Two Invariant Surface-Tensors Determine CSL of Massive Body Wave Function

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Abstract Decoherence of massive body wave function under Continuous Spontaneous Localization is reconsidered. It is shown for homogeneous probes with wave functions narrow in position and angle that decoherence is a surface effect. Corresponding new surface integrals are derived as the main result. Probe's constant density and two completely geometric surface-dependent invariant tensors encode full dependence of positional and angular decoherence of masses, irrespective of their microscopic structure. The two surface-tensors offer a new insight into CSL and a flexible approach to design laboratory test masses.

1 Introduction

Spontaneous decoherence and collapse models, reviewed e.g. by [1, 2] share the form of modified von Neumann equation of motion for the quantum state $\hat{\rho}$:

$$\frac{d\hat{\rho}}{dt} = -\frac{\mathrm{i}}{\hbar}[\hat{H}, \hat{\rho}] + \mathcal{D}\hat{\rho}, \tag{1}$$

where $\hat{H}$ is the many-body Hamiltonian of masses m_a with positions $\hat{\mathbf{x}}_a$ and momenta $\hat{\mathbf{p}}_a$, resp., for $a = 1, 2, \ldots$. The term of spontaneous decoherence takes this generic form:

$$\mathcal{D}\hat{\rho} = -\int\int D(\mathbf{r}-\mathbf{r}')[\hat{\varrho}(\mathbf{r}), [\hat{\varrho}(\mathbf{r}'), \hat{\rho}]]\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}', \tag{2}$$

containing the mass density operator at location $\mathbf{r}$:

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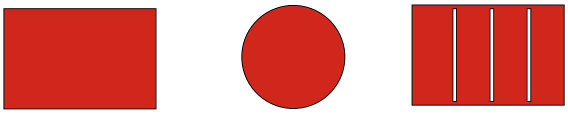

Fig. 1 For a shape (e.g. a cuboid) lacking rotational symmetry, both position and angle are localized since both of them alter the surface (left). For a sphere, angle does not alter the surface, hence position is localized but angle is not (middle). If we carve N transversal gaps into the cuboid (right), to multiply the surface then we enhance the localization rate by a factor about $N+1$ in the longitudinal direction (horizontal, in our case).

$$\hat{\varrho}(\mathbf{r}) = \sum_a m_a \delta(\mathbf{r} - \hat{\mathbf{x}}_a). \tag{3}$$

The non-negative decoherence kernel $D(\mathbf{r}-\mathbf{r}')$ is model dependent.

In a conference talk [4], I compared some characteristic features of the two leading proposals, the Continuous Spontaneous Localization (CSL) of Ghirardi, Pearle, and Rimini, and the model of Penrose and myself [5, 6] called DP-model after the two independent proponents. I claimed and gave examples (Fig. 1) for CSL in particular that the *surfaces of homogeneous massive bodies are the only subjects of localization*. My observation has been waiting for mathematical formulation until now.

In recent literature, the central mathematical object is the *geometric factor* of decoherence:

$$\mu_{\mathbf{k}} = \sum_a m_a \mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{r}_a}, \tag{4}$$

defined in the c.o.m. frame, introduced by [7], also discussed by [8] in this volume. This object is the Fourier-transform of the classical mass density in the c.o.m. frame:

$$\mu(\mathbf{r}) = \sum_a m_a \delta(\mathbf{r} - \mathbf{r}_a). \tag{5}$$

Usually, the contribution of the geometric factor is evaluated in the Fourier-representation. I am going to show that working in the physical space instead of Fourier's is not only possible but even desirable.

In Sect. 2 we recapitulate the decoherence of c.o.m. motion in terms of the geometric factor. For constant density probes, Sect. 3 derives a new practical expression of the decoherence in terms of a simple surface integral, the method is applied for angular (rotational) decoherence in Sect. 4. Possible generalizations towards probes with unsharp edges and for wider superpositions are outlined in Sect. 5, while Sect. 6 is for conclusion and outlook.

