Unitary equivalence of Schrödinger and Heisenberg pictures survives in nonlinear quantum mechanics

Lajos Diósi*

Wigner Research Center for Physics, H-1525 Budapest 114 , P.O.Box 49, Hungary and Eötvös Loránd University, H-1117 Budapest, Pázmány Péter stny. 1/A, Hungary

(Dated: September 9, 2026)

The equivalence of Schrödinger and Heisenberg pictures is often thought to break down in nonlinear quantum mechanics due to the absence of a state-independent unitary propagator. We show that for Hamiltonians depending on the state through instantaneous expectation values, the equivalence is preserved. While the unitary intertwiner becomes initial-state dependent, it remains well-defined and ensures identical physical predictions in both pictures. We illustrate this with the analytically solvable examples of a nonlinear spin precession and a mean-field harmonic force, as well as in local field theory with mean-field coupling.

arXiv:2609.07391v1 [quant-ph] 7 Sep 2026

I. INTRODUCTION

Nonlinear modifications of quantum mechanics have been proposed repeatedly, both as fundamental alternatives to the linear Schrödinger dynamics and as effective descriptions emerging from coarse-graining, mean-field limits, feedback, or semiclassical couplings. Fundamental examples include the logarithmic Schrödinger equation introduced by Białynicki-Birula and Mycielski [1], and the general Hamiltonian framework of Weinberg [2]. Well-known is the Møller–Rosenfeld semiclassical gravity [3–5] whose nonrelativistic limit is the widely discussed Newton–Schrödinger equation [6–8]. As the common feature, the nonlinear Hamiltonians in these theories depend on the state via the instantaneous expectation (mean) values of certain observables. In Ref. [1], there is a potential term which is proportional to the logarithm of the mean value of the position eigenstate projector $|q\rangle\langle q|$. The simplest Hamiltonian in Ref . [2] assumes Pauli spin precession in a field proportional to the mean value of $\hat\sigma_3$. In semiclassical gravity [3–5], the Hamiltonian depends on the classical metric which depends on the mean value of the energy-momentum tensor of the quantized matter. In the Newton–Schrödinger equation [6–8], the classical Newton potential is sourced by the mean values of the spatial mass density field.

Despite this long history, a recurring view sometimes stated explicitly (most recently in Ref. [9]), sometimes treated as 'folklore', is that once the dynamics becomes nonlinear, the familiar equivalence of the Schrödinger and Heisenberg pictures becomes problematic or even meaningless. (State dependent Hamiltonians lead to several well-known anomalies, see, e.g., Refs . [10, 11]. These anomalies do not occur as long as we consider only the nonlinear dyxnamics —which is what we are doing here—without taking any quantum measurements.) If one insists that the Heisenberg picture must be defined via a state-independent unitary propagator $\hat U(t)$ that evolves the Schrödinger state $|t\rangle$, then nonlinear dynamics seems to obstruct the construction at the first step: there is no universal $\hat U(t)$ implementing evolution for all initial states. Consequently, it is frequently asserted (at least implicitly) that in nonlinear quantum mechanics one should work only in a Schrödinger-like formulation or, at best, in the interaction picture where $\hat U(t)=\exp(-it\hat H_0)$ defined by the linear part $\hat H_0$ of the total Hamiltonian. The Heisenberg picture is therefore questioned as an alternative representation. The purpose of the present short paper is to show that this conclusion is too strong.

In Sec. II, the initial-state-dependent unitary operator $\hat U(t,|0\rangle)$ is introduced that evolves the Schrödinger state $|t\rangle$. We prove that this unitary generates the Heisenberg operators $\hat A_t$ the same way as $\hat U(t)$ does it in standard linear quantum mechanics. Concrete examples of the Heisenberg picture are shown in Sec. III.

II. SCHRÖDINGER AND HEISENBERG DYNAMICS, THEIR EQUIVALENCE

Nonlinear quantum mechanics is traditionally defined in the Schrödinger picture, so we start with the nonlinear Schrödinger equation of the time-dependent statevector $|t\rangle$:

$$\frac{d}{dt}|t\rangle = -i\hat H(O_t)|t\rangle, \tag{1}$$

where the Hamiltonian depends on the instantaneous expectation value $O_t=\langle t|\hat O|t\rangle$ of a certain observable $\hat O$. Note that O_t may stand for the expectation values of a list of different observables, even for the continuum of expectation values of a quantum field at all locations. For the expectation value of a generic observable $\hat A$ (including $\hat O$), we introduce the notation A_t:

$$A_t=\langle t|\hat A|t\rangle. \tag{2}$$

The corresponding Heisenberg dynamics, whose equivalence with Schrödinger's is yet to be confirmed, goes like this. The quantum state is $|0\rangle$ constantly, and the observables (including $\hat O$) evolve with the nonlinear Heisenberg

