

Centre of mass decoherence due to time dilation: paradoxical frame-dependence

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Abstract. The recently proposed centre-of-mass decoherence of composite objects due to gravitational time-dilation [Pikovski *et al*, Nat.Phys. 11, 668 (2015)] is confronted with the principle of equivalence between gravity and observer's acceleration. In the laboratory frame, a positional superposition $|x_1\rangle + |x_2\rangle$ can quickly decohere whereas in the free-falling frame, as I argue, the superposition can survive for almost arbitrary long times. The paradoxical result is explained by the so far unappreciated feature of the proposed model: the centre-of-mass canonical subsystem is ambiguous, it is different in the laboratory and the free-falling frames, respectively.

As long as the centre-of-mass motion of the composite object is non-relativistic, a simple Galilean-covariant Hamiltonian represents the Pikovski *et al* theory with exactly the same physical predictions. We emphasize the power of this Hamiltonian to understand essential features of the Pikovski *et al* theory and to moderate a few divergent statements in recent works.

1. Introduction¹

Pikovski, Zych, Costa and Brukner (PZCB) have shown that the centre-of-mass of a composite object undergoes universal position-decoherence in the presence of gravitational field [1]. They proposed an approximate relativistic correction of order $1/c^2$ to the non-relativistic many-body Hamiltonian. It results in the coupling

$$-\frac{H_i}{mc^2} \left(\frac{p^2}{2m} - mgx \right) \quad (1)$$

between the internal Hamiltonian H_i and the difference between kinetic and gravitational energies of the centre-of-mass. Here m is the total mass, x, p are the centre-of-mass canonical coordinate and momentum, resp., g is the gravitational acceleration. If the object is prepared at rest in superposition $|x_1\rangle + |x_2\rangle$ of two positions x_1, x_2 at large vertical difference Δx then the internal degrees of freedom will quickly decohere the said positional superposition. The universal mechanism, stemming from time-dilation in Earth gravity, decoheres composite systems into the position basis. This happens within standard physics, as emphasized by the authors.

¹ This section, apart from last paragraph, coincides with [2] rejected by *Nature Physics*.



I unearth that this decoherence is not independent of the frame of the observer. Changing the laboratory frame for the free-falling one, the coupling in question becomes

$$-\frac{H_i}{mc^2} \frac{p^2}{2m} \quad (2)$$

by the power of the equivalence principle. We can (though don't need to) take the same H_i in both the laboratory and the free-fall frames since its corrections would be relativistic to contain further $1/c^2$ -factors. If I solve the Schrödinger equation in the free-falling frame, the wave function exhibits centre-of-mass kinetic energy decoherence, position decoherence is completely absent. In the room temperature example of the authors, a gram-scale object prepared in the positional superposition $|x_1\rangle + |x_2\rangle$ at $\Delta x = 10^{-6}$ cm gets decohered after about a millisecond — in the laboratory frame. It does not get decohered in the free-falling frame as long as the kinetic energy decoherence remains marginal. Indeed, the dispersion of p in the initial superposition must be much bigger than $\hbar/\Delta x$ but otherwise it can be kept so small that the kinetic energy decoherence would remain ignorable for any conceivable period. Hence the superposition may survive practically forever in the free-falling frame. I find it fairly paradoxical. Although decoherence is frame-dependent relativistically, it is impossible that a superposition decays in one frame but it never decays in the other.

The reason of such paradoxical frame-dependence is simple but surprising. When we change from laboratory to free-fall frame, the laboratory momentum transforms like $p \Rightarrow p + (m + c^{-2}H_i)gt + \mathcal{O}(1/c^4)$ — it drives out from the state space of the laboratory centre-of-mass subsystem! In other words, the split of the composite system into centre-of-mass and internal canonical subsystems is frame-dependent. The notion of *canonical* centre-of-mass, unlike the notion of centre-of-mass coordinate, is frame-dependent. Quantum mechanics uses the canonical notion. Although in the laboratory frame and, alternatively, in the free-falling frame we are supposed to measure the state of *the* centre-of-mass, the latter is defined differently in the two frames, allowing for the particular inequivalence of decoherence calculated in the two frames respectively. This is a remarkable feature encoded in the theory of PZCB.