2 Center-of-mass Decoherence

The standard CSL model [1] introduces two universal parameters, collapse rate $\lambda = 10^{-17}s^{-1}$, localization $\sigma = 10^{-5}$ cm, and it contains the nuclear mass m_N. The decoherence kernel $D(\mathbf{r}-\mathbf{r}')$ is a Gaussian whose nonlocal effect can be absorbed by a Gaussian smoothening of the mass density $\hat{\varrho}(\mathbf{r})$. The key quantity is the σ-smoothened mass distribution operator:

$$\hat{\varrho}_\sigma(\mathbf{r}) = \sum_a m_a G_\sigma(\mathbf{r} - \hat{\mathbf{x}}_a), \tag{6}$$

where $G_\sigma(\mathbf{r})$ is the central symmetric Gaussian distribution of width σ. Then the decoherence term (2) becomes a single-integral:

$$\mathcal{D}\hat{\rho} = -\frac{4\pi^{3/2}\lambda\sigma^3}{m_N^2}\int[\hat{\varrho}_\sigma(\mathbf{r}),[\hat{\varrho}_\sigma(\mathbf{r}),\hat{\rho}]]\mathrm{d}\mathbf{r}. \tag{7}$$

Inserting Eq. (6), Fourier-representation yields this equivalent form:

$$\mathcal{D}\hat{\rho} = -\frac{\lambda\sigma^3}{2\pi^{3/2}m_N^2}\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}\sum_{a,b} m_a m_b[\mathrm{e}^{\mathrm{i}\mathbf{k}\hat{\mathbf{x}}_a},[\mathrm{e}^{-\mathrm{i}\mathbf{k}\hat{\mathbf{x}}_b},\hat{\rho}]]\mathrm{d}\mathbf{k}. \tag{8}$$

We are interested in the c.o.m. dynamics of the total mass $M = \sum_a m_a$:

$$\frac{d\hat{\rho}_{\mathrm{cm}}}{dt} = -\frac{\mathrm{i}}{\hbar}[\hat{H}_{\mathrm{cm}},\hat{\rho}_{\mathrm{cm}}] + \mathcal{D}_{\mathrm{cm}}\hat{\rho}_{\mathrm{cm}}, \tag{9}$$

where $\hat{\mathbf{X}}$, $\hat{\mathbf{P}}$ will stand for the c.o.m. coordinate and momentum. To derive the c.o.m. decoherence term (and also the rotational decoherence term later on in Sect. 4), substitute

$$\hat{\mathbf{x}}_a = \hat{\mathbf{X}} + \mathbf{r}_a + \hat{\boldsymbol{\varphi}} \times \mathbf{r}_a \tag{10}$$

in (8), where $\mathbf{r}_a$ are the constituent coordinates in the c.o.m. frame in rigid body approximation; $\hat{\boldsymbol{\varphi}}$ is the vector of angular rotation, assuming $\langle\hat{\boldsymbol{\varphi}}\rangle, \Delta\varphi \ll \pi$. Then Eq. (8), by taking trace over the rotational degrees of freedom, reduces to the following c.o.m. decoherence term:

$$\mathcal{D}_{\mathrm{cm}}\hat{\rho}_{\mathrm{cm}} = -\frac{\lambda\sigma^3}{\pi^{3/2}m_N^2}\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}|\mu_{\mathbf{k}}|^2\left(\mathrm{e}^{\mathrm{i}\mathbf{k}\hat{\mathbf{X}}}\hat{\rho}_{\mathrm{cm}}\mathrm{e}^{-\mathrm{i}\mathbf{k}\hat{\mathbf{X}}} - \hat{\rho}_{\mathrm{cm}}\right)\mathrm{d}\mathbf{k}, \tag{11}$$

where we recognize the presence of the geometric factor $\mu_{\mathbf{k}}$. At small quantum uncertainties, when $\Delta\mathbf{X} \ll \sigma$, we use the momentum-diffusion equation as a good approximation:

$$\mathcal{D}_{\rm cm}\hat\rho_{\rm cm} = -\frac{\lambda\sigma^3}{2\pi^{3/2}m_N^2}\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}|\mu_\mathbf{k}|^2[\mathbf{k}\hat{\mathbf{X}},[\mathbf{k}\hat{\mathbf{X}},\hat\rho_{\rm cm}]]\mathrm{d}\mathbf{k}. \tag{12}$$

This equation describes position-decoherence, together with momentum-diffusion, both of them being non-isotropic in the general case. We are going to concentrate on the evaluation of the tensorial coefficient of decoherence on the r.h.s. of (12).