* diosi.lajos@wigner.hu; www.wigner.hu/~diosi

equation

$$\frac{d}{dt}\hat{A}_t = i[\hat{H}(O_t^H), \hat{A}_t], \tag{3}$$

with initial condition $\hat{A}_0 = \hat{A}$. Also $\hat{O}_t$ satisfies the above equation, and the state dependence of the Hamiltonian is defined by $O_t^H = \langle 0|\hat{O}_t|0\rangle$. For the expectation value of a generic Heisenberg observables $\hat{A}_t$ (including $\hat{O}_t$), we introduce the notation A_t^H:

$$A_t^H = \langle 0|\hat{A}_t|0\rangle. \tag{4}$$

Observe that we used notations A_t in Schrödinger picture and A_t^H in Heisenberg picture, respectively. The proof of $A_t = A_t^H$ is just the task along the below equivalence proof.

We are going to show that the Heisenberg picture is unitary equivalent with the Schrödinger picture, similarly to standard linear quantum mechanics. There, the picture-change unitary is state independent; here, for each initial condition $|0\rangle$ we obtain a unitary $\hat{U}(t;|0\rangle)$, so the equivalence holds trajectory-wise (or initial-state-wise) rather than by a single universal representation change. We shall deliberately omit the $|0\rangle$-dependence from the notation $\hat{U}(t)$ since $|0\rangle$ is a constant all over our calculations.

A. From Schrödinger picture to Heisenberg

To begin with the Schrödinger picture, we define the unitary operator by

$$\frac{d}{dt}\hat{U}(t) = -i\hat{H}(O_t)\hat{U}(t), \tag{5}$$

with $\hat{U}(0) = 1$. The quantity $O_t = \langle t|\hat{O}|t\rangle$ is given via the solution of the Schrödinger eq. (1) so that O_t and finally $\hat{U}(t)$ will depend on the initial state $|0\rangle$, as we previously anticipated. Observe now that the unitary $\hat{U}(t)$ 'solves' the Schrödinger eq. (1):

$$|t\rangle = \hat{U}(t)|0\rangle. \tag{6}$$

Note, however, that the definition (5) of $\hat{U}(t)$ had assumed the knowledge of the solution $|t\rangle$. We define the Heisenberg observables just like we do it in the standard linear quantum mechanics:

$$\hat{A}_t = \hat{U}^\dagger(t)\hat{A}\hat{U}(t). \tag{7}$$

This transformation $\hat{A} \Rightarrow \hat{A}_t$ is the inverse picture transformation corresponding to $|0\rangle \Rightarrow |t\rangle$ in Eq. (6), like in standard quantum mechanics. And like there, we get the same expectation values in both Schrödinger and Heisenberg pictures:

$$A_t^H = A_t. \tag{8}$$

This ensures the equivalence of physical predictions between the two pictures. Last but not least, it is yet to be shown that $\hat{A}_t$ satisfies the Heisenberg eq. of motion. Using eq. (7) and then eq. (5), we can write:

$$\frac{d}{dt}\hat{A}_t = i[\hat{H}(O_t), \hat{A}_t]. \tag{9}$$

From eq. (8), we have $O_t = O_t^H$, and this makes the above equation coincide with the Heisenberg eq. (3).

B. From Heisenberg picture to Schrödinger

Alternatively, we can begin with the Heisenberg picture. We define the unitary operator by

$$\frac{d}{dt}\hat{U}^H(t) = -i\hat{H}(O_t^H)\hat{U}^H(t) \tag{10}$$

with $\hat{U}^H(0) = 1$. The quantity $O_t^H = \langle 0|\hat{O}_t|0\rangle$ is given via the solution $\hat{O}_t$ of the Heisenberg eq. (3) so that O_t^H and finally $\hat{U}^H(t)$ will depend on the initial state $|0\rangle$. This unitary 'solves' the Heisenberg eq. (3):

$$\hat{A}_t = \hat{U}^{H\dagger}(t)\hat{A}_0\hat{U}^H(t). \tag{11}$$

Recall that the definition (10) of $\hat{U}^H(t)$ assumes the knowledge of the solution $\hat{O}_t$. We define the Schrödinger observables by $\hat{A} = \hat{A}_0$ and the Schrödinger state by

$$|t\rangle = \hat{U}^H(t)|0\rangle, \tag{12}$$

like in the standard linear quantum mechanics. And like there, we get the same expectation values in both Schrödinger and Heisenberg pictures, cf. (8). Finally, we ought to inspect that the state $|t\rangle$ in Eq. (12) satisfies the Schrödinger equation. Since the expectation values O_t and O_t^H coincide, we have $\hat{U}^H(t) = \hat{U}(t)$, which proves that the state $|t\rangle$ in Eq. (12) coincides with the solution (6) of the Schrödinger equation (1).

