On the conjectured gravity-related collapse rate $E_\Delta/\hbar$ of massive quantum superpositions

Lajos Diósi[1, a)]
Wigner Research Center for Physics, Budapest, Hungary[b)]

(Dated: 9 November 2021)

Roger Penrose and the author share the proposal that the spatial superposition $|x_1\rangle + |x_2\rangle$ of a massive object collapses into its localized components $|x_1\rangle$ or $|x_2\rangle$ with the characteristic time $\hbar/E_\Delta$ where E_Δ is the gravitational self-energy excess of the superposition versus the localized states. Underlying arguments of such radical departure from standard quantum mechanics and different derivations of the rate equation are briefly recapitulated and discussed.

arXiv:2111.04604v1 [quant-ph] 8 Nov 2021

I. THE COLLAPSE RATE

Microobjects, from elementary particles to giant molecules, can exist in superpositions of different locations. As to more massive objects, however, the violation of standard quantum mechanics has been conjectured from certain purely theoretical speculations. The proposal concerned here has been surviving three and a half decades in a status of pure speculation. The overlap between Penrose's and my results is the claim that the spatial superposition $|x_1\rangle+|x_2\rangle$ becomes unstable for large masses and a random collapse

$$|x_1\rangle + |x_2\rangle \Rightarrow \begin{cases} |x_1\rangle & \text{with probability } 0.5 \\ |x_2\rangle & \text{with probability } 0.5 \end{cases} \tag{1}$$

happens at rate

$$\frac{1}{\tau} = \frac{E_\Delta}{\hbar} \ , \tag{2}$$

where τ is the mean lifetime of the superposition and E_Δ is the difference between gravitational self-energies before and after the collapse (1), respectively, times an unspecified numeric constant.

After decades of missing experimental evidences pro or contra, the advent of quantum controlled laboratory technique opened the era of testability. It is worthwhile to revisit the theoretical background, the diverse arguments that seem to converge to the above collapse rate.

Secs. II,III attempt to outline the proposals of Penrose and myself, respectively. Sec.IV discusses my occasional selection of related issues, followed by personal remarks in Sec.V.

II. COLLAPSE FROM CONJECTURED KILLING VECTOR AMBIGUITY

Let me try biefly interpreting Penrose's concept and arguments[1–3], leading him to the rate (2).

a)e-mail: diosi.lajos@wigner.hu; homepage: www.wigner.hu/~diosi
b)Also at Department of Physics of Complex Systems, Eötvös Loránd University, Budapest, Hungary

Consider the center of mass stationary state $|x_1\rangle$ of a massive object located at x_1 and the stationary state $|\gamma_1\rangle$ of the geometry corresponding to the state $|x_1\rangle$. The composite state $|x_1\rangle \otimes |\gamma_1\rangle$ is also stationary. Now take the same stationary state just shifted from x_1 to x_2 and consider the superposition:

$$|x_1\rangle \otimes |\gamma_1\rangle + |x_2\rangle \otimes |\gamma_2\rangle \ . \tag{3}$$

One would expect that this superposition is also stationary. Penrose argues that it can not be. Independently of the details of how γ_1 and γ_2 represent the two geometries, they have their own Killing vectors to define stationarity but they have no single common Killing vector to define stationarity of the superposition. The equivalence principle of general relativity (general covariance, in other terms) *"forbids a meaningful precise labelling of individual points in a space-time. [...] there is generally no precise meaningful pointwise identification between different space-times"* — says Penrose and adds: *"all that we can expect will be some kind of approximate pointwise identification"*.

The *"measure of this degree of approximation"* is obtained by Penrose in the Newtonian non-relativistic limit of general relativity. The time coordinate for the two geometries γ_1, γ_2 can now be taken the common t, the Killing vector becomes equivalent to "$\partial/\partial t$" while it remains ambiguous because the point-wise identification of the spatial coordinates x remains ambiguous. Penrose argues that this ambiguity corresponds to the ambiguity of free falls determined by the ambiguity of local accelerations $g = -\nabla\Phi$ where Φ is Newton's potential. If so, then the plausible measure of the ambiguity (uncertainty) will be proportional to the volume integral of the squared difference of local accelerations:

$$\Delta \propto \int |g_1(x) - g_2(x)|^2 d^3x \ . \tag{4}$$

One expresses g_1 by the mass distribution ρ_1 in state $|x_1\rangle$:

$$g_1 = -\nabla\Phi(x) = -G\nabla \int \frac{\rho_1(x')}{|x-x'|} d^3x' \ , \tag{5}$$

and similarly for g_2. The ambiguity, or "uncertainty" Δ, divided by G, takes this form:

$$E_\Delta = \text{const.} \times G \iint \frac{(\rho_1(x) - \rho_2(x))(\rho_1(x') - \rho_2(x'))}{|x - x'|} d^3x d^3x'$$
$$= \text{const.} \times (U(x_1 - x_2) - U(0)) \ , \tag{6}$$

where $U(x_1 - x_2)$ is the Newton interaction potential between ρ_1 and ρ_2.

