Principle of least decoherence in semiclassical gravity (1986-2017-?)

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Semiclassical Gravity 1962-63: sharp metric

Sharp classical space-time metric (Møller, Rosenfeld 1962-63):

$$\mathrm{G}_{ab} = \frac{8\pi G}{c^4}\langle\Psi|\hat{\mathrm{T}}_{ab}|\Psi\rangle$$

Schrödinger equation on background metric g:

$$\frac{d}{dt}|\Psi\rangle = -\frac{i}{\hbar}\hat{H}[\mathrm{g}]|\Psi\rangle$$

That's our powerful effective hybrid dynamics for (g_{ab}, Ψ), but

- with fundamental inconcistencies
- that are unrelated to relativity and even gravitation
- just related to quantum-classical coupling
- that makes Schrödinger eq. nonlinear

Hybrid dynamics of (g_{ab}, Ψ) invalidates statistical interpretation of Ψ.
Way out: metric cannot be sharp, must have fluctuations $\delta\mathrm{g}_{ab}$.

δg_{ab}: Early conjectures, DP spontaneous collapse

Alternative motivations for $\delta g_{ab} \neq 0$:

- Search for "some" quantum-gravity (Unruh)
- Search for "some" quantum-mechanics without Schrödinger cats (D, Penrose)

No direct derivations, just heuristic arguments, thought experiments.
Those that will fit to "rigorous" derivation (Tilloy & D 2017):

- Quantum-gravity metric uncertainty (Unruh 1984)
- Semiclassical metric uncertainty (D, D & Lukács 1986-87:)
- Time-like Killing-vector uncertainty (Penrose 1996)
- DP theory of spontaneous decoherence/collapse (1986-87, 1996)

Quantum-gravity uncertainty of metric 1984

Unruh's quantum-gravity relativistic thought experiment (1984):
Heisenberg uncertainty relation between metric and Einstein tensors:

$$\delta\bar{g}_{00}\delta\bar{G}^{00} \geq \frac{\hbar G}{c^4 VT}$$

Bar means average over volume V and time T.
Newtonian limit $g_{00} = 1 + 2\Phi/c^2$:

$$\delta g_{00} = 2\delta\Phi/c^2, \qquad \delta G^{00} = 2\nabla^2\delta\Phi/c^2$$

c cancels from Unruh's relativistic bound which reduces to

$$(\delta\overline{\nabla\Phi})^2 \geq \frac{\hbar G}{VT}$$

That looks like D. 1987 semi-classical uncertainty, derived without reference to relativity.

Semiclassical gravity uncertainty of metric 1986-87

Semiclassical gravity in Newton limit ($g_{ab} \to \Phi,\ \hat{T}_{ab} \to \hat{\varrho}$):

$$\nabla^2\Phi = 4\pi G\langle\Psi|\hat{\varrho}|\Psi\rangle$$

Schrödinger-Newton Equation:

$$\frac{d}{dt}|\Psi\rangle = -\frac{i}{\hbar}\left(\hat{H} + \int \Phi\hat{\varrho}dV\right)|\Psi\rangle$$

D. non-relativistic (Newtonian) thought experiment (1987): ultimate precision of measuring classical Φ:

$$(\delta\overline{\nabla\Phi})^2 = \text{const} \times \frac{\hbar G}{VT}$$

Equivalent with Penrose (1996) ultimate precision of space-time: general relativistic arguments but same Newtonian proposal.

Spontaneous decoherence/collapse from $\delta\Phi$ 1986

DP ultimate precision of Φ (of space-time)

$$(\delta\overline{\nabla\Phi})^2 = \text{const} \times \frac{\hbar G}{VT}$$

Intuition: $\delta\Phi$ undermines unitarity, can decohere Schrödinger cats!
Technical step: let $\delta\Phi$ be stochastic, of correlation

$$\mathbb{E}\left[\delta\Phi_t(\mathbf{x})\delta\Phi_\tau(\mathbf{y})\right] = \text{const} \times \frac{\hbar G}{|\mathbf{x}-\mathbf{y}|}\delta(t-\tau)$$

Underlies DP spontaneous decoherence/collapse theory (1986):

- For atomic d.o.f.: ignorable non-unitary effects
- For massive d.o.f.: non-unitary effects accumulate as to kill cats

Great!
But: vague justification of $\delta\Phi$-spectrum (no matter P, D, or Bill)
News: after 30 yy we get it exactly!

Decoherent Semiclassical Gravity 2016-17: unsharp metric

- Assume $\hat{\mathrm{T}}_{ab}$ is spontaneously measured ("monitored")
- Let T_{ab} be the measured value (called "signal" in control theory)
- Replace Møller-Rosenfeld 1962-63 by

$$\mathrm{G}_{ab} = \frac{8\pi G}{c^4}\mathrm{T}_{ab}$$

i.e.: source Einstein eq. by the noisy signal (do "feed-back")

- Complete Schrödinger eq. by stochastic terms for collapse:

$$\frac{d}{dt}|\Psi\rangle = -\frac{i}{\hbar}\hat{H}[\mathrm{g}]|\Psi\rangle + \mathrm{stoch.\ collapse\ terms}$$

- Tune monitoring by Principle of Least Decoherence

D 1990, Kafri, Taylor & Milburn 2014, Tilloy & D 2016-17,

cf. also Derekshani 2014, Altamirano, Corona-Ugalde, Mann & Zych 2016.

