~~Intrinsic Time in Markovian Open Systems~~

Total Disentanglment by Markovian Open System Dynamics

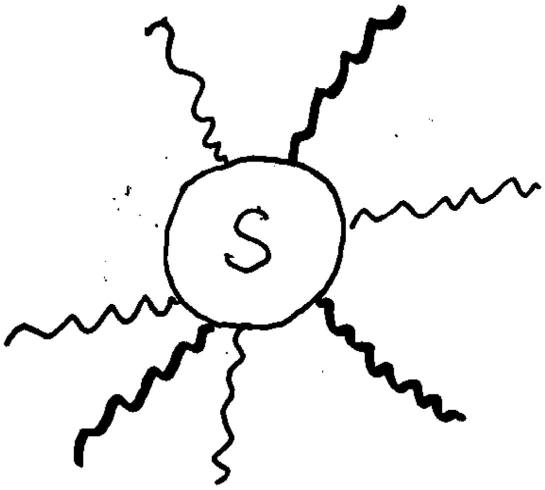
(Lajos Diósi, Budapest)

- von Neumann-measurement
- Disentanglment
- – " – of qbits
- – " – of particles
  - Wigner function ≥0
  - P-function ≥0

in finite time

- Conclusion

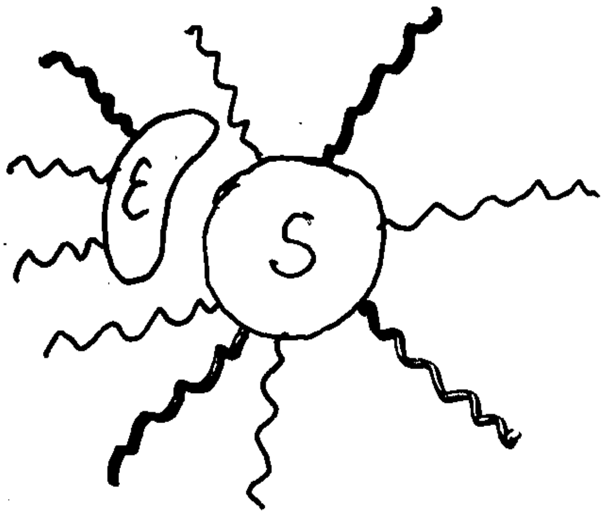
Piombino, 3 Sept 2002



$t < 0$

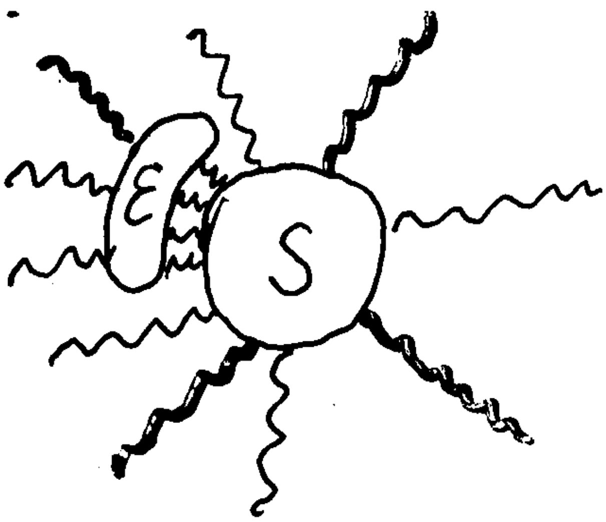
classical correlation

quantum correlation



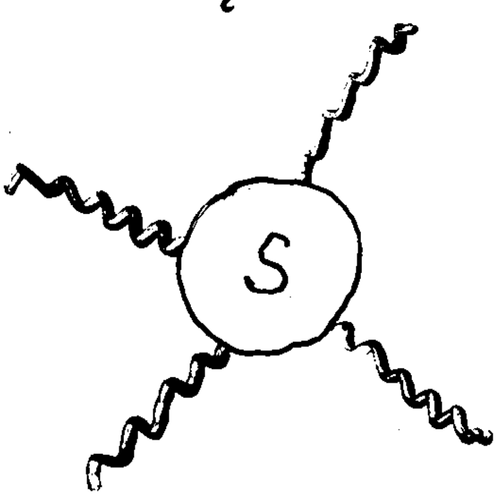
$t = 0$

System
Environment
Rest Of Universe



$$\frac{d}{dt}\hat{\rho}_S = \mathcal{L}\hat{\rho}_S$$

$$t \in [0, t_D']$$



$t > t_D$

$$\hat{\rho}_{S+ROU} = \sum_\lambda p_\lambda \hat{\rho}_S^\lambda \otimes \hat{\rho}_{ROU}^\lambda$$

$$p_\lambda \geq 0 ; \sum_\lambda p_\lambda = 1$$

Von Neumann-measurement

$$\hat{\rho} \longrightarrow \sum_\lambda \hat{P}^\lambda \hat{\rho} \hat{P}^\lambda \qquad \hat{P}^\lambda > 0,\ \sum \hat{P}^\lambda = \hat{I},\ [\hat{P}^\lambda, \hat{P}^\mu] = 0$$

attributes objective classical properties to the post-measurement state.