III. EXAMPLES

We demonstrate the construction of the Heisenberg picture using different nonlinear Schrödinger equations. In two simple examples, we provide closed analytic expressions for the Heisenberg variables and for the unitary map from the Schrödinger picture to the Heisenberg picture. This is facilitated because the state-dependent c-number functions O_t satisfy closed, analytically solvable differential equations. Analytic solutions for the Heisenberg picture in more complex nonlinear models may also exist; here we restrict ourselves to the simplest yet nontrivial analytically tractable examples. Our first choice is the nonlinear Hamiltonian $\frac{1}{2}\langle\hat{\sigma}^x\rangle\hat{\sigma}^z$ of a two-level system (qubit), a variant of Gisin's example [10]. Second, a free particle is considered under the simplest state-dependent directional force $\propto \langle\hat{q}\rangle$ as well as in the state-dependent potential $\propto (\hat{q}-\langle\hat{q}\rangle)^2$. This Galilean-invariant harmonic potential was already discussed in the foundations of quantum mechanics [12] and was rediscovered independently in a relevant special case [13, 14] of the Newton–Schrödinger equation. Our third example is a nonrelativistic boson field with generic local mean-field coupling.

A. Spin-$\frac{1}{2}$ in 'mean-field'

In the Pauli representation, the generic nonlinear Hamiltonian of a spin-$\frac{1}{2}$ system is the following:

$$\hat{H}(\boldsymbol{\sigma}_t) = \tfrac{1}{2}\boldsymbol{\omega}(\boldsymbol{\sigma}_t)\hat{\boldsymbol{\sigma}}, \tag{13}$$

where the vector $\boldsymbol{\omega}$ is a certain function of the Bloch vector $\boldsymbol{\sigma}_t = \langle t|\hat{\boldsymbol{\sigma}}|t\rangle$ at time t. The Schrödinger eq. (1) yields the closed rotation equation $\dot{\boldsymbol{\sigma}} = \boldsymbol{\omega}(\boldsymbol{\sigma})\times\hat{\boldsymbol{\sigma}}$ for the Bloch vector, not solvable analytically unless we choose a specific $\boldsymbol{\omega}$. Thus, we consider the following nonlinear spin Hamiltonian [10]:

$$\hat{H}(\sigma_t^x) = \tfrac{1}{2}\omega\sigma_t^x\,\hat{\sigma}^z, \tag{14}$$

i.e., the precession frequency about the z axis is proportional to the instantaneous x–component $\sigma_t^x = \langle t|\hat{\sigma}^x|t\rangle$ of the Bloch vector.

In the Schrödinger picture, the evolution is given by Eq. (1) with $O_t \equiv \sigma_t^x$ and $\hat{O} \equiv \hat{\sigma}^x$, namely

$$\frac{d}{dt}|t\rangle = -\frac{i}{2}\omega\,\sigma_t^x\,\hat{\sigma}^z\,|t\rangle. \tag{15}$$

Using (15) and the commutators/anti-commutators of the Pauli matrices, one obtains the following closed nonlinear equations for the Bloch vector components:

$$\dot{\sigma}_t^x = -\omega\,\sigma_t^x\,\sigma_t^y, \qquad \dot{\sigma}_t^y = \omega\,(\sigma_t^x)^2, \qquad \dot{\sigma}_t^z = 0. \tag{16}$$

Hence $\sigma_t^z = \sigma_0^z$ is constant, and $(\sigma_t^x)^2 + (\sigma_t^y)^2 = \rho^2$ is conserved where

$$\rho \equiv \sqrt{(\sigma_0^x)^2 + (\sigma_0^y)^2} = \sqrt{1-(\sigma_0^z)^2}\,. \tag{17}$$