The bottom line of the derivation is that the energy E_Δ should be considered the energy ambiguity of the superposition (3) and, as for usual unstable quantum states, E_Δ leads to decay at mean lifetime τ defined in (2).

Later, Penrose supports the role of acceleration g in the uncertainty measure (4) by an alternative reasoning[4]. Consider a mass M in free fall and compare its wavefunctions in the Earth system and in the free-falling system, respectively. The former has a phase factor $\exp(-iMgt^3/6\hbar)$. A key part of the new arguments invokes relativistic quantum field theory where the two wavefunctions would belong to two different vacua, i.e., to non-equivalent Hilbert spaces. Although alternative vacua are irrelevant in the given non-relativistic situation, as noticed by Penrose, his train of thought may still hit the target. Despite, perhaps, the unnoticed triviality of the strange-looking phase. It comes from the nonzero time-dependent kinetic energy in the Earth based frame: $Mgt^3/6$ is the integral of $Mgt^2/2$.

III. COLLAPSE FROM CONJECTURED GEOMETRIC AMBIGUITY

Curvature of space-time geometry is sourced by the energy-momentum of matter which is quantized obviously. Hence quantum uncertainties of matter's behavior should impose uncertainties of geometry as well. This unsharpness of geometry is thus unavoidable and depends on $\hbar$, but its details depend on the model that couples quantized matter and quantized (or perhaps classical) geometry. Independently of the model, we might nonetheless estimate the scale of uncertainties transferred from matter to gravity. The concept is this. The uncertainty of the geometry coincides with the optimum testability of geometry, using quantized material instruments. In particular, considering a network of quantized free falling test bodies to measure the geometry, one expects that there is a finite optimum of measurement precision.

The measure of this precision and the rate (2) is obtained in the Newtonian non-relativistic limit of general relativity[5,6]. Let us analyse how precisely the free fall of a single test mass M encodes the the local acceleration $g = -\nabla\Phi$. Let the standing initial wave packet of M have a certain size $\sim r$ and volume $V \sim r^3$. Under free fall, r is approximately retained over a period $T \sim Mr^2/\hbar$. Hence, the test mass encodes the average acceleration field $\bar{g}$ over the volume V and time T. The acquired momentum $M\bar{g}T$, part of the total one, has an uncertainty $\hbar/r$. Hence $\bar{g}$ is encoded at the precision

$$\delta\bar{g} \sim \frac{\hbar}{MrT} \ . \tag{7}$$

To improve precision, one can not increase M unlimitedly because M's Newton potential contributes to $\bar{g}$ and imposes a further uncertainty

$$\delta\bar{g} \sim \frac{GM}{r^2} \ , \tag{8}$$

because of M's position uncertainty r. The optimum value of the test mass M is reached when the above two uncertainties coincide. Then the optimum precision of the measurement reads:

$$\delta\bar{g} \sim \sqrt{\frac{\hbar G}{VT}} \ . \tag{9}$$

The factor $1/\sqrt{VT}$ suggests that the uncertainties of g at different locations and different times are independent. One can determine the corresponding structure and scale of uncertainties $\delta\Phi$ of the Newton potential. They remain independent at different times but become correlated at different locations. One can inspect that (9) is satisfied if we choose the following correlation:

$$\langle \delta\Phi(x,t)\delta\Phi(x',t') \rangle = \text{const.} \times \frac{\hbar G}{|x - x'|}\delta(t - t') \ . \tag{10}$$

Thus we have estimated the due uncertainty of the Newton potential (i.e.: of the space-time geometry in the Newtonian limit). It means an uncertainty that is present even in empty space. It yields the instability and the decay of the massive superposition (3) because it dephases the two components[7]. The time evolution of $|x_1\rangle$ contains a phase factor

$$\exp\left(-\frac{i}{\hbar}\int_0^t \delta\Phi(x,t')\rho_1(x)d^3x dt'\right) \equiv e^{-i\chi_1(t)} \tag{11}$$

and $|x_2\rangle$ contains $e^{-i\chi_2}$ with ρ_2 in place of ρ_1. One forms the expectation value of the squared difference of the two phases. Using the correlation (10) yields

$$\langle (\chi_1(t) - \chi_2(t))^2 \rangle = \text{const.} \times \frac{E_\Delta}{\hbar} t \ , \tag{12}$$

where E_Δ happens to be the expression (6). Therefore the decay (i.e.: dephasing) rate of the superposition (3) coincides with (2).