3 Invariant Surface-Tensor for C.O.M. Decoherence

As we see, the geometric factor $\mu_\mathbf{k}$ itself does not matter but its squared modulus does. We consider the approximation (12) which allows for a spectacular simple geometric interpretation of the relevant structure:

$$\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}|\mu_\mathbf{k}|^2(\mathbf{k}\circ\mathbf{k})\mathrm{d}\mathbf{k} = (2\pi)^3\int \nabla\mu_\sigma(\mathbf{r})\circ\nabla\mu_\sigma(\mathbf{r})\,\mathrm{d}\mathbf{r}. \tag{13}$$

We can recognize $\mu_\sigma(\mathbf{r})$ as the σ-smoothened mass density in the c.o.m. frame.[1] This latter form becomes amazingly useful if the bulk is much larger than σ and possesses constant density ϱ when averaged over the scale of σ. If, furthermore, we assume the density drops sharply from ϱ to zero through the surface then $\nabla\mu_\sigma(\mathbf{r})$ is vanishing everywhere but in about a σ-layer around the surface. Let $\mathbf{n}$ stand for the normal vector of the surface at a given point $\mathbf{r}$ and let h be the height above the surface, then

$$\nabla\mu_\sigma(\mathbf{r}+h\mathbf{n}) = -\varrho\mathbf{n}g_\sigma(h), \tag{14}$$

$g_\sigma(h)$ is the central Gaussian of width σ. The volume integral can be rewritten, with good approximation, as an integral along h and a subsequent surface integral:

$$\begin{aligned}(2\pi)^3\int \nabla\mu_\sigma(\mathbf{r})\circ\nabla\mu_\sigma(\mathbf{r})\mathrm{d}\mathbf{r} &= (2\pi)^3\varrho^2\oint \mathbf{n}\circ\mathbf{n}\left(\int g_\sigma^2(h)\mathrm{d}h\right)\mathrm{d}S\\ &= \frac{(2\pi)^3\varrho^2}{2\pi^{1/2}\sigma}\oint(\mathbf{n}\circ\mathbf{n})\mathrm{d}S. \end{aligned}\tag{15}$$

If the prove has cavities in it, and the characteristic sizes of the probe and cavities keep to be much larger than σ, then the surface integral must be extended for the surfaces of the cavities as well. Using Eqs. (13) and (15), the decoherence term (12) obtains the attractive form

[1]Previous works, like e.g. [10] and Supplemental Material (S11) of [11], used the double-integral :

$$\pi^{3/2}\sigma^{-3}\iint \exp\left(-|\mathbf{r}-\mathbf{r}'|^2/(4\sigma^2)\right)\nabla\mu(\mathbf{r})\circ\nabla\mu(\mathbf{r}')\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}',$$

without deriving the equivalent single-integral as of the r.h.s. of Eq. (13).

$$\mathcal{D}_{\mathrm{cm}}\hat{\rho}_{\mathrm{cm}} = -\frac{2\pi\lambda\sigma^2\varrho^2}{m_N^2}\oint[\mathbf{n}\hat{\mathbf{X}}, [\mathbf{n}\hat{\mathbf{X}}, \hat{\rho}_{\mathrm{cm}}]]\mathrm{d}S. \tag{16}$$

This is our main result. It shows that the c.o.m. decoherence is completely determined by the constant density ϱ and the shape of the body, through the *surface-tensor*

$$\mathrm{S}_{\mathrm{cm}} =: \oint(\mathbf{n}\circ\mathbf{n})\mathrm{d}S. \tag{17}$$

In CSL, at small quantum uncertainties $\Delta\mathbf{X} \ll \sigma$, the c.o.m. decoherence of homogeneous sharp-edged bulks is a *surface effect*!

Recall that the main result (16) remains valid if the probe has cavities and we integrate over the surfaces of the cavities as well. This allows us to multiply the CSL decoherence by carving cavities inside the otherwise homogeneous probe, CSL decoherence can be multipled (cf. Fig. 1). This explains the reason of enhanced decoherence in layered structures, proposed by [9].

The heating rate, coming from the decoherence term in (12), is defined by the Heisenberg derivative $\Gamma_{\mathrm{cm}} = \mathcal{D}_{\mathrm{cm}}(\hat{\mathbf{P}}^2/2M)$. Now easy is to write it in a more explicite form than before. Reading $\mathcal{D}_{\mathrm{cm}}^\dagger = \mathcal{D}_{\mathrm{cm}}$ off from (16), one immediately obtains

$$\Gamma_{\mathrm{cm}} = \frac{2\pi\lambda\sigma^2\varrho^2}{m_N^2}\frac{S}{M} = \frac{2\pi\lambda\sigma^2\varrho}{m_N^2}\frac{S}{V}, \tag{18}$$