For $\rho > 0$, we can parametrize the transverse components by an azimuthal angle $\phi(t)$:

$$\sigma_t^x = \rho\cos\phi(t), \qquad \sigma_t^y = \rho\sin\phi(t). \tag{18}$$

Substitution into (16) yields a single scalar equation

$$\dot{\phi}(t) = \omega\rho\cos\phi(t). \tag{19}$$

It is explicitly solvable:

$$\sin(\phi(t)-\phi(0)) = \tanh(\rho\omega t), \tag{20}$$

whence σ_t^x, σ_t^y follow from (18). The unitary propagator (5) is in this case a z–rotation with a state-dependent angle which turns out to be $-\phi(t)+\phi(0)$:

$$\hat{U}(t) = \exp\left[-\frac{i}{2}\big(\phi(t)-\phi(0)\big)\hat{\sigma}^z\right]. \tag{21}$$

Indeed, the definition (5) of $\hat{U}(t)$, with our Hamiltonian (14), reads:

$$\frac{d}{dt}\hat{U}(t) = -\frac{i}{2}\omega\,\sigma_t^x\,\hat{\sigma}^z\,\hat{U}(t). \tag{22}$$

Since eqs. (18) and (19) yield $\omega\sigma_t^x = \rho\omega\cos\phi(t) = \dot{\phi}(t)$, the solution (21) is confirmed, with the initial condition $\hat{U}(0) = 1$. The Schrödinger solution is $|t\rangle = \hat{U}(t)|0\rangle$ in agreement with (6). Finally, the Heisenberg observables, defined by (7), become the following:

$$\begin{aligned}
\hat{\sigma}_t^x &= \cos(\phi(t)-\phi(0))\,\hat{\sigma}^x - \sin(\phi(t)-\phi(0))\,\hat{\sigma}^y\\
\hat{\sigma}_t^y &= \cos(\phi(t)-\phi(0))\,\hat{\sigma}^y + \sin(\phi(t)-\phi(0))\,\hat{\sigma}^x\\
\hat{\sigma}_t^z &= \hat{\sigma}^z
\end{aligned} \tag{23}$$

The Heisenberg observables $\hat{\boldsymbol{\sigma}}_t$ satisfy the equivalence condition (8):

$$\langle 0|\,\hat{\boldsymbol{\sigma}}_t\,|0\rangle = \langle t|\,\hat{\boldsymbol{\sigma}}\,|t\rangle = \boldsymbol{\sigma}_t\ , \tag{24}$$

and the standard commutator-anticommutator relationships of the Pauli matrices hold for the components of $\hat{\boldsymbol{\sigma}}_t$ in the Heisenberg picture as well.

B. 'Mean-field' directional force

Consider a free particle ($m=1$) of momentum $\hat{p}$ and position $\hat{q}$, subject to a directional force proportional to its mean position $q_t = \langle t|\hat{q}|t\rangle$. The state-dependent Hamiltonian reads:

$$\hat{H}(q_t) = \tfrac{1}{2}\hat{p}^2 + \omega^2 q_t\hat{q}. \tag{25}$$

This time we start in the Heisenberg picture. The nonlinear Heisenberg equation (3) yields the equation of motion of the canonical variables:

$$\dot{\hat{q}}_t = \hat{p}_t, \qquad \dot{\hat{p}}_t = -\omega^2 q_t. \tag{26}$$

The expectation values obey the closed canonical equations of a classical harmonic oscillator of frequency ω. The elementary solutions are

$$q_t = q_0\cos\omega t + \frac{p_0}{\omega}\sin\omega t, \qquad p_t = p_0\cos\omega t - \omega q_0\sin\omega t, \tag{27}$$

depending on the initial values $q_0 = \langle 0|\hat{q}|0\rangle$ and $p_0 = \langle 0|\hat{p}|0\rangle$. Now we can solve the Heisenberg equations (26). They are driven by the c-number force $-\omega^2 q_t$ whose time-dependence is known from (27). Integrating them yields the explicit Heisenberg operators:

$$\hat{p}_t = \hat{p} + (p_t - p_0), \qquad \hat{q}_t = \hat{q} + t\hat{p} + (q_t - q_0 - p_0 t). \tag{28}$$