For a related criticism on [1], see also ref. [3].

2. Debates, clarifications

In 2015, PZCB were neither aware nor interested in the behaviour of their theory in situations different from what they considered the important ones exclusively. Indeed, the claim [2] of paradoxical frame dependence originated from such applications rather than from — as the authors stated in [4] — conceptual oversight. In their concept, transformation between different frames means the laboratory and free-falling observers describe the same system and the same detector. No doubt then, presence or loss of interference fringes are invariant. Just frame dependence of interference fringes originates from another, not less sensible and standard concept of frame transformation: the laboratory observer applies laboratory detector while the free-falling observer applies free-falling detector. No doubt again, for a free-falling mass the laboratory detector sees PZCB decoherence but the free-falling detector shows perfect fringes. Using different detectors means different experiments, as [4] pointed it out correctly but that should not have been a reason to brush off the surprising outcomes. Recognition and resolution of the paradox could have been the *productive* alternative. For the unsupported (free-falling) mass two major points should have deserved attention.

First, the centre-of-mass reduced dynamics turned out to be unitary in the free-fall frame and heavily non-unitary in the laboratory frame. The offered resolution was the following [2]. The usual invariance of the split

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{cm}} \otimes \mathcal{H}_i \quad (3)$$

of the total system into centre-of-mass and internal degrees of freedom is lost due to the coupling (1) between centre-of-mass and internal degrees of freedom. Without this coupling \mathcal{H}_{cm} is the same in both frames, q, p and centre-of-mass density matrices transform unitarily from one frame to the other. In the PZCB theory, however, \mathcal{H}_{cm} becomes different for laboratory and free-fall frames, respectively. Ref. [4], constantly disregarding the application of the theory in a different situation from ref.'s [1], failed to appreciate the point: the frame-dependence of the split (3) explains why the *reduced dynamics of the centre-of-mass are not unitary equivalent in both frames*.

Second, it turned out that measuring the same (identically prepared) object, a free-falling detector should see perfect fringes while a laboratory detector may see no fringes at all. That is paradoxical enough unless we consider a concrete model of detection. This happened finally with the lucid proof of Pang, Chen and Khalili [5]. These authors considered the superposition of two centre-of-mass wave packets of a free-falling object, prepared at respective vertical positions x_1, x_2 and at common horizontal momentum p . The two wave packets reach the vertical screen through distance L at arrival time Lm/p . If the screen is free-falling together with the mass, the wave packets form the following interference pattern on the screen:

$$\text{const} \times \left(1 + V \cos \left[\frac{p(x_1 - x_2)/L}{\hbar} x_{\text{scr}} \right] \right), \quad (4)$$

where visibility V reaches 1 in ideal experiment. (We set $x_{\text{scr}} = 0$ to height $(x_1 + x_2)/2$). Now let the screen move at vertical speed v_{scr} relative to the mass. The fringes get shifted on the screen:

$$\text{const} \times \left(1 + V \cos \left[\frac{p(x_1 - x_2)/L}{\hbar} \left(x_{\text{scr}} - v_{\text{scr}} \frac{Lm}{p} \right) \right] \right). \quad (5)$$

This shift does not influence fringe visibility. Now let us insert the relativistic correction H_i/c^2 of the mass. The dispersion ΔE of the internal energy H_i means dispersion of arrival times at the screen (i.e: *dispersion of time dilation* according to ref. [1]) resulting in reduction of fringe visibility by the factor

$$\exp \left(-\frac{1}{2} \left(v_{\text{scr}} \frac{(x_1 - x_2)\Delta E}{\hbar c^2} \right)^2 \right). \quad (6)$$

This decoherence effect is special relativistic, depends on the relative transverse velocity of the mass and the screen, but unrelated to gravity. If we insert $v_{\text{scr}} = gt$ corresponding to the screen supported in the laboratory on Earth, and understand that $t = Lm/p$ is the (mean) time passed by from preparation of the superposition until its detection then we expect the above decoherence factor coincide with the prediction of the PZCB theory:

$$\exp \left(-\frac{t^2}{2} \left(\frac{g(x_1 - x_2)\Delta E}{\hbar c^2} \right)^2 \right). \quad (7)$$

In [7] PZCB literally distanced their decoherence effects from those found by ref. [5] on the moving screen (cf. Fig. 1).