IV. DISCUSSION

Starting point for both of us, as shown in the previous two sections, was the the inapplicability of standard quantization in general relativity. But each of us could implement his concept in the Newtonian limit only. While Penrose kept, correctly, interpreting the non-relativistic proposal in the context of general relativity, I was happy to recognize that the Newtonian limit is rich and self-contained, though one should not forget its roots and embedding in general relativity.

Our original derivations, outlined in Secs.II and III, are bearing conceptual and even technical similarities as well as important disparities. The "uncertainties" responsible for the decay of massive superpositions was thought coming from the ambiguous Killing vectors (Sec.II) or from the limited testability of the geometry (Sec.III). Are the two concepts compatible, complementary, or hopeless contradictory? The answer needs further studies beyond the scope of the present work. I deliberate on two related things.

A. Exact derivation

The proposed collapse rate (2) is based on dimensional considerations hence it contains a numeric constant which is left undefined. Interestingly, a semiclassical concept and its mathematical realization[8] by Tilloy and myself confirms the heuristic proposal and makes the constant unique.

The concept looks radically different from that in Sec.III but it is related to it intrinsically. Assume that the distribution of quantized masses is measured everywhere continuously, by hypothetic detectors which are hidden from, i.e., not part of the physical word. They are yielding the classical mass distribution ρ as the outcomes. This ρ is random, like measurement outcomes in quantum systems used to be. The postulated presence of such universal and spontaneous measurements serves the coupling of quantized matter to gravity. The classical valued ρ, used in $\nabla^2\Phi = -4\pi G\rho$, yields the classical Newton potential Φ which is fed back to the Schrödinger equation of the quantized masses.Now, both the continuous measurement and the stochastic potential Φ cause decoherence in the quantized material system. Weak (imprecise) measurement causes low decoherence at the price of high stochasticity of the outcome ρ yielding high decoherence by the feedback. At optimum measurement precision the total decoherence is the lowest, irrelevant for atomic systems but relevant for massive ones! Under it, the superposition (3) will decay exactly at the rate (2) where E_Δ is defined by (6) with the unique prefactor const. $= 1/2$.

B. Footprint of Planck scale uncertainty?

It would be reassuring to see that the proposed non-relativistic "uncertainty", whether the Killing vector's (Sec.II) or the geometry's (Sec.III), is the non-relativistic limit of the corresponding Planck scale uncertainty. Penrose[4] talks about *"decay after Planck-scale difference geometry measure"* and even conjectures that the decay, according to the formula (2), happens when *"two space-times in superposition differ from one another by an amount of order unity [...] measured in Planck units"*. To estimate space-time differences, symplectic measure in linearized gravity is mentioned cursorily. However, the whole suggestion about connections to Planck scale is missing any quantitative support, be it heuristic or approximate.

Interestingly, a certain heuristic support existed even before Refs.[5,7] and was already noticed in them. Unruh[9] proposed unusual (non-canonical) commutators between components of the metric tensor g and the Einstein tensor G:

$$[\mathrm{g}^{\nu\mu}(x), \mathrm{G}^{\rho\sigma}(x')] = \text{const.} \times \ell_{Pl}^2 \delta_\nu^\rho \delta_\nu^\sigma \delta^{(4)}(x, x') \ , \tag{13}$$

with the Planck length $\ell_{Pl} = \sqrt{\hbar G/c^3}$ (here x, x' stand for space-time coordinates). Unruh's motivation was a heuristic non-canonical uncertainty relation between the 00 components averaged over four-volume $V^{(4)}$:

$$\delta\bar{\mathrm{g}}_{00}(x)\delta\bar{\mathrm{G}}^{00}(x') \geq \frac{\ell_{Pl}^2}{V^{(4)}} \ , \tag{14}$$

now a consequence of (13). The Newtonian limit of this relativistic bound leads to the limit (9) of Sec.III. We insert $\mathrm{g}_{00} = 2c^{-2}\Phi/c^2$ and $\mathrm{G}^{00} = (1/2)c^{-2}\nabla^2\Phi$, as well as $V^{(4)} = cVT$. Quite remarkably, the velocity of light c cancels:

$$\delta\bar{\Phi}\delta\nabla^2\bar{\Phi} \geq \frac{\hbar G}{VT} \ . \tag{15}$$