where S is the total surface (including cavities' internal surfaces) and V is the total volume. Note that Γ_{cm} is the same if we start from the general dynamics (11) not restricted by $\Delta\mathbf{X} \ll \sigma$. [It does not matter if we calculate the Heisenberg derivative of the quadratic $\hat{\mathbf{P}}^2$ by $\mathcal{D}_{\mathrm{cm}}$ in (11) or, alternatively, by the $\hat{\mathbf{X}}$-quadratic approximation of $\mathcal{D}_{\mathrm{cm}}$ in (12).] Interestingly, c.o.m. heating is inverse proportional to the size of the bulk. Recall the total heating rate

$$\Gamma = \mathcal{D}\sum_a \frac{\hat{\mathbf{p}}^2}{2m_a} = \frac{3\hbar^2\lambda}{2m_N^2\sigma^2}M, \tag{19}$$

always much larger than the c.o.m. heating. For a sphere of radius R we get $\Gamma_{\mathrm{cm}}/\Gamma = 3(\sigma/R)^4$.

Examples. Consider the longitudinal motion of a cylinder, Eq. (16) reduces to

$$\mathcal{D}_{\mathrm{cm}}\hat{\rho}_{\mathrm{cm}} = -\frac{2\pi\lambda\sigma^2\varrho^2}{m_N^2}S_\perp[\hat{x}, [\hat{x}, \hat{\rho}_{\mathrm{cm}}]], \tag{20}$$

where $S_\perp$ is the total surface perpendicular to the motion (i.e.: the area of both faces of the cylinder). At a given constant density ϱ, the decoherence is independent of the length of the cylinder. It can be squeezed to become a plate or elongated to become a rod. This invariance of the decoherence offers a fair guidance when we design laboratory probes. However, the same invariance may raise conceptual questions as well. With increasing length of the rod while decoherence rate remains constant, CSL might leave the longitudinal superposition of our massive rod with counter-intuitive long coherence times. An other remarkable feature of the surface-tensor S is that spontaneous decoherence in one direction can be decreased by tilted edges instead of perpendicular ones. If the faces of the above cylinder are replaced by cones of apex angle θ then the two factors $\mathbf{n}\hat{\mathbf{X}}$ in Eq. (16) get a factor $\sin(\theta/2)$ each while the surface of the cones becomes $\sin^{-1}(\theta/2)$-times larger than $S_\perp$. The spontaneous longitudinal decoherence becomes suppressed by the factor $\sin(\theta/2)$. E.g.: sharp pointed needles become extreme insensitive to longitudinal CSL.

4 Rotational Decoherence

Rotational decoherence of objects under CSL has recently been discussed by [12, 13]. Derivation of our main result (16) on decoherence of lateral superpositions tells us how to express this time the decoherence of angular superpositions in terms of a surface integral. We outline the steps, without the details. After substituting $\hat{\mathbf{x}}_a$ by Eq. (10) into Eq. (8), we trace over the c.o.m. motional d.o.f., yielding

$$\mathcal{D}_{\mathrm{rot}}\hat{\rho}_{\mathrm{rot}} = -\frac{\lambda\sigma^3}{2\pi^{3/2}m_N^2}\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}\sum_{a,b} m_a m_b [\mathrm{e}^{\mathrm{i}\mathbf{k}(\mathbf{r}_a+\hat{\varphi}\times\mathbf{r}_a}, [\mathrm{e}^{-\mathrm{i}\mathbf{k}(\mathbf{r}_b+\hat{\varphi}\times\mathbf{r}_b}, \hat{\rho}_{\mathrm{rot}}]]\mathrm{d}\mathbf{k}. \tag{21}$$

If $\Delta(\varphi\times\mathbf{r}_a)\ll\sigma$ for all a, we approximate the integral as follows:

$$\int \mathrm{e}^{-\mathbf{k}^2\sigma^2}\sum_{a,b} m_a m_b \mathrm{e}^{\mathrm{i}\mathbf{k}(\mathbf{r}_a-\mathbf{r}_b)}[\hat{\boldsymbol{\varphi}}\mathbf{k}\mathbf{r}_a, [\hat{\boldsymbol{\varphi}}\mathbf{k}\mathbf{r}_b, \hat{\rho}_{\mathrm{rot}}]]\mathrm{d}\mathbf{k}, \tag{22}$$

where we define the triple scalar product by $\mathbf{abc} = \mathbf{a}(\mathbf{b}\times\mathbf{c})$. This integral is equivalent to the following volume integral:

$$(2\pi)^3\int [\hat{\boldsymbol{\varphi}}\mathbf{r}\nabla\mu_\sigma(\mathbf{r}), [\hat{\boldsymbol{\varphi}}\mathbf{r}\nabla\mu_\sigma(\mathbf{r}), \hat{\rho}_{\mathrm{rot}}]]\mathrm{d}\mathbf{r}. \tag{23}$$