3. Shortcut to the Pikovski *et al* theory

Let us start from standard non-relativistic quantum mechanics of a free composite object:

$$H = \frac{p^2}{2m} + H_i. \quad (8)$$

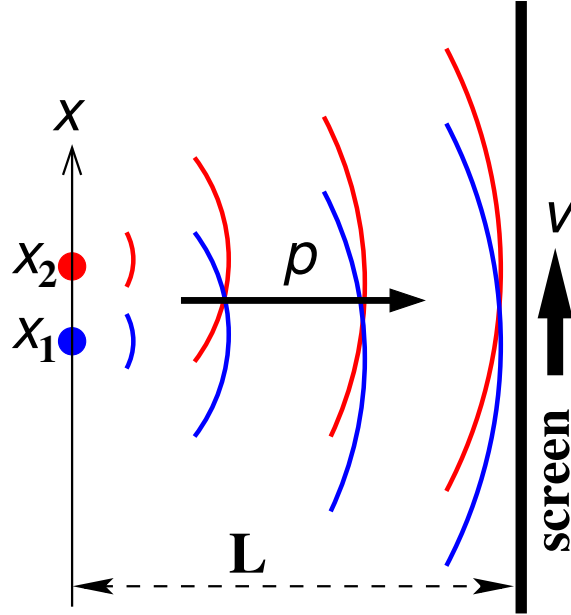


Figure 1. Interference of vertical superposition of two centre-of-mass wave packets in inertial motion, traveling horizontally at momentum p up to the screen which is in upward motion at velocity v_{scr} . Vertical shift of the fringes is $-v_{\text{scr}}$ times the arrival time. Since the arrival time Lm/p becomes $L(m + c^{-2}H_i)/p$ due to special relativistic time dilation, fringe visibility becomes corrupted by the dispersion of the internal energy H_i .

The non-standard ansatz is the following: we take the relativistic correction H_i/c^2 of the mass into the account:

$$H = \frac{p^2}{2(m + c^{-2}H_i)} + H_i. \quad (9)$$

This couples the centre-of-mass and internal degrees of freedom. It is important to emphasize that we ignore other possible relativistic corrections of the Hamiltonian. The obtained theory is a non-relativistic theory with a relativistic correction of the mass only. The Hamiltonian is Galilean-covariant, more precisely, it is covariant under the trivial extension of the Galilean group [6]. Moreover, it satisfies the Newtonian equivalence principle: an accelerated frame is equivalent with gravity. In particular, if we change the inertial (free-falling) reference frame for upward accelerated one (equivalent to laboratory frame) then, after canonical transformation, the Hamiltonian reads:

$$H = \frac{p^2}{2(m + c^{-2}H_i)} + (m + c^{-2}H_i)gx + H_i, \quad (10)$$

which, indeed, captures the presence of gravity g . The gravitating mass contains the same relativistic correction that we added to the inertial mass. The above non-relativistic theory, with its simple Galilean and Newtonian transformation properties, contains all physics of the PZCB theory as long as the centre-of-mass velocities can be ignored on the scale of light's velocity c , so that we can safely ignore the usual $\propto (p/mc)^2$ relativistic corrections of the Hamiltonian.

PZCB [1] proposed a fruitful relativistic correction to spatial dynamics of non-relativistic composite objects. The small parameter of usual relativistic corrections is $(p/mc)^2$. However, in ref. [1] there is a second small parameter (H_i/mc^2) . *The two corrections are independent.*

The latter correction, without the usual ones, captures all the novelties due to relativistic time dilation of the internal ‘clock’ of the composite object. If the spatial motion is non-relativistic, that has been the case in most, if not all proposals so far, then the above Galilean-Newtonian Hamiltonian theory is equivalent with the PZCB theory [1].

