PERMANENT STATE REDUCTION: MOTIVATIONS, RESULTS, AND BY-PRODUCTS [1]

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Abstract

We enlist various motivations underlying the idea that von Neumann collapses occur permanently, spontaneously and even universally. In addition to the conceptual value of permanent reduction theory, we mention its important by-product promoting new quantum Monte-Carlo simulations of irreversible quantum processes, especially in laser optics.

1 INTRODUCTION AND CONTENTS

In the last two decades, serious efforts were concentrated to find the proper mathematical formalism unifying the unitary evolution with permanent collapses. Up to now, things have become well understood at least in the Markovian regime though the universality of permanent reduction has merely been modeled on heuristic grounds. We attempt to classify main contributions that have led to our present-day theory of permanent state reduction. We also emphasize that, as for the fundamental purpose of the project of permanent reduction, we see a spectacular convergence between it, on one hand, and the so-called consistent history concept, on the other hand. This is not very surprising since both projects were initiated to eliminate the fundamental dichotomy of the ordinary quantum theory.

Contents:
1 C-numbers from ψ: Reduction

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2 C-NUMBERS FROM ψ: REDUCTION

However well is quantum mechanics understood its famous dichotomy has remained unresolved after all. The quantum state ψ evolves according to the Schrödinger equation:

$$\dot{\psi} = -iH\psi. \tag{1}$$

The knowledge of ψ in itself does not still provide suitable experimental "c-number" predictions. One has to turn to the von Neumann reduction theory. An apparatus interacts with the system in question and changes the original state of it:

$$\psi \to \psi_z \tag{2}$$

with probability $p_z = |\langle\psi_z|\psi\rangle|^2$, where z is the c-number recorded by the apparatus. The possible final states $\{\psi_z\}$ form a complete orthogonal system. The reduction process above is nonlinear whereas the initial and final statistical (density) operators are related linearly:

$$\rho \to \sum_z P_z \rho P_z \tag{3}$$

with projectors $P_z = |\psi_z\rangle\langle\psi_z|$.

Let us see what we have done starting with the general ambition to eliminate the above fundamental dichotomy of the standard quantum mechanics. What have we invented for our original purpose and what for others ?

3 PERMANENT REDUCTION—EQ. FOR $\psi(t)$

Jumps (2) are not acceptable in Nature, let us interpolate them by equivalent processes. This was the main motivation to construct the detailed time evolution equation for $\psi(t)$ interpolating the von Neumann reduction (2). Let us see the basic results in historical order.

Bohm and Bub [1] proposed deterministic evolution equation where randomness were only put into initial conditions. Nonlinear Wiener process was introduced by Pearle, later he found a a clever gambling game analogy, too [2]. Gisin [3] pointed out that the stochastic average of $|\psi(t)\rangle\langle\psi(t)|$, i.e. $\rho(t)$, must all time obey to linear master equation, and he constructed the first such nonlinear Wiener process. Diósi [4] showed how a unique nonlinear Wiener process follows from an arbitrarily given master equation of Lindblad class. And finally, Gisin and Percival [5] presented the Ito equations of the unique Wiener process. This has been known as Quantum State Diffusion (QSD) theory (see Appendix A).

In addition the the first mentioned motivation, it is worthwhile to invoke two further ones.

Environmental Reduction. In everyday experiences, Nature shows spontaneous classicality. Logically, all c-numbers emerge in corresponding von Neumann reductions. Macroobjects are never isolated and their environment acts formally as apparatus [6]. This action is permanent and spontaneous. The macrosystem's density operator $\rho(t)$ is uniquely calculable, at least in principle. The stochastic process for the unraveling $\rho(t)$ into pure state stochastic process $\psi(t)$ can be done in many ways. The QSD theory yields a unique one.

Universal Reduction. On philosophical grounds we can hardly accept that classicality is only due to the particular, e.g., thermal environment. A certain — dynamically not specified — universal "environment" is to be assumed that guarantees the observed generality and universality of permanent reduction. Then, postulating the form of the ensemble evolution equation, the pure state evolution, representing classicality, follows from QSD theory automatically. For the state of art, see Appendix B.

4 PERMANENT REDUCTION—EQS. FOR $\psi(t)$ AND $z(t)$

A typical class of quantum measuring apparatuses acts permanently, i.e., performs unsharp continuous detection. (Also universal permanent reduction can be represented formally as if Nature would apply certain apparatuses performing permanent detection.) The joined statistics of the continuously reduced state $\psi(t)$ and of the classical record $z(t)$ represents both practical and fundamental interests. The task that follows is twofold. Firstly, one has to construct plausible mathematical model of continuous (unsharp) von Neumann reductions and, secondly, the simplest form of the corresponding stochastic equations for $\psi(t)$ and $z(t)$ has to be found.