Applying the arguments and approximations as in Sect. 3, we rewrite this volume integral as a surface integral:

$$\frac{(2\pi)^3\varrho^2}{2\pi^{1/2}\sigma}\oint [\hat{\boldsymbol{\varphi}}\mathbf{rn}, [\hat{\boldsymbol{\varphi}}\mathbf{rn}, \hat{\rho}_{\mathrm{rot}}]]\mathrm{d}S. \tag{24}$$

Using this form for the integral in Eq. (21), the rotational decoherence term takes the following ultimate form:

$$\mathcal{D}_{\mathrm{rot}}\hat{\rho}_{\mathrm{rot}} = -\frac{2\pi\lambda\sigma^2\varrho^2}{m_N^2}\oint[\hat{\boldsymbol{\varphi}}\mathbf{rn},[\hat{\boldsymbol{\varphi}}\mathbf{rn},\hat{\rho}_{\mathrm{rot}}]]\mathrm{d}S. \tag{25}$$

The rotational decoherence is determined by the constant density ϱ and the *rotational surface-tensor*:

$$\mathrm{S}_{\mathrm{rot}} =: \oint(\mathbf{r}\times\mathbf{n})\circ(\mathbf{r}\times\mathbf{n})\mathrm{d}S, \tag{26}$$

where, as before, $\mathbf{r}$ is the coordinate of a surface point in the c.o.m. frame and $\mathbf{n}$ is the corresponding normal vector to the surface. Remember, the validity of (25) was limited by $\Delta(\varphi\times\mathbf{r}_a)\ll\sigma$ for all a. In terms of the locations $\mathbf{r}$, the condition becomes $\Delta(\varphi\times\mathbf{r})\ll\sigma$ for all surface points $\mathbf{r}$.

Calculation of the spontaneous heating rate of the rotational degrees of freedom is straightforward, yielding

$$\Gamma_{\mathrm{rot}} = \frac{2\pi\lambda\sigma^2\varrho}{m_N^2}\mathrm{Tr}(\mathrm{I}^{-1}\mathrm{S}_{\mathrm{rot}}), \tag{27}$$

where $\mathrm{I}=\int(\mathbf{r}\circ\mathbf{r})\mathrm{d}\mathbf{r}$ is the inertia tensor of the probe.
Examples. Consider the rotation of a long cylindric rod of length L and radius $R\ll L$, around a perpendicular axis $\mathbf{n}_{\mathrm{rot}}$ through its center. All along the rod —except for its short middle part of size $\sim R$— the expression $\mathbf{rn}_{\mathrm{rot}} = r\sin(\Phi)$ is a good approximation where $r\in(-L/2, L/2)$ is the axial coordinate and Φ is the azimuthal angle of the surface position $\mathbf{r}$. Using this approximation, we can easily evaluate the axial element of the rotational surface-tensor $\mathrm{S}_{\mathrm{rot}}$ that controls the angular decoherence (25):

$$\oint(\mathbf{rn}_{\mathrm{rot}})^2\mathrm{d}S = \frac{\pi R L^3}{12}. \tag{28}$$

As another example, consider our cylinder rotating around its axis of symmetry: CSL predicts zero decoherence (cf. Fig. 1). But we introduce a small elliptical eccentricity $e\ll 1$ of the cross section. In leading order, we have $\mathbf{rn}_{\mathrm{rot}} = \frac{1}{2}Re^2\sin(2\Phi)$, yielding the following contribution of the shape to the strength of angular decoherence:

$$\oint(\mathbf{rn}_{\mathrm{rot}})^2\mathrm{d}S = \frac{e^4}{4}\pi R^2 L, \tag{29}$$

that is $e^4/4$ times the volume of the cylinder. Recall that $e^2 = 2\Delta R/R$ where ΔR is the small difference between the main diameters of the elliptic cross section. The obtained result may raise the same conceptual problem that we mentioned for the longitudinal superposition of the massive rod/needle: azimuthal superpositions of massive cylinders of low eccentricity may become practically insensitive to CSL.