Note that the PZCB coupling (1) corresponds to the Hamiltonian (10) expanded up to terms linear in the small parameter (H_i/mc^2). Although the relativistic consistency of higher orders of (H_i/mc^2) may be questionable, the full Hamiltonian (10) is convenient for its exact (extended) Galilean symmetry.

4. Epilogue

The Pikovski *et al* theory [1] is an *original, thoughtful and highly motivating* proposal for the effective relativistic coupling between centre-of-mass and internal degrees of freedom, with all its consequences like spatial gravitational decoherence, and with all its interpretations in context of special and general relativistic time dilation and of quantum foundations. The PZCB theory had been conceived as an *approximate Lorentz invariant* theory, up to order of $1/c^2$. This approach has long masked the fact that there is an *exact extended-Galilean-invariant* reduction of PZCB theory, fully incorporating relativistic time dilation of internal motion and being equivalent to PZCB theory *as long as the centre-of-mass motion is non-relativistic*. Interestingly, in ref. [8] Zych and Brukner were fully aware of this theory but, apparently, they did not accept it as a relevant option. Recognition would have waived desperate divergences in previous debates, would help ongoing discussions as well.

Bondar, Okon and Sudarsky [3] criticized the PZCB theory for it violates Bargman’s mass superselection rule [9]. The PZCB refutation [4] was this: the theory is derived from a relativistic invariant theory, it is not Galilean-invariant hence not subject to mass superselection [9]. The precise resolution of the conflict is that mass superposition is allowed by a trivial extension of the Galilean symmetry, see [6], under which (10) is exact covariant. This was clear for the authors of [8] but, unfortunately, the refutation did not mention it.

A composite object supported at rest in the laboratory frame, rather than in free-fall, was the exclusive object of the original derivation of PZCB centre-of-mass decoherence [1]. A recent work [10] claimed that the decoherence effect becomes fully canceled by the supporting potential. Fortunately, PZCB [11] point out correctly that [10] constructs the mistaken supporting potential which turns out to be a gravitational field $-g$ just canceling with Earth’s gravity g . Now, the mistake of [10] emerged from the desire to replace the non-relativistic supporting potential well $V(x)$ by some relativistically covariant supporting field. Had one started from the Hamiltonian (10), an additional supporting potential $V(x)$ would have been added consistently and the PZCB decoherence would have been derived correctly and in agreement with [1].

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Appendix A. Derivation of PZCB Hamiltonian from special relativity

We start with the standard Lagrange function of classical pointlike mass in special relativity, in Minkowski coordinates:

$$L = -mc\sqrt{c^2 - \dot{x}^2} \quad (\text{A.1})$$

yielding the Hamilton function:

$$H = c\sqrt{m^2c^2 + p^2}. \quad (\text{A.2})$$

Consider the same motion in Rindler coordinates \tilde{t}, \tilde{x} (without restriction of generality, we can take a single spatial dimension). Substituting

$$t = \frac{1}{c}(\tilde{x} + c^2/g) \sinh(g\tilde{t}/c), \quad x = (\tilde{x} + c^2/g) \cosh(g\tilde{t}/c) - c^2/g, \quad (\text{A.3})$$

the Lagrange function reads:

$$\tilde{L} = -mc\sqrt{(c + g\tilde{x}/c)^2 - \dot{\tilde{x}}^2}, \quad (\text{A.4})$$

yielding the Rindler Hamilton function:

$$\tilde{H} = c\sqrt{m^2(c + g\tilde{x}/c)^2 + \tilde{p}^2}. \quad (\text{A.5})$$

The quantized motion of relativistic point-like masses needs field theory, but the quantum mechanical model exists at small velocities $p \ll mc$, with the following Hamiltonian in Minkovski coordinates:

$$H = mc^2 + \frac{p^2}{2m} \left(1 - \left(\frac{p}{2mc} \right)^2 \right), \quad (\text{A.6})$$

and in Rindler coordinates:

$$\tilde{H} = mc^2 + \frac{1}{2} \left\{ \frac{\tilde{p}^2}{2m} + mg\tilde{x}, 1 - \left(\frac{\tilde{p}}{2mc} \right)^2 \right\}. \quad (\text{A.7})$$