One can directly verify the canonical commutator $[\hat{q}_t, \hat{p}_t] = i$ in the Heisenberg picture. To derive the unitary $\hat{U}(t)$, we can skip the defining equation (5) because $\hat{U}(t)$ is already defined by the above unitary transformation of the canonical variables $(\hat{q}, \hat{p})$ from the Schrödinger picture to $(\hat{q}_t, \hat{p}_t)$ in the Heisenberg picture. The propagator $\exp(-it\hat{H}) = \exp(-\frac{1}{2}it\hat{p}^2)$ acts first, then the position and momentum operators are shifted by $q_t - q_0 - p_0 t$ and $p_t - p_0$ respectively. The unitary $\hat{U}(t)$ —that generates the Heisenberg observables via eq. (7) as well as the Schrödinger state via eq. (6)— is given by:

$$\hat{U}(t) = \exp\bigl[-i(p_t - p_0)\hat{q}\bigr] \exp\bigl[i(q_t - q_0 - p_0 t)\hat{p}\bigr] \exp(-it\hat{H}), \tag{29}$$

up to a c-number phase. This $\hat{U}(t)$ depends on the initial state $|0\rangle$ only through the initial expectation values q_0, p_0 of the solutions (28). The construction is valid for any initial state $|0\rangle$, demonstrating that unitary equivalence is preserved despite the non-trivial nonlinearity.

It is even easier to construct the Heisenberg picture for a slightly different nonlinear Schrödinger equation:

$$\hat{H}(q_t) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2(\hat{q} - q_t)^2, \tag{30}$$

where the directional force is centered around the instantaneous mean position. We exploit the Galilean invariance of the nonlinear dynamics. It coincides with the standard (linear) dynamics of the central harmonic oscillator in the specific inertial frame where $q_0 = p_0 = 0$, the nonlinear Hamiltonian becomes the linear Hamiltonian $\hat{H}_0 = \frac{1}{2}(\hat{p}^2 + \omega^2\hat{q}^2)$ of the central harmonic oscillator. The Heisenberg picture is generated by the unitary $\exp(-it\hat{H}_0)$. We can transform it into the general frame by Galilean transformations of the coordinate and momentum, yielding

$$\begin{aligned}
\hat{q}_t &= \hat{q}\cos\omega t + \frac{\hat{p}}{\omega}\sin\omega t + q_0 + p_0 t \\
\hat{p}_t &= -\omega\hat{q}\sin\omega t + \hat{p}\cos\omega t + p_0 \\
\hat{U}(t) &= \exp\bigl[-ip_0\hat{q}\bigr] \exp\bigl[i(q_0 + p_0 t)\hat{p}\bigr] \exp(-it\hat{H}_0).
\end{aligned} \tag{31}$$

C. Local mean-field coupling

In the Schrödinger picture, let $\hat{\mathcal{A}}(\mathbf{x})$ and $\hat{\mathcal{B}}(\mathbf{x})$ be local observables of a boson field theory, satisfying the locality (microcausality) condition:

$$[\hat{\mathcal{A}}(\mathbf{x}), \hat{\mathcal{B}}(\mathbf{y})] = 0, \qquad (\mathbf{x} \neq \mathbf{y}). \tag{32}$$

Consider the expectation value of a certain local field:

$$\mathcal{O}_t(\mathbf{x}) = \langle t|\hat{\mathcal{O}}(\mathbf{x})|t\rangle \tag{33}$$

which enters the local field $\hat{\mathcal{H}}(\mathbf{x}, \mathcal{O}_t(\mathbf{x}))$ of Hamiltonian density, yielding the nonlinear Hamiltonian:

$$\hat{H}[\mathcal{O}_t] = \int \hat{\mathcal{H}}(\mathbf{x}, \mathcal{O}_t(\mathbf{x}))d\mathbf{x}. \tag{34}$$

The nonlinear Schrödinger equation reads

$$\frac{d}{dt}|t\rangle = -i\hat{H}[\mathcal{O}_t]|t\rangle. \tag{35}$$

For a given $|0\rangle$, there is a unique solution $|t\rangle$, and hence a unique solution $\mathcal{O}_t$, and a unique solution $\hat{H}[\mathcal{O}_t]$. Therefore, there exists a unique solution of (5) whose unitarity is most clearly displayed by the time-ordered explicit form:

$$\hat{U}(t) = T\exp\left(-i\int_0^t \hat{H}[\mathcal{O}_\tau]d\tau\right). \tag{36}$$

Remember, the correct notation would be $\hat{U}(t;|0\rangle)$, just the initial-state-dependence is suppressed in our notation. Using this initial-state-dependent unitary, we define the Heisenberg fields:

$$\hat{\mathcal{A}}_t(\mathbf{x}) = \hat{U}^\dagger(t)\hat{\mathcal{A}}(\mathbf{x})\hat{U}(t). \tag{37}$$

No matter that the unitary map depends on the initial state $|0\rangle$, the map preserves the algebra of observables:

$$[\hat{\mathcal{A}}_t(\mathbf{x}), \hat{\mathcal{B}}_t(\mathbf{y})] = [\hat{\mathcal{A}}(\mathbf{x}), \hat{\mathcal{B}}(\mathbf{y})]. \tag{38}$$

It follows that the Heisenberg fields preserve the locality (microcausality) (32) of the Schrödinger fields.

