorists with the present theory, as well as their predictions about the role of gravity in solving MP seem to be verified. New ideas (e.g. coarse-graining, decoherence) in Quantum Cosmology have shown to be related to earlier results in measurement theory.

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