If the mass has internal degrees of freedom of Hamiltonian H_i then we can introduce the relativistic correction H_i/c^2 to the inertial mass. If we retain terms linear in H_i , the above two Hamiltonians take the following forms:

$$H = mc^2 + H_i + \frac{p^2}{2m} \left(1 - \left(\frac{p}{2mc} \right)^2 - \left(\frac{H_i}{mc^2} \right)^2 \right), \quad (\text{A.8})$$

$$\tilde{H} = mc^2 + H_i + \frac{p^2}{2m} \left(1 - \left(\frac{p}{2mc} \right)^2 - \left(\frac{H_i}{mc^2} \right)^2 \right) + \frac{1}{2} \left\{ mg\tilde{x}, 1 - \left(\frac{\tilde{p}}{2mc} \right)^2 + \left(\frac{H_i}{mc^2} \right)^2 \right\}. \quad (\text{A.9})$$

Appendix B. Reduced dynamics in laboratory and in free-fall frames²

We are going to determine the centre-of-mass reduced density matrix $\rho_{\text{cm}}(t)$ using an uncorrelated initial state:

$$\rho_{\text{cm}}(t) = \text{Tr}_i \left[e^{-itH/\hbar} \left(\rho_{\text{cm}}(0) \otimes \frac{e^{-H_i/k_B T}}{Z} \right) e^{itH/\hbar} \right]. \quad (\text{B.1})$$

Let our Hamiltonian be

$$H = \frac{p^2}{2(m + c^{-2}H_i)} + (m + c^{-2}H_i)gx + H_i \quad (\text{B.2})$$

which yields the coupling used by the authors in the leading order of $1/c^2$. We use this non-perturbative form just for convenience of calculations, without attributing any physical significance to the higher order terms. [The Hamiltonian of ref. [1] must contain a further

² As appeared in v2 of [2].

potential $V(x)$, to support the two wave packets that are superposed initially.] In the free-falling frame we determine $\rho_{\text{cm}}(t)$, using the Hamiltonian

$$H = \frac{p^2}{2(m + c^{-2}H_i)} + H_i \quad (\text{B.3})$$

obtained from (B.2) in the free-fall coordinate $x - gt^2/2$. The laboratory and free-falling frames coincide at $t = 0$, we assume the same initial state, cf. (B.1), in both frames, respectively. It turns out that $\rho_{\text{cm}}(t)$ may show quick positional decoherence in the laboratory frame and practically no decoherence ever in the free-falling frame.

We introduce the following centre-of-mass Hamiltonian in function of the internal energy eigenvalues E :

$$\begin{aligned} H(E) &= \frac{p^2}{2(m + c^{-2}E)} + (m + c^{-2}E)gx \\ &\equiv K(E) + (m + c^{-2}E)gx \end{aligned} \quad (\text{B.4})$$

where $K(E)$ stands for the kinetic part. The reduced state of interest (B.1) can be expressed in this form:

$$\rho_{\text{cm}}(t) = \sum_E \frac{e^{-E/k_B T}}{Z} e^{-itH(E)/\hbar} \rho_{\text{cm}}(0) e^{itH(E)/\hbar}. \quad (\text{B.5})$$

The following useful identity holds for the operator part:

$$\begin{aligned} \langle x_1 | e^{-itH(E)/\hbar} \rho_{\text{cm}}(0) e^{itH(E)/\hbar} | x_2 \rangle &= \\ &= e^{-it(m+c^{-2}E)g(x_1-x_2)/\hbar} \times \\ &\times \langle x_1 - \frac{1}{2}gt^2 | e^{-itK(E)/\hbar} \rho_{\text{cm}}(0) e^{itK(E)/\hbar} | x_2 - \frac{1}{2}gt^2 \rangle. \end{aligned} \quad (\text{B.6})$$