5 Outlines of Generalizations

That in CSL the c.o.m and rotational decoherences are surface effects for homogeneous probes has been explicitly shown in Sects. 3 and 4 for ideal sharp edges and for spatial superpositions much smaller than σ. Both of the latter restrictions can be relaxed and $\mathcal{D}_{\rm cm}$ still remains a surface integral.

The case of unsharp edges is not much different from the ideal case. Let $H(h)\varrho$ be the profile of how the density drops from the constant ϱ down to zero through a thin layer defining the surface where the layer's thickness is small w.r.t. the sizes of the probe. Then the following generalization of Eq. (14) helps:

$$\nabla\mu_\sigma(\mathbf{r}+h\mathbf{n}) = \varrho\mathbf{n}\int g_\sigma(h-h')\mathrm{d}H(h'). \tag{30}$$

The rest of constructing the surface integral is the same as for Eq. (14) which described the special case where H was the (descending) step function.

The case of not necessarily small quantum positional uncertainties was described by Eq. (11). It takes an equivalent closed form in coordinate representation:

$$\mathcal{D}_{\rm cm}\hat\rho_{\rm cm}(\mathbf{X},\mathbf{Y}) = -\frac{\lambda\sigma^3}{\pi^{3/2}m_N^2}(2\pi)^3\int\left[\mu_\sigma(\mathbf{r}+\mathbf{X})\mu_\sigma(\mathbf{r}+\mathbf{Y})-\mu_\sigma^2(\mathbf{r})\right]\mathrm{d}\mathbf{r}\;\hat\rho_{\rm cm}(\mathbf{X},\mathbf{Y}). \tag{31}$$

The relevant structure is the integral, which we write as

$$(2\pi)^3\int\left[\mu_\sigma(\mathbf{r}+\mathbf{X}-\mathbf{Y})-\mu_\sigma(\mathbf{r})\right]\mu_\sigma(\mathbf{r})\mathrm{d}\mathbf{r}. \tag{32}$$

As long as the quantum uncertainty $|\mathbf{X}-\mathbf{Y}|$ is much smaller than the sizes of the probe, but not necessarily smaller then σ, the integral is vanishing everywhere in the bulk except for a thin layer of thickness $\sim|\mathbf{X}-\mathbf{Y}|$ below the surface. Accordingly, we incline to anticipate CSL decoherence remains a surface effect and, investing some harder mathematical work, $\mathcal{D}_{\rm cm}$ as well as $\mathcal{D}_{\rm rot}$ would take a form of surface integral, generalizing (16) and (25) beyond their quadratic approximations in $\hat{\mathbf{X}}$ and $\hat\varphi$.

6 Concluding Remarks

We have discussed CSL for constant density test masses and proved that spontaneous decoherence of both translational and rotational motion is determined by the density ϱ and by two invariant surface-tensors of the bodies:

$$\mathrm{S}_{\rm cm} = \oint(\mathbf{n}\circ\mathbf{n})\mathrm{d}S,$$

$$\mathrm{S}_{\mathrm{rot}} = \oint (\mathbf{r} \times \mathbf{n}) \circ (\mathbf{r} \times \mathbf{n}) \mathrm{d}S.$$

These two fully encode the relevant features of the probe's geometry. Previously, these features were encoded by the so-called geometric factor

$$\mu_{\mathbf{k}} = \varrho \int \mathrm{e}^{-\mathrm{i}\mathbf{k}\mathbf{r}} \mathrm{d}\mathbf{r},$$

an integral over the probe's volume and a function of the wave number $\mathbf{k}$. In case of general heavily inhomogeneous test masses the necessity of using the geometric factor is certainly doubtless. But for homogeneous probes, the surface-tensors should take over the role.

Important is the new insight into the physics of CSL in motion of a general massive bulk as a whole. First, microscopic structure is totally irrelevant, only the σ-smoothened density matters. Furthermore, displacements of homogeneous regions are not decohered at all. Only the displacements of inhomogeneities are decohered. The sharper the inhomogeneity, the stronger the decoherence it induces. In a constant density probe, the only inhomogeneous part is its surface, hence is CSL decoherence a surface effect for it—that we have here exploited. The same is true for layered probes where mass density jumps—through surfaces (walls) between the layers—contribute to the decoherence tensors. Inhomogeneities other than the said two-dimensional inhomogeneous regions around surfaces may rarely be sharp and fat enough to contribute to c.o.m. or rotational decoherence. Decoherence of probes with smooth material inhomogeneities may remain dominated by the said surfaces, our method of surface-tensors might extend for them!

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