Let us see a deliberate choice of results. To my knowledge, the idea of restricting Feynman-path integrals for a tube along $z(t)$ is due to Mensky [7]. Barchielli *et al.* [8, 9] introduced Gaussian "tubes" and derived generating functionals for the distribution functions of the processes $\psi(t), z(t)$. Caves and Milburn [10] invented the feed-back mechanism in the path-integral formalism. Diósi [11] derived 2 separate Ito-equations: one for $\psi(t)$ and one for $z(t)$. The first one turned out to be the Gisin nonlinear equation [3]. Recently, Diósi, Gisin, Halliwell, and Percival [12] proved that the standard QSD corresponds to permanent reduction onto the (approximate) eigenstates of the Lindblad generators accompanied by a certain feed-back *à la* Caves and Milburn [10]. The classical record $z(t)$ is given by $z(t) = \langle L \rangle + \dot{w}(t)$ where $w(t)$ is the same standard (complex) Wiener process that drives $\psi(t)$ in QSD.

5 CONSISTENT HISTORIES

The motivation for the reinterpretation of quantum mechanics in terms of CH [13–16] is the same that the motivation for the permanent reduction project: both models explain the spontaneous emergence of the classical from the quantum. The CH theory constructs families of repeated (or even continuous) reductions, consistent according to classical logics. It is not at all surprizing that the CH and the permanent reduction theories are closely related. Actually, it can be shown that the samples $\psi(t)$ of QSD are histories [12] and in most cases they are consistent as well. At least in those cases where the closed quantum mechanical system splits into a reservoir and a Markovian subsystem in it, the well-developed Ito-calculus of permanent reduction offers explicit equations for the CH theory, too.

6 BY-PRODUCTS AND CONCLUSION

Even if permanent reduction would not have been successful in clarifying fundamental problems it could have, nevertheless, initiated a number of more practical results. Seemingly, the relevance of nonlinear stochastic wave equations were realized first in the cited works of fundamental purposes in the eighties. Nonlinear stochastic equations, based mostly on independent researches (see, e.g., Refs. [17–20]) have now become extensively used in quantum optics. It is not worthless to note that the full technical apparatus for wave function Monte Carlo simulations [21] in any Markovian system has been granted, e.g., from 1988 [4] or, in terms of QSD, from the nineties. Several good examples could be invoked to show that practical by-products of the researches on permanent reduction might well be fertilizing other fields such as quantum optics and communication.

A. QUANTUM STATE DIFFUSION

Consider a system whose state is subjected to permanent reduction and assume that we know the Lindblad master equation governing the evolution of the density operator:

$$\dot{\rho} \equiv \mathcal{L}\rho = -i[H,\rho] + L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\} \tag{4}$$

where L denotes the (vector of) Lindblad generator(s). According to the QSD theory, the following Ito stochastic equation belongs to the Liouville superoperator $\mathcal{L}$:

$$\dot{\psi} = (\mathcal{L}|\psi\rangle\langle\psi|)\,\psi + \dot{w}^*\,(L - \langle\psi|L|\psi\rangle)\,\psi \tag{5}$$

where $w(t)$ is the (vector of) standard complex Wiener process(es):

$$\dot{w}(t)\dot{w}^*(t') = \delta(t-t') \quad , \qquad \dot{w}(t)\dot{w}(t') = 0 \quad . \tag{6}$$

The QSD process (5) has, among others, two noticeable properties. From the viewpoint of fundamental principles: the above QSD is the only pure state diffusion process free of any additional parameters or functions once $\mathcal{L}$ has been defined. It depends only on $\mathcal{L}$; it is invariant under the redefinition of the Lindblad generator(s) L. From a more practical viewpoint: the above pure state diffusion process put as much of the dynamics as possible into the nonlinear drift term hence the stochasticity is kept minimum. In other words, the nonlinear drift of QSD offers the best nonlinear Schrödinger equation to approximate the short time behaviour of the density operator.

B. UNIVERSAL PERMANENT REDUCTION

A well-known first attempt to model Universal Permanent Reduction is the GRW-theory [22]. Let us illustrate the status of Universal Permanent Reduction project by a model which relates permanent reduction to gravity [23]. A compact definition of the model can be achieved if we specify the Lindblad generators. They are chosen to be proportional to the square root of the gravitational constant G:

$$L_{\mathbf{k}} = \sqrt{\frac{G}{4\pi k}} \sum_m m \exp\left(i\mathbf{k}\mathbf{x}_m\right) \tag{7}$$

where $m, \mathbf{x}_m$ are the masses and coordinate operators, resp., of the objects belonging to our system; summation should be taken for all masses. The vector $L_{\mathbf{k}}$ is labelled by the wavenumber $\mathbf{k}$. A short distance cutoff must be applied at $k \approx 10^5 cm^{-1}$ [24]. It can be shown that the corresponding QSD equations reduces to the ordinary quantum mechanics for microscopic masses. For more massive objects, permanent reduction become effective and plausible classicality emerges, e.g., in unique position distributions. Gravitational permanent reduction will hopefully find new justification in the frame of quantum cosmology.

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