Principle of Least Decoherence — Example

Quantum control of path $\hat{x}_t$ of Schrödinger particle
Purpose: Generate harmonic potential $\frac{1}{2}R\hat{x}^2$ semiclassically at minimum "cost of" decoherence.
Free parameter: precision γ of monitoring.

- Monitoring $\hat{x}_t$ causes spatial decoherence with coeff. γ and yields signal x_t with noise intensity $1/\gamma$:

$$x_t = \langle\Psi_t|\hat{x}_t|\Psi_t\rangle + \delta x_t, \qquad \mathbb{E}\delta x_t \delta x_s = \gamma^{-1}\delta(t-s)$$

- Feedback $\hat{H}_{fb} = R x_t \hat{x}_t$ yields potential $\frac{1}{2}R\hat{x}_t^2$ as desired, at increased decoherence: $\gamma + 4\gamma^{-1}(R/\hbar)^2$
- Minimum decoherence singles out optimum precision

$$\gamma = 2|R|/\hbar$$

If $\hat{x} \Rightarrow \hat{\mathrm{T}}_{ab}$: problems even with Lorentz invariante monitoring.
But the Newtonian limit works out well (Tilloy & D 2016-17)!

PLD singles out DP for semiclassical gravity

Spontaneous monitoring of mass density $\hat{\varrho}_t(\mathbf{r})$ yields signal

$$\varrho_t(\mathbf{r}) = \langle\Psi_t|\hat{\varrho}_t(\mathbf{r})|\Psi_t\rangle + \delta\varrho_t, \qquad \mathbb{E}\delta\varrho_t(\mathbf{r})\delta\varrho_s(\mathbf{y}) = \gamma^{-1}(\mathbf{x},\mathbf{y})\delta(t-s)$$

Free parameter: precision kernel γ of monitoring.
Signal feeds gravity via $\nabla^2\Phi = 4\pi G\varrho$:

$$\Phi(\mathbf{x}) = -G\int\frac{d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\;\varrho(\mathbf{y}) \equiv (R\varrho)(\mathbf{x})$$

$$\mathbb{E}\delta\Phi_t(\mathbf{r})\delta\Phi_s(\mathbf{y}) = (R\gamma^{-1}R)(\mathbf{x},\mathbf{y})\delta(t-s)$$

Feedback $\hat{H}_{fb} = \int\hat{\varrho}\Phi dV \equiv (\hat{\varrho}R\varrho)$ induces Newton interaction $\frac{1}{2}(\hat{\varrho}R\hat{\varrho})$ as desired, at the price of enhanced decoherence: $\gamma + 4\hbar^{-2}R\frac{1}{\gamma}R$.
Minimum decoherence (Fourier-mode-wise) singles out $\gamma = -2R/\hbar$.
PLD uncertainty of Φ (metric) is unique and coincides with DP's:

$$\mathbb{E}\delta\Phi_t(\mathbf{r})\delta\Phi_s(\mathbf{y}) = \frac{\hbar G}{2|\mathbf{x}-\mathbf{y}|} \quad\Leftrightarrow\quad \mathbb{E}(\delta\overline{\nabla\Phi})^2 = \frac{\hbar G}{2VT}$$

Summary of Decoherent Semiclassical Gravity 2016-17-

- Spontaneous monitoring of $\hat{\varrho}_t(\mathbf{x})$ yields noisy signal $\varrho_t(\mathbf{x})$
- to source classical Newton field $\Phi_t(\mathbf{x})$
- that we feed back to induce Newton pair-potential.
- PLD singles out the unique consistent hybrid dynamics of (Φ, Ψ)
- which turrns out to be the DP-theory.

Averaging over the stochastic Φ (metric) obtains standard Newton interaction plus spontaneous DP-decoherence:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H} + \frac{G}{2}\iint \frac{d\mathbf{x}d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\hat{\varrho}(\mathbf{x})\hat{\varrho}(\mathbf{y}), \hat{\rho}\right] - \frac{G}{2\hbar}\iint \frac{d\mathbf{x}d\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}[\hat{\varrho}(\mathbf{x}),[\hat{\varrho}(\mathbf{y}),\hat{\rho}]]$$

Double goal achieved:

- Consistent semiclassical theory of gravity
- Theory of G-related spontaneous collapse (cats go collapsed)

Concluding remarks

Møller-Rosenfeld (sharp) Semiclassical Gravity is quantum-nonlinear, with related fundamental problems and particular effects:

- superluminality, conflict with statistical interpretation of Ψ (problems)
- self-attraction (main effect for tests)

These fundamental problems and self-attraction are missing in (unsharp) Decoherent Semiclassical Gravity. But new problems and effects arise:

- non-conservation of energy, momenta, etc. (problems)
- decoherence, spontaneous heating (effects for tests)
- need of submicron cutoff against diverging decoherence (major open problem
- submicron breakdown of Newton force (effect for tests)

PLD and Decoherent Semiclassical Gravity wouldn't have been realized without ...

background in standard quantum control—monitoring, feedback, etc.— and its various formalisms —master eqs., Ito-stochastic eqs., path integrals, time-ordered exponentials, double-time-superoperators (Keldysh), etc.

Basic references

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