IV. SUMMARY, AND CLOSING REMARKS

We have shown that the standard equivalence between the Schrödinger and the Heisenberg pictures survives in nonlinear quantum mechanics, provided the nonlinearity enters through expectation values. The key is to recognize that while a universal, state-independent unitary intertwiner $\hat{U}(t)$ between the two pictures no longer exists, a trajectory-wise unitary $\hat{U}(t;|0\rangle)$ can always be constructed for any given initial state. (The equivalence of the interaction picture follows by similar arguments.) The time-derivative $d\hat{U}/dt$ satisfies the same differential equation as in the linear theory. As demonstrated by our examples, the nonlinear Heisenberg equations remain an equivalent and transparent alternative to the nonlinear nonlinear Schrödinger equations.

This work was motivated originally by the author's disagreement with the statement in Ref. [9] that nonlinear dynamics does not preserve the microcausality (locality) condition of quantum fields because the evolution is non-unitary. The contrary argument [15], which states that the evolution of fields is initial-state-dependent *unitary*, also ensures the equivalence of the Schrödinger and Heisenberg pictures. Very recently, Ref. [16] has already recognized this equivalence in a particular nonlinear quantum model, but the authors did not need to set out to prove that as a general rule.

ACKNOWLEDGMENTS

This research was supported by the EU COST Actions (Grants CA23115, CA23130). The author thanks an AI assistant for helpful discussions.

[1] Iwo Bialynicki-Birula and Jerzy Mycielski, "Nonlinear wave mechanics," Annals of Physics **100**, 62–93 (1976).
[2] Steven Weinberg, "Testing quantum mechanics," Annals of Physics **194**, 336–386 (1989).
[3] Christian Møller, "Les theories relativistes de la gravitation," Colloques Internationaux CNRS **91**, 353 (1962).
[4] Leon Rosenfeld, "On quantization of fields," Nuclear Physics **40**, 1 (1963).
[5] Claus Kiefer, "Quantum gravity: general introduction and recent developments," Annalen der Physik **518**, 129–148 (2006).
[6] L. Diósi, "Gravitation and quantum-mechanical localization of macro-objects," Physics Letters A **105**, 199–202 (1984).
[7] Roger Penrose, "On gravity's role in quantum state reduction," General Relativity and Gravitation **28**, 581–600 (1996).
[8] Mohammad Bahrami, André Großardt, Sandro Donadi, and Angelo Bassi, "The schrödinger–newton equation and its foundations," New Journal of Physics **16**, 115007 (2014).
[9] Stephen Hsu, "Relativistic covariance and nonlinear quantum mechanics: Tomonaga-schwinger analysis," Physics Letters B **872**, 140053 (2026).
[10] N. Gisin, "Weinberg's non-linear quantum mechanics and supraluminal communications," Physics Letters A **143**, 1–2 (1990).
[11] Lajos Diósi, "Nonlinear schrödinger equation in foundations: summary of 4 catches," Journal of Physics: Conference Series **701**, 012019 (2016).
[12] Lajos Jánossy, "The physical aspects of the wave-particle problem," Acta Physica Academiae Scientiarum Hungaricae **1**, 423–467 (1952).
[13] Lajos Diósi, "Notes on certain Newton gravity mechanisms of wavefunction localization and decoherence," Journal of Physics A: Mathematical and Theoretical **40**, 2989–2995 (2007).
[14] Huan Yang, Haixing Miao, Da-Shin Lee, Bassam Helou, and Yanbei Chen, "Macroscopic quantum mechanics in a classical spacetime," Phys. Rev. Lett. **110**, 170401 (2013).
[15] Lajos Diósi, "Comment on" relativistic covariance and nonlinear quantum mechanics: Tomonaga-schwinger analysis"," arXiv preprint arXiv:2602.06845 (2026).
[16] Yubao Liu, Wenjie Zhong, Yanbei Chen, and Yiqiu Ma, "Semiclassical gravity phenomenology under the causal-conditional quantum measurement prescription. ii. heisenberg picture and apparent optical entanglement," Physical Review D **111**, 062004 (2025).