With a deliberate (though justifiable) symmetrization $\delta\bar{\Phi}\delta\nabla^2\bar{\Phi} \Rightarrow \delta\nabla\bar{\Phi}\delta\nabla\bar{\Phi} = (\delta\bar{g})^2$, the obtained uncertainty of the acceleration field coincides with (9).

Much later, and only recently, Ref.[10] proposed a special relativistic construction of conform metric uncertainty around the Minkowski metric, whose Newtonian limit confirms the uncertainty (10) in Sec.III. Introducing a perturbative conformal factor $1 + h$ (at $|h| \ll 1$) where the uncertainties h are proportional to ℓ_{Pl}, one chooses the following relativistic invariant correlation:

$$\langle h(x)h(y)\rangle = \text{const.} \times \frac{\ell_{Pl}^2}{(2\pi)^4} \int e^{-ik(x-y)} \frac{\theta(-k^2)}{-k^2} d^4k \ , \tag{16}$$

ignoring the issues of regularizing $\theta(-k^2)/k^2$. One writes h into the form $h = 2\Phi/c^2$, anticipating that Φ plays the role of the (uncertain) Newton potential for non-relativistic matter. The limit $c \to \infty$ does converge, and does exactly to the expression (10).

V. PERSONAL

"Ah, you are working with Penrose, aren't you?" — I was asked a few times. No, we used to work on our own. There used to be definite parallelisms and divergences between our struggles in the field of unknowns. Roger's interest was a gift. After many well-defined theoretical tasks that his talent famously solved, one earned his Nobel Prize 2021, he challenged the not-so-well-defined problem. How is Schrödinger's cat bending the spacetime? We agree that the cat should collapse after a time $\sim \hbar/E_\Delta$. We disagree on how this happens, smoothly, suddenly, or some other way. The hard task is: gravity-related collapse dynamics that conserves energy and momentum. Worth of research. If we are on the right track at all ...

Acknowledgments. I thank Dr. Maaneli Derakhshani for useful discussions. I acknowledge support from the Foundational Questions Institute and Fetzer Franklin Fund, a donor advised fund of Silicon Valley Community Foundation (Grant No. FQXi-RFP-CPW-2008), the National Research, Development and Innovation Office for "Frontline" Research Excellence Program (Grant No. KKP133827) and research grant (Grant. No. K12435), the John Templeton Foundation (Grant 62099).

[1] R. Penrose, *Shadows of the Mind: A Search for the Missing Science of Consciousness* (Oxford University Press, 1994).
[2] R. Penrose, "On gravity's role in quantum state reduction," General Relativity and Gravitation **28**, 581–600 (1996).
[3] R. Penrose, "John bell, state reduction, and quanglement," in *Quantum (Un)speakables: From Bell to Quantum Information*, edited by R. Bertlmann and A. Zeilinger (Springer Science & Business Media, 2002) pp. 319–331.
[4] R. Penrose, "On the gravitization of quantum mechanics 1: Quantum state reduction," Foundations of Physics **44**, 557–575 (2014).
[5] L. Diósi, *A quantum-stochastic gravitation model and the reduction of the wavefunction*, Ph.D. thesis, Budapest (1986), in Hungarian, https://wigner.hu/ diosi/prints/thesis1986.pdf.
[6] L. Diósi and B. Lukács, "In favor of a newtonian quantum gravity," Annalen der Physik **499**, 488–492 (1987).
[7] L. Diósi, "A universal master equation for the gravitational violation of quantum mechanics," Physics Letters A **120**, 377–381 (1987).
[8] A. Tilloy and L. Diósi, "Principle of least decoherence for newtonian semiclassical gravity," Phys. Rev. D **96**, 104045 (2017).
[9] W. Unruh, "Steps towards a quantum theory of gravity," in *Quantum theory of gravity. Essays in honor of the 60th birthday of Bryce S. DeWitt*, edited by S. Christensen (Adam Hilger Limited, 1984) pp. 234–242.
[10] L. Diósi, "Planck length challenges non-relativistic quantum mechanics of large masses," Journal of Physics: Conference Series **1275**, 012007 (2019).