The diagonal form

$$\hat{\rho} = \sum_\lambda \hat{P}^\lambda \hat{\rho} \hat{P}^\lambda$$

in itself wouldn't be enough since $\hat{\rho}$ may still be q-correlated with ROU.

This is not in vN's book:

$$\hat{\rho}_U \longrightarrow \sum_\lambda (\hat{P}^\lambda \otimes \hat{I}_{ROU}) \hat{\rho}_U (\hat{P}^\lambda \otimes \hat{I}_{ROU})$$

but should be understood!

vN measurement diagonalizes & disentangles*

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* to the extent needed for objective classical prop's.

Local maps that disentangle

Definition: $\hat{\rho}$ is disentangled from the RestOftheUniverse if

$$\hat{\rho}_U = \sum_\lambda p_\lambda \hat{\rho}^\lambda \otimes \hat{\rho}^\lambda_{ROU}$$

$$p_\lambda > 0, \ \sum_\lambda p_\lambda = 1$$

Theorem: $\hat{\rho} \to \mathcal{M}\hat{\rho}$ disentangles all $\hat{\rho}$ from the RestOftheUniverse iff

M. Horodecki, Shor (2002)

$$\mathcal{M}\hat{\rho} = \sum_\lambda \hat{\rho}_\lambda \, \mathrm{tr}\left(\hat{P}^\lambda \hat{\rho}\right)$$

$$\hat{P}^\lambda \geq 0, \ \sum_\lambda \hat{P}^\lambda = \hat{I} \quad \text{i.e.: } \{\hat{P}^\lambda\} \equiv \text{POVM}$$

Markovian disentanglement of spin-$\frac{1}{2}$

$\hat\rho(0)$ arbitrary, maybe entangled with ROU

"free" spin in isotropic bath

$$\frac{d}{dt}\hat\rho = \frac{1}{4}\hat{\vec\sigma}\hat\rho\hat{\vec\sigma} - \frac{3}{4}\hat\rho, \qquad t\geq 0$$

$$\rightsquigarrow \hat\rho(t) \equiv \mathcal{M}(t)\hat\rho(0)$$

$$= \frac{1}{2}\left[\hat{I} + e^{-t}\hat{\vec\sigma}\,\mathrm{tr}\left[\hat{\vec\sigma}\hat\rho(0)\right]\right]$$

$\mathcal{M}(t)$ totally disentangles from ROU

iff

$$\mathcal{M}(t)\bullet = \sum_\lambda \hat\rho^\lambda\, \mathrm{tr}(\hat{P}^\lambda \bullet)$$

Try this: $\lambda = (\alpha, s)$; $\alpha = x, y, z$; $s = \pm 1$

$$\hat\rho^{(\alpha,s)} = \frac{1}{2}\left(\hat{I} + 3se^{-t}\hat\sigma_\alpha\right)$$

$$\hat{P}^{(\alpha,s)} = \frac{1}{6}\left(\hat{I} + s\hat\sigma_\alpha\right)$$

Condition $\hat\rho^{(\alpha,s)} \geq 0 \Rightarrow t \geq \log 3$

Total disentanglement for $t \geq t_0 \log 3$

Markovian disentanglment of a particle

$\hat{\rho}(0)$ is arbitrary

free particle in frictionless bath $\quad \dfrac{d\hat{\rho}}{dt} = -i\left[\dfrac{\hat{p}^2}{2m}, \hat{\rho}\right] - \dfrac{D}{2}\left[\hat{x},\left[\hat{x},\hat{\rho}\right]\right]; \quad t \geq 0$

$t_0 = \sqrt{m/D}$ = decoherence time

One expects that $\hat{\rho}(t)$ gets disentangled for $t \gtrsim t_0$.

Proof: • $W(x,p;t) \geq 0 \quad$ if $\quad t > 3^{1/4} t_0$

• $P(x,p;t) \geq 0 \quad$ if $\quad t > 1.9\cancel{7}\, t_0$

• $\mathcal{M}(t): \hat{\rho}(0) \to \hat{\rho}(t)$ disentangles

D. 1987
Halliwell, Zoupas 1995
Kiefer & D. 2001

Khalfin & Tsirelson 1992