We can ignore the unitary evolution $\exp[-itK(E)/\hbar]$ if both the coherent momentum and momentum uncertainty of the initial state $\rho_{\text{cm}}(0)$ are suitably small. This can simply happen to a massive object initially at rest, as we see later. So we insert the identity (B.6) with $K(E) = 0$ into the expression (B.5) of $\rho_{\text{cm}}(t)$; we obtain:

$$\begin{aligned} \langle x_1 | \rho_{\text{cm}}(t) | x_2 \rangle &= \\ &= \sum_E \frac{e^{-E/k_B T}}{Z} e^{-it(m+c^{-2}E)g(x_1-x_2)/\hbar} \times \\ &\times \langle x_1 - \frac{1}{2}gt^2 | \rho_{\text{cm}}(0) | x_2 - \frac{1}{2}gt^2 \rangle. \end{aligned} \quad (\text{B.7})$$

We evaluate the pre-factor using the Gaussian approximation of thermodynamic fluctuations. The result is an explicit positional decoherence pre-factor:

$$\begin{aligned} \langle x_1 | \rho_{\text{cm}}(t) | x_2 \rangle &= \\ &= e^{-\frac{1}{2}[c^{-2}\Delta E g(x_1-x_2)t/\hbar]^2} \times \\ &\times e^{-it(m+c^{-2}\bar{E})g(x_1-x_2)/\hbar} \langle x_1 - \frac{1}{2}gt^2 | \rho_{\text{cm}}(0) | x_2 - \frac{1}{2}gt^2 \rangle \end{aligned} \quad (\text{B.8})$$

where $\bar{E}, \Delta E$ are the mean and the fluctuation, respectively, of the internal energy. We read out the decoherence time:

$$\tau_{\text{dec}} = \frac{\hbar c^2}{\Delta E g |x_1 - x_2|}. \quad (\text{B.9})$$

[This result is equivalent with the one obtained by [1] for the superposition resting in a potential $V(x)$.] The squared fluctuation is proportional with the heat capacity of the object:

$$(\Delta E)^2 = k_B T^2 \frac{d\bar{E}}{dT}. \quad (\text{B.10})$$

Let us apply the result (B.8-B.10) to an initial superposition of two Gaussian wave packets of width σ each, standing at x_1 and x_2 , respectively. For concreteness, we choose $\sigma = 10^{-8}$ m and $|x_1 - x_2| = 10^{-6}$ m, guaranteeing the perfect separation of the two wave packets. [This is a faithful physical representation of the symbolic superposition $|x_1\rangle + |x_2\rangle$.] Also we take $m = 1$ g and $T = 300$ K. The influence of the kinetic Hamiltonian $K(E)$ is completely ignorable as long as $t \ll m\sigma^2/\hbar \sim 10^{10}$ s. Hence, for any conceivable period, the results (B.8-B.10) are correct.

Guessing the heat capacity of the 1 g object by $(d\bar{E}/dT) \sim 1$ J/K, we get $\Delta E \sim 10^{-4} - 10^{-5}$ erg from eq. (B.10). Therefore eq. (B.9) yields $\tau_{\text{dec}} \sim 0.1 - 0.01$ ms. [If one calculates ΔE from the microscopic model of ref. [1], eq. (B.9) recovers its decoherence time ~ 1 ms.]

Let us change the reference frame for the free-falling one. Using the Hamiltonian (B.3), the counterpart of eq. (B.7) reads:

$$\rho_{\text{cm}}(t) = \sum_E \frac{e^{-E/k_B T}}{Z} e^{-itK(E)/\hbar} \rho_{\text{cm}}(0) e^{itK(E)/\hbar}, \quad (\text{B.11})$$

where $K(E) = \frac{1}{2}p^2/(m + c^{-2}E)$ and p is the momentum operator in the free-fall frame. We showed above that the chosen initial superposition with the “wide” standing wave packets remain practically static against the kinetic Hamiltonian $K(E)$. Accordingly, we can set $K(E) = 0$ for them and we end up with the trivial result:

$$\rho_{\text{cm}}(t) = \rho_{\text{cm}}(0). \quad (\text{B.12})$$

The said superposition of the 1g object remains static practically forever in the free-fall reference frame.

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